Robust Control Analysis of Labor Markets

Diploma Thesis
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Abstract

This Diploma Thesis wants to analyze, how search- and job-acceptance behaviour of workers is affected, if they have misspecified believes concerning the model environment. To capture features like knightian uncertainty aversion, and model misspecification I make use of Hansen Sargent robust control techniques and combine them with search theoretic models of the labor market. I introduce two different ways of doing so. I start out with an easy or naive version, where the worker are uncertain concerning the firm or the government behavior, i.e. future wages or government benefits turn sour. I therefore develop techniques, how the wage distribution is skewed by the agent. In the end, I present a search theoretic model with explicit search intensity and finite lived agents, making use of theses skewed wage distributions.

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1 Introduction

Economic Models are meant to be a faithful mirror of the real economy. They are created to answer the big economic questions of this time. There is a great need for precision, as the big wheels of the world are steered following recommendations of these models. On the other hand, there is the great need of understandability. At least the people who make the recommendations must understand them. As result, they cannot be precise. We just try to make them as precise as they can get. So we cannot live without them, neither can we entirely trust them.

There is always need for new models, that are understandable on the one side while performing better against the data on the other side. This Diploma Thesis tests a new approach, by bringing two building blocks of Economic Theory together. On the one hand search theoretic models of the labor market and on the other hand robust control theory. So we have knightian uncertainty on the micro level, having various consequences for the macro level. The thing which makes this interesting is a very simple one. Typically search theoretic models dealing with exogenous probability distributions assume rational expectations. As a consequence, the subjective probability distribution assumed by the agent, and the "empirical" probability distributions coincide. So the agents personal believes about her own individual standing in the labor market are removed from the model. So what is done here is that the a treats her knowledge about the labor market as approximation and develops rules which are consistent with other models, which are also consistent with her experiences, which makes him kind of cautious. So I don't want to explain directly some economic phenomena already observed, I want to provide a set of instruments being able to explain consequences of uncertainty and model misspecification on the labor market. I think it is important to combine these features, because there is rarely one field in economics having such a high potential of being affected by uncertainty. Almost no one of the participants in the labor market has perfect insight of the mechanisms which are actually working around them. Although the drastic consequences of this amount of uncertainty are felt there has been only view research done dealing explicitly with uncertainty and up to now, there has been no model framework capturing the effects of knightian uncertainty in the labor market.

The basic intuition of the models provided here is actually quite simple. I start out with a typical search model, and then I include a second so called “evil agent”. The evil agent derives utility from disturbing the original model, or more precisely, selected state or control variables in a restricted set. So she doesn’t directly try to hurt the regular agent, she just tries to skew the original model. The result of these disturbances is that that the regular agent becomes cautious. So given that the “evil agent” is allowed to disturb the wage, the regular agent starts to assume, that the wage may be lower than expected, to insure against possible losses. In search models this has usually two consequences. The value of being unemployed sinks, since the future job opportunities turn sour the value of an offered job fades away. So the consequences are not obvious. The reservation wage for example may increase, may decrease of may even remain unchanged. In the version of the model, where I skew the wage distribution the basic intuition is not definite, since there are two ways skewing the wage distribution. In any case the “empirical” wage distribution remains unchanged whereas the subjective wage distribution is skewed. The evil agent can interpret a wage as a payment, or she can interpret it as an event. In the case that she treats is just as event, the resulting wage distribution flattens out. In the more realistic case that she treats it as a payment, the wage distribution is shifted asymmetrically to the right. In the first case excess unemployment is generated, because the agent puts very much probability mass on the high wages. Therefore the reservation wage increases and the agent doesn’t accept moderate wage offers any more. In the case that she treats it as a payment, the reservation wage falls and the agent accepts low wage offers, since she believes, that the probability getting a high wage offer is very low.

The two player interpretation for this kind of game remains formal. The other possible interpretation is that there is only a single agent, since this kind of technique itself is designed to describe the behavior of a single person. So it is actually up to your own, if you want to interpret the model as a single player, or a two player game. It is easier to understand what is actually happening inside the model framework, if one chooses the two player perspective, but the robust control technique is designed to explain a single agent uncertain about the model framework.

I will proceed as follows. In section 2, I will present the development of the two kinds of models. In Section 2.1 the development of robust control theory, in section 2.2 the history of search theoretic models. In section 3 I will apply the naïve approach to a set of search models. Section 4 presents a method to solve the distribution skewing mechanism; section 5 applies it to a search model with finite lived agents and explicit search intensity. In section 6 I will summarize and discuss the results and draw final conclusions in the end.

2 Summary of the Literature

The question of the role of risk and uncertainty in economic questions has always been a question of major interest in economics. The special kind of uncertainty I'm interested here is knightian uncertainty, introduced by Knight in (1921). The thing making knightian uncertainty special is, that in the absence of probabilities it is extremely difficult to capture economic decisions in mathematical models. Using rational expectations here, meaning the mathematical expectation using all information available, would imply, that a distinction between risk with known probabilities, and uncertainty is irrelevant. But phenomena like the Elsberg Paradox imply that using rational expectations doesn't capture human behavior in decision making appropriately for this kind of model.

On the other hand, search theory has been invented to explain phenomena like why workers choose to remain unemployed? What determines the length of employment spells? Why do we observe phenomena like unfilled vacancies and unemployed workers at the same time? What determines the length of employment spells? There are many other questions search theoretic models have answered already. One of the biggest achievements of search theoretic models is that these phenomena can be sufficiently explained using micro-founded models with rational expectations. To further extend the set of economic phenomena these kind of models can explain, I think it is time to incorporate uncertainty non rational expectations.

2.1 Robust Control Theory

A way to incorporate such kind of behavior in economic models is robust control theory. Robust control theory incorporates knightian uncertainty in the way, that it interprets abnormal risk sensibility as aversion to knightian uncertainty. The some kind of extreme perturbations of the original model are reactions; so the brilliant idea how unknown probabilities in mathematical models describing human behavior can be incorporated, is that only the reaction of the agent concerning the risk is modeled, not the special kind of risk itself. There are various ways to do so, another way of doing so would be to use subjective expected utility and to proceed with some concept of revealed likelihoods. The reason why I use robust control theory is that I want to capture the effects of model misspecification and the resulting caution. I think that rarely anybody participating at the labor market understands fully what is actually happening there. Everybody has some idea, what is happening around him, but nobody is fully sure. If everybody reacts concerning his/ her own restricted knowledge, these perturbations on the micro-level have effects for the aggregate market.

Where does robust control theory actually come from? Robust control theory comes from engineering. It has been introduced by engineers in the 1960's based on the results of standard control theory using e.g. bellman equations or Lagrange multiplier problems invented by engineers in the 1950's. RC techniques allow the engineer to control plats perturbed by unknown disturbances. Think of a helicopter rotor. It has to run smooth no matter if it rains, or it is especially windy. It is also not possible to estimate the special kind of disturbance before, because the disturbances are always changing. So RC techniques allow the engineer to build a rotor, which always runs sufficiently smooth, no matter what the actual weather is. It would be possible to build a smooth running rotor cheaper, given a specific weather one can forecast somehow, but it is not possible to construct a single rotor being able to deal with all kinds of weathers.

2.1.1 A short introduction to Robust Control Theory

So what does robust control theory in an economic framework? A robust control agent (RC agent)aims to minimize a loss function (or maximize an objective function, depending on the notion chosen), similar to a rational expectations agent. The main difference is, that the RC agent entertains an approximating model representing a set of possible law of motions of the economy, whereas the RE agent entertains a reference model, of which she -knows- that it is true. So the RC agent has only a rough idea about the environment around her. So being uncertain about he model, she has a set of possible solutions in mind and now tries to develop robust decision rules yielding an satisfactory outcome no matter what the actual model actually does. So given an RC firm manager knows that her profits (π) are somewhere between π_{low} and π_{high}, he behaves as if he knew that the profits are π_{low} to make sure the company doesn't become insolvent when facing the real profit in the end of the period. This behavior differs from the behavior of a Bayesian agent. The Bayesian would try to estimate the real model, so she would combine her data with her priors over the expected probability distribution and therefore minimize the amount of resulting uncertainty. So the decision the RC agent is not optimal in the common sense, he makes strategic mistakes and derives a sub optimal payoff. So, the central question before starting to formulate any RC model is: Why doesn't a RC agent aim to become Bayesian agent? Or more elaborated: What is the justification for using robust control techniques from the economic point on view?

Loosely speaking: the objective! Robust control refers to the control of unknown models with unknown dynamics subject to unknown disturbances. The aim is, to design decision rules, which are robust, which deliver maximum utility in a set of in the end unknown models, when all assigned probability weights are unknown. So the RC agent has by assumption not the possibility to estimate the real model, because the aim is to design utility maximizing decision rules being robust to a set of unknown models.

The way RC theory is implemented follows usually a max-min approach following Gilboa & Schmeidler (1989). The agent minimizes a loss function, while the -evil agent- maximizes the degree of model misspecification. The second evil agent often leads to misunderstandings. Seen from the formal point of view, the evil agent is not just a metaphor for the caution of the agent. He is a fully self governed player in a two agent game. Her job is also not to minimize the utility of the regular agent, her job is just to maximize a linear quadratic function objective function subject to the transversality condition and a restriction to the absolute degree of model misspecification. So her actual job is to maximize the degree of model misspecification given a restriction to the absolute degree of model misspecification. The results of her endeavors are not always obvious. It is intended, that the evil agent will find the worst model possible for the agent. But as Sims (2001) has already mentioned, it may happen that the actions of the evil agent drop out of the model and her presence simply increases the degree of complexity.

In the following passage I will describe the basic setup of a typical linear quadratic robust control problem, where the objective function is quadratic and the model linear. Hansen and Sargent have shown that the problem can be represented in the following way.

$$max_{\{u\}_{t=0}^{\infty}} min_{\{\nu\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t (x_t' Q x_t + u_t' R u_t + 2 x_t' U u_t) \right\} \tag{1}$$

subject to

$$x_{t+1} = A x_t + B u_t + C \left(e_t + \nu_t \right) \tag{2}$$

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^{t+1} \nu'_{t+1} \nu_{t+1} \leq \eta_0 \right\} \tag{3}$$

where: x_t is the $(n \times 1)$vector of states, u_t is the $((n \times 1)$vector of controls, β is the discount factor, t denotes time, E is the expectations operator,Q, R, U, A, B and C are $(n \times n)$ matricies.$\nu\,(n \times 1)$ are the evil agents pertubations, $e\,(n \times 1)$ are the errors.

The matrices R and Q are assumed to be symmetric, the roots of the characteristic polynomial of the transversality equation have to be inside the unit circle. Note that x_0 is given. Notice that the distortions generated by the "evil agent" are actually nothing else then an additional control variable. Many researchers like Giordani and Söderlind argue, that the random disturbances have to be present, to make sure that the disturbances are embedded in the white noise. This is said to be important for the model uncertainty to make sure that the agent interprets it as parameter instability. I personally don't think that the random disturbances have to be present in the model. In the original concept developed by the engineers, the random and unpredictable disturbances are responsible for problem they want to solve. In economic models we try to model human behavior. We have to assume, that the regular agent doesn't solve for the disturbances, because we want to study, what she does if she has no idea what the disturbances look like. So I think it doesn't matter if there are actually white noise disturbances, because we have to assume anyway, that she doesn't solve for the perturbations.

There exists some kind of confusion about the time indices for the evil agent. If one lets the evil agent decide about the $t+1$ disturbances ν_{t+1} in t as Hansen and Sargent propose, then the regular agent would know the disturbance already in t and react accordingly. If one lets the evil agent decide in t about the disturbance in t then he would not know the decisions of the regular agent and is somehow unable to find the worst model possible. As I am presenting research done mainly by Hansen and Sargent at the moment, I will adopt the notion of Hansen and Sargent in this chapter, and switch to my notion when presenting my models. Both notions are time consistent by construction. For further details see Hansen & Sargent(2001) “Time Consistency of robust control?”. To come back to the basic model, it is possible to include the problem of the evil agent into the objective function of the regular agent. The basic problem then looks as follows:

$$max_{\{u\}_{t=0}^{\infty}} min_{\{\nu\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t (x_t' Q x_t + u_t' R u_t + 2 x_t' U u_t - \theta \nu_{t+1}' \nu_{t+1}) \right\} \tag{4}$$

subject to

$$x_{t+1} = A x_t + B u_t + C\left(e_t + \nu_t\right) \tag{5}$$

It is important whether the evil agent can be included in the objective function or not, because this is in the end the decision criterion between the two kinds of robust control concepts making sense from the economic point of view. The concept mainly used is the $\mathcal{H}_\infty$ norm. The other concept making economic sense is the $\mathcal{H}_2$ norm. Loosly spoken the problem is formulated as an $\mathcal{H}_\infty$ problem, if the evil agent is included in the objective function. It is formulated as an $\mathcal{H}_2$ problem, if the disturbances appear as exogenous transformations of the core model. There are formal definitions of the $\mathcal{H}_\infty$ and the the $\mathcal{H}_2$ norm, but it would take too long to make clear what it exactly means.

There are three basic types of games: a multiplayer game in sequences, the Stackelberg multiplier game and the Markov multiplier game. In the first game, both agents determine simultaneously for a sequence of control variables, which makes it possible to interpret it as a single player game. In Stackelberg game, first the regular agent commits to a sequence of controls, and then the evil agent commits to a sequence of her control. In the Markov game both agents are allowed to determine their controls every period again. Only in the first type of the game, the typical reaction of the regular agent tends to be more cautious then expected by the researcher. In the other two games the extreme reaction to the degree of uncertainty disappears, because the regular agent knows that the evil agent will try to wreck the model. Hansen and Sargent have shown under which conditions all three concepts are compatible to each other and under which conditions they differ.

Anyway, when solving such kinds of models, the choice of the parameters θ or η_0 is crucial. In the basic (Game 1) model, the behavior of the whole model is typically heavily influenced by these parameters, since they determine the size of the set of possible alternative models. The worst model possible often lies on the boundary of this set, and although highly unlikely, determines the reaction of the RC agent. This feature is often seen as the weakness of robust control. I think this critique is not justified. The decisions of a RC agent achieve a stable error tolerance, even if the approximating model he has in mind is completely wrong and all possible estimates of the real model are completely misspecified.

2.1.2 Overview of the existing literature

As mentioned above Robust Control theory has its roots in engineering. There is actually no date of invention. Robust Control emerged by combining standard control theory (being developed by mathematicians and engineers like Richard Bellman in the 1940's) and Robust Design. Robust Design has its roots in the early developments of automatic control mechanisms. They had to develop feedback amplifiers being able to deal with large variations in supply voltage. This problem was key in the telephone industry, and has been solved in 1934 by H. S. Black in "Stabilized feedback amplifiers". In response to this seminal paper a whole robust design literature with leading scientists like H.W. Bode and later I. M. Horowitz evolved. In the 1960's the former mainly graphically described robust design problems were now described using differential equations, which made it possible to apply standard control mechanisms and formulate the solutions as optimization problems. New solution mechanisms like state space solutions, the Kalman - Filter and (combining both) the LGQ mechanism were developed. These approaches found their margins in the late 1970's, as M. G. Safonov and M. Athans showed. The next significant achievement was the $\mathcal{H}_\infty$

loop. The seminal article by G. Zames was -Feedback and optimal sensitivity: Model reference transformations, multiplicative seminorms, and approximative inverse.- (1981). This article led to the development of the $\mathcal{H}_\infty$ theory. The great achievement of H-inf theory was that it was now possible to -develop systematic design methods that were guaranteed to give stable closed loop systems for systems with model uncertainty-. So, in the beginning, this new approach was a new development in robust design, and had only very view to do with control theory.

Interestingly, the Maxmin principle was introduced in engineering, following a game theoretic theorem of the mathematician John von Neumann. He has discovered it in 1928 and it has been used in game theory since by then. In the 1980's robust $\mathcal{H}_\infty$ design problems were formulated using the Maxmin principle and turned into what is known today as $\mathcal{H}_\infty$ robust control.

The literature of Robust Control theory in economics is actually quite short. The Maxmin approach has been practically introduced in classical economics in 1989 by Gilboa and Schmeidler in "Maxmin Expected Utility with Non-unique Prior". They did not intend to introduce Robust Control theory in economics, they just used a pretty old game theoretic approach to design utility functions. For nearly one century, there has been no major publication adopting this approach, until Lars Peter Hansen, Thomas J. Sargent and Thomas D. Tallarini adopted it in "Robust Permanent Income and Pricing" in 1997 to study consumption/savings profiles with RC agents. This was actually the first attempt to use Robust Control theory in economics. This paper was followed by a series of papers studying either macroeconomic topics or solution methods of RC models. Macroeconomic covered up to now where monetary policy, Asset Pricing and the aggregate effects of uncertainty on the micro - level. This year they are going to publish a monograph currently called "Robust Control and Model uncertainty in Macroeconomics".[1] These publications provoked, that a whole literature of Robust Control in economics aroused. The reactions to this -new- approach are ambiguous. While researchers like Christopher Sims are

[1] available as unpublished monograph on http://home.uchicago.edu/ lhansen/rgamesb.pdf

focussing on finding pitfalls of this new approach, a growing group of researchers recognizes Robust Control theory as a promising new alternative for normative and descriptive analysis of economic behavior.

2.2 Search Theoretic Models of the Labor Market

I will now present a brief history of the literature of search theory in labor market economics. I will loosely follow a recently published survey by Rogerson, Shimer and Wright. Since it is not possible to do justice to every seminal article of this broad literature, I will concentrate on selected contributions[2].

2.2.1 A brief overview of the literature

The study of search in labor market economics began with a static model by George Stigler in "The Economics of Information", (1961). His intention was to find a general model for the economics of information acquisition. The labor market was just one application of his theory, which he has studied in "Information in the labor market", 1961. His main interests were devoted to returns of search activities on the stock market, like the value of identifying potential buyers. Other possible fields of interest were the search for investment possibilities, advertising and the search for the quality of goods available or search for the best technology available for your company.

Sequential labor search models, as we know them today have been invented in 1970. There have been several seminal papers published almost simultaneously, one by McCall (1970), one by Mortensen (1970) and one by Reuben Gronau (1971). Similar papers have been published before, but they are focussing on problems like the housing market (like Simon, 1955) or asset markets (like Karlin 1962), but for some reasons there was almost a ten year break between the pioneering work of Stigler and the rise of search theory.

The approach of McCall is closest to the basic approach known today. Gronau already quotes McCall and Mortensen, but his study is by far more

[2] I will leave out the seminal papers on efficiency wages, since efficiency wages are not part of the discussion afterwards

sophisticated.[3] While McCall was interested in the microeconomics of search, Mortensen was interested in deriving a relationship between unemployment and the Phillips Curve. Gronau was interested finding a micro founded model explaining the aggregate fluctuations of the labor market.

These papers were followed by a large number of other papers studying search on the labor market more deeply. Most of this literature was theoretical. In 1975 the first empirical implications of these models were tested. A first survey of the literature by McCall appeared in 1976. He already quotes more than 40 serious papers dealing with labor market search. To that time, the bulk of the literature was formulated in discrete time. To avoid the problem of simultaneous arriving wage offers, it became good practice to set up the models in continuous time by the end of the 1970's, there is no precise date or paper provoking that change, one can just observe, that the ratio of continuous time models published has somehow increased. For some reasons, discrete time models are still published today. In the end of the 1970's Jovanovic published his work about the interactions between search, matching and learning, Burdett invented on the job search as it is known today and finally Mortensen and Burdett have published their work concerning labor supply under uncertainty. For completeness I have to add here, that on the job search was already included in the pioneering work of Gronau and part of the survey of McCall, but Burnett was the first who hasn't made more effort than necessary, hit the point and whose empirical implications could be successfully tested. The studies before were mathematically highly complicated while unable to reach a satisfactory degree of explanation. Albrecht and Bo Axell started to study the effects of heterogeneous preferences of workers in 1984. The result were heterogeneous reservation wages and cut of values of firms. In 1985, Pissarides published a paper, studying the determinants of arrival rates, match formation, match dissolution and wages. He therefore extended the existing framework. The result was a search model with random matching and bargaining. The model is build up on two basic pillars. On the one side the idea of implementing a matching

[3] All three models have been developed at the same time

function, which has been invented by Peter A. Diamond (1981), and on the other side he uses generalized Nash Bargaining. So there is no need of assuming an exogenous wage distribution or productivity distribution. The only need is to determine the workers bargaining power. This is done either directly, via defining some parameter, or indirectly, by defining a set of rules determining the bargaining process. The next important survey of the literature has been published in 1986 by Dale T. Mortensen. The literature was already quite large at that point of time. Mortensen cited nearly 80 papers in his survey. Although search theory was -still a young actor on the stage of economic analysis- it has already become a standard tool for the analysis of labor markets at that point of time. The results of empirical studies published in the late seventies and early eighties revealed the amazing explanatory power search theory already had in this early stage. The next new invention of great importance was putting the Beveridge Curve on central stage. This has been done by Oliver Blanchard and Peter Diamond in 1989. Their approach made it popular, to formulate search models as general equilibrium models. The Beveridge curve, describing the relationship between unemployment and vacancies, was known already known before as the second big relation in Macroeconomics after the Phillips Curve. Blanchard and Diamond incorporated the Beveridge Curve in a typical search model after studying the data carefully. The model developed was seen as "highly instructive on some major issues about the operation of the labor market."(Robert E. Hall)[4] Following the good practice constructing search models as Equilibrium Models, Dale Mortensen and Christopher Pissarides formulated an approach, which became the Benchmark Search Model or the "Mortensen-Pissarides" model. Unemployment was now an equilibrium phenomenon, the cycles and volatility of job creation and destruction could be explained amazingly well. Another fundamental extentions were the endigenization of the Job Separation rate and the fact that the equilibrium was build out of the Beveridge Curve and the Job Destruction rate. One extension to this model, was to integrate it into an RBC model, as done e.g. by Monika Merz (1999). This extension

[4]can be found in the discussion, located at the end of the same paper, see References

made it it also possible to track the cycles over the business cycle. To this time, the development of search models was on top level. Most of the features being fundamental for a diligent policy analysis were developed. A realistic human capital accumulation process has been developed, multiple equilibria could be studied, technological process could be incorporated and the numerical methods for analyzing realistic images of existing policy environments existed.

At this stage of development, the science of economic search theory was almost completely developed. Models build in 1998 are still used today in small variations. New Models developed today are studying minor features like mismatch or try to develop policy recommendations. The last model published by Mortensen, just shows, that a recalibration of the well known "Mortensen-Pissarides equilibrium search model" is able to explain the observed volatility of the job finding rate. The progress of search theory is stagnating. There are no major features on the labor market which remained unexplained.

3 The Naïve Approach

To give a basic impression what robust control does to a search model, I will implement a $\mathcal{H}_\infty$ evil agent in various search models presented in the survey article by Rogerson, Shimer and Wright (2005).[5] In contrast to the usual approach, the evil agent will be allowed to influence a single variable instead of the whole model. The precise of definition what I'm actually doing is variable uncertainty not model uncertainty. Although using the notion "model uncertainty" is not wrong, since I select parts of the model which are uncertain. Other models using the notion of model uncertainty also need to have parts in the model which are not uncertain. So I use robust control, to create uncertainty concerning less parts of the model then usual in the literature up to now. I also forgo to embed the disturbances in white noise, because of reasons mentioned above. This section is designed to create a basic intuition. Since the random disturbances

[5] What I'm actually doing here, is to take some of the models presented in the paper and implement robust control techniques.

complicate the analysis without need I leave them away.[6]

3.1 On Basic Job Search

3.1.1 Basic Job Search in Discrete Time

The simplest model one can imagine consists of a single unemployed worker, who has the option to take a job offered forever, or remain unemployed and wait for another job offer. In the well known version she receives an offer each period, drawn from some distribution (i.i.d.). The agent's objective function is given by $max_{\{x\}_{t=0}^{\infty}} E_0 \left\{\sum_{t=0}^{\infty} \beta^t x_t\right\}$, where x_t is the wage less then the feared amount of cheating , $w_t - \nu_t$ if he's employed, in case of unemployment x_t denotes the unemployment benefit b. The problem of the evil agent is given by $min_{\{\nu\}_{t=0}^{\infty}} E_0 \left\{\sum_{t=0}^{\infty} \beta^t \nu_t^2\right\} \leq \eta_0$, with respect to the model, where η is the parameter restricting the power of the evil agent. The corresponding Maxmin problem is given by:

$$max_{\{x\}_{t=0}^{\infty}} min_{\{\nu\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t x_t + \theta \nu_t^2 \right\} \tag{6}$$

s.t

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \nu_t' \nu_t \leq \eta_0 \right\} \tag{7}$$

Where θ is inversely related to η. So, if $\eta \rightarrow 0, \theta \rightarrow \infty$. The problem is stationary and formulated in discrete time. The interpretation of the change I have made is that the worker doesn't trust the wage arrived. So she knows the offer, but is not sure, whether the firm chats or not. Although pretty simple, I think that this problem is of serious relevance, as the perfect contract doesn't exist. It is never possible to write a contract where everything is perfectly governed. So this is a new possibility to think about the economic consequences of the remaining uncertainty. Now observe that the resulting system of Bellman equations is given by:

[6] Something similar has already been done in the Hansen & Sargent monograh, chapter 5 and by C. Sims in "Pitfalls of a minimax approach to model uncertainty"(2001)

$$W(w,\nu) = w^e + \beta W(w,\nu)\ ;\ w^e = w - \nu \tag{8}$$

$$U = min_{\{\nu\}} \left\{ b + \theta\nu^2 + \beta \int_0^\infty max\left\{U, W(w,\nu)\right\} dF(w) \right\} \tag{9}$$

Where $W(w,\nu)$ is the discounted stream of wages from the accepted job. The Bellman equation named U represents the value of being unemployed. The next step when solving such a model is deriving the reservation wage. The crucial question when deriving the reservation wage here is whether the reservation wage is interpreted as the wage offer being equal to the value of being unemployed or the anticipated payment equal to the value of being unemployed. When assuming that the reservation wage is the wage offer equal to the value of being unemployed, nothing much happens. The entire uncertainty cancels out. The results remain unchanged, while the level shift, the uncertainty has caused remains unnoticed. The same thing happens, if the wage and the unemployment benefit are assumed to be uncertain (See Appendix). The most interesting case is, when the reservation wage is seen as the anticipated payment being equal to the value of being unemployed.

$$w_R =$$
$$min_{\{\nu\}} \left\{ b + \theta\nu^2 - \frac{\beta}{1-\beta}\nu[1 - F(w_r)] + \frac{\beta}{1-\beta}\int_{w_R}^\infty [1 - F(w)]\, dF(w) \right\} \tag{10}$$

As the basic game I play here dictates, the evil agent knows the action of the RC agent when she determines her ν.

$$\frac{\partial}{\partial\nu} =^! 0 \ \Leftrightarrow\ \nu^* = \frac{\beta\left[1 - F(w_R)\right]}{2\theta(1-\beta)} \tag{11}$$

Therefore the expression of the reservation wage is given by:

$$w_R = b + \theta\nu^{*2} - \frac{\beta}{1-\beta}\nu^* \left[1 - F\left(w_r\right)\right] + \frac{\beta}{1-\beta}\int_{w_R}^{\infty}\left[1 - F\left(w\right)\right] dF(w) \quad (12)$$

A detailed derivation of the results and the conditions under which they hold is contained in the Appendix. As obvious, the disturbances converge to zero, when $\theta \to \infty$. As obvious, the resulting reservation wage is no necessarily greater then before. The shrinking the expected earnings has two different effects. On the one hand, the value of the job offer sinks, the offered job is therefore less attractive. On the other hand, the value of being unemployed here is much less attractive since all future job offers also become less attractive. Let's take a closer look at the optimal ν. It depends mainly on two different things, the discount factor and θ. The uncertainty level chosen $\eta \geq 0$ is typically quite small, since the agent is assumed to have already a pretty good picture of the economy, θ is therefore positive and typically quits large. As β is necessarily $\in [0,1]$ and typically close to one, the resulting value for ν is actually ≥ 0. The resulting reservation wage is surprisingly lower then before. If $\theta \to \infty$, the reservation wage is given by:

$$w_R = b + \frac{\beta}{1-\beta}\int_{w_R}^{\infty}\left[1 - F\left(w\right)\right] dF(w) \quad (13)$$

Which is just the reservation wage of the original model. The intuitive explanation for this is also straight forward. Since every expected wage offer in the future turns sour, the value of being unemployed melts down faster then the actual wage offered. Regard that the term (evil agents utility) represents the utility of the "evil agent". When interpreting this solution it can be either regarded as utility of the "evil agent" or the final result of the uncertainty creation mechanism in the single worker interpretation. It doesn't influence the utility of the worker. However, this basic relationship determines the outcomes of this class of models.

3.1.2 Basic Job Search in Continuous Time

When switching from the discrete time version to the continuous time framework, one typically considers a short time period of length Δ. When having formulated the whole model in such a way, one can make Δ infinitely small and obtain the continuous time formulation. Since this is actually a standard procedure, I will present the derivation of the continuous time version in the Appendix and work directly with the continuous time Bellman Equations. So after other standard operations, I mean substituting β by $\frac{1}{1+\Delta r}$ and assuming the worker gets a job offer with probability α, we arrive at the following system of Bellman equations.

$$rW(w,\nu) = w^e \; ; \; w^e = w - \nu \tag{14}$$

$$rU = min_{\{\nu\}} \left\{ b + \theta\nu^2 + \alpha \int_0^\infty max \left\{0, W(w,\nu) - U\right\} dF(w) \right\} \tag{15}$$

If U is the value of being unemployed, then rU is the flow value. After the usual rearrangements we yield following results for the optimal level of ν and the reservation wage:

$$w_R = min_{\{\nu\}} \left\{ b + \theta\nu^2 - \frac{\alpha}{r}\nu \left[1 - F\left(w_r\right)\right] + \frac{\alpha}{r} \int_{w_R}^\infty \left[1 - F\left(w\right)\right] dF(w) \right\} \tag{16}$$

$$\frac{\partial}{\partial\nu} \overset{!}{=} 0 \;\Leftrightarrow\; \nu^* = \frac{\alpha\left[1 - F\left(w_R\right)\right]}{2\theta} \tag{17}$$

Everything included in the basic model, as presented in Rogerson, Shimer and Wright is still in there. It explains why unemployed workers sometimes choose to remain unemployed. An increase in b for example has still the consequence, that the reservation wage increases and thus the distribution of observed wages is higher. In contrast to the discrete time version, ν^* doesn't depend on the discount factor any more, since there are no periods any more. It just depends on the probability of a job offer and the level of uncertainty. So, the more likely it is that she will receive a job offer, the more distrust will focus on the single offer. The general trade-off is still valid. As mentioned before, if every single wage melts down, the value of being unemployed also melts down. In the end, the final reservation wage is lower then before. This becomes more obvious, when focusing on the underlying wage distribution she has in mind. The way she thinks of the future wages arriving is equal to a right shift of the whole wage distribution. So, when regarding these results, what happens to the implied dynamics of the aggregate labor market? The answer is that nothing changes up to now. Thing is, that the wage offer she receives has to be at least $w_R + \nu$, because she will drop ν anyway and $w_R + \nu$ is just equal to the reservation wage without uncertainty. This is valid for both models presented up to now. In the last model presented, this relation is obvious. Remember not to regard the uncertainty creation mechanism when looking at the utility of the regular agent. This is the utility of the "evil agent", not the utility of the regular agent. The change in the model just changes the perspective the agent has on the labor market, but remains unobserved by everybody else. So if this would be the mechanism of a real labor market, generating data that econometricians could analyze, every analysis of welfare would be redundant. The econometrician would only be able to describe superficially the way the agent acts. She cannot conclude on the internal perspective of the agent since it doesn't leave any traces in the data.

3.2 Vitalizing the Labor Market

Although the models in the previous section are interesting, because they answer fundamental questions, they are still very unrealistic because they allow for a single flow: from unemployment to employment. A model with the claim of being realistic,has at least to allow for the flows observed. Given participation, there are at least two different flows the model above doesn't allow for. The flow from employment to unemployment and the flow from employment back to employment. There is nothing like a flow from unemployment to unemployment. The following sections generalize the model presented above such that they allow for these flows.

3.2.1 From Employment to Unemployment

The easiest way to incorporate this kind of flow, is to assume, that there is some exogenous force that pushes workers out of employment. They face a layoff risk determined by the parameter λ. The other interpretation for the same mechanism: a Poisson process determined by the parameter λ delivers the decision whether a worker is laid off or not. This change only affects the formulation of the Bellman equation describing the flow value of being employed.

$$rW(w,\nu) = w^e + \lambda\left[U - W(w,\nu)\right] \;;\; w^e = w - \nu \tag{18}$$

$$rU = min_{\{\nu\}}\left\{b + \theta\nu^2 + \alpha\int_0^\infty max\left\{0, W(w,\nu) - U\right\} dF(w)\right\} \tag{19}$$

I also deliver the derivation for the new Bellman equation for flow value of the wage, because I think it is not obvious on the first glance. So in my modified version of the model, there is also not very much changing. In contrast to the basic continuous time model, there are two new elements showing up in v-star: the interest rate and the lay-off probability. The formulation for the reservation wage has changed a little more, since the worker now considers the probability of being laid off.

$$w_R = min_{\{\nu\}}\left\{b + \theta\nu^2 - \frac{\alpha}{r+\lambda}\nu\left[1 - F\left(w_r\right)\right] + \frac{\alpha}{r+\lambda}\int_{w_R}^\infty \left(w - w_R\right) dF(w)\right\} \tag{20}$$

$$\frac{\partial}{\partial\nu} \overset{!}{=} 0 \;\Leftrightarrow\; \nu^* = \frac{\alpha\left[1 - F\left(w_R\right)\right]}{2\theta(r+\lambda)} \tag{21}$$

As in the previous models, the detailed derivations of the results are given in the Appendix. The change made here is actually not very deep. The only thing that has really changed, is the fact that one doesn't keep a job forever and the implied change of the effective discount rate. The discount rate is $\frac{1}{1+\Delta r}$, the effective discount rate is $\frac{1}{r+\lambda}$. The interest rate vanished before, now it shows up again, because the positive lay-off probability makes the agent think about the time after the lay off. As obvious, the basic relations in the basic continuous time model framework are still valid here. The workers own reservation wage, in the expected payment interpretation, is still lower then before. The view from the outside is still unchanged with respect to the unmodified model. As this is the simplest model suitable for a serious analysis of labor markets, I will discuss a modified version of this model in chapter 5.

As lay-offs are typically not a pure random event, as a worker in real life has the option to quit her job, there is a great need to endogenize lay-offs. One can do so, via letting a wage for a given job change with probability λ. The easiest case is to let the distribution of the new wage to be independent of the prior wage. $F(w'|w-2) = F(w'|w-1) = F(w)$. The realistic case, is to assume that the worker normally makes some kind of career. One assumes that $F(w'|w-2)$first order stochastically dominates $F(w'|w-1)$. I will only discuss the simple case here, because of two reasons. Our picture of the labor market is still very simple and the solution to the more realistic case is quite complicated. Although the derivation of the solution is that complicated, it doesn't tell much more about labor market dynamics then the easy version. The second reason is that I'm mainly interested in the change of the model framework when the worker becomes uncertain about the wage. Since the influence of the uncertainty is the same in both models, there is no possible further insight in the topic I'm interested.

Again, the interesting change happens in $rW(w, \nu)$. The change to the version before is straightforward. With probability λ, the agent decides, whether the new wage satisfies him. If not she decides to quit the job and leaves into unemployment.

$$rW(w,\nu) = w^e + \lambda \int_0^\infty max\left\{W(w,\nu)' - W(w,\nu), U - W(w,\nu)\right\} dF(w'|w) \tag{22}$$

$$rU = min_{\{\nu\}} \left\{ b + \theta\nu^2 + \alpha \int_0^\infty max\left\{0, W(w,\nu) - U\right\} dF(w) \right\} \tag{23}$$

Where $w^e = w - \nu$

Again the derivation of the solution is presented in the Appendix.

$$w_R = min_{\{\nu\}} \left\{ b + \theta\nu^2 + \frac{\alpha - \lambda}{r + \lambda} \int_{w_R}^\infty (w - w_R - \nu)\, dF(w) \right\} \tag{24}$$

And:

$$\frac{\partial}{\partial \nu} \overset{!}{=} 0 \;\Leftrightarrow\; \nu^* = \frac{\alpha - \lambda}{2\theta(r + \lambda)} \left[1 - F\left(w_R\right)\right] \tag{25}$$

The result is very similar to the ones obtained before. There are two changes of interest. If $\lambda > \alpha$, then the reservation wage is lower then the unemployment benefit and ν^* turns out to be negative. Although the latter fact has no practical consequences, as the observed reservation wage is again the same as in the standard framework, this is a fundamental change in the subjective perception of job offers. The worker now overestimates the value of a job, because it is more likely to find a better job when she works. Since she want's to be well-insured against sudden unforeseen drops of the wage, it is now better to be working when this happens. Before being unemployed had one advantage, it gave him security against sudden wage drops. Now being unemployed is something that she can still do, when the unlikely event that she drops out of employment happens.

3.2.2 From Employment to Employment

The last flow on the labor market, which is not part in the models presented up to now, is the flow from employment back to employment. So, the next model must offer the worker a possibility to look for a new job when employed. In the framework presented here, I allow for exogenous lay offs and endogenous on the job search. Again, the interesting change happens in the Bellman equation describing the flow value of a job offer. Since this framework turns out to be quite complicated to handle, the version with endogenous job transitions and exogenous layoffs is presented here.

$$rW(w,\nu) = w^e + \alpha_1 \int_0^\infty max\left\{W(w,\nu)' - W(w,\nu)\right\} dF(w'|w) + \lambda \left[U - W(w,\nu)\right] \tag{26}$$

$$rU = min_{\{\nu\}} \left\{ b + \theta\nu^2 + \alpha_0 \int_0^\infty max\left\{0, W(w,\nu) - U\right\} dF(w) \right\} \tag{27}$$

Where $w^e = w - \nu$

Surprisingly, when the wage is uncertain, all uncertainty drops out of the model. The solution is just the same as in the traditional model

$$w_R = min_{\{\nu\}} \left\{ b + \theta\nu^2 + (\alpha_0 - \alpha_1) \int_{w_R}^\infty W(w,\nu) dF(w) \right\} \tag{28}$$

As the reservation wage still includes ν and depends on the wage equation, it is necessary to reduce the sollution untill it just depends on parameters.

$$w_R = min_{\{\nu\}} \left\{ b + \theta\nu^2 + (\alpha_0 - \alpha_1) \int_{w_R}^\infty \left(\frac{[1 - F(w_R]}{r + \lambda + \alpha_1 [1 - F(w_R]} \right) dF(w - \nu) \right\} \tag{29}$$

Then:

$$\frac{\partial}{\partial \nu} \stackrel{!}{=} 0 \;\Leftrightarrow\; \nu^* = 0 \tag{30}$$

It remains to mention the interesting predictions made by the model. So workers typically move up the wage distribution during an employment spell. Not because they are skilled (since skills are not part of the model), but because they have the ability to switch to a better job during an employment spell. This framework is a basic framework which invites every researcher to discuss all kinds of microeconomic features on the labor market. It suitable to discuss aggregate labor supply, but it is not suitable to discuss general equilibrium relationships of the labor market since it is a partial equilibrium model. So it is a perfect framework to discuss the microeconomics of learning and human capital. It is also suitable to study the effects of welfare states on labor supply or to evaluate the efficiency of labor market policies. It is straightforward to include a government who pay's the unemployment benefits by taxing employed, as done e.g. by Ljunquist and Sargent (1998,2005).

3.3 Discussion

The models presented up to now are all very similar. They describe the microeconomics of a searching individual. The effects of uncertainty have only "real" consequences in this kind of model, if the unemployment benefit is uncertain. In all models presented up to now, this causes a direct and sharp drop in the reservation wages. The effects of uncertain wages are more subtle. From the outside perspective nothing changes, since the wage offers accepted are the same. The only change happens in the inside perspective of the worker. There is always the tradeoff between the drop in the expected payment and the drop in the value of being unemployed. The payment she wants to have in the end is lower then before. Although the fact that there are no real changes, if wages are uncertain holds only up to the point, as long one doesn't include any other variables, it shows that the individual perspective of the aggregate labor market and the aggregate perspective of individual labor supply can differ. Another surprising effect: in the models presented, there is no hard data generated which supports this difference. I think it is also noteworthy, that the usual relationships, which characterize the usual implications of the model, remain unchanged. So I don't try to revolutionize labor economics, I just incorporate the effects of uncertainty in the conventional wisdom.

3.4 Bargaining, Matching and General Equilibria

In this section I try to incorporate the effects of uncertainty in a typical Pissarides type Matching and Bargaining models in a general Equilibrium. In this type of Models, the job arrival probabilities are endogenized and given by a matching technology. The matching function m is typically assumed to be continuous, nonnegative, increasing and concave. It also typically assumed to have constant return to scale. The usual arguments used are the number of vacancies and the number of unemployed. So a matching function doesn't help to understand how workers and employers come together, it is a black box generating matching probabilities in a simple way, having approximately the same properties like the ones observed in the data. The Framework presented here is almost entirely different then the frameworks presented above. We have now 3 agents in the model. One representative and rational employer, one robust control worker and one "evil agent" who makes the worker uncertain about the wage. The worker and the employer bargain about the wage. Both have exogenously assumed bargaining power, θ for the worker and $(1-\theta)$ for the employer. The value of a certain wage is given by $W(w)$, the value of being unemployed is denoted by U, the profit function of the employer is denoted by $J(\pi)$ and the value of a vacancy is denoted by V. Usually generalized Nash Bargaining solution is assumed, implying to the following wage determination program. The steps between the results can be found in the Appendix.

$$w \in arg\, max\, [W(w^e) - U]^{\theta}\, [J(\pi) - V]^{1-\theta} \tag{31}$$

The condition a solution to this must fulfill is given by:

$$\theta\, [W(w^e) - U]^{\theta - 1}\, \frac{\partial W(w^e)}{\partial w} = (1-\theta)\, [J(\pi) - V]^{-\theta}\, \frac{\partial J(w^e)}{\partial w} \tag{32}$$

The employer's situation is quite similar to the worker's situation. She has a stream of output y, has to pay the wages. Her profits also depend on the vacancies. The value of a vacancy depends on a lump sum cost or flow cost k the profits and the match probability α_e. Free entry dictates that $V \stackrel{!}{=} 0$. The worker can either work or choose to stay unemployed. If she decides to work then she gets the bargained wage w and is laid off with probability λ. If she is unemployed, she finds a new job with probability α_w. In that time she gets her unemployment benefit b. As usual, the value of a job offering the reservation wage is just as big as the value of being unemployed.

$$rV = -k + \alpha_e \left[J(\pi) - V \right] \tag{33}$$

$$rU = min_{\{\nu\}} \left\{ b + \theta_1 \nu^2 + \alpha_w \left[W(w^e) - U \right] \right\} \tag{34}$$

$$rW(w^e) = w^e + \lambda \left[U - W(w^e) \right] \tag{35}$$

$$rJ(\pi) = \pi + \lambda \left[V - J(\pi) \right] \tag{36}$$

Note that there is a slight change in notation: θ is now the bargaining power while θ_1 limits the destructive power of the "evil agent". The solution to this standard framework is not as simple as the solution to the models before. The most puzzling thing is that the uncertainty feeds back into every mechanism in the model and when solving for the "optimal" degree of uncertainty, this expression gets quite complicated. One solves the Generalized Nash Bargaining using the Bellman equations for the two agents and the conditions on $W(w_R)$ and V. The resulting equation describes the relation between π, the equilibrium wage and the reservation wage.

$$w = w_R + \nu + \theta \left(y - w_R - \nu - \pi_R \right) \tag{37}$$

This equation in combination with the equation describing the value of a vacancy helps to determine the surplus generated in this economy. The surplus S is not only informative and tells us that this is no zero-sum game, it also helps to determine the matching probabilities. Bargaining implies:

$$S \equiv J(\pi) - V + W(w^e) - U \tag{38}$$

$$S = \frac{k}{\alpha_e(1-\theta)} \tag{39}$$

given this equation one can solve for:

$$k = \frac{\alpha_e(1-\theta)(y - b + \theta_1\nu^2)}{r + \lambda + \alpha_e\theta} \tag{40}$$

$$\alpha_w = \frac{\alpha_e(1-\theta)(y - b + \theta_1\nu^2) - k(r+\lambda)}{k\theta} \tag{41}$$

Given these expressions with α_w and α_e, it is possible to solve for the reservation wage depending on the equilibrium wage, the optimal distortion ν and the parameters. This minimization problem can be solved easily.

$$\begin{aligned} w_R = min_{\{\nu\}} \{ & b - \theta_1\nu^2 + \\ & \left(\frac{\alpha_e(1-\theta)(y-b)}{k\theta(r+\lambda)} + \frac{\alpha_e(1-\theta)\theta_1\nu^2}{k\theta(r+\lambda)} - \frac{1}{\theta}\right)(w - \nu - w_R) \end{aligned} \tag{42}$$

To simplify the notation, define: $A_1 = \frac{\alpha_e(1-\theta)(y-b)}{k\theta(r+\lambda)}$

The surprising result is, that there are two different optimal distortions. So as a result of including uncertainty, this economy has suddenly two equilibria.

$$\begin{aligned} & \frac{\partial}{\partial\nu} \stackrel{!}{=} 0 \;\Leftrightarrow \\ & \nu_{1,2} = \frac{1}{6}(2w - w_R - \frac{2}{A_1}) \pm \sqrt{\left(-\frac{1}{6}(2w - w_R - \frac{2}{A_1}\right)^2 + \frac{\theta A_1(y-b) - 1}{3\theta_1 A_1\theta}} \end{aligned} \tag{43}$$

So there are two possible reservation wages, two implied equilibrium wages two implied surpluses and so on. The effects of uncertainty are ambiguous here, not just because of the two equilibria, but because it depends on the degree of uncertainty where those equilibria are located. It is possible, that both are above the regular one, it is possible, that one is far above and one is slightly below and it is also possible, that both are below. The story that this tells is, that effects of uncertainty are not obvious. It is good practice in economics, to solve a model as if there was full information and in the end to relax this assumption in the end. The effects of this kind of uncertainty are often linear. Although this approach is already a quite naïve implementation of robust control techniques, it shows that the common practice may be to naïve to fully illuminate the various consequences of uncertainty.

The next step of complicating this kind of model is to assume match specific productivity. Surprisingly the solution to this approach turns out to be easier then the model before. The only difference to the model before is, that one now assumes, that every worker has a specific productivity. Most of the results of the model before can be used with only slight modifications. So the Bellman equations describing the flow value of unemployment and vacancy have changed accordingly to:

$$rV = -k + \alpha_e \int_{y_R}^{\infty} J\left[y - w(y)\right] dF(y) \tag{44}$$

$$rU = min_{\{\nu\}} \left\{ b + \theta_1 \nu^2 + \alpha_w \int_{y_R}^{\infty} W\left[w^e(y)\right] - U \; dF(y) \right\} \tag{45}$$

where $w^e(y) = w(y) - \nu$

The reservation productivity is determined by the condition, that the surplus of the reservation productivity must be equal to zero. I will take a short cut here, because I'm just interested, what the implementation of the uncertainty does to the model framework, so I will not show all results, just the ones changed in response to the distortion. Given, that the Surplus of the reservation productivity equals zero, it is possible to conclude, that the reservation productivity is characterized by:

$$y_R = min_{\{\nu\}} \left\{ b + \theta_1 \nu^2 + \frac{(\alpha_w - \nu)}{\alpha_e (1 - theta)} \right\} \tag{46}$$

Therefore, the optimal level of distortions is characterized by:

$$\frac{\partial}{\partial \nu} \stackrel{!}{=} 0 \;\Leftrightarrow\; \nu^* = \frac{\theta k}{2\theta_1 \alpha_e (1 - \theta)} \tag{47}$$

With these result, it is possible to derive all other variables characterizing the solution. When looking closer on the solution for v-star, the first thing one recognizes is, that it is positive. Therefore the reservation productivity increases and therefore unemployment and the equilibrium wage increases, since the distribution of productivities is exogenously given. Surprisingly, there is suddenly only a single equilibrium although the model has become more complicated then before. I will not discuss the case of endogenous separations, because the solution to these models with robust control type uncertainty is by far more complicated then intended in this chapter.

3.5 Models with Competitive Search and Direct Matching

I this subsection, I want to analyze the effects of uncertainty on competitive search models and direct matching models. Both models have in common, that the employers post wages and workers can only work to posted wages. The other interpretation is that a market maker, who brings workers and employees together, manages a submarket with the property that every match happens at the posted wage. Anyway as this has no practical consequences for the model, I will proceed using the first interpretation.

3.5.1 A one-shot model with wage posting and uncertainty

So, if an employer posts high wages, she will get a lot of workers. The worker on the other side sees the high wage and knows that the probability to get the job will be quite low. So in the core of the model is the matching function$m(u, v)$, which is determined by the queue length $q = \frac{u}{v}$. So the worker applies only, if the wage times the matching probability outperforms the utility of being unemployed. So, the employer has the problem to find the optimal queue length, whereas the worker mistrusts the employer and thinks that she will use the incomplete contract to cheat. The other thing which is uncertain is the matching probability of the worker $\alpha_w^e(q)$. The worker wants to ensure against unexpected fluctuations in labor demand.$\alpha_w^e(q) = \alpha_w(q) - \nu$

$$U \leq min_{\{\nu\}} \left\{(1 - \alpha_w^e(q))b + \theta\nu^2 + \alpha_w^e(q) * w^e\right\} \tag{48}$$

$$V = max_{q,w} \left\{-k + \alpha_e(q)(y - w)\right\} \tag{49}$$

Please note, that the notatoin has swiched back to the original notation. θ is now the limiter of the pertubations again. The employer considers the behavior of the worker when solving for the optimal q, but not her influence on the value of being unemployed. Therefore the optimal wage posted depends only on q. So the optimal q follows the relation:

$$V = max_q \left\{-k + \alpha_e(q)(y - b) - q(U - b)\right\} \tag{50}$$

The evil agent takes the behavior of the employer himself as given and solves for the optimal distortionν^*.

$$\frac{\partial}{\partial \nu} =^! 0 \;\Leftrightarrow\; \nu^* = \frac{w - b}{2\theta} \tag{51}$$

This result leads to the interesting property, that there are 2 wages compatible with this distortion. So we have a case with multiple equilibria again. The location of these new equilibria depends heavily on the parameterization.

$$w_{1,2} = b - 2\theta\alpha_w(q) \pm 2\sqrt{\theta^2\alpha_w^2 - r\theta\alpha_e'(q) + y\theta\alpha_e'(q)} \tag{52}$$

To develop an intuition for the model framework as it is presented here, note that the solution of the model depends heavily on the elasticity of $\alpha_{e(q)}$, even if there is no uncertainty at all. If there is now uncertainty implemented essentially one thing happens. The employer regards the change in the matching behavior and changes the wages offered accordingly. Now, since the change in the matching behavior has become nonlinear there may be several wages maximizing the profits. So although this is a simple one-shot-model this helps to understand, how employers can react on uncertainty on the workers side.

3.5.2 A Dynamic Model with Direct Matching and Uncertainty

The next model I want to discuss, is a dynamic direct matching model. So this is similar to the basic Pissarides model, just with uncertainty. This model is very similar to the Matching and Bargaining Model with General Equilibrium. Here bargaining has been replaced with the direct matching mechanism of the previous model. In here, the matching probability of the workers is no longer assumed to be uncertain. So the list of Bellman equations actually needs no further explanation, since the equations are already common.

$$rU = min_{\{\nu\}} \left\{ b + \theta \nu^2 + \alpha_w(q) \left[W(w^e) - U \right] \right\} \tag{53}$$

$$rW = w^e - \lambda \left[U - W(w^e) \right] \tag{54}$$

$$rV = -k + \alpha_e(q) \left[J(\pi) - V \right] \tag{55}$$

$$rJ(\pi) = y - w + \lambda \left[V - J(\pi) \right] \tag{56}$$

As always, $\pi = y - w$. The solution of the model presented here follows roughly the solution of the one shot model presented in the previous section and is surprisingly short. First thing one uses is the free entry condition, so $V = 0$. The next thing is, that one can already solve for the optimal distortion.

$$k = \frac{\alpha_e(q)(y-w)}{r+\lambda} \tag{57}$$

by using this condition, one can solve directly for ν

$$\frac{\partial}{\partial \nu} \stackrel{!}{=} 0 \;\Leftrightarrow\; \nu^* = \frac{q\alpha_e(q)}{2\theta(r+\lambda)} = \frac{\alpha_w(q)}{2\theta(r+\lambda)} \tag{58}$$

Those two equations give an expression relating the wage to the matching probability of the workers. Then substitute this equation into the profit Bellman equation. The employer can now substitute for the wage and optimize over q. After some rearrangements one can show, that the optimal solution for the wage is just given by:

$$w = \frac{(rU+\nu^*)\alpha_w(q) - (r+\lambda)(b - rU + \theta\nu^{*2})}{\alpha_w(q)} \tag{59}$$

To get a complete sollution, the employer has yet to be considered. She maximizes:

$$J(\pi) = max_{\{q\}} \left\{ \frac{(b - rU + \theta\nu^{*2})(r+\lambda) - (rU + \nu - y)\alpha_w(q)}{(r+\lambda)\alpha_w(q)^2} \right\} \tag{60}$$

when differentiaing one yields:

$$\frac{\partial}{\partial q} \stackrel{!}{=} \Leftrightarrow\; 0 = -\frac{b - rU + \theta\nu^{*2}\alpha_w'(q)}{\alpha_w(q)^2} \tag{61}$$

Together with $q\alpha_w(q) = \alpha_e(q)$ the solution is entirely characterized. So the central question here is, what has changed due to the uncertainty. There is a single equilibrium, the solution has become simpler and there are explicit solutions for α_e and α_w. The wage is higher then before, unemployment too. In the end nothing has happened which was not known before.

3.6 Discussion

The changes in the previous Models were mainly characterized by surprising and counterintuitive changes. All changes were "real" in contrast to the partial equilibrium models in the beginning of this chapter. While being reasonable in every single framework, the results seem to contradict each other among the different models. One says, that the equilibrium and reservation wage will fall, the other that there suddenly is no single equilibrium wage any more and the next states that the equilibrium wage will raise. In the end there is no clear message of this chapter. The only thing all models have in common, that the employers could perfectly react on the change of the workers behavior, but this is not surprising as the models were designed that way. The outcome of the model was heavily influenced by the wage determination mechanism. Slight changes in the framework resulted in massive changes in the model outcome, although the outcomes of wage posting and bargaining are similar in the original framework. Before implementing robust control techniques in this class of models, I would recommend to closely watch the data and the institutional framework in the economy of interest.

3.7 Heterogeneous Agents

The last model I want to discuss in this chapter is a model with agents preferring different levels of leisure. I discuss this in a separate subsection, since it doesn't match the characteristics of both subsections before. In the model framework presented here, there are two groups of agents, L_1 and L_2 with different preferences concerning their free time, where $L_1 + L_2 = L$. Let L_2 be the group who likes free time very much. As consequence, the unemployment benefit must differ. So group 1 receives $b = b_1$ and group 2 recieves $b = b_2 > b_1$. Therefore the reservation wages must differ. So group 1 has a lower reservation wage then group2. This has the consequence, that there is a group accepting all wages on the market and another group, accepting only a view wages. The general model environment is a general equilibrium environment. The workers perspective is quite common except for two differences. There are two different wages and the fraction of firms paying w_2 is given by sigma. The worker can work or stay unemployed. If he's unemployed she gets unemployment benefit b. With probability α_w she gets a job. If she gets a job offer this pays with probability σ the wage w_2 and with probability $1-\sigma$ one which pays w_1. So group one will get a job with probability α_w and group two will get a job with probability $\sigma\alpha_w$. When employed, the worker faces a exogenously given layoff probability λ. The difference to the standard framework is the same as in all models presented above. The workers don't trust the employers and fear that they might cheat. This creates some kind of uncertainty concerning the wage.

$$rU_1 = min_{\{\nu\}} \left\{b_1 + \theta\nu^2 + \alpha_w(1-\sigma)\left[W_1(w_1^e) - U_1\right] + \alpha_w\sigma\left[W_1(w_2 - U_1\right]\right\} \tag{62}$$

$$rU_2 = min_{\{\nu\}} \left\{b_2 + \theta\nu^2 + \alpha_w\sigma\left[W_2(w_2 - U_2\right]\right\} \tag{63}$$

$$rW_1(w_1^e) = w_1^e + \lambda\left[U_1 - W_1(w_1^e)\right] \tag{64}$$

$$rW_1(w_2^e) = w_2^e + \lambda\left[U_1 - W_1(w_2^e)\right] \tag{65}$$

$$rW_2(w_2^e) = w_2^e + \lambda\left[U_2 - W_2(w_2^e)\right] \tag{66}$$

The resulting equilibrium wages can be derived from that. Since $W_2\,(w-2)$ must equal U_2 , w_2 is simply the unemployment benefit $b_2+\nu$. Wage one can then be derived from the condition $W_1\,(w_1) = U_1$. In the end it can be shown, what w_1 is here given by:

$$w_1 = min_{\{\nu\}} \left\{\frac{(r+\lambda)b_1 + \alpha_w\sigma b_2}{r+\lambda+\alpha_w\sigma} + \frac{r+\lambda}{r+\lambda+\alpha_w\sigma}\theta\nu^2 + \nu\right\} \tag{67}$$

$$\frac{\partial}{\partial\nu} \stackrel{!}{=} 0 \;\Leftrightarrow\; \nu^* = -\frac{r+\lambda+\alpha_w\sigma}{2\theta(r+\lambda)} \tag{68}$$

The employer's perspective is entirely different then usual. The employers are divided into two groups, one paying w_1 and one paying w_2. They maximize the discounted sum of profits. The profit Π is given by $y - w_i i = 1, 2$. So when she pay's low wages, the profit per worker will be high but she will get only a small fraction of workers. If she pays high wages, the profit per worker will be quite low, but she will get a large fraction of the workforce. The profits of the firms depend on the steady state unemployment rates. The unemployment rates can be derived from the change on the unemployment rate and then implementing that $\dot{u} = 0$.

So they are given by:

$$u_1 = \frac{\lambda}{\alpha_w + \lambda} \tag{69}$$

$$u_2 = \frac{\lambda}{\sigma\alpha_w + \lambda} \tag{70}$$

The discounted sum of profits is then given by:

$$\Pi_1 = \alpha_e \frac{L_1 u_1}{L_1 u_1 + L_2 u_2} \frac{y - w_1}{r + \lambda} \tag{71}$$

$$\Pi_2 = \alpha_e \frac{y - b_2}{r + \lambda} \tag{72}$$

When thinking about finding the equilibrium, it is obvious, that two equilibria are just given by $\sigma = 0$ or σ. This passes unchanged from the standard version. The only mentionable difference to the standard version is just, that workers in L_2 will derive an unexpected surplus from working, as they expect the employers would cheat. There exists also an interior equilibrium. For it's existence it is necessary, that there is no incentive to change the employer group. Therefore profits must be equal. There are several where this is possible, depending on the output of the worker. In the standard version it is now possible, to set up an equation proportional to the difference of Π_1 and Π_2. As this version of the model is a little more complicated then the standard version, I directly subtract the two profit functions, plug in the solutions of the unemployment rates and the wages. After some rearrangements it turns out that the solution is characterized by:

$$\underline{y} = \frac{4b_2L_2\theta + L_1u_1(1 - 4b_1\theta + 4b_2\theta)}{4L_2u_2\theta} \tag{73}$$

$$\overline{y} = \underline{y} + \frac{L_1u_1\alpha_w(r + \alpha_w + 4b_1r\theta - 4b_2r\theta + \lambda + 4b_1\theta\lambda - 4b_2\theta\lambda)}{4L_2u_2\theta(r+\lambda)(r + alpha_w + \lambda)} \tag{74}$$

It can be shown, that both results converge to the standard result for $\theta \to \infty$.

So, what tells us the application of uncertainty in this model framework? The answer is not clear. Depending on the parameterization, nearly every change is possible. The only clear change is that the L_2 group will now derive a surplus from working, but they don't internalize this surplus fully, since they don't expect it.

I won't discuss efficiency wages, because the incorporation of uncertainty concerning the wage turned out to complicate the models to a degree, that it is not possible to develop a basic intuition. I therefore decided not to discuss efficiency wages here.

3.8 Discussion of the Chapter

This Chapter was designed to give a basic intuition, what robust control can do to Search Theoretic Models of the Labor Market. I divided this chapter in three subsections. The first subsections treated partial equilibrium models. The change of the behavior in this chapter seemed to be feasible. They showed the indirect consequences of the uncertainty, and that the consequences may be different then expected on the first glance. The change to the models in the second subsection seemed to be uncontrolled. They reacted uncontrollable on the different wage determination mechanisms. There was no clear statement, what uncertainty and the knightian uncertainty aversion implemented by the robust control techniques do in a general equilibrium environment. The other possible interpretation of the subsection is that wages may react very sensitive to a change in the wage determination process. Anyway, it is not possible for me to give a distinct answer, what knightian uncertainty about the wage do to a well established general equilibrium environment. The reader may judge himself, to what extend the outcomes of this subsection are regarded as useful. The results of the last subsection were also not clear, but actually quite interesting. While one group of the population clearly gained, the changes concerning the other group were not clear. So it is possible, that one group gains, whereas the other group looses. So what to conclude? The chapter was made to look what may happen, to give a first hint where it pay's to take a closer look and to develop an intuition what changes when wages become uncertain. All of these claims could be satisfied.

4 Over the Distribution

4.1 Introduction

This chapter deals with the question, how a robust control type "evil agent" perturbs an uncertain wage distribution. In general, she has actually two options. To spread the disturbances over time, as commonly done by Hansen & Sargent and others, or she has the option to spread the uncertainty over the distribution. The latter thing is tried to be done here in this chapter. The way how robust control techniques are implemented in economic reasoning here therefore differs rigorously from the way it is done e.g. by Hansen & Sargent. In the previous chapter, I used a very uncommon and naïve way to implement of robust control techniques. I left the errors embedding the distortions out, to simplify the solution and I mainly focused on a variable, the wage, which is in general quite certain. I accepted these shortcomings because of reasons mentioned above, but I have to admit, that the economic questions which can be answered with this approach are not very interesting. The approach may be somewhat helpful to discuss incomplete contracts or underhand dealings, but I don't believe that these topics are of major macroeconomic importance. But what happens to the wage distribution, if the worker is cautious? Anyway, nobody has perfect information about the part of the labor market relevant to her or him and there is often good reason to mistrust the information given by public authorities. The actual labor demand on the labor market relevant often fluctuates heavily, so there is good reason to be careful. It is also realistic to assume that the worker has prior knowledge, but believes that her knowledge may be misspecified. The next crucial assumption necessary to justify the use of robust control techniques is the assumption that the agent has no way to estimate the real environment. This remains always an artificial assumption in an economic model environment, but matches the situation of a searching worker actually pretty good.

4.2 The Actual Problem

The next question is, what the mathematical problem is, the “evil agent” has to solve. This section deals with the problem how to set up the actual mathematical problem the agent has to solve. Remember what the evil agent actually wants. The evil agent doesn’t want to minimize the utility of the agent, the evil agent want to perturb the model, or elements of the model. The RC agent doesn’t know, what the evil agent actually does and maximizes the minimum utility. So the evil agent derives her “utility” from perturbing the model. So what does that mean now for the problem here? The goal of the evil agent here is to perturb the distribution in a way that the difference between the original distribution and the perturbed distribution is maximized given a limited destructive potential. The next question is how the evil agent treats the variable of interest. Thing is that the regular agent minds about the size of the variable, she is not just interested what the variable is. So in general she will tend to overestimate variables that harm him, e.g. inflation, she will tend to underestimate variables good for him, e.g. wages, and she will be simply more uncertain about variables, where she is just interested about the location. All of these problems are just slight modifications of one single problem, the last one. Before trying to solve the modified versions of the problem, let’s focus on the solution of the basic problem.

First thing useful is to collect properties of the problem. Let us call the original cumulative distribution Function $F(w)$ and the perturbed distribution $G(w)$, where w is the variable of interest. F and G are increasing. Both must have the following properties:

$$G \in \mathcal{M} = \{G : [0, \infty) \rightarrow [0, 1]\,, lim_{w \rightarrow \infty} G(w) = 1\}$$

$$F \in \mathcal{N} = \{F : [0, \infty) \rightarrow [0, 1]\,, lim_{w \rightarrow \infty} F(w) = 1\}$$

The basic problem can be characterized by:

$$sup_w |F(w) - G(w)| \leq N \tag{75}$$

such that

$$N'N \leq \eta \tag{76}$$

The thing which makes this problem kind of special is, that the solution of this problem is the distribution function $G(w)$. As the solution of the general problem has turned out to be complicated, I decided to simplify the problem. "If need may be, discretize and calculate". So I decided to discretize the problem, which turns the whole problem into a system of n variables with n equations. Let $G(w_i) = \alpha_i$ and $F(w_i) = \pi_i$, the problem then turn into the problem:

$$max \left| \sum_{j=1}^{n} \pi_j - \sum_{j=1}^{n} \alpha_j \right| \leq N \tag{77}$$

such that

$$N'N \leq \eta \tag{78}$$

To visualize the problem.

But before thinking about the way to solve the problem, let's think a while

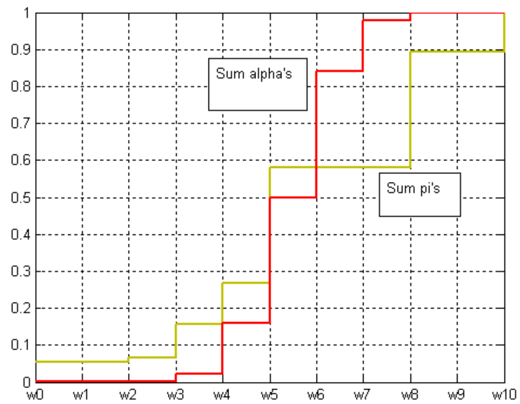

Figure 1: The Problem to Solve

how the solution should look like. A reader familiar with robust control theory should expect some kind of tube around the function of interest. One of the borders of the tube should be the worst case, the other border should be the opposite reaction, and within the tube should be the function using rational expectations and the function using the approximating model. The phenomena that the limit of possibilities available to the evil agent can be visualized using some tube exists, if she has to spread her devastations along the time. Here it would be a simple shift of the distribution as described in chapter 3. Since she uses here her whole destructive power to perturb a distribution function for all points in time, the thing gets a little more complicated. Here she has a certain absolute destructive power she can spread over the distribution. Moreover, if she changes the probability of event i, she also has to adjust the distance for all elements greater then i. To visualize what may be possible, I summarize different solution possibilities in Picture 4.2

There are now several ways to solve this problem. By looking at the picture, it has become intuitively clear, that not every perturbed distribution exhausting the limit of the "evil agent" is automatically a solution to the problem. When looking closely at the problem, one notices, that there are yet too few conditions to find an analytic solution. One can rewrite the problem in matrix form, then

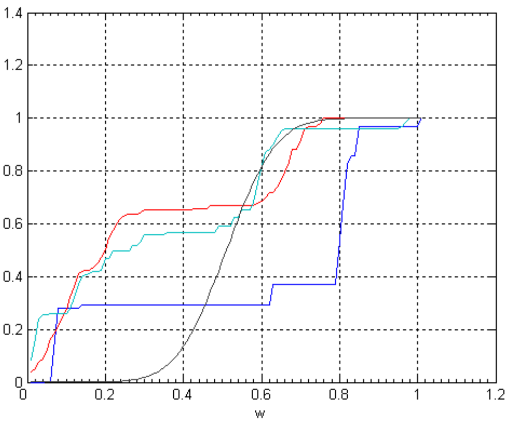

Figure 2: Different Potential Solutions

switch from the minimization problem to a set of equations problem. To do so define a distance vector V, then rewrite the problem such that $G(w)$ becomes the result of the maximization problem.

$$\begin{pmatrix} G(w_1) \\ \vdots \\ G(w_n) \end{pmatrix} = max \begin{vmatrix} F(w_1) - \nu_1 \\ \vdots \\ F(w_n) - \nu_n \end{vmatrix} \tag{79}$$

such that

$$N'N \leq \eta \tag{80}$$

where

$$\begin{pmatrix} \nu_1 \\ \vdots \\ \nu_n \end{pmatrix} = N \tag{81}$$

Let's check whether this system has a straightforward solution. So we have $2n$ unknown variables, but only $n+2$ restrictions. So we have a unique solution for $n = 2$ and an under-determined system for all $n > 2$. It is not necessary to mention that this situation is highly unsatisfying. I will not argue that the derivation of an analytic solution is not possible. By further transformations of the system and finding additional restrictions there may be a general analytic solution of the problem. There may also be analytic solutions for special cases, e.g when assuming a specific original cumulative distribution function like a cumulative exponential distribution function. But special case solutions are also not satisfying, since I want to find a general mechanism solving any problem of this kind.

The alternatives to analytic solutions are either taking numerical solutions or brute force solutions. Numerical solutions appear on center stage, when there is a system of equations where there is no analytic solution to a potentially solvable system or finding the analytic solution is too complicated. Brute Force methods provide solutions, even when numerical methods don't help. So when developing a general solution method I decided to take the brute force approach. Any numerical approach would be only able to solve one specific problem, so when realizing that the original distributions function taken is either unrealistic or incompatible with the model, where it is implemented it would have to be developed from the very beginning. A brute force approach solves the problem no matter what the original distribution is, finds every solution, if there may be several and has no problems in finding complicated corner solutions. The disadvantages of brute force solutions are the high computational power necessary and the lack of insight in the reasoning of the evil agent.

4.3 A Brute Force Algorithm

So to solve the problem I decided to develop a brute force solution mechanism based on random numbers. The basic idea is, to generate random numbers, transform them such that they add up to a cdf and collect them. When there is a large number of potential cdf's, the most disastrous fulfilling the capacity condition is selected. So the result get better the more cdf's are generated, and the lower n, the number of events, is. So that was the basic idea in short, I proceed by describing how the program written to solve the Problem. The challenge is, to be persistent against the "curse of dimensionality". The term "curse of dimensionality" has been coined by the well known engineer and applied mathematician Richard Ernest Bellman, the inventor of the dynamic programming. It refers to the massive increase of hypervolume as a function of dimensionality. An example: When discretizing some continuous function, one splits the function into a set of units. When one tries to spread the units now over an input space, one is interested, that the average distance to each point is relatively small, to ensure that the calculations done are somehow precise enough. It is now intuitively clear, that the number of units necessary to ensure a stable average distance to each point increases exponentially when increasing the number of dimensions of the input space. To make the problem clearer, I refer to an example given by Leo Breiman: 100 units cover the one dimensional unit interval $[0, 1]$ on the real line pretty well. But when one spreads 100 units on the corresponding 10- dimensional unit hypersquare, 100 units are isolated points on an empty map. To reach a similar coverage, one needs 10^{20} units, which is already quite an undertaking.[7] Related on my problem here, this means that it is not possible to give a precise approximation of the perturbed cdf and to be close to the real solution at the same time, since computation power is still a binding constraint to me.

So what do I actually do? The Program I've written is called chapter4.m and is organized in four subparts. In the first step I define the basic Settings. The second part generates and collects possible solutions if the location of the variable of interest is of importance. In the end the most disastrous cdf is selected. The third part does the same as the second part, if the size of the variable of interest matters in a positive way. The last part collects and displays the result in the end. The description of the first part is a good opportunity to introduce all features of the program. The first setting (regard) determines, how the variable shall be treated. What it actually does is to determine whether parts 2 or 3 are used to solve the problem. As you might have noticed, I left

[7] The example is taken from: Leo Breiman. "Random forests". Machine Learning, 45(1):532, 2001.

out the possibility where the variable of interest matters in a negative way (like inflation). Since it is neither necessary to understand what the evil agent does, nor any application of it is needed in this thesis, I decided to leave this possibility out. It would be anyway just a trivial modification of part 2. So possible settings for *regard* are 0 (uses part 2) and 1(uses part 3). The next possible setting is called k and determines the number of possible cdf's which will be generated. Closely related to k is the setting w, which sets the number of units used to approximate the cdf's. So when k and w get large, the time necessary for computations gets long. To give an example, if k=100000 and w=10 it takes approx five hours on my computer to solve the problem. The settings "s", "m" and "B" are just location parameters. So s determines the distance between the units. The settings m and B are the moments of the distribution used, where m is the first moment and B is the second moment. The setting *eta* regulates the destructive capacity of the evil agent. To make clear, how the restriction has to be understood: For every unit, there is a difference between the original and the perturbed cdf. The differences are collected in an $(w+1 \times 1)$ vector v. Now $v'v$ has to be smaller or equal to *eta*. The settings *mue* and *moe* influence the way how the cdf's are generated and are my solution to the curse of dimensionality problem. To understand why they "solve" the problem, let me just explain, how the cdf's are generated. The first thing I do is to generate numbers, by using the pseudo-random number generator with standard seed, to generate an AR1 process[8].

$$z_i = \phi z_{i-1} + \gamma(r_1 - .25) \tag{82}$$

Where $z_1 = 1.5 * (r_1 - .25)$ and r_1 is a uniform distributed pseudo random variable with bounds $[0;1]$, $i \in [1, w]$. The resulting numbers are then cut back such that they are in the interval $[0, 1]$, because it must be possible to interpret the result of the process as likelihood. As I don't want to simulate or estimate time series, there is no need to care about unit rooters in the characteristic

[8]the design of the formulas describing the program are abutted on the design of the actual program, so that there should be no problem understanding the Matlab Code.

polynomial of the AR1 process. It is just a process which has shown to be able generating a large number of very different cdf's. What I do next is:

$$v_i = \frac{x_i}{\sum_{i=1}^{w} x_i} \tag{83}$$

Where $x_i = max(min(z_i, 1), 0)$

The resulting variable v can be interpreted as event likelihood, in the sense that $v_i = P(w_i)$. To get to a discretized cumulative distribution function, they just have to be added up in the way, that:

$$\pi_i = \sum_{1}^{i} v_i \tag{84}$$

Where π_i corresponds to a candidate $G(w_i)$ of the problem presented in the beginning[9]. So, what are "mue" and "moe" now? These settings are the parameters determining the AR1 process. To be precise: $\phi = mue$ and $\gamma = moe$. This makes it possible that one can generate cdf's in the neighborhood of a certain cdf. Why does that solve the curse of dimensionality problem? Given that one has generated cdf's in a broad area, and the solution seems to be imprecise, these two settings enable the researcher to generate a large number of cdf's in the direct neighborhood of the prior solution to get a more accurate result.

So this is actually just an application of the oldest idea how to overcome the curse of dimensionality problem: by learning. It would have been nice to develop an automated learning mechanism here, but the constraint concerning the computational power is too strict to allow that. The last setting is the selection of the original cumulative density function called p. One can choose every cdf available. I decided to work basically with a normal cdf with mean 0.5 and standard deviation of 0.1 following Ljungquist and Sargent (1998).

The second part and the third part work very similar. It would have been able to build a single part doing both jobs. The reason why I set up two different,

[9]Note that the Notation used has changed in this chapter, so π is no profit any more, whereas ν still denote the disturbations.

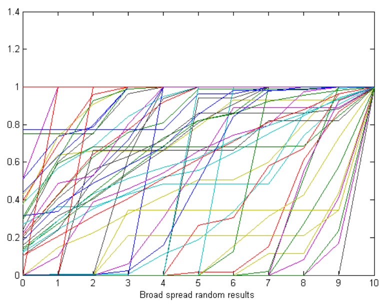

Figure 3: Broad Spread Random Solutions

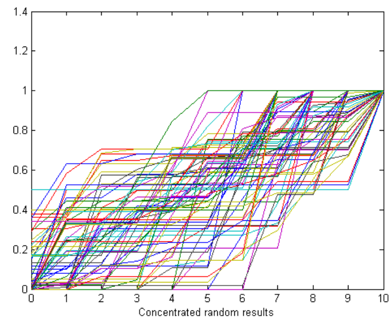

Figure 4: Concentrated Random Solutions

but almost identical parts is just, that the program runs faster that way. Both parts are actually a collection of various “for-loops”. There is a one for-loop being repeated k times generating many solutions and a for-loop picking the out the best solution. The for loop generating and collecting the cdf’s has also two parts. The first part is the cdf generation described above. The second part picks out the cdf’s which might be a solution to the actual problem. First thing done is to save the generated cdf. Then the absolute difference to the original distribution function is measured. The differences, here written in a vector called G, are also saved to be able to check them afterwards. Then the capacity check will be performed as described above. All differences which survive the capacity check are saved in a matrix called A. The basic difference of this for loop in the third part is that there is an additional check which makes clear that the original cdf will be underestimated. The second part starts, by building a long vector with the sum of the differences of each single cdf. The difference to the capacity criterion is, that the capacity criterion is quadratic which throws all cdf’s out, which have too large single deviations. The second for loop then picks out the most disastrous cdf having survived the capacity criterion. This part is identical for both parts. So the second part delivers the most devastating way to perturb the original cdf. The third part delivers the most devastating way to underestimate the original cdf.[10]

[10]There are actually two way’s to make the evil agent care about the size of the wage, the

The fourth part is actually very simple. It just takes the original and the perturbed cdf , computes the probability density function of the original cdf by first differences to make them comparable. Remember, that the perturbed cdf was made by summing up the pdf. In the end it plots the results.

As announced, this is a way to solve the problem via brute force. I won't conceal the problems this approach has. The first disadvantage is the immense computing power necessary to run the program. The second disadvantage the program has is, that it doesn't accept the original function itself to be a solution of the problem, although there is no single case known, where this turned out to be a solution. This is actually a problem of Matlab, because it treats a resulting zero-vector as "undefined function or variable".[11] Third disadvantage is, that the learning mechanism is actually the user, so the user must know how to adjust "mue" and "moe" in the proper way. Another disadvantage is, that if the number of units gets large it still takes a long time to solve the problem, since there are several readjustments necessary to reach a satisfying solution. The last disadvantage is that due to the random character of the program, there are no well defined conditions, how close the result of the program must be to the real solution. It would be nicer to have a numerical algorithm that tries every combination and picks out the best one, then it would be possible to have a well defined average distance to the values of the true solution. But remember, for 2 digit precision and 10 units this would already require to check up to 10^{19} combinations[12] if they could be a solution. To find a good solution with 20^5 random combinations deems quite efficient to me.

first thing is to give weights to the singe ν's, the other to impose a restriction. I imposing a restriction, because it is more efficient.

[11] To make sure, that the original cdf is also a candidate sollution, the original sollution will be assumed whenever a candidate drops out.

[12] This is the maximum number of candidate sollutions possible, for usual sollutions like the ones given there are of course less, because of the restriction and the property that cdf's are increasing.

4.4 The results

Before looking at the results, lets think a while about them to be able to check, whether they are reasonable. I will start out with the simplest ones in the first case, using the second part of the program. If the agent is extremely fearful, and the original cdf puts the whole probability mass on one of the first/last unit, the perturbed cdf will put the whole probability mass on the last/first unit. The second obvious thing: If eta is small, the perturbations will be more or less equally distributed over the original distribution function. If they would be concentrated on one side, this would require large single deviations. In the second case there are also at least two obvious things. The first obvious thing is, that the perturbed cdf will be always more pessimistic, close to an asymmetric right shift of the original cdf. The second obvious thing is that the perturbed cdf will always put more probability weight on the lower units and less on the high units.

When regarding the results, it becomes clear, that they seem to fulfill the expectations. They look precisely what they should look like. Moreover a single solution existed for every cdf-eta combination tested. Cdf's with similar performance were very close to the one selected. When regarding the pictures, it becomes clear, what the RC agent expects. So let's look first at the results of the first case. For small eta's, the perturbed distribution starts to flatten out in an asymmetrical way.

So the first thing which happens is that the RC agent puts more probability weight on cases which are highly unlikely in the original distribution. The interesting thing is, that the perturbed cdf cuts the original cdf. So to use the space under the original cdf, the agent prefers to have small deviations in the center of the distribution. When increasing eta further, then the resulting perturbed cdf looks very close to an uniform distribution. Every event deems him to be almost equally likely, so she already mistrusts the information she has to a degree, that it is almost not possible to tell anything about the process. This changes again when dealing with a large etas. In the end, the agent mistrusts

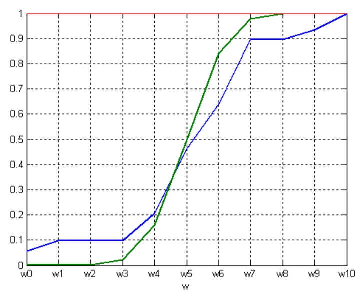

Figure 5: Result eta=0.1

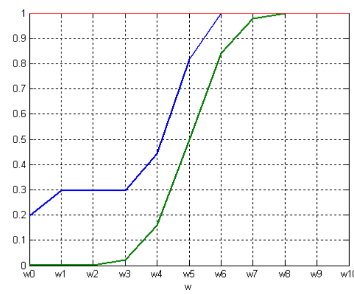

Figure 6: Result eta=0.5

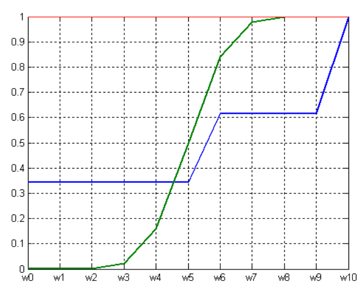

Figure 7: Result eta=0.1

her prior information completely and assumes that the opposite is true. The whole probability weight is shifted to the outside. It is not necessary to mention that this would wreck every model in which this distribution would be implemented. The agent would always tend to extreme decisions. One if the most interesting things is the u-shaped degree to which the agent makes use of her prior information. At the beginning, she is just skeptical that the information is true. She insures himself by shifting a bit of the probability weight to the outside. When the uncertainty gets bigger, she just forgets almost everything she knows. Every event seems almost as likely just as likely as every other event. When the uncertainty increases to an extreme point, she makes use of the information again, she assumes that the opposite is true.

In the second case the reasoning of the agent becomes simpler. She has just the possibility to become more skeptical. One might get the idea, that a simple right shift of the whole distribution may be the optimal solution, but this can be clearly rejected by simple reasoning. The deviations to the original distribution are measured in a vertical way. A symmetric right shift on an horizontal axis would require an asymmetrical shift in the horizontal direction. The first deviations would have to be zero and the last deviations would require being zero, while the deviations in the center on the distributions would have to be quite large. As the capacity criterion is quadratic, there must be a solution which makes more damage using the whole capacity.

So the actual solutions are all asymmetric right shifts of the original distribution. The agent becomes more skeptical in an almost linear way. An agent, who is extremely uncertain, believes that she will for sure get the worst wage possible. The potential economic consequences of that will be discussed in Chapter 5.

4.5 Discussion

The solution to the actual mathematical problem has turned out to be complicated. There are also no obvious analytical solutions to the discretized ver-

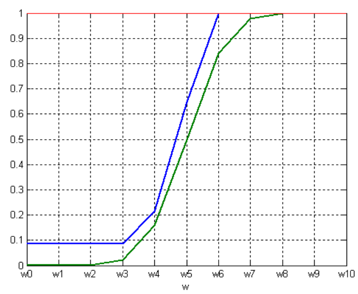

Figure 8: Result eta=0.1

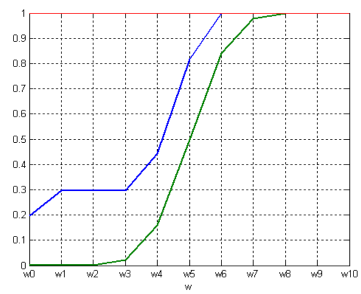

Figure 9: Result eta=0.5

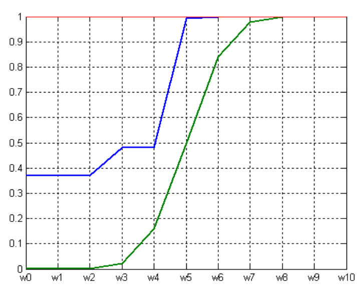

Figure 10: Result eta=1

sion of the problem. There may be numerical solutions to specific problems, since the misspecification increases in a linear way and the capacity criterion is quadratic. But there are so many possible combinations, that the numerical approach doesn't seem to be promising. A more efficient way to solve the problem would be to write an automated learning algorithm, such that the total number of tested combinations decreases even further, but such a project would be far beyond the scope of this diploma thesis. The results in the end deem promising to me. They are understandable, although no formal decision rules are available. It is possible to develop an intuition for the way the RC agent perturbs the original cdf.

5 Perturbed Believes

This Chapter is designed to show, what the perturbed believes concerning the original distribution function do within a labor search model. In other words, the basic idea of this Chapter is, to use the results from Chapter 4 within a slightly more sophisticated search model environment. The results in the end are that reservation wages and unemployment rate decrease compared to zero uncertainty. Unemployment duration will also be shorter then without uncertainty. Search intensity changes, but the direction is not obvious on the first glance. The basic model framework I modify, is a slightly more sophisticated version of the partial equilibrium models discussed in Section 3.2, but is still far from the research frontier in search theory. To be more precise, I used a simplified version of Ljunquist and Sargent (1998). The reason why I tired to keep the model simple is that I want to show the undistorted fundamental changes due to the perturbed believes of the worker in this class of models and what they might mean for the implied dynamics of the aggregate economy. I don't want to focus on details like the change in individual skill aggregation or include features that partly absorb the effects I want to show. The reason why I used a partial equilibrium framework here is, that one of the results in Chapter 3 was, that the relations within these models appear to be stable if robust control techniques are implemented. The robust control type used here is H_2 robust control, as the changes appear completely outside the objective function. If there would be a straightforward analytic solution to the problem discussed in Chapter 4 it would be possible to set up the model as H_∞ robust control problem. But the disadvantages of using H_2 robust control are very limited. The only major disadvantage I could find is, that there are no expressions describing the reasoning of the evil agent within the model framework. This has also the consequence that it is not possible to hand to exact utility of a wage to the evil agent. So there is only one of the two agents acting inside the model(the worker). The changes due to his uncertainty happen exogenously to her. I will proceed here as follows, I will first present the model framework used here, then I'm going to describe the way I solved the model. Before concluding I present the results of the model.

5.1 The Model

As mentioned before, the actual model is very simple. There are two agents, an evil agent and a RC agent(the worker). The actions or if you like, the consequences of the deeds of the evil agent are solved outside the model namely in the program of chapter 4. So there is only one representative agent in the model.

5.1.1 The Economy

First thing is, that the model has been set up in discrete time. The agent behaves rational within the framework presented to him and represents a continuum of workers. The workers are assumed to have geometrically distributed life spans. The life span is interpreted as entering and leaving the labor market. The population is assumed to be constant, so the number of born workers equals the number of agents who left the labor market. The subjective survival probability is assumed to be indexed by $\alpha \in [0, 1]$. When unemployed, workers can search for a new job. Their search intensity is denoted by $s_t \geq 0$. The resulting probability to get a job offer is $\pi(s_t) \in [0, 1]$. Searching is exhausting, so he has to pay search costs $c(s_t)$. He can either accept the job offer, or he can choose to stay unemployed. The job offer comes from an unknown and time varying distribution. The long run average of this distribution is $F(w)$ and is assumed to be the original or here approximating distribution, which the agent fears to be misspecified. As he has no further information, the agent insures against that, by acting as if the true wage distribution was $G(w)$. This means that he makes strategic mistakes, and he knows that he probably acts suboptimal. So this is a pure search model, there is no matching in the model. While being unemployed, the worker receives an unemployment benefit b. The unemployment benefit is assumed to be constant over time. When employed he gets the wage accepted till he is laid off again. There is no search while employed. There are at all three possibilities what can happen when he's employed. He can either stay employed, he may be laid off with probability $\lambda \in [0, 1]$ or he can decide to quit the job. The objective function the representative worker has is:

$$E_0 \sum_{t=0}^{\infty} \beta^t (1 - \alpha)^t y_t \tag{85}$$

Where E is the expectations operator and his income y_t is b when unemployed and w_t when employed. The dicount factor is denoted by β. The model is set up using the usual set of value functions. The value of being employed is denoted by $V(w)$, the value of being unemployed is denoted by $V_b(s)$. So the state variable is w_t the control variable is s_t. Note that the set of Bellman equations is denoted by:

$$V(w) = max_{accept,reject} \left\{ \left(w + \beta(1 - \alpha) \left[(1 - \lambda) V(w) + \lambda V_b(s) \right] \right), V_b(s) \right\} \tag{86}$$

$$V_b(s) = max_{\{s\}_{t=0}^{\infty}} \{-c(s) + b + \beta(1 - alpha) \\ \left[(1 - \pi(s))\, V_b(s) + \pi(s) \int_0^{\infty} max\, [V(w), V_b(s)]\, dG(w)\right] \quad (87)$$

s.t

$$G(w): \; max\, |F(w) - G(w)| \quad (88)$$

The optimal search intensity is denoted by s^*. The resulting reservation wage is denoted by w_R.

What I'm trying to do, is to solve for the steady state of the economy. So I can assume, that the true wage distribution is indeed $F(w)$, but the worker acts according to $G(w)$.

As mentioned above, this is a simplified version of Ljungquist & Sargent (1998). For completness, I have to mention what has been left out: The two central points I left out are the skill accumulation mechanism and the government who can decide to quit the unemployment benefit, must finance the benefit via taxes and gives every worker and unemployment benefit according to his last wage. For further details I have to refer to the original paper.

5.1.2 The Calibration

I calibrated the model following basically the recomandations made by Ljungquist and Sargent. So β is assumed to be 0.9985. The probability for dying and being laid off are set to α =0.0009 and λ =0.009. So the worker works on average for 42.7 years, she is laid off 10 times on average, so the average span a worker spends in a job is 4.3 years. The benefit b is set to 0.2, the return to search and the costs of search are calibrated in the following way:

$$\pi(s) = s^{0.3} \;;\; c(s) = 0.5\, s$$

The original wage distribution is assumed to be normal with mean 0.5 and variance of 0.1. There are 11 wages in the economy, the wages are normalized on the unit interval, so $w \in [0, 0.1, 0.2, \ldots, 0.9, 1]$. The probabilities for the single wages are therefore discrete and add up to one. The subjective probabilities for the job, summarized in the function $G(w)$ add up to one by construction.

5.2 The Solution Method

I decided to solve the model in via a simple Value function iteration. The basic principle is to determine the perturbed wage distribution $G(w)$ first with the program presented in Chapter 4. The rest of the Model is solved by value function iteration, taking the perturbed and the original distribution function's as given.

While Ljungquist & Sargent used iterative methods to solve their model via numerical simulations, I decided to solve my simplified version via simple Value Function Iteration. The reason why I solved the model numerically was, because the analytic solution of this simple model has already turned out to be enormously complicated. The other reason is, that there are only numerical solutions to the perturbed wage distribution. The reason why I preferred Value Function Iteration to other numerical solution techniques is that it works precise even if the model is highly non-linear.

What I did is, I took the value functions as given, started out by some pair of arbitrary $[V^0(w), V_b^0(s)]$ and took the resulting value as new stating value, or more formally the k'th iteration step is given by:

$$\begin{aligned} V^{k+1}(w) = max_{accept,reject} \{(w + \beta(1-\alpha) \\ \left[(1-\lambda) V^k(w) + \lambda V_b^k(s)\right], V_b^k(s) \end{aligned} \quad (89)$$

$$\begin{aligned} V_b^{k+1}(s) = max_{\{s\}_{t=0}^{\infty}} \{-c(s) + b + \beta(1-alpha) \\ \left[(1-\pi(s)) V_b^k(s) + \pi(s) \int_0^{\infty} max\left[V^k(w), V_b^k(s)\right] dG(w)\right] \end{aligned} \quad (90)$$

The contraction mapping property implies that the error for each iteration step is given by:

$$\left|V^k - V^*\right| \leq \frac{1}{1-\beta(1-alpha)}\left||V^{k+1} - V^k\right|$$

where V^* is the real value of the value function[13].

To solve the Bellman equation describing the value of being unemployed the maximization problem had to be solved for every iteration step. When the difference between steps k and k+1 reached the error-tolerance, the iteration stops and the other variables are computed recursively. As there are only very view wages, there is almost never an existing wage making the worker indifferent between working and being unemployed. The reservation wage is therefore computed from the Value Functions available. We always have a value of a wage below Vb and one above Vb. Then the relative position of Vb between the other two wages determines the reservation wage. The error made should be negligible. Aggregate variables depending on the reservation wage will be computed with the virtual reservation wage, individual variable with the first wage where the worker is willing to work. It has turned out, that the approximation error using strictly eleven wages is too big to study changes on the macro level.

To do that, I wrote a Matlab program again. The program can be found in the Appendix and is called "chapter5.m". The solution of the program is obtained in six steps, steps zero to step five. In step zero, parameter values and the settings for the core of the chapter4 program are set. The wages are defined by the setting of the chapter4 program. There is also an additional switch, where one can decide to skip the perturbation and pass just the original distribution to step $4(R-ON)$. Step 1 consists of the core of the capter4.m code. I won't describe it again, it is already described I detail in chapter 4. I use the second type of perturbations to solve the model. The option to switch to type one is still included. The difference to the original code is just, that the superfluous matrices are deleted in the end and the final result is passed to step 3. Step 2 is also quite simple. First thing done there is to transform the units

[13]Compare: Kenneth L. Judd, 1998. "Numerical Dynamic Programming"Chapter 12

of step 1 in wages for step 4 and 5. Second thing done is to extract the discrete wage probabilities from the original distribution, if the $R-ON$ switch is turned off. Last thing done is to define the grid of the search intensity. Step 3 is also simple. The initial values for the Value Functions are defined here. $V^0(w)$ and $V_b^0(s)$ are assumed to be zero. $V^0(w)$ is an (11×1) zero vector and $V_b^0(s)$ is just a zero scalar. The second thing done is to define the tolerance level, start the counter for the while-loop in Step 4 and to define the initial error. Step 4 includes the actual value function iteration. The whole step is actually just a big while loop. The Bellman equations from the model are entered unchanged. The initial values are inserted and the new values are just the result of the Bellman equations. For the second Bellman equation, the optimal level of s_t has to be determined. Then, the error is determined, the values for the Bellman equations are updated and the whole while loop starts again. It stops when the error is lower then the error tolerance. The while loop converges at rate $\beta * (1-\alpha)$. So it takes a large number of iterations till convergence is reached [14].

Step 5 calculates the other interesting variables like the unemployment rate, the average unemployment hazard rate and the average unemployment duration recursively from the reservation wage first. Then it returns the results to the user.

5.3 The results

The goal of this thesis is to find out, how the job search and job acceptance behavior is affected, if the worker is knightian uncertain. So when presenting the results now, I compare the results when solving the model without perturbations with the results of the models with perturbations of different degrees. So I solve the model several times, the only difference is the level of uncertainty (eta).

The results without perturbations are now considered as benchmark results. They do not need to be realistic; they are a fixed point to compare the different steady state solutions with different levels of uncertainty. To compare to

[14](It takes usually ≈1200 steps till convergence is reached, so it converges pretty slow.)

outcomes of the different models, I will focus on a compact set of five different factors. The unemployment rate, the virtual reservation wage, the search level, hazard rate and average unemployed duration. I will also compare the values of the different states.

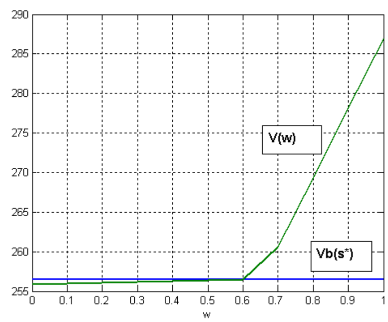

Figure 11: Value Functions eta=0.1

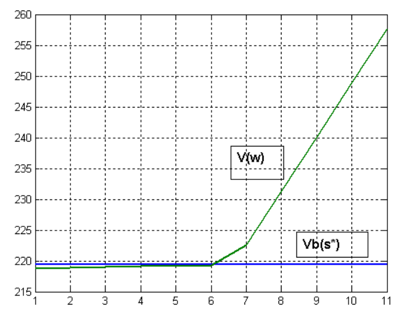

Figure 12: Value Functions eta=0.5

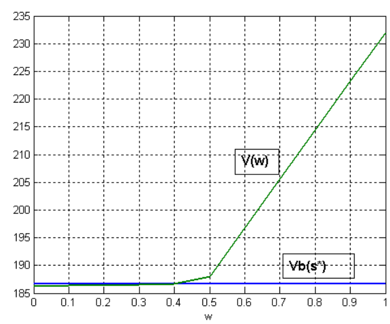

Figure 13: Value Functions eta=1

The Benchmark case represents an economy with moderate frictional unemployment (7.12%). The unemployment benefit is actually quite low (0.2) compared to the real average wage, which is 0.5 by construction. The observed average wage is higher, of course (0.72). The wage distribution is not assumed to be realistic. It is a centered symmetrical distribution with low variance. I followed the suggestion by Ljungquist & Sargent, because it is a good starting point to compare different degrees of perturbations. The virtual reservation wage is actually quite high (0.6) while the average search level seems to be moderate to low. The results of the model with the different settings are summarized in Figure 5.1.

	Eta=0	Eta=0.1	Eta=0.2	Eta=0.3	Eta=0.4	Eta=0.5	Eta=0.6	Eta=0.7	Eta=0.8	Eta=0.9	Eta=1
w_R	0.6012	0.6008	0.6003	0.5053	0.5033	0.5018	0.5035	0.4080	0.4047	0.4035	0.4056
s*	0.3889	0.3796	0.3616	0.3297	0.3233	0.3123	0.3241	0.2484	0.2426	0.2384	0.2448
U_bar	0.0712	0.0713	0.0718	0.0255	0.0253	0.0252	0.0253	0.0164	0.0163	0.0163	0.0163
Hazard	0.1558	0.1568	0.1580	0.4790	0.4868	0.4928	0.4861	0.8212	0.8297	0.8328	0.8274
Duration	6.4166	6.3764	6.3303	2.0875	2.0542	2.0292	2.0570	1.2178	1.2052	1.2008	1.2086
$V_b(s)$	256.55	252.02	243.16	227.83	224.87	219.38	225.13	188.57	185.65	183.53	186.77
$V(w^e)$	257.37	252.83	243.94	228.56	225.59	220.08	225.85	189.16	186.23	184.10	187.35

Figure 14: Summary of the Solutions

When increasing the degree of uncertainty slightly, there are only small changes happening. The first thing one observes is that the values of the different value functions decrease. This is not surprising, as the worker knows that her decisions are suboptimal. So she knows that she might be better of when she wouldn't be uncertain. She is just afraid of the worst case scenario she can imagine, and maximizes her utility with respect to that. So the decrease in utility is owed to the uncertainty aversion. The reason why she forgoes parts of her aggregate utility is that the reservation wage has decreased because the wage distribution she has in mind tells her, that it is extremely unlikely to get a job with high income. So she is willing to accept an income slightly below the income then before. This has two consequences. As she believes, that the best wage she can get will be lower, she accepts job offers she has not accepted before. This has again another consequence, as the wage has decreased the expected return to search has also decreased. She therefore searches less then before.

When increasing the uncertainty further to eta =0.3, the changes to the model appear to be quite drastic. The reservation wage has fallen such that the individual worker accepts another wage, the wage .6. She believes, that gaining any wage higher then .6 is practically impossible and so she expects

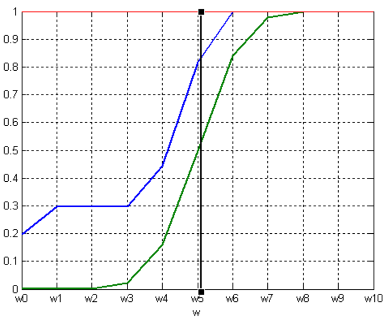

Figure 15: Res. Wage eta=0.5

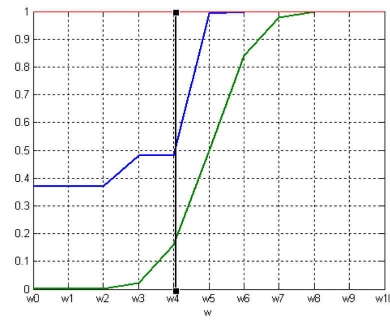

Figure 16: Res. Wage eta=1

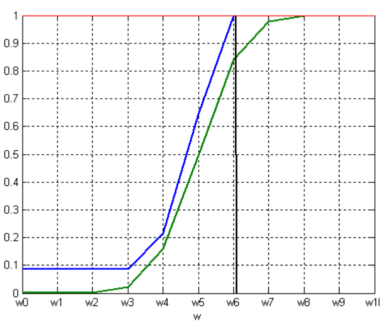

Figure 17: Res. wage eta=0.1

a wage of exactly 0.6. Practically these wages are possible, and so the actual observed average wage is higher then 0.6. Already at this point the assumed level of uncertainty becomes unfeasible. The worker believes that there is no possibility of getting a higher wage then .6 although she clearly recognizes that there must be wages higher then 0.6. She should somehow underestimate the probability that she gets a higher wage. Anyway, the unemployment rate has dropped drastically. The probability of leaving unemployment in a given period has strongly increased. The effects observed get even stronger, when increasing the uncertainty further. A worker tends to accept wages she has not accepted before. The expected value of working decreases very strongly, since the value of being unemployed depends of the expected value of working next period, the value of being unemployed also decreases. The key result can be summarized in one sentence: The worker tends to accept lower wages then before, while the intensity of search also decreases.[15]

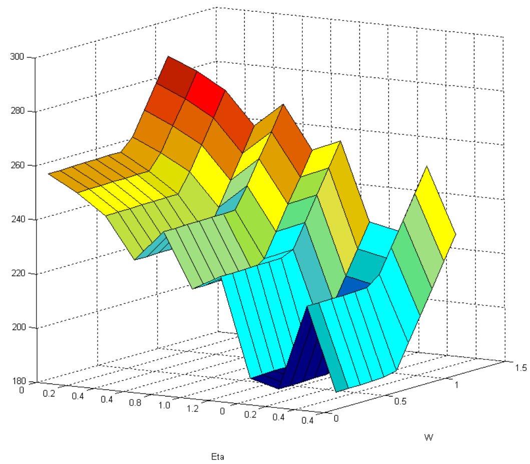

Figure 18: Graphical Summary of the Solutions

It has turned out that it doesn't make sense to try type one uncertainty in the context of this model. As the uncertainty causes to put high probabilities on unlikely events, it turns out that a massive part of the probability mass will be shifted to the high wages, which makes the worker super-optimistic. Frictional unemployment reaches levels of twenty percent and above.

5.4 Discussion

What does the incorporation of uncertainty in a common search theoretic model framework tell us in the end? The key question here is, does it make sense to incorporate this kind of uncertainty in this search model. I think the answer is yes. The results where feasible, the wreckage done to the model using high degrees of uncertainty was predictable, but when using small degrees of uncertainty, the perturbation of the model were moderate and the behavior seems to be closer to human behavior then in the rational expectations case. So here

[15]The slight fluctuations account for the random nature of the solution procedure

the slight decrease of average wages and average unemployment rates seem to be feasible. The model seems to capture the uncertainty while searching for a job and seems to be able to explain abnormal behavior of unemployed workers. So it is already felt that the extension might increase the explanatory power of search theoretic models. There is a criticism that the use of robust control techniques leads automatically to an increase of the formal level to a degree, where hardly anybody is able to understand what is actually happening. This can not be shared. The results also differ substantially from the results of the rational expectations solution, while the fundamental relationships driving the results of the model where only slightly perturbed. The reasoning of the worker doesn't seem to be weird. A short example, given that the labor demand differs indeed from the long run average and suddenly behaves according to the feared wage distribution G(w), rational expectations workers would have to suffer a period of high unemployment while RC workers would just observe a slight decrease of the average wage.

6 Summary and Final Conclusions

The aim of this chapter is to summarize and discuss the results of the previous chapters. In the end I will draw some concluding remarks.

6.1 Summary

The central question of this thesis was how the job search and acceptance behavior is influenced by using robust control techniques of model uncertainty. In the very beginning, I gave a short overview about the agenda and the problem itself. This introduction was followed by an overview over the existing relevant literature. This had to be done in two separate subsections, since this is the first time robust control techniques are used in search theoretic models. Chapter three of this thesis applied a in a naïve way robust control techniques to various search theoretic models. This was done tracking closely the models presented in the literature survey of Rogerson, Shimer and Wright(2005). The main message was that the approach chosen here seems to be promising in partial equilibrium models and needs further refinement for models with general equilibrium. The results of the partial equilibrium models are comparable to the results of the model presented in chapter five. Chapter four developed a method to solve the problem, how distribution functions are perturbed, when spreading the whole disastrous capacity of the evil agent over the wage distribution. The result was a brute force mechanism, able to solve the problem for any numerically given cumulative distribution function. Chapter five applies the results of Chapter four to a common search theoretic model with partial equilibrium. The key result was, that the worker tends to accept lower wages then before, while the intensity of search also decreases. Chapter 6 concludes.

So this diploma thesis tried to bring two workhouses of economic theory together. Search Theoretic Models of the Labor Market on the one side and Robust Control on the other side. The result in the end seems to be a very promising approach. In the end, the worker was able to deal with fluctuating and uncertain labor demand while the utility she lost was found to be relatively small, while the rational expectation worker who was unable to anticipate the sudden drops in labor demand and lost substantial parts of her aggregate welfare. Given that there are indeed unforeseen fluctuations, it seems natural to develop a certain amount of model uncertainty over time.

6.2 Conclusion

After all, the analysis done here was intended to remain on a basic level, to be able to shed light on the fundamental changes of the models discussed. Although using robust control methods, there was never the danger not to see the wood for the trees. It was always easy to develop an intuition what is actually happening, even if the actual decision rule wasn't available or the analysis of the model was complicated. There are two basic differences to the usual approach making use of robust control techniques. The first basic difference is, that the amount of uncertainty available has been spread over the distribution and not over time. The second basic difference is that I tried to focus on a specific variable or parameter that I wanted to be uncertain. One of the conceptual pitfalls of the rest of the literature up to now is, to spread the uncertainty uniformly over the whole model. Christopher Sims formulated his doubts in the following way: "Is it really uncertainty about the actual values of coefficients in log-linear local approximations to a particular model?" Regarding the models discussed in Chapter three, several things remain to be stated. No model presented there claims to be able to explain observed behavior. Although simplifying to an extreme degree, the conclusions which could be drawn concerning serious implementations of robust control techniques were remarkable. The Chapter has shown, that the implementation of robust control techniques is straightforward in partial equilibrium models, while one needs to be very careful concerning the wage determination process. Simplifying assumptions leading to simple but intuitive outcomes may wreck the whole model when incorporating model uncertainty. This is interesting, because normally the robust control is blamed for wrecking models without need, here it might be the just the different way around. In chapter four, I tried to solve one specific perturbation problem. The solution given was a brute force mechanism working with randomly generated possible solutions. It would have been good to have analytic solutions, to understand the reasoning of the evil agent when perturbing the distribution. It would have been especially good to be able to understand the precise curvature of the perturbed wage distribution. As the solution algorithm has, up to now, always returned a single solution there might be at least a numerical algorithm finding some kind of a decision rule. Although the partial equilibrium model discussed in Chapter 5, allowed to study only a quite limited but fundamental changes, it gave clear insight in the basic changes when including uncertainty concerning the wage distribution. The shortcomings of the brute force algorithm feed back in the interpretation. It is not possible to set up the model as H-inf problem. So there is no way to hand the precise preference structure concerning the wage to the evil agent. There may be additional undiscovered insights. The approach followed here is a tractable way, to find out, what uncertainty concerning the variables that matter does in economic model environments. It seems to be promising to incorporate this kind of uncertainty in micro founded search models of the labor market, because there is a large potential of possible uncertainty. The last question has to remain unanswered in the end: Is this approach just a tool simulating effects of model or variable uncertainty or is it a fully matured method? I have no answer, but I know, that they can be used as a faithful mirror, being able to answer questions concerning economically relevant human behavior.

7 References

1. Albrecht, James W., and Bo Axell. 1984. "An Equilibrium Model of Search Unemployment." Journal of Political Economy, 92(5): 82440.

2. Astrom, K. J. 1999."Model Uncertainty and Robust Control" URL:http://www.control.lth.se/.

3. Bellman, R. 1957. "Dynamic Programming." Princeton University Press, New Jersey,.

4. Bellman, R.E. 1961. "Adaptive Control Processes". Princeton University Press, Princeton, NJ.

5. Black, H. S. 1934. "Stabilized feedback amplifiers". Bell System Technical Journal, 13:1 18.

6. Blanchard, Olivier, and Peter Diamond. 1989. "The Beveridge Curve." Brookings Papers on Economic Activity, 1: 160.

7. Bode, H. W. 1945. "Network Analysis and Feedback Amplifier Design". Van Nostrand, New York.

8. Burdett, Kenneth. 1978. "A Theory of Employee Job Search and Quit Rates." American Economic Review, 68(1): 21220.

9. Burdett, Kenneth. 1979. “Unemployment Insurance Payments as a Search Subsidy: A Theoretical Analysis.” Economic Inquiry,

10. Burdett, Kenneth, and Dale T. Mortensen. 1980. “Search, Layoffs, and Labor Market Equilibrium.” Journal of Political Economy,

11. Diamond, Peter A. 1981. ”Mobility Costs, Frictional Unemployment, and Efficiency.” Journal of Political Economy, 89(4): 798812.

12. Diamond, Peter A. 1982a. ”Aggregate Demand Management in Search Equilibrium.” Journal of Political Economy, 90(5): 88194.

13. Doyle J. C. 1978. “Guaranteed margins for LQG regulators.” AC-23:756757.

14. Doyle J. C., Glover K., Khargonekar P.P, and Francis B. A. 1989. “State-space solutions to standard $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control problems. AC-34:831847.

15. Ellsberg, D. 1961. “Risk, Ambiguity and the Savage Axioms”. Quarterly Journal of Economics, Vol. 75, pp. 643669.

16. Gilboa, I., and D. Schmeidler, 1989, “Maxmim expected utility with non-unique prior”, Journal of Mathematical Economics 18, 141153.

17. Gronau, Reuben. 1971. ”Information and Frictional Unemployment.” American Economic Review, 61(3): 290301.

18. Hansen, L. P., and T. J. Sargent, 1999, “Robust permanent income and pricing”, Review of Economic Studies 66, 873907.

19. Hansen, L. P., and T. J. Sargent, 2002, Robust control and model uncertainty in macroeconomics, Stanford University, unpublished book manuscript.

20. Horowitz, I. M. and Shaked, U. 1975. “Superiority of transfer function over state-variable methods in linear time-invariant feedback system design.” AC-20:8497.

21. Jovanovic, Boyan. 1979a. ”Job Matching and the Theory of Turnover.” Journal of Political Economy, 87(5): 97290.

22. Kalman, R. E. 1960. "A New Approach to Linear Filtering and Prediction Problems," Transaction of the ASMEJournal of Basic Engineering, pp. 35-45.

23. Knight, F., 1921, Risk, uncertainty and profit (London School of Economics: London).

24. Ljungqvist, Lars, and Thomas J. Sargent. 1998. "The European Unemployment Dilemma." Journal of Political Economy, 106(3): 51450.

25. Lippman, Steven A., and John J. McCall. 1976a. "The Economics of Job Search: A Survey." Economic Inquiry, 14(2): 15589, 34768.

26. Ljungqvist, Lars, and Thomas J. Sargent. 2004. "Recursive Macroeconomics Theory". Cambridge: MIT Press.

27. McCall, John J. 1970. "Economics of Information and Job Search." Quarterly Journal of Economics, 84(1): 11326.

28. Merz, Monika. 1999. "Heterogeneous Job-Matches and the Cyclical Behavior of Labor Turnover." Journal of Monetary Economics, 43(1): 91124

29. Mortensen, Dale T. 1970. "A Theory of Wage and Employment Dynamics," in Microeconomic Foundations of Employment and Inflation Theory. E. S. Phelps et al., eds. New York: W. W. Norton, 12466.

30. Mortensen, Dale T. 1986. "Job Search and Labor Market Analysis," in Handbook of Labor Economics. O. Ashenfelter and R. Layard, eds. Amsterdam: North Holland, 849920

31. Mortensen, Dale T. 2000. "Equilibrium Unemployment with Wage Posting: BurdettMortensen Meets Pissarides," in Panel Data and Structural Labor Market Models. H. Bunzel, B. J. Christianesen, N. M. Keifer, and D. T. Mortensen, eds. Amsterdam: Elsevier, 28192.

32. Mortensen, D., and C.A. Pissarides (1994). "Job Creation and Job Destruction in the Theory of Unemployment," Review of Economic Studies, 61:397415.

33. Nash, John. 1950. "The Bargaining Problem." Econometrica, 18(1): 15562.

34. Pissarides, Christopher A. 1985. "Short-Run Equilibrium Dynamics of Unemployment Vacancies, and Real Wages." American Economic Review, 75(4): 67690.

35. Pissarides, Christopher A. 2000. "Equilibrium Unemployment Theory". Cambridge: MIT Press.

36. Rogerson, R., Shimer, R. and Wright R. 2005. "Search-Theoretic Models of the Labor Market: A Survey." Journal of Economic Literature Vol. XLIII, pp. 959988.

37. Safonov, M. G. and M. Athans. 1977. "Gain and phase margins for multiloop lqg regulators." AC-22:173179.

38. Sims, C. A., 2001a, "Pitfalls of a minimax approach to model uncertainty", Princeton University Discussion Paper.

39. Stigler, George. 1961. "The Economics of Information." Journal of Political Economy, 69(3): 21325.

40. von Neumann, J. and O. Morgenstern. 1944. "Theory of Games and Economic Behavior." Princeton, NJ: Princeton University Press.

8 Appendix

The Appendix has three parts. One part for the missing derivations of Chapter 3. Two other parts for the Matlab code of Capters 4 and 5. I start with the missing derivations.

8.1 Proofs and additional derivations for Chapter 3

Additional derivations basic job search in discrete time Basic Model was:

W(w,ν) $= w^e + \beta W(w,\nu)$ $w^e = w - \nu$

U=$\min_{\{\nu\}} \left\{ b + \theta\nu^2 + \beta \int_0^\infty max \left\{ U, W(w,\nu) \right\} dF(w) \right\}$

The reservation wage is defined by: $U =^! W(w_R)$

$$\Rightarrow \; w_R - \beta w_R = min_{\{\nu\}} \left\{ b(1-\beta) + \theta\nu^2(1-\beta) + \beta \int_0^\infty max \left\{ 0, w - w_R - \nu \right\} dF(w) \right\}$$

$$\Rightarrow \; w_R(1-\beta) = min_{\{\nu\}} \left\{ b(1-\beta) + \theta\nu^2(1-\beta) + \beta \int_{w_R}^\infty \left\{ w - w_R - \nu \right\} dF(w) \right\}$$

$$\Rightarrow \; w_R = min_{\{\nu\}} \left\{ b + \theta\nu^2 + \tfrac{\beta}{(1-\beta)} \int_{w_R}^\infty \left\{ w - w_R \right\} dF(w) - \tfrac{\beta}{(1-\beta)} \int_{w_R}^\infty \nu dF(w) \right\}$$

$$\Rightarrow \; w_R = min_{\{\nu\}} \left\{ b + \theta\nu^2 + \tfrac{\beta}{(1-\beta)} \int_{w_R}^\infty \left\{ w - w_R \right\} dF(w) - \tfrac{\beta}{(1-\beta)} \nu[1 - F(w_R] \right\}$$

Which corresponds to the sollution given in Chapter 3

Additional derivations for basic job search in continuous time Starting from the modified discrete time model, regarding a short time period Δ

$$rW(w,\nu) = w^e \ ; \ w^e = w - \nu$$

$$rU = min_{\{\nu\}} \left\{ b\Delta + \Delta\theta\nu^2 + \frac{\alpha}{1+\Delta r} \int_0^\infty max\left\{U, W(w,\nu)\right\} dF(w) + \frac{1-\alpha\Delta}{1+\Delta r} U \right\}$$

$$\Rightarrow \ rU\Delta \frac{r-\alpha\Delta}{1+\Delta r} = min_{\{\nu\}} \left\{ b + \theta\nu^2 + \frac{\alpha}{1+\Delta r} \int_0^\infty max\left\{U, W(w,\nu)\right\} dF(w) \right\}$$

Now let $\Delta \to 0$

$$\Rightarrow \ rU = min_{\{\nu\}} \left\{ b + \theta\nu^2 + \alpha \int_0^\infty max\left\{0, W(w,\nu) - U\right\} dF(w) \right\}$$

The basic model was then finally given by:

$$rW(w,\nu) = w^e \ ; \ w^e = w - \nu$$

$$rU = min_{\{\nu\}} \left\{ b + \theta\nu^2 + \alpha \int_0^\infty max\left\{0, W(w,\nu) - U\right\} dF(w) \right\}$$

The reservation wage is still defined by: $U \stackrel{!}{=} W(w_R) = w_R/r$

$$\Rightarrow \ w_R = min_{\{\nu\}} \left\{ b + \theta\nu^2 + \frac{\alpha}{r} \int_{w_R}^\infty \left\{w - w_R - \nu\right\} dF(w) \right\}$$

$$\Rightarrow \ w_R = min_{\{\nu\}} \left\{ b + \theta\nu^2 + \frac{\alpha}{r} \int_{w_R}^\infty \left\{w - w_R - \nu\right\} dF(w) - \frac{\alpha}{r} \int_{w_R}^\infty \nu dF(w) \right\}$$

$$\Rightarrow \ w_R = min_{\{\nu\}} \left\{ b + \theta\nu^2 + \frac{\alpha}{r}\int_{w_R}^{\infty} \{w - w_R - \nu\}\, dF(w) - \frac{\alpha}{r}\int_{w_R}^{\infty} \nu dF(w) \right\}$$

$$\Rightarrow \ w_R = min_{\{\nu\}} \left\{ b + \theta\nu^2 - \frac{\alpha}{r}\nu \left[1 - F(w_r)\right] + \frac{\alpha}{r}\int_{w_R}^{\infty} \left[1 - F(w)\right] dF(w) \right\}$$

Which is just the equation given in Chapter 3.

with exogenous layoffs Basic model was just given by:

$$rW(w,\nu) = w^e + \lambda \left[U - W(w,\nu)\right] \ ; \ w^e = w - \nu$$

$$rU = min_{\{\nu\}} \left\{ b + \theta\nu^2 + \alpha \int_0^{\infty} max \{0, W(w,\nu) - U\}\, dF(w) \right\}$$

Incorporating $U = W(w_R) \Rightarrow \ U = \frac{w_R}{r}$

$$w_R = min_{\{\nu\}} \left\{ b + \theta\nu^2 + \alpha \int_0^{\infty} max \{0, W(w,\nu) - U\}\, dF(w) \right\}$$

$$\Rightarrow \ w_R = min_{\{\nu\}} \left\{ b + \theta\nu^2 + \frac{\alpha}{r+\lambda}\int_{w_R}^{\infty} max \{w - \nu) - w_R\}\, dF(w) \right\}$$

$$\Rightarrow \ w_R = min_{\{\nu\}} \left\{ b + \theta\nu^2 + \frac{\alpha}{r+\lambda}\int_{w_R}^{\infty} max \{w - w_R\}\, dF(w) \frac{\alpha}{r+\lambda}\nu[1 - F(w_R)] \right\}$$

which corresponds to equation [20]

With endogenous transitions to unemployment As usual: start off with the basic model.

$$rW(w,\nu) = w^e + \lambda \int_0^\infty max \left\{ W(w,\nu)' - W(w,\nu), U - W(w,\nu) \right\} dF(w) \; ; \; w^e = w - \nu$$

$$rU = min_{\{\nu\}} \left\{ b + \theta\nu^2 + \alpha \int_0^\infty max \left\{ 0, W(w,\nu) - U \right\} dF(w) \right\}$$

applying the usual criteron for the reservation wage:

$$rU = w_R + \lambda \int_0^\infty max \left\{ W(w,\nu)' - W(w_R) \right\} dF(w)$$

Using that:

$$rU = w_R + \lambda \int_0^\infty max \ W(w,\nu)' - W(w_R) \} dF(w)$$

´ and

$$rU = min_{\{\nu\}} \left\{ b + \theta\nu^2 + \alpha \int_0^\infty max \left\{ W(w,\nu) - W(w_R) \right\} dF(w) \right\}$$

gives:

$$\Rightarrow \; w_R = min_{\{\nu\}} \left\{ b + \theta\nu^2 + \frac{\alpha - \lambda}{r + \lambda} \int_{w_R}^\infty max \left\{ w - \nu - w_R \right\} dF(w) \right\}$$

Which corresponds to the sollution given.

Additional formulas for Job to Job transitions

$$rW(w,\nu) = w^e + \alpha_1 \int_0^\infty max \left\{ W(w,\nu)' - W(w,\nu) \right\} dF(w'|w) + \lambda \left[U - W(w,\nu) \right]$$

Where: $w^e = w - \nu$

$$rU = min_{\{\nu\}} \left\{ b + \theta\nu^2 + \alpha_0 \int_0^\infty max \left\{ 0, W(w,\nu) - U \right\} dF(w) \right\}$$

applying the usual reservation wage criterion:$U = W(w_R)$, moreover I consider the easiest case.

$$\Rightarrow \; w_R = min_{\{\nu\}} \left\{ b + \theta\nu^2 + (\alpha_0 - \alpha_1) \int_{w_R}^{\infty} (W(w,\nu))\, dF(w) \right\}$$

since this still includes $W(w,\nu)$.

$$rW(w^e) = w^e + \alpha_1 \int_0^{\infty} \{W(w,\nu)' - W(w,\nu)\}\, dF(w'|w) + \lambda\, [W(w_R) - W(w,\nu)]$$

$$\Rightarrow \; rW(w^e) + \lambda W(w^e) + \alpha_1 W(w^e)[1 - F(w)] = w^e + \alpha_1 \int_0^{\infty} \{W(w,\nu)'\}\, dF(w'|w) + \lambda W(w_R)$$

$$\Rightarrow \; W(w^e)(r + \lambda + \alpha_1[1 - F(w)]) = w^e + \alpha_1 \&int_0^{\infty} \{W(w,\nu)'\}\, dF(w'|w) + \lambda W(w_R)$$

Now take the derivative w.r.t w^e.

$$\frac{\partial}{\partial w^e} \;\; \Rightarrow \;\; \frac{\partial W(w^e)}{\partial w^e}(r + \lambda + \alpha_1[1 - F(w)]) = 1$$

then integrate over the expression again...

$$\int_0^{\infty} \frac{1}{r + \lambda + \alpha_1[1 - F(w)]} dw^e = W(w^e)$$

and insert this equation back into the prior result...

$$w_R = min_{\{\nu\}} \left\{ b + \theta\nu^2 + (\alpha_0 - \alpha_1) \int_{w_R}^{\infty} \int_0^{\infty} \frac{1}{r + \lambda + \alpha_1[1 - F(w)]} dw^e dF(w) \right\}$$

$$\Rightarrow\ w_R = min_{\{\nu\}}\left\{b+\theta\nu^2+(\alpha_0-\alpha_1)\int_0^\infty \frac{[1-F(w)]}{r+\lambda+\alpha_1[1-F(w)]}d(w-\nu)\right\}$$

additional equations for the bargaining model One of the questions which remains open is, why bargaining implies that:$S=\frac{k}{\alpha_e(1-\theta)}$. There are two things that one has to use. First thing is, that $V=0$, the other thing that $\frac{\partial J(\pi)}{w}=-1$. When taking the first order condition of the bargaining equation, one gets:

$$\theta\left[W(w^e)-W(w_R)\right]^{\theta-1}\frac{\partial W(w^e)}{\partial w}=(\theta-1)\,J(\pi)^{-\theta}$$

$$\Rightarrow\ \theta J(\pi)=(1-\theta)(W(w^e)-W(w_R))$$

$$\Rightarrow\ J(\pi)=(1-\theta)S$$

Using that $rV=-k+\alpha_e[J(\pi)-V]\ \Rightarrow\ k=\alpha_e J(\pi)$. Now insert the result from bargaining in there.

Next unusual thing is, how to get from the reservation wage equation to the two degree of uncertainty. Remember:$A_1=\frac{\alpha_e(1-\theta)(y-b)}{k\theta(r+\lambda)}$

$$w_R=min_\nu\left\{b-\theta_1\nu^2+\left(\frac{\alpha_e(1-\theta)(y-b)}{k\theta(r+\lambda)}+\frac{\alpha_e(1-\theta)\theta_1\nu^2}{k\theta(r+\lambda)}-\frac{1}{\theta}(w-\nu-w_R)\right)\right\}$$

$$\frac{\partial}{\partial\nu}=0\ \Leftrightarrow\ -2\theta_1\nu+2A_1\theta_1\nu w-A_1\theta_1\nu^2+1\theta-A_1\theta_1 2\nu w_R=0$$

$$\Leftrightarrow\ 0=3\theta_1\theta\ A_1\theta\nu^2-(2w-w_R-\frac{A}{2})\theta_1A_1\theta\nu-\theta\ A_1(y-b)-1$$

When applying the well known p-q formula, one gets:

$$\nu_{1,2} = \frac{1}{6}(2w - w_R - \frac{2}{-}A_1) \pm \sqrt{(-\frac{1}{6}(2w - w_R - \frac{2}{A_1})^2 + \frac{\theta A_1(y-b)-1}{3\theta_1 A_1 \theta}}$$

As the sollution of the model with match specific productivity, the one-shot model and the dynamic model with direct matching are straightforward, I will directly jump to the heterogenous leisure model.

additional equations for the heterogenous leisure model The first interesting thing is the derivation of w_1. The system of equations is given by:

$$\mathrm{rU}_2 = min_{\{\nu\}}\left\{b_2 + \theta\nu^2 + \alpha_w\sigma\left[W_2(w_2 - U_2\right]\right\}$$

$$\mathrm{rW}_1(w_1^e) = w_1^e + \lambda\left[U_1 - W_1(w_1^e)\right]$$

$$\mathrm{rW}_1(w_2^e) = w_2^e + \lambda\left[U_1 - W_1(w_2^e)\right]$$

$$\mathrm{rW}_2(w_2^e) = w_2^e + \lambda\left[U_2 - W_2(w_2^e)\right]$$

The corresponding conditions to the reservation wage condition are here:$W_1(w_1^e) = U_1$,$W_1(w_1^e) = U_1$. This implies, that: $w_2 = b_2 + \nu$ and $W_1(w_1^e) = w_1^e/r$. To derive w_1 it is necessary to know what $W_1(w_2^e)$is.

$$rW_1(w_2^e) = w_2^e + \lambda\left[W_1(w_1^e) - W_1(w_2^e)\right]$$

$$\Leftrightarrow \; (r+\lambda)W_1(w_2^e) = w_2^e + \lambda W_1(w_1^e)$$

$$\Leftrightarrow \; W_1(w_2^e) = \frac{w_2^e}{r+\lambda} + \frac{\lambda w_1^e}{r+\lambda}$$

It is now possible to substitute this equation in the flow value for unemployment of the type 1 workers.

$$w_1^e = min_{\{\nu\}} \left\{ b_1 + \theta\nu^2 + \alpha_w\sigma \left[\frac{w_2^e}{r+\lambda} + \frac{\lambda w_1^e}{r+\lambda} - \frac{w_1^e}{r} \right] \right\}$$

$$\Leftrightarrow w_1 = min_{\{\nu\}} \left\{ \frac{(r+\lambda)b_1 + \alpha_w\sigma b_2}{r+\lambda+\alpha_w\sigma} + \frac{r+\lambda}{r+\lambda+\alpha_w\sigma}\theta\nu^2 + \nu \right\}$$

8.2 Matlab Code for Chapter 4 and 5

```
clear all
%_____________________________________________________%
% MATLAB CODE FOR CH.4 BRUTE FORCE ALGORITHM          %
%                                                     %
%                                                     %
% August 2006                                         %
% Robust Control Analysis of Labor Markets            %
% Humboldt-University of Berlin                       %
% Marc Vahlert                                        %
%_____________________________________________________%

%Settings

regard=0;        % regard wage as pure event: 0  regard wage as payment: 1
k=100;           %no of trials, caution may get computational intensive!
w=10;%+1          # possible wages arriving%
s=.1;            %steplength%
m=.5;            %settings of the distribution%
B=.1;
eta=100;         %measure of uncertainty%
mue=.0;          % adjustment parameters
moe=1;
p=[normcdf(0:s:s*w,m,B)]';  %original cdf

% option 1


if regard<.5
for n=1:k
    z(1,1)=1.5*(randn(1,1)-.25);           %  AR1 random numbers
    for i=2:w+1
        z(i,1)=mue*z(i-1,1)+(randn(1,1)-.25)*moe;
    end
    x=max(min(z,1),0);                     % cut numbers back to [0,1]
    if ones(1,max(size(z)))*x>0
        for i=1:max(size(z))
            v(i,1)=x(i,1)/(ones(1,max(size(z)))*x); % generate probabilites
        end
    else
        v(1,1)=p(1,1);
        for i=2:length(p)
            v(i,1)=p(i,1)-p(i-1,1);         % compute original probabilites
        end
    end
    V(n,:)=v;
    rho(1,1)=v(1,1);
    for i=2:max(size(z))
        rho(i,1)=ones(1,i)*v(1:i,1);        %adding them up to cdf's
    end
    RHO(n,:)=rho;
    G(:,n)=abs(p-rho);
    if G(:,n)'*G(:,n)<=eta                  % capacity criterion
       A(:,n)=G(:,n);
    else A(:,n)=zeros(1,w+1);
    end
end
```

```
sel=ones(1,w+1)*A;                          % computing destructive power

%pick out most desaterous nu%

sel_star=zeros;
A_star=zeros;
for t=1:max(size(sel));
    if sel(:,t)>sel_star;
        sel_star=sel(:,t);
        A_star=A(:,t);
        RHO_star=RHO(t,:);
        V_star=V(t,:);
        counter=t;
    end
end
end

% option 2

if regard>.5                         % in general the same as option 1
    for n=1:k
    z(1,1)=1.5*(randn(1,1)-.25);
    for i=2:w+1
        z(i,1)=mue*z(i-1,1)+(randn(1,1)-.25)*moe;
    end
    x=max(min(z,1),0);
    if ones(1,max(size(z)))*x>0
        for i=1:max(size(z))
            v(i,1)=x(i,1)/(ones(1,max(size(z)))*x);
        end
    else
        v(1,1)=p(1,1);
        for i=2:length(p)
            v(i,1)=p(i,1)-p(i-1,1);
        end
    end
    V(n,:)=v;
    rho(1,1)=v(1,1);
    for i=2:max(size(z))
        rho(i,1)=ones(1,i)*v(1:i,1);
    end
    RHO(n,:)=rho;
    G(:,n)=rho-p;
    if G(:,n)>=0;                     % additional criterion on w
    if G(:,n)'*G(:,n)<=eta
       A(:,n)=G(:,n);
    else A(:,n)=zeros(1,w+1);
    end
    end
    end

 %pick out most desaterous nu%

sel=ones(1,w+1)*A;
```

```
sel_star=zeros;
A_star=zeros;
for t=1:max(size(sel));
    if sel(:,t)>sel_star;
        sel_star=sel(:,t);
        A_star=A(:,t);
        RHO_star=RHO(t,:);
        V_star=V(t,:);
        counter=t;
    end
end
end

%presenting results

beep;
ppdf(1,1)=p(1,1);
for i=2:length(p)
    ppdf(i,1)=p(i,1)-p(i-1,1);
end
hpdf=V_star';
PDF=[ppdf,hpdf];
one=ones(w+1,1);
cpr=[RHO_star',p,one];
plot(cpr);
disp('Hit any key when ready to watch the pdf-s...');
pause;
plot(PDF);
```

```
clear all;
%___________________________________________________%
% MATLAB CODE FOR CH.5 VALUE FUNCTION ITERATION     %
%                                                   %
%                                                   %
% July 2006                                         %
% Robust Control Analysis of Labor Markets          %
% Humboldt-University of Berlin                     %
% Marc Vahlert                                      %
%___________________________________________________%


% STEP 0
% Define parameters
%___________________________________________________%

% For the Model

betta = .9985;           % Discount Factor
alfa = .0009;            % survival Prbability
betta2=(1-alfa)*betta;  % alpha and beta always show up together!
lambda=.009;             % layoff Probability
b_I=0.2;

% For the expected wage distribution

%Settings
R_ON=1;         % 1: use Robust control wage distribution 0: Off
regard=1;       % regard wage as neutral event: 0  regard wage as payment: 1
k=25000;        %no of trials, caution may get computational intensive!
w=10;%+1         # possible wages arriving%
s=.1;           %steplength%
m=.5;           %setting of the distribution%
B=.1;
eta=1;          %measure of uncertainty%
mue=.75;        % adjustment parameters
moe=.35;

% Step 1
%determine the expected wage distribution
%___________________________________________________

% option 1

if R_ON >.5                  % for comments see chapter4.m
p=[normcdf(0:s:s*w,m,B)]';
if regard<.5
for n=1:k
    z(1,1)=1.5*(randn(1,1)-.25);
    for i=2:w+1
        z(i,1)=mue*z(i-1,1)+(randn(1,1)-.25)*moe;
    end
    x=max(min(z,1),0);
    if ones(1,max(size(z)))*x>0
        for i=1:max(size(z))
            v(i,1)=x(i,1)/(ones(1,max(size(z)))*x);
```

```
        end
    else
        v(1,1)=p(1,1);
        for i=2:length(p)
            v(i,1)=p(i,1)-p(i-1,1);
        end
    end
    V(n,:)=v;
    rho(1,1)=v(1,1);
    for i=2:max(size(z))
        rho(i,1)=ones(1,i)*v(1:i,1);
    end
    RHO(n,:)=rho;
    G(:,n)=abs(p-rho);
    if G(:,n)'*G(:,n)<=eta
       A(:,n)=G(:,n);
    else A(:,n)=zeros(1,w+1);
    end
end

sel=ones(1,w+1)*A;

%pick out most desaterous nu%

sel_star=zeros;
A_star=zeros;
for t=1:max(size(sel));
    if sel(:,t)>sel_star;
        sel_star=sel(:,t);
        A_star=A(:,t);
        RHO_star=RHO(t,:);
        V_star=V(t,:);
        counter=t;
    end
end
end

% option 2

if regard>.5
    for n=1:k
    z(1,1)=1.5*(randn(1,1)-.25);
    for i=2:w+1
        z(i,1)=mue*z(i-1,1)+(randn(1,1)-.25)*moe;
    end
    x=max(min(z,1),0);
    if ones(1,max(size(z)))*x>0
        for i=1:max(size(z))
            v(i,1)=x(i,1)/(ones(1,max(size(z)))*x);
        end
    else
        v(1,1)=p(1,1);
        for i=2:length(p)
            v(i,1)=p(i,1)-p(i-1,1);
        end
    end
```

```
    V(n,:)=v;
    rho(1,1)=v(1,1);
    for i=2:max(size(z))
        rho(i,1)=ones(1,i)*v(1:i,1);
    end
    RHO(n,:)=rho;
    G(:,n)=rho-p;
    if G(:,n)>=0;
    if G(:,n)'*G(:,n)<=eta
       A(:,n)=G(:,n);
    else A(:,n)=zeros(1,w+1);
    end
    end
    end
%pick out most desaterous nu%

sel=ones(1,w+1)*A;

sel_star=zeros;
A_star=zeros;
for t=1:max(size(sel));
    if sel(:,t)>sel_star;
        sel_star=sel(:,t);
        A_star=A(:,t);
        RHO_star=RHO(t,:);
        V_star=V(t,:);
        counter=t;
    end
end
end
ppdf(1,1)=p(1,1);
for i=2:length(p)
    ppdf(i,1)=p(i,1)-p(i-1,1);
end
hpdf=V_star';
PDF=[ppdf,hpdf];
one=ones(w+1,1);
cpr=[RHO_star',p,one];

% delete obsolete variables to free memory for value function iteration

clear A;
clear RHO;
clear V;
clear sel;
clear G;
clear x;
clear z;
clear v;
end

% STEP 2
% Defining grids for wage and search level
%___________________________________________________

Wmin = 0.00;
```

```
Wmax = s*w;
wag = [Wmin:s:s*w];
n = w+1;

if R_ON<.5
    p=[normcdf(0:s:s*w,m,B)'];
    v(1,1)=p(1,1);
    for i=2:length(p)
        v(i,1)=p(i,1)-p(i-1,1);
    end
end

% Define Grid for Search Intensity

Smin=0;
Smax=1;
S_fine=0.0001;
S=[Smin:S_fine:Smax]';
NoSlevels=((Smax-Smin)/S_fine)+1;


% STEP 3
% Define initial settings & Value functions
%___________________________________________________

% Start off with Zero Vectors

V_new = zeros(1,max(size(wag)));      % allow for every w-h combination
Vb_new = zeros(1,max(size(wag)));     % allow sollution for every h-level

V_p1=V_new;
Vb_p1=Vb_new;

% Set max_diff such that the loop enters at least once

max_diff = 100;

% Set tolerance for approximation

tol = 0.001;
Count=1;


% Step 4
% Solve Value Functions & determine reservation wage
%___________________________________________________
while ( abs(max_diff)>tol)

% Calulate the  Value Function

% Solve equation 1 V(w)

acceptV=wag+betta2.*((1-lambda)*V_p1+lambda*Vb_p1);
rejectV=Vb_p1;
V=max(acceptV',rejectV')';
```

```
% computing the expected value of V(w) from the perspective of Vb(s)

if R_ON>.5
Ve=V_star*V';
end
if R_ON<.5
Ve=v'*V';
end

% Solve Equation 2 Vb(s)

for i=1:max(size(S))
    s=S(i,:);
    Vb(i,:)=(-.5)*s+b_I+betta2.*((1-s^.3).*Vb_p1+(s^(.3))*Ve);
end

% Maximize over S

Vb_star=zeros(1,max(size(wag)))-10000;
s_star=zeros(1,1);
for i=1:length(S);
    if Vb(i,:)>Vb_star
        Vb_star=Vb(i,:);
        s_star=S(i,:);
    end
end

% Determine remaining error loog stops if erros is smaller than tollerance

diff1=(V_p1-V)*ones(max(size(wag)),1);
diff2=(Vb_star-Vb_p1)*ones(max(size(wag)),1);
max_diff=diff1+diff2;
VB(Count,:)=Vb(1,1);                % evolution of Vb while Solving
DIFF(Count,:)=max_diff;
V_p1=V;                             % Update Value function 1
Vb_p1=Vb_star;                      % Update Value function 2
Count=Count+1;
end

% Determine reservation wage

sel=acceptV-rejectV;
ct=1;
Wr=0;
while Wr<=0
    if sel(1,ct)>0
       if ct-1<.5
          k=sel(1,ct)
       end
       k=sel(1,ct)-sel(1,ct-1);
       plus=(1-sel(1,ct)/k)/length(wag);
       Wr=wag(1,ct-1)+plus;
     end
ct=ct+1;
end
```

```
% Step 5
% Solve other variables recursively & present results
%___________________________________________________

cpr=[rejectV;acceptV];

plot(cpr')
disp('Hit any key when ready to watch Vb(s)');
pause;
plot(Vb)
disp('Virtural aprox. Reservation wage');
Wr
disp('search level');
s_star
disp('Steady State Unemployment Rate');
% unemployment rate %
u_bar=lambda/(lambda+s_star^.3*(1-normcdf(Wr,m,B)))
% Hazard rate %
disp('average Hazard Rate');
H=s^.3*(1-normcdf(Wr,m,B))
% duration %
disp('average unemployment duration');
D=1/H
```

Declaration of Authorship

I hereby confirm that I have authored this diploma thesis independently and without use of other than the indicated resources.

Berlin August 31, 2006

Marc Vahlert