

Risk Premium and FPCA for Temperature

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Chair of Statistics

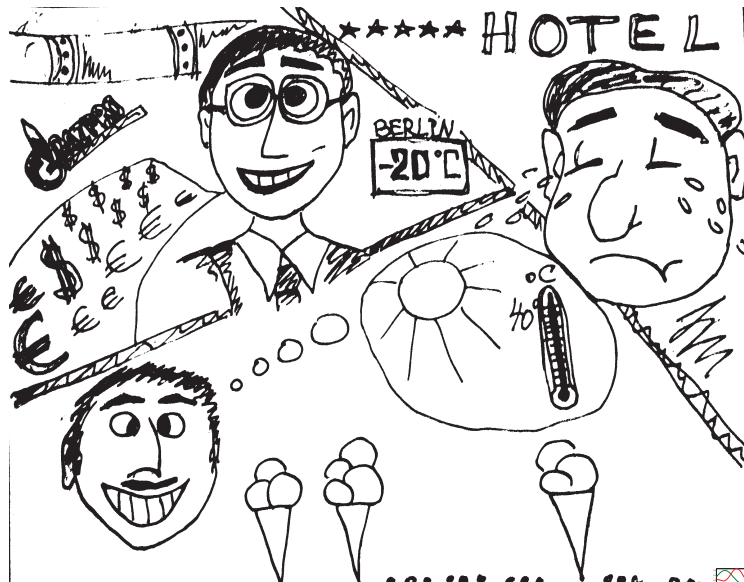
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Future Contracts on Temperature

- transfer weather related risks,
- number of agents limited,
- weather non-tradable, no absence of arbitrage,
- geographically separated markets, spatial relationship.





Figure 1: Contracts on 9 cities in Europe are traded on CME



Pricing Model for Average Temperature

□ Decompose the temperature:

- ▶ seasonal function

$$\Lambda(t) = a + bt + \sum_{j=1}^k \left\{ c_j \sin\left(\frac{2j\pi}{365}\right) + d_j \cos\left(\frac{2j\pi}{365}\right) \right\},$$

- ▶ p -dimensional Ornstein-Uhlenbeck process $\mathbf{X}(t)$:

$$d\mathbf{X}(t) = A\mathbf{X}(t)dt + \mathbf{e}_p \sigma(t)dB(t),$$

- ▶ $B(t)$ is Brownian motion, \mathbf{e}_p p th column of I_p , $\sigma(t)$ seasonal variation.



Seasonality

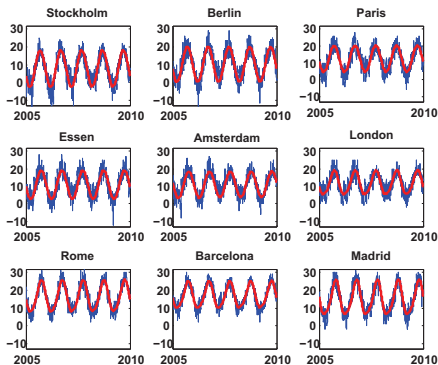


Figure 2: Daily average temperatures (blue) and seasonality $\Lambda(t)$ (red).



Seasonal Variation

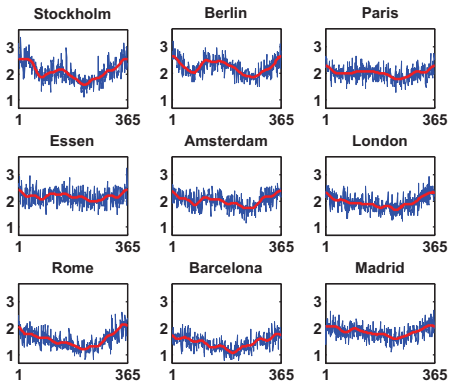


Figure 3: Estimated daily temperature variations (blue) and smoothed by Fourier series (red).



Pricing Model for Average Temperature

Price of a future on Cumulative Average Temperature index (CAT):

$$\begin{aligned} F_{CAT(t, \tau_1, \tau_2)} &= \int_{\tau_1}^{\tau_2} \Lambda(u) du + \mathbf{a}(t) \mathbf{X}(t) \\ &+ \int_t^{\tau_1} \theta(u) \sigma(u) \mathbf{a}(u) \mathbf{e}_p du \\ &+ \int_{\tau_1}^{\tau_2} \theta(u) \sigma(u) \mathbf{e}_1^\top A^{-1} [\exp \{A(\tau_2 - u)\} - I_p] \mathbf{e}_p du. \end{aligned} \quad (1)$$

with $\mathbf{a}(t) = \mathbf{e}_1^\top A^{-1} [\exp \{A(\tau_2 - t)\} - \exp \{A(\tau_1 - t)\}]$

- MPR $\theta(t)$ unknown for specific locations.



Pricing a Weather Location

How can FDA help pricing an arbitrary location?

- ▣ $\theta(t)$ varies across locations,
- ▣ Connect $\theta(t)$ to a known risk factor!
- ▣ Spatial model to interpolate for other locations.



Outline

1. Motivation ✓
2. Functional Principal Components (FPCA)
3. Bayesian Geographically Weighted Regression (BGWR)
4. Empirical Findings
5. Outlook



- RP for $t = \tau_1$, i th location and j th contract month (RP_{ij}) assuming (1),
- a functional regression set up with scalar response:
 $w(t) \stackrel{\text{def}}{=} \mathbf{e}_1^\top A^{-1} [\exp \{A(\tau_2 - t)\} - I_p] \mathbf{e}_p$ and
 $\theta_w(t) = \theta(t)w(t)$:

$$RP_{ij}(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \theta_w^{ij}(u) \sigma_{ij}(u) du + \varepsilon_{ij}.$$

- No "registration" problem: domain $t \in [1, 365]$ fixed.
- Regress RP on PC scores of $\sigma(t)$.



Functional Principal Components

1. Decompose

$$\sigma_{ij}(t) = \{\sigma_{ij}(t) - \bar{\sigma}_i(t)\} + \bar{\sigma}_i(t),$$

$\sigma_{ij}(t)$ variation curve for i th location and j th month,

$\bar{\sigma}_i(t)$ average curve for i th location.

$$\begin{aligned} RP_{ij} &= \int_{\tau_1}^{\tau_2} \theta_w^{ij}(u) \bar{\sigma}_i(u) du \\ &+ \int_{\tau_1}^{\tau_2} \theta_w^{ij}(u) \underbrace{\{\sigma_{ij}(u) - \bar{\sigma}_i(u)\}}_{\text{FPCA for } \sigma_{ij}} du + \varepsilon_{ij}. \end{aligned}$$

2. Perform FPCA: derive scores for temperature variation



FPC Scores

- PC scores for functions $\sigma_{ij}(t)$ $i = 1, \dots, 9$ (9 cities),
 $j = 1, \dots, 7$ (7 traded months):

$$c_{ijk} = \int \xi_{ik}(t) \{ \sigma_{ij}(t) - \bar{\sigma}_i(t) \} dt,$$

c_{kij} scores for k largest eigenvalues, $\xi_l(t)$ orthonormal eigenfunctions of *Cov* operator.

- Collect scores capturing the variance in the data in matrix C .



3. Regress the response RP_{ij} at $t = \tau_1$ on the PC scores Ramsay (2008):

$$RP_{ij} = \beta_0^i + \int \sum_{k=1}^K \beta_k^i \xi_{ik}(t) \{ \sigma_{ij}(t) - \bar{\sigma}_i(t) \} dt + \varepsilon_{ij}$$

for i th location and j th month,

- Recover functional form of MPR: $\theta_w^{jj}(t) = \sum_k \beta_k^i \xi_{ik}(t)$
- Need spatial model for regression on PC scores.



Spatial Specification: BGWR

Why BGWR (LeSage 1998)?

- ▣ distance based weights,
- ▣ nonconstant variance over space and regime shifts,
- ▣ prior info about spatial dependence,
- ▣ valid inference in small samples.



BGWR: the Model

$$W_i^{\frac{1}{2}} RP = W_i^{\frac{1}{2}} C \beta_i + \varepsilon_i,$$

C – matrix of FPCA scores,

$$W_i = \text{diag} \left\{ \exp \left(\frac{-d_{i1}}{h} \right), \dots, \exp \left(\frac{-d_{in}}{h} \right) \right\},$$

$d_{il}, l = 1, \dots, n$ – distances to i th city,

h – decay bwth by CV,

$$\varepsilon_i \sim N(0, \sigma_i^2 V_i), \quad V_i = \text{diag}(\nu_{1i}, \dots, \nu_{ni}),$$

$$\nu_{ij} \sim IG(r, r), \quad r \text{ dispersion in } V_i.$$



BGWR: the Model

$$\beta_i = (\omega_{i1} \otimes I_k \dots \omega_{in} \otimes I_k) \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_n \end{pmatrix} + u_i,$$

$$\omega_{ij} = \frac{\exp\left(\frac{-d_{ij}}{h}\right)}{\sum_{j=1}^n \exp\left(\frac{-d_{ij}}{h}\right)}, \quad \omega_{ii} = 0,$$

$$u_i \sim N \left\{ 0, \sigma_i^2 \left(C W_i^{\frac{1}{2}} V_i^{-1} W_i^{\frac{1}{2}} C \right)^{-1} \right\},$$

$$\sigma_i^2 \sim IG(1, 1).$$



BGWR: Posteriors

$$p(\beta_i | \dots) \propto N \left\{ \hat{\beta}_i, \sigma_i^2 \left(CW_i^{\frac{1}{2}} V_i^{-1} W_i^{\frac{1}{2}} C \right)^{-1} \right\},$$

$$\hat{\beta}_i = \left(CW_i^{\frac{1}{2}} V_i^{-1} W_i^{\frac{1}{2}} C \right)^{-1} \left(CW_i^{\frac{1}{2}} V_i^{-1} W_i^{\frac{1}{2}} RP \right),$$

$$p(\sigma_i^2 | \dots) \propto IG \left\{ \frac{m}{2} + 1, \frac{1}{2} \sum_{j=1}^m \left(\frac{e_{ij}^2}{\nu_{ij}} \right) + 1 \right\}, \quad m \text{ obs with nonzero } \omega,$$

$$p(\nu_{ij} | \dots) \propto IG \left\{ \frac{1}{2} + r, \frac{1}{2} \frac{e_{ij}^2}{\sigma_i^2} + r \right\},$$

$$e_{ij} = W_i^{\frac{1}{2}} RP_j - W_i^{\frac{1}{2}} C_j \beta_i.$$



Estimation via GS

Draw from conditional posteriors of β_i, σ_i, V_i :

- start with $\beta_i^0, \sigma_i^0, V_i^0$,
- for $i = 1, \dots, n$ obtain:
 - ▶ β_i^1 from $p(\beta_i | \sigma_i^0, V_i^0, \beta_{-i}^0)$,
 - ▶ V_i^1 from $p(V_i | \sigma_i^0, \beta_{-i}^0, \beta_i^1)$,
- sample σ_i^1 from $p(\sigma_i | V_i^1, \beta_i^1)$,
- start from the beginning with the new values.

After N steps and M burn-in we get $N - M$ samples from the joint density.



Temperature Data

City	First Date	Last Date	First F_{CAT} Trade
Amsterdam	19730101	20090930	20030401
Berlin	19480101	20090930	20030401
Barcelona	19730101	20090930	20050401
Essen	19700101	20090930	20050401
London	19730101	20090930	20030401
Madrid	19730101	20090930	20050401
Paris	19730101	20090930	20030401
Rome	19730101	20090930	20050401
Stockholm	19730101	20090930	20030401

Table 1: Average Temperatures without 29th February. Source Bloomberg and DWD.



Volatility Functions $\sigma_i(t)$

Fit to data

- ▣ Seasonality Λ_t ,
- ▣ AR(p) process with seasonally heteroscedastic errors,
- ▣ estimate RP_{ij} ,
- ▣ estimate $\sigma_i(t)$, $t \in [1, 365]$ using residual standard deviation for each day of year and smooth by Fourier series.



RP

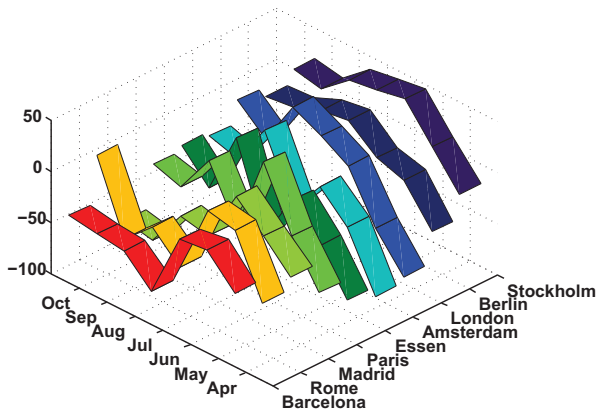


Figure 4: Average RP for Traded Locations computed according to (1)
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Volatility Functions $\sigma_i(t)$

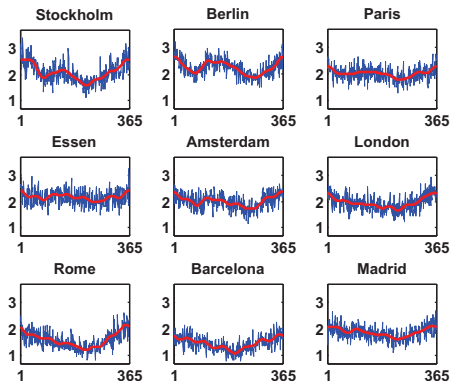


Figure 5: Estimated seasonal volatilities (blue) and smoothed by Fourier series of order 9 (red)



Eigenfunctions

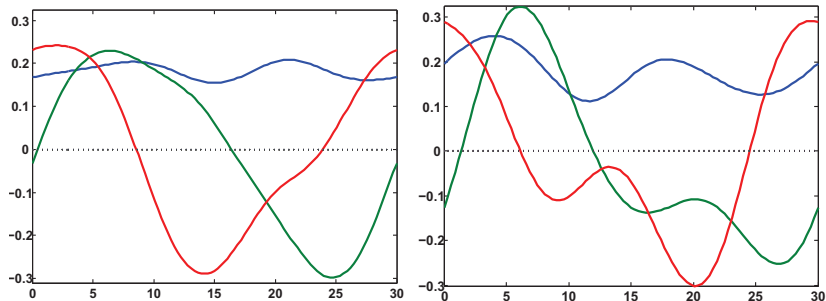


Figure 6: FPCA weight functions for Berlin (left) and Amsterdam (right): eigenfunction ξ_1 : 79.8%, 59.6% (blue), ξ_2 : 15.3%, 29.4% (green), ξ_3 : 3.4%, 7.8% (red)



Eigenfunctions

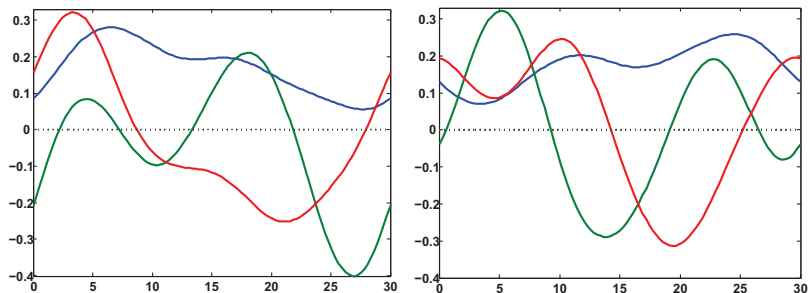


Figure 7: FPCA weight functions for London (left) and Paris (right): eigenfunction ξ_1 : 64.3%, 85.4% (blue), ξ_2 : 18.0%, 7.3% (green), ξ_3 : 10.0%, 4.1% (red)



FPCA Scores

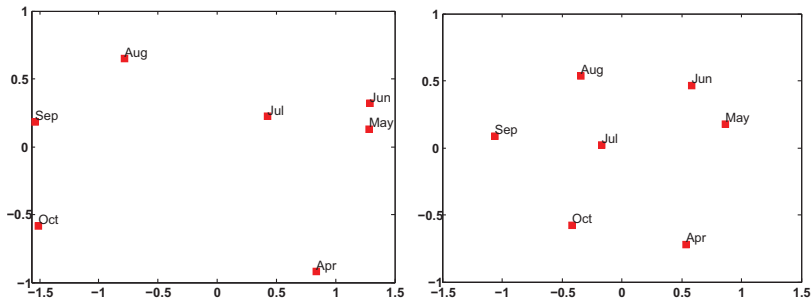


Figure 8: FPCA scores for Berlin (left) and Amsterdam (right)



FPCA Scores

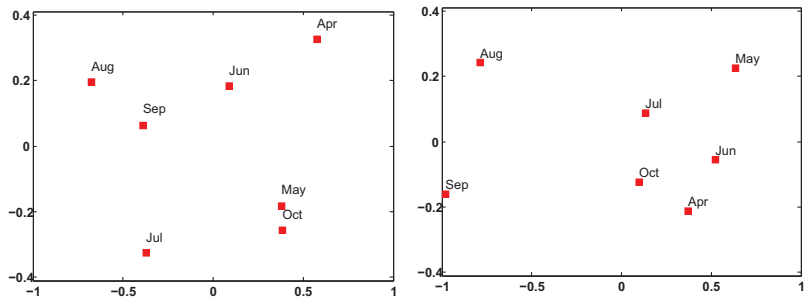


Figure 9: FPCA scores for London (left) and Paris (right)



BGWR Estimation Results: $\hat{\beta}^i$

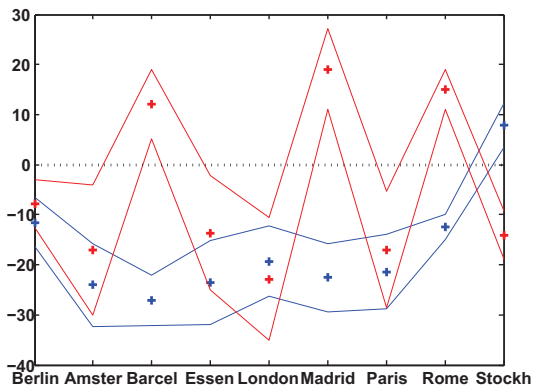


Figure 10: Estimated Parameters of BGWR β_0 (blue), β_1 (red) with 95% CI for April corresponding to intercept and first PC.
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BGWR Estimation Results: $\hat{\beta}^i$

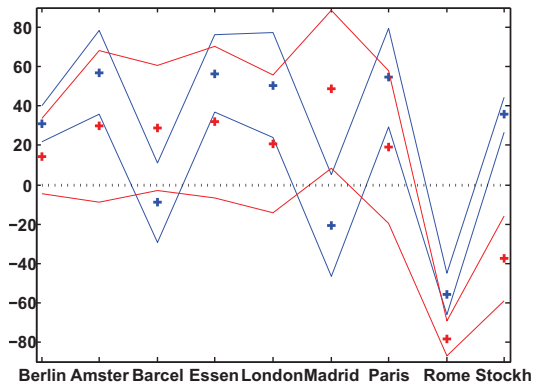


Figure 11: Estimated Parameters of BGWR β_2 (blue), β_3 (red) with 95% CI for April corresponding to second and third PC.
Risk Premium and FPCA for Temperature



BGWR Estimation Results

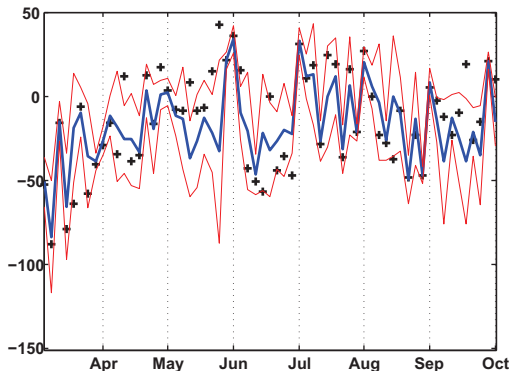


Figure 12: RP (black) and Fitted Values (blue) with 95% CI (red) returned by BGWR (RMSE=22.01)



Recovered $\theta_w(t)$: Berlin and Amsterdam

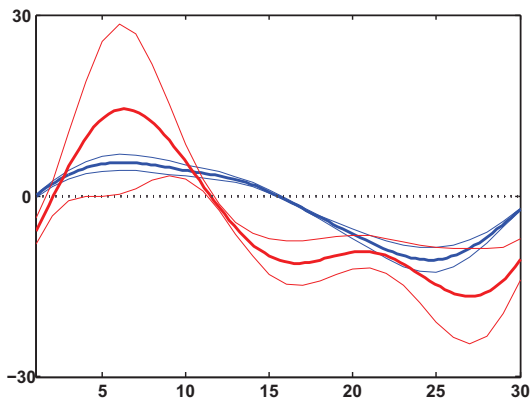


Figure 13: Recovered $\theta_w(t)$ for April in Berlin (blue) and Amsterdam (red)



Recovered $\theta_w(t)$: London and Paris

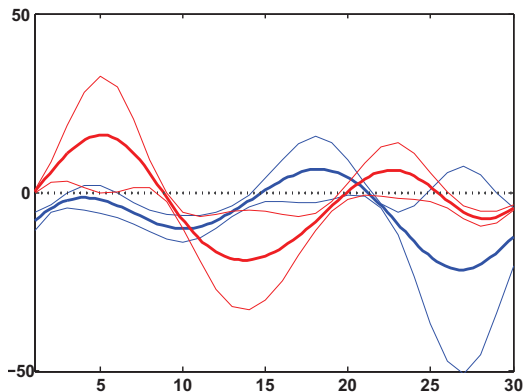
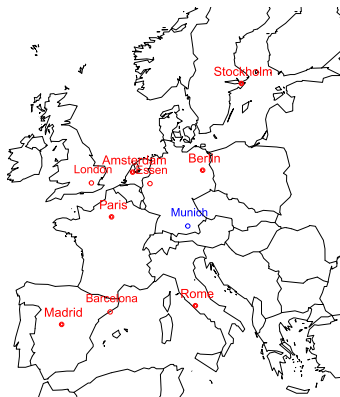


Figure 14: Recovered $\theta_w(t)$ for April in London (blue) and Paris (red)



Out-of-Sample Forecast: Munich

- A beer producer in Munich transfers risk via CAT futures.
- What **RP** he'd pay for Munich?



Out-of-Sample Forecast: Munich

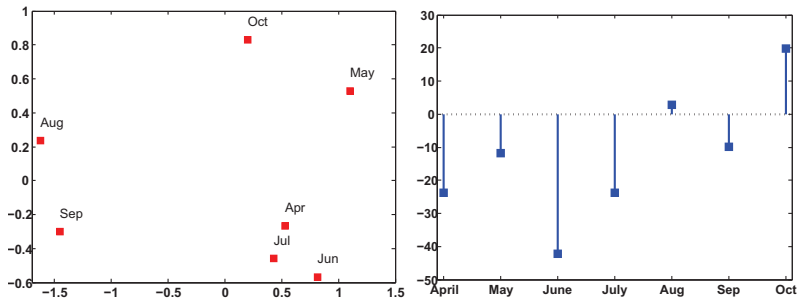


Figure 15: FPCA scores for Munich and resulting forecast for RP.



Outlook

- Expand the model to further locations (USA, Asien, Australien).
- RP interpolation
- Alternative spatial specifications.
- Other pricing models, e.g. equilibrium approach.



Literature



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