Modelling time-varying CoVaR

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  - e.g. Value at Risk (VaR): an individual firm's maximum loss that will not be exceeded over a certain time period with some high probabiliy
- Adrian/Brunnermeier (2009) propose to extend VaR to CoVaR: *Co*ntribution, *Co*nditional, *Co*ntagion VaR.

Definition: CoVaR

Let $VaR^{k}_{p,t}$ denote the pth return quantile of entity k at time t.

1. Contribution CoVaR:
   $CoVaR^{system|i}_{p,t} = VaR^{system}_{p,t} | Y^{i}_{t} = VaR^{i}_{p,t}$
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3. Network CoVaR:
   $CoVaR^{i|j}_{p,t} = VaR^i_{p,t} | \, Y^j_t = VaR^j_{p,t}$
   Which institutions are most connected in times of distress?

$\Rightarrow$ Quantity of interest: $\Delta CoVaR^{k|l} = CoVaR^{k|l} - VaR^k$

Outline

1. Analysis of Adrian/Brunnermeier (2009) (A&B 09)
2. Extensions
3. Toy example: CoVaR of Eurostoxx given S&P 500
4. Outlook

A&B 09: Data

- Sample period: 01/01/1986 to 31/12/2008
- Panel data set: weekly equity returns and quarterly balance sheet data on 1340 financial institutions
- Regressors: Set of state variables/economic fundamentals/indicators: volatility, liquidity spread, slope of the yield curve...

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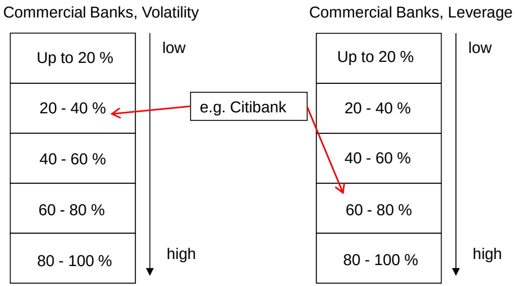
- Regressors: Set of state variables/economic fundamentals/indicators: volatility, liquidity spread, slope of the yield curve...
- 100 characteristic portfolios formed quarterly, according to
  - sectors: commercial banks, investment banks, insurance companies, other security broker-dealers (4 sectors)
  - sorts of maturity mismatch, size, leverage, market-to-book ratio, equity volatility (5×5 quintile portfolios)

Illustration: Quintile portfolios



A&B 09: Estimation of CoVaR

- Calibrate model for system VaR (system return=sum of portfolio returns):

$$VaR_{p,t}^{system,i} = \alpha_p^{system,i} + \beta_p^{system,i} M_{t-1} + \gamma_p^{system,i} Y_t^i,$$

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- Compute (Contribution) CoVaR by plugging in preestimated portfolio VaR:

$$\widehat{CoVaR}_{p,t}^{system|i} = \widehat{\alpha}_p^{system,i} + \widehat{\beta}_p^{system,i} M_{t-1} + \widehat{\gamma}_p^{system,i} \widehat{VaR}_{p,t}^{system,i}$$

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- $\Delta\widehat{CoVaR}_{p,t}^{system|i} = \widehat{CoVaR}_{p,t}^{system|i} - \widehat{VaR}_{p,t}^{system}$

Discussion

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- can be used for medium-term macroprudential regulation as well as for risk estimation on a more frequent basis

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- natural extension of VaR framework taking interdependences into account
- can be used for medium-term macroprudential regulation as well as for risk estimation on a more frequent basis
- data issues:
  - each institution shows up in 5 out of 100 portfolios that are also part of the 'system'
    $\Rightarrow$ simultaneity when estimating Contribution/Exposure CoVaRs
  - given institution might 'wander' through portfolios over time
    $\Rightarrow$ simultaneity when estimating Network CoVaRs
- extension of quantile specification

Adressing data issues

1. construct sector portfolios that are constant over time
2. concentrate on Network CoVaR to detect risk dependencies across institutions/markets/countries

Extensions of model specification

Replace return by VaR in model equation.

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1. linear specification:

$$CoVaR^{j|i}_{p,t} = \alpha^{j|i}_p + \beta^{j|i}_p M_{t-1} + \gamma^{j|i}_p VaR^i_{p,t},$$

$$VaR^i_{p,t} = \alpha^i_p + \beta^i_p M_{t-1}$$

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Replace return by VaR in model equation.

2. partially linear specification

$$CoVaR_{p,t}^{j|i} = \alpha_p^{j|i} + \beta_p^{j|i} M_{t-1} + g^{j|i}(VaR_{p,t}^{i}),$$

$$VaR_{p,t}^{i} = \alpha_p^{i} + \beta_p^{i} M_{t-1}$$

Extensions of model specification

Replace return by VaR in model equation.

3. dynamic nonparametric specification

$$CoVaR_{p,t}^{j|i} = f_p^i \left(VaR_{p,t}^i(X_t) \right); \quad \text{e.g. } X_t = Y_{t-1}$$

$\Rightarrow$ Toy example: Eurostoxx CoVaR as function of S&P VaR

Toy example: Nonparametric Estimation of Eurostoxx CoVaR given S&P VaR

- data: 10 years of daily index returns (1999-2009)
- estimate S&P VaR function using a local linear approach with one lag as regressor, $X_t^i = Y_{t-1}^i$:

$$\min_{\beta^i \in \mathbb{R}^2} \sum_{t=1}^{n} \rho_p \left(Y_t^i - \beta_0^i - \beta_1^i (X_t^i - x) \right) K \left(\frac{X_t^i - x}{h} \right);$$

$\rho_p(u) = u \cdot (p - I(u < 0))$, $\widehat{VaR}^i_{p,t}(x) = \widehat{\beta}_0^i$

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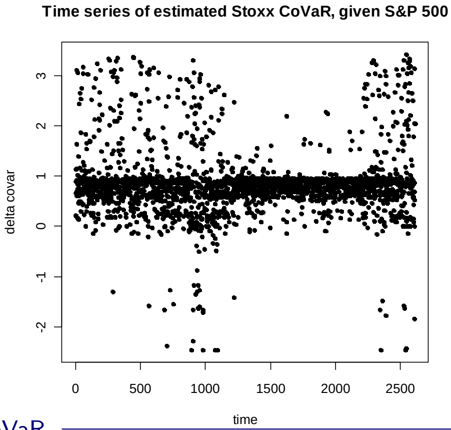
- estimate the model

$$CoVaR_{5\%,t}^{Stoxx} = f \left(VaR_{5\%,t}^{S\&P} (Y_{t-1}^{S\&P}) \right)$$

using a second local linear quantile estimator.

Toy example: Estimated Eurostoxx ΔCoVaR time series



Toy example: Summary statistics of Eurostoxx ΔCoVaR

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
-2.463	0.618	0.846	0.845	0.929	3.429

Table 1: Summary statistics $\Delta CoVaR^{Stoxx|S\&P}_{0.05,t}$ (in %)

$\Rightarrow$ including estimated S&P 500 VaR as regressor leads to positive $\Delta CoVaR$

$\Rightarrow$ risk spillover from American to European equity market

Outlook

- extend toy example to different data set and different specifications of the VaR functions (e.g. include state variables)
- construct portfolios without simultaneity problems
- panel forecast regression

Theory: Nonparametric regression with nonparametrically generated regressors

- model in general:

$$Y_t = f_0(r_0(S_t)) + \epsilon_t,$$

  where S_t is a $(p \times 1)$-vector of covariates, $f_0 : \mathbb{R}^d \longrightarrow \mathbb{R}$, $r_0 : \mathbb{R}^p \longrightarrow \mathbb{R}^d$ and ϵ_t is error term with $E[\epsilon_t | S_t] = 0$.
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nonparametrically in a two-stage procedure.
- in the present context: estimate conditional quantile function of Y_t, instead of conditional mean:
$$VaR_{0,p}(r) = quant[Y_t|r_0(S_t) = r]; \quad quant[\epsilon_{p,t}|S_t] = 0$$
where $r_0(S_t)$ is the VaR function of a different return series.