Locally Adaptive Multiplicative Error Models

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Transaction Time Points

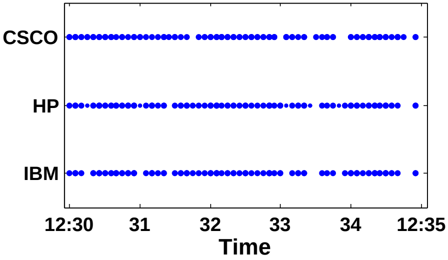


Figure 1: Buy/sell transaction time points for CSCO, HP and IBM stocks on Mai 1, 2009 from 12:30 until 12:35

Objectives

(i) Modelling key market microstructure processes
- Durations: time between buy/sell transactions or quotes, until price changes or until order executions
- Relationship with stock price, bid-ask spread, trading volume

(ii) Finding the longest time period of parameter homogeneity for a process at given time
- Change point detection
- Optimal buy/sell trading times

(iii) Forecasting and financial applications

Statistical Challenges

- Financial data described by a (partially) unobserved process
- Time-varying parameters and structural breaks
- Modern nonparametric statistical techniques
- Flexible statistical framework for modelling and forecasting financial processes

Economic Implications

- Modelling trading activity for buying and selling
- Intensity of demand for and supply of liquidity
- Assessing alternative risk and volatility measures
- Time-varying market liquidity intensity
- Financial applications

Outline

1. Motivation ✓
2. Multiplicative Error Models (MEM)
3. Local Parametric Approach
4. Data
5. Modelling and Forecasting
6. Conclusions

Multiplicative Error Models (MEM)

- Proposed by Engle (2002), MEM(p, q):

$$y_t = \mu_t \varepsilon_t, \qquad \mathsf{E}\left[\varepsilon_t \,|\mathcal{F}_{t-1}\right] = 1 \qquad (1)$$

$$\mu_t = \omega + \sum_{i=1}^{p} \alpha_i y_{t-i} + \sum_{j=1}^{q} \beta_j \mu_{t-j}, \qquad \omega, \alpha_i, \beta_j \geq 0 \qquad (2)$$

- y_t - squared (de-meaned) log return: GARCH(p, q) models
- y_i - financial duration: linear ACD(p, q) models

Autoregressive Conditional Duration (ACD) Models

- Linear ACD model - linear news impact function, ACD$(1,1)$ with $\theta^* = [\omega^*, \alpha^*, \beta^*]^\top$ and $\widetilde{\theta} = \arg\max_{\theta \in \Theta} L(\theta)$:

$$\mu_i = \omega + \alpha y_{i-1} + \beta \mu_{i-1}, \qquad \omega > 0, \ \ \alpha, \beta \geq 0 \tag{3}$$

$$\mu_i = \omega + (\alpha \varepsilon_{i-1} + \beta)\, \mu_{i-1} \tag{4}$$

(i) Exponential-ACD model (EACD) - $\varepsilon_i \sim Exp(1)$

(ii) Weibull-ACD model (WACD) - $\varepsilon_i \sim W(1, s)$

Financial Duration Processes

- Trade and quote durations
  - Trade durations (buy and sell) - trading intensity
  - Quote durations - intensity of liquidity supply

- Price durations
  - Price durations - alternative risk and volatility measures
  - Directional change durations - optimal trading times

- Volume durations
  - Volume durations - time-varying market liquidity
  - Excess volume durations - intensity of demand for liquidity

Local Parametric Approach

- Firstly proposed by Spokoiny (1998)

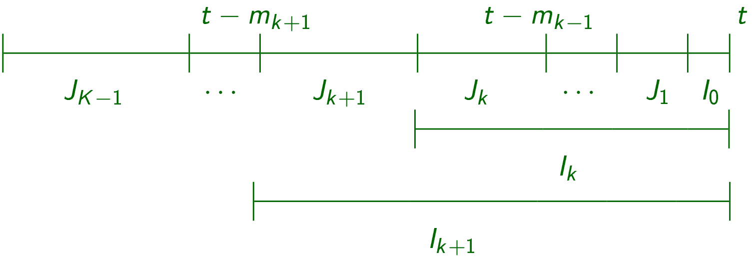
- Four steps for a given parametric model:
  - Risk bound, $\mathcal{R}_r(\theta^*)$
  - Propagation condition
  - Small modelling bias (SMB) condition
  - Stability and oracle property

- Algorithm:
  1. Interval selection, I_k
  2. Local change point (LCP) detection test
  3. Critical values for the LCP detection test, $\mathfrak{z}_k$

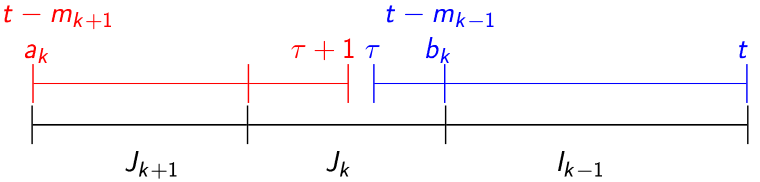
Interval Selection, I_k

- Set of intervals $\{I_k\}_{k=0}^{K}$, $I_0 \subset I_1 \cdots \subset I_K$, length $m_k = |I_k|$

- Example: fix t, $I_k = [t - m_k, t]$, $m_k = m_0 c^k$, $c > 1$



Local Change Point (LCP) Detection Test



- Hypotheses at step $k = 1, \ldots, K$ and fixed t:
  H_0 : y_i follows a given parametric model over $I_k = I_{k-1} \cup J_k$
  H_1 : Non-consistency of the given parametric model over I_k;
  Change point within $J_k = I_k \backslash I_{k-1}$

- Test statistics and critical value

$$T_{I_k, J_k} = \sup_{\tau \in J_k} \left\{ L\left(\widetilde{\theta}_{[a_k, \tau+1]}\right) + L\left(\widetilde{\theta}_{[\tau, t]}\right) - L\left(\widetilde{\theta}_{[a_k, t]}\right) \right\} \leq \mathfrak{z}_k \quad (5)$$

Risk Bound and Propagation Condition

Risk bound with $L\left(\widetilde{\theta}, \theta^*\right) = L\left(\widetilde{\theta}\right) - L\left(\theta^*\right)$ and power r

$$\mathsf{E}_{\theta^*}\left|L\left(\widetilde{\theta}, \theta^*\right)\right|^r \le \mathcal{R}_r\left(\theta^*\right) \tag{6}$$

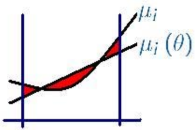
Propagation ($\mathfrak{z}_k$ selection): $\widehat{\theta}_k$ is close to $\widetilde{\theta}_k$ at step k;
Index of the largest interval of homogeneity, $\widehat{k}$

$$\mathsf{E}_{\theta^*}\left|L\left(\widetilde{\theta}_k, \widehat{\theta}_k\right)\right|^r \le \rho_k \mathcal{R}_r\left(\theta^*\right) \tag{7}$$

$$\widehat{k} = \max_{k \le K}\left\{k : T_s \le \mathfrak{z}_s\right\}, \quad s = 1, \ldots, k, \quad \widehat{\theta}_k = \widetilde{\theta}_{\widehat{k}} \tag{8}$$

SMB Condition and Stability Property

SMB condition: 'true' model μ_i well approximated by $\mu_i(\theta)$



$$\Delta_{I_k}(\theta) = \sum_i \mathcal{K}\{\mu_i, \mu_i(\theta)\} \leq \Delta \qquad (9)$$

Stability property: attaining the quality of estimation

$$L_{I_{k^*}}\left(\widetilde{\theta}_{k^*}, \widehat{\theta}_{\widehat{k}}\right) \mathbf{I}\left\{\widehat{k} \geq k^*\right\} \leq \mathfrak{z}_k \qquad (10)$$

Data

- Data
  - NYSE TAQ database, Mai 1 to July 31, 2009
    Analyzing 3 stocks: CSCO, HP, IBM
  - y_i : trade duration process (daily sampling period: 10:00-15:00)

Stock	$\bar{y}$	$\widehat{\sigma}_y$	$y_{max.}$	BP_y
CSCO	2.0	1.7	40.0	60770
HP	5.6	7.8	164.0	35411
IBM	2.6	2.9	59.0	67743

Table 1: Descriptive statistics in seconds and Box-Pierce (BP) test statistics for CSCO, HP and IBM trade durations from Mai-July 2009

Estimated Parameter, $\widetilde{\theta} = [\widetilde{\omega}, \widetilde{\alpha}, \widetilde{\beta}]^\top$

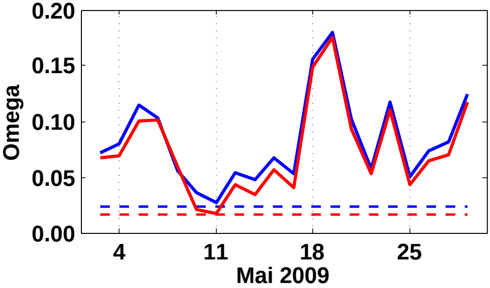


Figure 2: Estimated daily (solid) and monthly (dashed) $\widetilde{\omega}$ using the EACD$(1,1)$ and the WACD$(1,1)$ model for IBM

Estimated Parameter, $\widetilde{\theta} = [\widetilde{\omega}, \widetilde{\alpha}, \widetilde{\beta}]^\top$

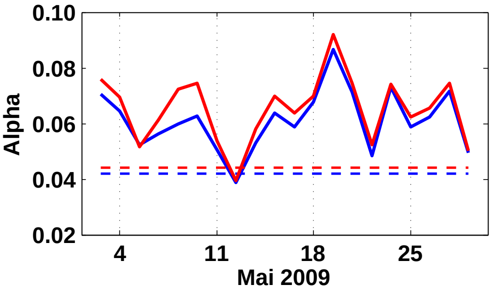


Figure 3: Estimated daily (solid) and monthly (dashed) $\widetilde{\alpha}$ using the EACD$(1,1)$ and the WACD$(1,1)$ model for IBM

Estimated Parameter, $\widetilde{\theta} = [\widetilde{\omega}, \widetilde{\alpha}, \widetilde{\beta}]^\top$

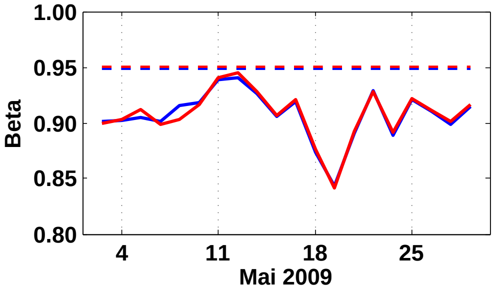


Figure 4: Estimated daily (solid) and monthly (dashed) $\widetilde{\beta}$ using the EACD(1,1) and the WACD(1,1) model for IBM

Empirical Findings

- Trade duration processes:
  - Overdispersion
  - High persistant processes
- HP has lower trading activity than CSCO and IBM
- Time-varying parameters of EACD(1, 1) and WACD(1, 1) models: local parametric approach

Further Steps

- Implementation of the local parametric approach
  - Interval selection, I_k
  - Critical values, $\mathfrak{z}_k$
- Analysing various financial processes and models
- Key market microstructure variables
- Selection of more stocks, different time horizons
- Forecasting and financial applications

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