Rational Overconfidence and Social Security

by

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Abstract: Two of the features that distinguish Social Security and many other state mandated pension plans around the world are that (i) a minimum level of savings for retirement is imposed on most citizens and (ii) that individuals cannot decide how their contributions are invested. Here, a rationale for these two features, based on rational overconfidence, is proposed. Rational overconfidence is present when equally informed agents hold diverse confident, rational beliefs. The fact that beliefs are diverse means that all of them cannot be correct, hence seen as a collective agents do not act optimally.

In the face of rational overconfidence, Pareto efficiency is no longer the natural criterion for comparing policies and we suggest ex-post welfare optimality in stead. This criterion makes amends for the possible inconsistencies of agents' beliefs.

Our results on social security are based on a methodology that places itself strictly between the traditional neoclassical approach and that championed by behavioral economics. This methodology does not deviate from the neoclassical assumption of rationality but only broadens it and can therefore readily be applied to many public policy issues.

JEL classification number: D01, D02, D60, D81, D84, F31, G11
Key words: Subjective Expectations, Rational Beliefs, Ex-post Welfare Optimality, Social Security, Rational Overconfidence, Portfolio Choice.

*This work has benefitted from discussions with Ken Arrow, Angiola Cottini, Carlo Devillanova, Peter Hammond, Mordecai Kurz, Maurizio Motolese, Hiro Nakata, Chang-Ho Yoon and Ho-Mou Wu. It was presented at the EEA 2008 (Milan) and SIE 2008(Perugia) meetings as well as at seminars at Korea University, Stanford University, Universitá Cattolica, and University of Essex. I thank members of the audience for useful comments.

The King's dilemma:
In the neighboring kingdom the bad king has announced that his daughter, the princess will be married to the one she likes the most among any suitable suitor that shows up next Sunday. Those who are turned down by her are fed to the dragon in the moat. The good king has three sons, Wit, Looks, and Charm who are the only ones who would be considered suitable suitors. Each of them is convinced that the princess will choose him, if he is allowed to go, no matter whether the other two princes show up or not and is sufficiently egoistic not to refrain from going out of care for his brothers. If the good king is Paretian, he should allow all of them to go, knowing that with certainty one of them will be married, while the two others will serve as dinner for the dragon (after all, much worse than being a bachelor prince). If he is concerned with ex-post efficiency he should let exactly one of them go. This decision makes none of the three princes subjectively better off and yet seen as a collective they are objectively better off.

1 Introduction

1.1 Basic Ingredients

Many developed nations have either government mandated pensions plans or social security plans, both of which guarantee a minimum income for most who reach retirement. In this study we argue that to explain such policies, which override individual freedom of choice, there is no need to resort to an assumption of individual irrationality. Rather, we use an approach to thinking about economic policy, introduced in Nielsen(2003), which has as its main building blocks the concepts of rational overconfidence (defined in Nielsen, 2008) and ex-post welfare optimality (Starr, 1973, Harris, 1978, Hammond, 1981)

The starting point of our analysis of government mandated savings plans and Social Security is the prevalence of diverse beliefs. Rational overconfidence is present when equally informed agents rationally use distinct models, each believed to be superior, for interpreting the available data. In the words of Varian(1985 and 1992) agents in our model have different opinions but not different information. In such a situation it becomes relevant to use the concept of ex-post welfare optimality rather than the Pareto criterion (which is based on individual subjective beliefs) for comparing different policies.

When beliefs are diverse, they cannot all be correct. Thus, when individuals take actions based on diverse beliefs, there must be some degree of inefficiency present, as seen from the point of view of society[1]. Knowing that such an inefficiency is present does not necessarily imply knowing how to remedy it. However, we think that there is in many instances in real life a commonly agreed upon core of knowledge, which may form a basis for a commonly agreed upon assessment of the consequences of various policies.[2] The central assumption of rational belief theory (as defined in Kurz, 1994) is exactly that such a core of objective knowledge exists and (hence) that all individual beliefs are compatible with this knowledge. Our approach then provides a framework that integrates the notion of commonly agreed upon information (as for example exposed in the rational beliefs approach to modelling expectations) along with the notion of ex-post welfare optimality. The result is a tool for policy analysis that may be used in situations where diversity of beliefs play an important role. We note that while agents in our models hold rational beliefs, this assumption is not indispensable for our results - we

[1] *The King's Dilemma* illustrates our point: The sum of individual expectations about rewards (= 3 marriages) is greater than the objective sum (= 1 marriage). See Remark 12 below for a complete analysis.

[2] In *The King's Dilemma* this knowledge, shared by the king and the princes, is about the rules of the contest for the princess. Any belief (about the princess' preferences) held by the King that embodies the known facts will lead to the same set of ex-post efficient decisions.

shall provide some simple examples below.

We assume that the commonly available data is in the form of an empirical distribution for the process of exogenous and/or endogenous variables. What gave rise to this empirical distribution cannot be precisely determined and as a consequence, agents may end up *rationally* using different probability distributions when they make investments/savings decisions for their retirement. Without government intervention, some save (invest) too much, while others too little. By mandating a minimum level of savings, the government makes sure that no one is saving too little (these savings may be required to be invested in safe assets, or a relatively safe portfolio, to avoid, on average, suboptimal returns). However, since we assume, realistically, that it is not possible to put a cap on savings, there may still be some who invest too much. The commonly accepted empirical distribution offers a natural neutral way of calculating the expected welfare of such a policy (and of alternatives to it).

The stylished facts that motivate our study are (i) The prevalence of government mandated pension or social security plans in many countries (ii) Diversity of beliefs, which we think cannot be explained by diversity of information, see f.i. Branch(2004), Chavas(2000), Evans and Gulamani(1984), Frankel and Froot(1987), Ito(1990), Kurz(2001), Kurz and Motolese(2007) and Souleles(2004) and (iii) Evidence of suboptimal portfolio choices for individual retirement plans, see f.i. Agnew(2002), Choi, Laibson and Madrian(2005), Munnell(1999) and Palma and Sunden(2004).

The above studies as well as many others reveal the shortcomings of the rational expectations approach to assessing the potential benefits of social security and other plans for mandated saving for retirement. At the other extreme is the position that individuals are (more) irrational (than the government), as witnessed in the behavioral economics literature. Our position is strictly between these two and an important contribution of the theory of rational beliefs is to make room for such a position. As we shall see, rational beliefs theory provides a tractable model consistent with the claim that *societal irrationality* may be present concurrently with *individual rationality* also in the context of expectations.

Our contribution has two aspects to it. Firstly, it is normative in proposing that in some situations (like in *The King's Dilemma*) the Pareto criterion should be substituted by an ex-post criterion for efficiency. Secondly, it is positive in suggesting that the prevalence of diverse rational beliefs explains observed suboptimal savings decision in the aggregate. We then argue that the implicit rationale for the government mandated savings for pensions we observe is that they, in the ex-post sense, offer an improvement in efficiency.

1.2 Some other explanations offered for the existence of Social Security

Gotardi and Kubler(2006) propose that social security is a remedy against the impossibility of private insurance contracts between unborn and living generations in an OLG model. Their analysis is however confined to the case of only two generations living concurrently. It is an open question whether their results would also be quantitatively important in a context where there are at each date many generations of agents who can trade insurance contracts with each other, implying that the economic weight of unborn generations is smaller relative to the whole economy.

Others, like Feldstein(1985 and 2005), hold that some agents are myopic and fail to make adequate provisions for the future. Feldstein(2005) at the same time proposes that agents be allotted individual social security accounts and be allowed to invest the savings in these accounts in portfolios of stocks and other assets. It seems puzzling though, that an agent who is deemed unable to plan for the optimal stream of consumption over his own lifetime at the same time is deemed capable of planning an investment strategy for the stock market, over the same horizon. Note also that an explanation based on myopia implicitly is an explanation based on an assumption that the government is less myopic than the public. We shall discuss some further related reasons proposed for mandatory savings below.

An alternative, positive explanation for the transfer to future old (now, voters) from future young (now non-voters) entailed in some social security programs is political power. This idea is formalized in Tabellini(2005), but the results obtained hinge on the assumption that the social security system embodies a substantial transfer from more to less well-off agents, something that according to Feldstein(2005) is not the case for the American Social Security program.

The so-called "Samaritan's dilemma" might provide a reason for why people are required to have a minimum savings level for retirement (but not why this level is increasing with their income). The explanation is based on the unwillingness of many societies to ignore abject poverty. This means that if the disposable income of a retired person is too low, this person will receive support from the government. Knowing this, some young agents may decide not to save up for retirement. If the government's threat to punish such persons by withholding support is not credible, an alternative may be to require everyone to have a minimal savings level.

While we explore a particular possible explanation for government mandated pension plans, this should not be seen as a rejection of all other explanations - many factors probably play a role in justifying the institutions we observe. However, we do wish to make a methodological point: In our model, agents are rational, but many are wrong. The government's authority is not based on being better informed, more rational (less myopic) than anyone else, but on being able to impose a savings level that *everyone* agrees is *on average* better i.e. in this sense objectively improving.

1.3 Outline

The results obtained in this study are of a qualitative nature and we have opted for simplicity in demonstrating the intuition behind them.[3] Since our policy analysis and conclusions are more novel in the context of production economies than for stochastic exchange economies, we employ two different models, one with and one without production. This also allows us to study separately two aspects to government mandated savings programs, namely that a minimum savings level for retirement is imposed and that, as is the fact for many programs, there are restrictions on where these savings can be placed. In the first model, in Section 3, which is concerned with the level of savings, we abstract from any (general) equilibrium (price) effects and assume that each individual, based on his beliefs about the common future level of wealth, decides on his savings in a single riskless productive asset.[4] Some beliefs are optimistic, while others are pessimistic, i.e. beliefs are inconsistent. Another simplification, we shall make, is to assume that (individual) pensions are fully funded, i.e. there is no transfer between generations for the policies we shall consider. In our models this is immaterial (we shall not distinguish between the two types of systems in our discussions), but in a broader context it is obviously far from an innocuous simplification. It means in particular that we ignore both a possible insurance motive behind social security, mentioned above, and the negative effect that a pay-as-you-go social security system has on national savings. In the second model (Section 4) which is concerned with how savings are invested there are endogenous prices, but no production. Aggregate consumption is thus exogenous, however individual contingent consumption plans, as induced by individual portfolio choices, are, due to incorrect expectations,

[3] For a more formal presentation in the context of the choice of exchange rate regime in a stochastic exchange economy, see Nielsen(2003) and Nielsen(2007a).

[4] The model has no general equilibrium properties, but since rational overconfidence is defined as a social phenomenon, we cannot consider an agent's actions in isolation.

ex-post suboptimal. Government policies may remedy this situation. In Section 5 we discuss a trade-off present when, as is realistic, the government has imperfect information about the agents' characteristics. We devote Section 6 to a discussion of behavioral welfare economics and other views of welfare economics as compared to our approach and conclude in Section 7.

2 Rational Overconfidence and Ex-post Welfare Optimality

2.1 Rational Beliefs

We shall assume that there is a continuum of agents, indexed by $i \in [0,1]$. To explain the idea of rational beliefs which originates with Kurz(1994), we use some particulars of the first of the two models presented below. In this model any individual agent can at any date, t choose to invest in a riskless asset with gross return, $r > 0$. The only uncertainty is about the future common wealth level of individuals (i.e. about the general wealth level of society), W_2. Note that we have by assumption (for simplicity) decoupled this general wealth level and aggregate savings (investments).

An agent i at date t, solves the problem (with the utility function u being strictly increasing, strictly concave, and s.t. the solution to the problem is interior):

$$\max_{S \in [0,W_1]} E_{it} u(W_1 - S, W_{2t+1} + Sr)$$

where E_{it} denotes his expectation over second period wealth, W_{2t+1} and W_1 is the non-stochastic wealth that he starts with.

The stochastic sequence, $\{W_{2t}\}_{t=1}^{\infty}$ is assumed to be stable, meaning that there is an empirical distribution, $\overline{\mu}$ for it (based on the limit frequency of all finite events).[5] To simplify, suppose that W_{2t} can take only two values, $W^L < W^H$. Based on many (past) observations it is known that the frequency of W^L is $\overline{P}_L$, so that $\overline{P}_H = 1 - \overline{P}_L$ is the frequency of W^H. It is also known that the empirical distribution for other events is the product of these frequencies. For instance, the frequency of W^L followed by W^H is $\overline{P}^L \overline{P}^H$. These frequencies are averages and not necessarily the true probabilities. However, if we assume stationarity of $\{W_{2t}\}_{t=1}^{\infty}$, then the knowledge about these frequencies means that it is known that at all dates t, the probability that $W_{2t} = W^L$ is $\overline{P}_L$. Stability is a weaker assumption than stationarity, since with it we can only conclude that *on average* the probability of W^L is $\overline{P}_L$. Specifically, if at a fraction $Q \in (0,1)$ of all dates $t \in \{1,2,3,\ldots\}$, the probability is P_L^0 while at the remaining fraction $1-Q$ of dates the probability is P_L^1, where $QP_L^0 + (1-Q)P_L^1 = \overline{P}_L$ (and if the dates where P_L^0 is the true probability are sufficiently dispersed) then, for all realizations of $\{W_{2t}\}_{t=1}^{\infty}$, the frequency of W^L will be equal to the observed one, namely $\overline{P}_L$.[6] Thus believing that there are such two one-period probabilities P^0 and P^1, each in force at different dates, rather than one, $\overline{P}$ at all dates, is consistent with the empirical observations, in the sense that if this were really the case, we would observe the frequencies we actually do observe. Such a belief is therefore called a rational belief. Formally, it is a deterministic sequence of probabilities $\{P_t\}_{t=1}^{\infty}$ taking values in $\{P_0, P_1\}$ with the just stated properties.[7]

[5] To simplify, we have assumed that the wealth process is stable. It is more realistic to assume that the stochastic growth rate is stable. Allowing for this would not change the basic nature of our conclusions though. By the limit frequency of the set A is meant the limit of $\frac{1}{T}\#\{t \leq T : w_{2t} \in A\}$ as the number, T of observations tends to infinity. Stability also means that there is an empirical distribution for any stochastic sequence of the format $\{f(W_{2t})\}_{t=1}^{\infty}$ for any real function f. Below, we shall use this fact for the case where f is a utility function.

[6] Almost all realizations, to be precise.

[7] See f.i. Nielsen(1996) for a formal statement of the construction just outlined.

We shall assume that agents live for two periods (overlapping or non-overlapping), but in dynasties such that each rational belief, in the format of the sequence, $\{P_{it}\}_{t=1}^{\infty}$, is not only that of the individual agent, i, but also that of the entire dynasty, which we also index by $i \in [0,1]$. Each dynasty is assumed to have a different such sequence, but at every date a fraction Q of the agents in $[0,1]$ use P^0 while the rest, a fraction $1-Q$, use P^1.[8] Thus beliefs are diverse at all dates. Note that this construction embodies the idea of Muth(1961), who did *not* assume that all agents have correct beliefs, that is what is now commonly referred to as rational expectations, but only that the *average belief* is equal to the "prediction of the theory" (Muth, 1961, p. 316).

Remark 1 *Endogenous Uncertainty*
If at any date t all agents either use P^0 or P^1, beliefs would not be diverse and the argument presented here in favor of mandatory savings would not carry through. In this case beliefs are correlated, in other words the distribution of beliefs varies over time. In many other studies using rational beliefs (see f.i. Kurz and Motolese, 2001) changes in the distribution of beliefs is of central importance, giving rise to so called *endogenous uncertainty.* With this types of uncertainty, agents implicitly or explicitly form expectations about future beliefs (akin to the Keynes' *Beauty Contest* story). These expectations about the future distribution of beliefs may then also be diverse, giving rise to a further layer of inefficiency, see Nielsen(2003) ■

Remark 2 *Alternative to dynasties*
In stead of assuming that short lived agents inherit the beliefs of the infinite lived dynasty they belong to, we could assume that beliefs, defined on infinite sequences, are at each date randomly assigned to agents. For every single young agent the relevant part of any belief assigned to him is its prediction for the next (and last) period of his life. Because there is to each agent ample data available on the past performance of the economy, it does make sense to assume that he or she can form a rational belief for the entire future. Because we assume a continuum of agents, we may assume that the distribution of belief is constant over time (see Nielsen, 2007 for more on this construction) ■

2.2 Rational Beliefs and Rational Overconfidence

Holding a non-stationary rational belief means being more confident about one's ability to predict the future than when holding the stationary rational belief. This is most easily seen by imaging the following situation: We are being told that for any date t Nature picks with probability Q the probability vector (P_L^0, P_H^0) for W_{2t} and with probability $1-Q$ the probability vector (P_L^1, P_H^1). If an agent thinks that he does not know the choice of Nature, the probability he should use in forecasting W_{2t+1} is $(\overline{P}_L, \overline{P}_H)$, the compound probability for the two states. If, in contrast, at date 0 the agent is being told the whole sequence of Nature's choices, that is for every date in the future, whether P^0 or P^1 is the true distribution, this extra information allows him to make a more precise forecast at every date. An agent holding a non-stationary rational belief in effect believes that he knows the entire sequence of Nature's choice of probability vectors and in this sense believes his forecasts to be more precise (having a smaller support) than that based on the stationary, empirical distribution[9]. We therefore say that his belief exhibits rational confidence. Since we assumed that every agent (dynasty) has a different sequence of one-period beliefs, at most one of them (we do not know who, if any) has the correct timing

[8]A full specification of this construction is provided in Nielsen(2008). The results obtained here would remain also in the case of complete diversity at all dates i.e. if we assumed that the distribution of rational beliefs is such that at any date every agent holds a different belief about the next period (while the distribution of beliefs is constant).

[9]Statistical analysis of the past performance of this belief may reveal that it did not perform well. However, there is no way to turn this observation into an assessment of how the particular belief will perform in the future. In other words there is no logically compelling reason to abandon or modify the belief as far as the future is concerned.

and hence the correct belief. Thus looked at collectively rational overconfidence is present: the vast majority of agents (or more precisely, dynasties) hold a belief which is rational, confident *and* not correct. We summarize with two definitions:[10]

Definition 1 *Confident Belief*
A rational belief is said to exhibit confidence if it is more precise (has a smaller support) than the empirical distribution ■

Definition 2 *Rational Overconfidence*
We say that rational overconfidence is present when there are two non-negligible groups of agents, with all members holding non-stationary rational beliefs, such that the belief held by any member of one group is held by no member of the other group ■

Note that if all agents hold the same confident rational belief we do not say that overconfidence is present. Since this belief may be correct, there is no objective justification for using the prefix "over".

Remark 3 *Entropy and confidence*
For any two probability vectors $p = (p_1, p_2, \dots, p_n)$ and $q = (q_1, q_2, \dots, q_n)$ which are not identical, we have $-\sum_{i=1}^{n} p_i \ln p_i < -\sum_{i=1}^{n} p_i \ln q_i$. This means that for $k = 0, 1$, $-(P_L^k \ln P_L^k + P_H^k \ln P_H^k) < -(P_L^k \ln \overline{P}_L + P_H^k \ln \overline{P}_H)$. On the left hand side of this inequality, we have the entropy of the probability (P_L^k, P_H^k). Multiplying by the frequency weights Q and $1-Q$ and summing over the k's we get

$$-\left(Q(P_L^0 \ln P_L^0 + P_H^0 \ln P_H^0) + (1-Q)(P_L^1 \ln P_L^1 + P_H^1 \ln P_H^1)\right) <$$
$$-\left(Q(P_L^0 \ln \overline{P}_L + P_H^0 \ln \overline{P}_H) + (1-Q)(P_L^1 \ln \overline{P}_L + P_H^1 \ln \overline{P}_H)\right) = \overline{P}_L \ln \overline{P}_L + \overline{P}_H \ln \overline{P}_H$$

On the right hand side of this inequality, we find the entropy of $(\overline{P}_L, \overline{P}_H)$, while on the left hand side we find the *average* entropy (over time) of the non-stationary rational belief. If we interpret lower average entropy as meaning more informative, this is another way of stating that a non-stationary rational belief is exhibiting more confidence ■

The concept of overconfidence originates from outside the economics literature, Oskamp(1982) and Svensson(1981) are early examples.[11] It plays a prominent role in the behavioral economics literature where it is invariably linked to some sort of irrationality.[12] In contrast, here overconfidence is not the outcome of irrationality but of the fact that it is, in most situations, possible to interpret the available data in many different ways.

2.3 Ex-post Welfare Optimality

In a situation where decisions are decentralized (to the agent level) and where expectations are inconsistent, these decisions cannot all be optimal. If decisions are, on the other hand, centralized, the decision maker would supposedly base them on one belief. Formally, while the concept of Pareto optimality and improvements (typically) is associated with the aggregate

$$\int_0^1 w_i E_i U_i(C_i) \mathrm{d}i$$

[10]For the formal versions of these definitions, see Nielsen(2008).

[11]Svensson(1981) observed a group of drivers of which more than half thought they were above the median in terms of driving skills. There are countless similar studies (where agents are asked to rank themselves in terms of competence, likeability etc,). Many of the studies share the problem, that the subjects may interpret the question differently - f.i., what does it mean to be a "skilled driver"?

[12]See f.i. Odean(1998) and Daniel, Hirschleifer and Subrahmanyam(1998).

where the integration is over agents (indexed by i), w_i refers to some weight given to agent i, while E_i is the expectation based on his belief, U_i is his utility and C_i is his consumption, from the point of view of ex-post efficiency, the maximization is of

$$\int_0^1 w_i E U_i(C_i)\mathrm{d}i = E\left\{\int_0^1 w_i U_i(C_i)\mathrm{d}i\right\}$$

where E refers to expectation according to some specific belief. In Section 3 we shall provide some arguments why using expectations based on the stationary rational belief is a natural choice. However, it is important to keep in mind, that no one can claim that this belief is correct, it is only one out of many possible rational beliefs.

Remark 4 *The ex-post concept*
Dreze(1970) was probably the first to note that, in the presence of uncertainty, Pareto optimal allocations may not be what Hammond(1981, p. 236) calls ex-post intertemporal Pareto efficient. In other words, the marginal rate of substitution for incomes in two different states may not be the same for all consumers unless the subjective probabilities of the states they use are the same. In Starr's (1973) formalization of this idea, an allocations that is Pareto efficient also if, at the date before its realization, the stochastic state is known is said to be ex-post Pareto optimal (for the given state).[13] Universal ex-post efficiency (Harris, 1978) then means that given any realization of the future state, the allocation of present and future resources is Pareto undominated. These definitions only make sense in a stochastic exchange economy like the one we study in Section 4 (or for very particular cases like the so called intratemporal production economies), see Harris(1978).[14] In Sandmo(1983) "ex-post" refers to the social welfare function being defined on realized utility levels. Hammond(1981) defines *ex-post welfare optimality* (called Allais Optimality by Mirrlees, 1974) as the outcome of the decision maker maximizing the expected value of a Bergsonian welfare function, where the expectation is based on one set of "social probabilities" (not necessarily that of any agent). Harris(1978) defines expected ex-post efficiency, as a stochastic allocation that is Pareto optimal if all agents use some given probability in forming expected utility. The problem with these definitions is that they provide no suggestions for what probabilities should be used in forming expectations. We place ourselves closest to the approach of Hammond(1981) but impose that the expectation (probability measure) used must have certain objective properties, a requirement which is especially important for production economies. Note that, in contrast with Hammond(1981), we do not consider the possibility of ignoring individual's tolerances towards risks, i.e. when measuring welfare we use the individuals' preferences over sure consumption. What makes the ex-post approach relevant to us is that it ensures consistency of expectations. Although we shall stick to the term "ex-post", "consistent expectations welfare optimality" would thus have been a, from our point of view, more appropriate name ■

3 Model and Optimal Policy

A young agent expecting to have a high income in the future (as middle aged) will save up less for retirement, expecting to make up for the shortfall later. If the macroeconomy is good, typically individual incomes are high. Thus someone who expects the macroeconomy to perform well, also expects to earn a high income and

[13]The usage of "ex-post" may cause confusion. Hammond(1981) calls an allocation *ex-post efficient* if, given any realized state, it is Pareto efficient, that is, given the realized state, there is no room for a Pareto improvement within the period. This efficiency criterion is implied by all the other efficiency concepts we consider here.

[14]Our production economy satisfies what Starr(1973) calls weak independence in production but not intratemporality.

so will be saving less for retirement.[15] In our model the macroeconomy is simply the wealth level that everyone of the identical agents have in the second, final period of their lives. If we assumed instead that the return of the savings for retirement are stochastic and beliefs about it differ our basic insights would not be altered, i.e. the ex-post optimal policy would, as in our model, be to require agents to have a minimum savings level.

In the two models we use, here and in Section 4, shocks are economy-wide and individual's beliefs as well as consequences of diverse policies are assessed by usage of the long-term average performance of the economy, i.e. the empirical distribution. In models of this kind the rational beliefs construct becomes useful in modelling diverse, dynamic beliefs. In Remarks 7 and 8 we briefly consider the opposite possibility, that shocks are idiosyncratic. Postulating a kind of law of large numbers, the distribution, across agents, of realized shocks is constant over time and known and it is this distribution we compare with, when declaring that beliefs are inconsistent and that taken as a whole actions are not rational.

The model:

At any date t any agent i, holding the belief $P^k, k = 0, 1$, solves the following problem:

$$\max_{S \in [0, W_1]} \sum_{j \in \{H, L\}} u(W_1 - S, W^j + Sr) P^k_j \tag{1}$$

where r is the return on the safe investment. We assume an interior solution throughout. Let S^k be the solution (independent of i) to this problem. We label the stochastic consumption resulting from the investment decisions of dynasty i, $\{(C^i_{1t}, C^i_{2t+1})\}^{\infty}_{t=1}$ (note that no one knows the true associated distribution, since it depends on the unknown, true probability distribution of $\{W_t\}^{\infty}_{t=1}$). Let $\overline{S}$ solve (1) after replacing P^k with $\overline{P}$ and let $\{(\overline{C}_{1t}, \overline{C}_{2t+1})\}^{\infty}_{t=1}$ be the resulting stochastic consumption stream. The allocation, where each agent consumes this stream, is referred to as $\overline{C}$. The government imposes a minimal savings level, S^* for all individuals and returns that saving plus interest in the second period of their lives.[16] With this required minimum savings level, the problem of an agent, who uses the belief $P^k, k = 0, 1$ becomes:

$$\max_{S \in [0, W_1 - S^*]} \sum_{j \in \{H, L\}} u(W_1 - (S^* + S), W^j + (S^* + S)r) P^k_j \tag{2}$$

with solution $\tilde{S}^k$. Then if $S^* \geq S^k$, $\tilde{S}^k = 0$, else $\tilde{S}^k = S^k - S^*$ and the effective savings level of an agent with belief $P^k, k = 0, 1$ is $\max(S^*, S^k)$.

The government's problem:

Let E^*_t denote expectations at date t, using the correct (but unknown!) distribution of W_{2t+1} and let $\overline{E}$ be expectations using the empirical distribution. Since this distribution is stationary, expectations are the same at all date so we do not use a subscript t here. There are three ways of arriving at the objective function, we shall propose as being the government's. One is, for any t, to find the expected value of the (unweighted) sum of agents' utility, using $\overline{E}$:

$$\overline{E}\{Qu[W_1 - \max(S^*, S^0), W_{2t+1} + \max(S^*, S^0)r] + (1 - Q)u[W_1 - \max(S^*, S^1), W_{2t+1} + \max(S^*, S^1)r]\} =$$

[15] Our model has two periods only. If we added one period (for retirement) markets would no longer automatically be complete, an added complication. With a two-period model we are basically assuming that a middle aged, via the riskless savings technology, transfers consumption to the period of retirement.

[16] The government could offer two different minimum savings levels, but as will become clear this will not improve on the outcome. Note also that in reality it is difficult if not impossible to put a cap on savings, since it is typically possible to store wealth.

$$\sum_{j=L,H} \{Qu[W_1 - \max(S^*, S^0), W^j + \max(S^*, S^0)r]$$
$$+(1-Q)u[W_1 - \max(S^*, S^1), W^j + \max(S^*, S^1)r]\} \overline{P}_j \qquad (3)$$

In this first approach, the government may attach different weights to different generations without this having any effect on its objective (since this objective is non-dynamic).

The next is to take the average (over time) of the expected (unweighted) sum of individual utility using an expectation, E_t, based on *any* rational belief about $\{W_{2t}\}_{t=1}^{\infty}$:

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} E_t\{Qu[W_1 - \max(S^*, S^0), W_{2t+1} + \max(S^*, S^0)r] +$$
$$(1-Q)u[W_1 - \max(S^*, S^1), W_{2t+1} + \max(S^*, S^1)r]\}$$

which, since we assumed that $\{W_{2t}\}_{t=1}^{\infty}$ is stable, is equal to (3).[17] Notice that this equality in particular holds if we use the expectation E_t^* at any date, i.e. the truth. Notice also, that the equality holds for any of the rational beliefs being used by the individual agents of this economy. Thus, like in *The King's Dilemma*, which of the individual agents' beliefs is used, is immaterial for the decision being made.

The third approach is to consider the average (over time) sum (unweighted) of individual *realized* utility:

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \{Qu[W_1 - \max(S^*, S^0), W_{2t+1} + \max(S^*, S^0)r]$$
$$(1-Q)u[W_1 - \max(S^*, S^1), W_{2t+1} + \max(S^*, S^1)r]\}$$

which, again by stability, with probability 1, is equal to (3).

In the first approach we do not compare the welfare of different generations, but use the stationary probability $\overline{P}$, which may be incorrect and which every single agent (by assumption) holds to be incorrect at any date t. In the second and third approaches, we give equal weight to all generations when constructing the objective. Note that in the special case of stationarity where $E_t = \overline{E}$ for all t (something that *could* be the case), the equivalence of the first and second approach is trivial, while the equivalence of the the second and third approach is a consequence of Birkhoff's Ergodic Theorem. Note also that, in the special case we are considering, using the stationary measure at every date amounts to using, at every date, the *average* of all agents' beliefs.

The reason for the equivalence of the three ways of constructing an objective function for the government is that the distribution of actions is constant over time (which follows from the fact that the distribution of beliefs is constant), which because of stability means that the average (over time and across agents) realized utility is equal to the expected average (across agents) utility when taking expectations using the empirical measure. It is noticeable that what beliefs agents have does not influence the objective of the government - in particular, the government need have no knowledge about the beliefs of agents other than that the distribution is constant.[18]

Obviously, with the objective function (3), it is then optimal for the government to set $S^* = \overline{S}$. This means that all agents in the economy save up at least $\overline{S}$ while others suboptimally save up more. Notice that in many periods some agents are able to state ex-post that they would have been better off without government intervention. Note also that in this economy each agent (viewed as a dynasty) would hold that the pension plan imposes an inferior consumption plan on himself, but that at the same time, it represents an objective improvement for the average population. Thus accepting the objectively improving policy implies a sacrifice

[17] This follows directly from Proposition 2 of Kurz(1994).

[18] This assumption can be weakened. Even in the case where the distribution does vary over time, under the assumption of structural independence (see Nielsen, 2007b), we retain the format of the objective.

(possibly in the expected sense, if acceptance is behind a "veil of ignorance"). If the agent (dynasty) cares about the average population he would find it desirable to have the freedom to choose his own savings level but to have a minimum savings level imposed on all other agents.

The important point of our approach is that the expected average utility in (3) is *objective*. In other words, every agent in the economy agrees that (3) expresses the average utility, over agents at any date, t and over dates, that will be observed for (almost) all realizations of the sequence $\{W_{2t}\}_{t=1}^{\infty}$. The policy solution is thus not arbitrary, but neither is it based on any superior knowledge possessed by any authority. Rather, it is based on what is commonly agreed on by all agents, namely the empirical distribution of $\{W_{2t}\}_{t=1}^{\infty}$. From these consideration a general principle suggests itself: *That any policy should respect the common content of diverse opinions*. Since diverse opinions are rooted in the same reality, such a common content is likely to be present in many situations.

From an analytic point of view it is useful to decompose into two the welfare properties of the allocation resulting from the policy solution. Firstly, every agent agrees on the welfare consequences of the policy. Secondly, there is no feasible policy that according to the beliefs of all agents lead to an improvement in terms of expected welfare.

Definition 3
We say that there is *universal agreement about welfare consequence* of a stochastic allocation if, for this allocation, the expected welfare achieved (for given welfare weights) is the same independently of which agent's belief is being used when taking expectations ■

Secondly, we propose a criterion for objective welfare improvements as well as a notion of objective welfare optimality:

Definition 4 *Objective Welfare Improvement and Objective Welfare Optimality*
For given welfare weights, we say that an allocation is objectively welfare improving over some given allocation, if it is feasible and if, according to all agents' beliefs, the weighted expected utility of it is higher than that of the other, given allocation. If there is no objective welfare improvement for a feasible allocation, this allocation is said to be objectively ex-post welfare optimal ■
This notion of an improvement embodies unanimity in evaluation. Obviously, weaker notions of the degree of approval could be adopted. To understand the usefulness of the two definitions, notice that the consumption stream, $\{C_{1t}^{ij}, C_{2t+1}^{ij}\}_{i\in[0,1],t=1,2,...}$ where $C_{1t}^{ij} = C_{1t}^{j}$ and $C_{2t+1}^{ij} = C_{2t+1}^{j}, \forall i$ is objectively welfare optimal (since, according to (the belief of) agent j, it achieves the highest possible welfare for the given weights). However, there is no universal agreement about the welfare consequences of this consumption stream.

We then suggest that the government should seek to achieve an allocation about which there is universal agreement about consequences and which is objectively ex-post welfare optimal. Such an allocation might then be said to be *objectively socially optimal*. The consumption stream $\overline{C}$ (where each agent invests $\overline{S}$) is such an allocation, however, in our model it cannot be achieved since the government cannot prevent agents from saving too much. We note that this allocation is expected ex-post efficient (in the sense of Harris, 1978) only when using the stationary distribution, but not when using the individual rational beliefs of agents (or any other rational belief that a decision maker may hold). We summarize the main conclusions of the analysis in the following

Proposition 1
If the decision maker weights all agents and generations equally, independently of the beliefs it uses, as long as this belief is rational, the optimal minimum savings level is $S^* = \overline{S}$. Moreover, all agents in the economy

agree that if the objective is to maximize average, over agents and over time, expected utility, S^* should be set equal to $\overline{S}$. This savings level will induce a consumption stream that is objectively improving over any other consumption stream, that can be attained with a minimum savings level (including the laissez faire consumption stream (where $S^* = 0$), and about which there is universal agreement about consequences. At the same time, all agents (viewed as dynasties) are subjectively worse off under this policy ■

Remark 5 *The distribution of beliefs*

We already noted that if all agents hold the same non-stationary, rational belief, we cannot say that overconfidence is present, in particular, we cannot say that there is an inefficiency present. But what if the vast majority of agents hold one non-stationary, rational belief, while the rest hold another such? We know for sure that in this case, if everyone saves according to their own beliefs, an inefficiency is present. However, there may be no agreed way to improve on the laissez-faire distribution. To understand why, observe that in the case of complete diversity, as we already noted before, every agent (dynasty) i thinks that everyone would do better if they acted according to his belief, which we denote B_i, i.e. if they used his investment strategy. In that case the average (over agents and time) realized welfare would, according to this belief, be:

$$Q \sum_{j \in \{H,L\}} u(W_1 - S^0, W^j + S^0 r) P_j^0 + (1-Q) \sum_{j \in \{H,L\}} u(W_1 - S^1, W^j + S^1 r) P_j^1 \tag{4}$$

which is greater than

$$\begin{aligned} &\sum_{j=L,H} \{Qu[W_1 - \max(\overline{S}, S^0), W^j + \max(\overline{S}, S^0) r] \\ &+ (1-Q) u[W_1 - \max(\overline{S}, S^1), W^j + \max(\overline{S}, S^1) r]\} \, \overline{P}_j \end{aligned} \tag{5}$$

- the welfare attained under the government's policy.

With many agents holding the belief B_i, they might not consider the government's policy to be objectively improving. Suppose that there are only two beliefs (about infinite sequences) present in the economy but that the one-period beliefs are like before, P^0 and P^1. Suppose that a mass $m \in (0,1)$ of agents (belonging to group 1) use the first belief B_i while the rest, of mass $1-m$, hold the other belief, B_j and furthermore, that the two beliefs are structurally independent in the sense that at a fraction Q^2 of all dates, both groups use P^0, at a fraction $Q(1-Q)$ of all dates the first group uses P^0 and the other group P^1, while at a fraction $(1-Q)Q$ of all dates the first group uses P^1 and the second P^0. Letting $U(S) = u[W_1 - S, W_2 + Sr]$, we get the expected average (across agents and dates) welfare from the government's policy, using the belief B_i:

$$\begin{aligned} QE^0\{mU(\max(\overline{S}, S^0)) + (1-m)[QU(\max(\overline{S}, S^0)) + (1-Q)U(\max(\overline{S}, S^1))]\} + \\ (1-Q)E^1\{mU(\max(\overline{S}, S^1)) + (1-m)[QU(\max(\overline{S}, S^0)) + (1-Q)U(\max(\overline{S}, S^1))]\} = \\ m[QE^0U(\max(\overline{S}, S^0)) + (1-Q)E^1U(\max(\overline{S}, S^1))] + (1-m)\overline{E}[QU(\max(\overline{S}, S^0)) + (1-Q)U(\max(\overline{S}, S^1))] \end{aligned}$$

where $E^i, i = 0, 1$ refers to expectations under the one-period belief, P^i. When m is large this average welfare is smaller than the expected welfare resulting from no government intervention (again using the belief B_i):

$$\begin{aligned} QE^0\{mU(S^0) + (1-m)[QU(S^0) + (1-Q)U(S^1)]\} + (1-Q)E^1\{mU(S^1) + (1-m)[QU(S^0 + (1-Q)U(S^1)]\} = \\ m[QE^0U(S^0) + (1-Q)E^1U(S^1)] + (1-m)\overline{E}\{QU(S^0) + (1-Q)U(S^1)\} \end{aligned}$$

So the first group of agents, using B_i, would not agree that the government's policy is improving over no intervention.

Note the importance of the rational belief, dynamic perspective for this reasoning. In our original model, at every date, like in the example we just studied, there are only two one-period beliefs. However contrary to the example we just studied, the beliefs (on infinite sequences) about the macroeconomic process of endowments are completely diverse. The rationale for forced savings in this model is that even though some people may "beat" (i.e. perform better using their subjective beliefs) the stationary, empirical process, over longer horizons, most people will not be able to do this.

To see that the notion of objective welfare improvements extends beyond the case of complete diversity consider the following variant of the model from Section 3: Suppose that $Q = 1/2$, that $1/2$ of all agents hold one belief (the rest the second belief), and that the two rational beliefs are structurally independent in the sense that at any date $1/2$ of the agents use the one-period belief P^0 while the other $1/2$ use P^1. In that case the objective of the government remains (3) (whatever of the two beliefs is being used) and consequently the optimal policy also remains unchanged ■

Remark 6 *Many wealth levels*

We assumed that all agents have the same wealth, the only difference among them being what beliefs they hold. If we instead assume that there are many stochastic wealth levels (in terms of the empirical distribution of W_2) then there are also many objectively optimal savings levels, say $\overline{S}_K > \overline{S}_{K-1} > \ldots > \overline{S}_1$. If these wealth levels are observable, the government will impose wealth dependent minimum savings rates. In fact, for the case of Social Security in the US, the mandatory contribution is increasing in income. If the government cannot observe the wealth levels, by the same reasoning as above, if it offers more than one savings level, no individual can do strictly better by choosing anything but the lowest of them (and then invest privately, if so desired). Thus the government will only impose one savings level, S^*. If it sets $S^* > \overline{S}_1$, it forces some agents to save more than is objectively optimal for them. Depending on how social welfare is calculated and for political reasons this may not be optimal or feasible. We shall return to this issue in Section 5 ■

Remark 7 *Pecuniary externalities in production economies*

In *The King's Dilemma* one may argue that there is a negative externality in the sense that the probability of a given prince being chosen by the princess decreases when another prince is allowed to go. However, *subjectively* there is no such negative externality (see also Remark 12 below). Note also, that in the model of savings and social security above there is, in any sense of the concept, no negative externality from an individual's investment decision and in this sense the case for imposing *social rationality* has been made more difficult.

Here we briefly consider the possibility of a pecuniary externality in a production economy.[19] Suppose individual agents do not consume the product they produce, but are selling it on a market with a downward sloping demand curve and that the return on their investment is stochastic, leading to a problem of the following format, for an agent with belief k:

$$\max_{I \in [0, W_1]} \sum_{j \in \{H, L\}} u(W_1 - I, I r_j^i p) P_j^k \tag{6}$$

with solution $I^k, k = 0, 1$ where $r_j^i, j \in \{L, H\}$ is the return of i's investment in the two states, taking values in $\{r_H, r_L\}$, and p is the price at which the product is being sold. Notice that we now assume idiosyncratic shocks rather than economy-wide ones. As before, at every date a fraction Q of agents use the belief (P_L^0, P_H^0) and a fraction $1 - Q$ use (P_L^1, P_H^1) where $Q(P_L^0, P_H^0) + (1 - Q)(P_L^1, P_H^1) = (\overline{P}_L, \overline{P}_H)$. Assume (using a kind of

[19] See Nielsen(2009) for a related model developed in more detail.

law of large numbers) that at every date among both groups of agents (those who use (P_L^0, P_H^0) and those who use (P_L^1, P_H^1)), a fraction $\overline{P}_L$ of the agents experience a low return, r_L. These assumptions together imply that if the price is constant, aggregate output is constant over time, in turn implying that the price will be constant as well, so justifying the format (6). Let $\overline{p}$ be the market clearing price when all agents use the stationary probability and hence invest $\overline{I}$. In an equilibrium we may have either over- or underinvestments, i.e. either $I^* \equiv QI^0 + (1-Q)I^1 > \overline{I}$ or $< \overline{I}$. In the case of overinvestments which leads to overproduction in the aggregate, the equilibrium price, p^* is less than $\overline{p}$ and a negative pecuniary externality is present.[20] This case may be interpreted as a situation with "excess entry". It has been argued that empirically there is excess-entry, i.e. too many start-ups, in the sense that the failure rate of such new businesses is very high (see f.i. Wu and Knot, 2006 and the references therein).

In either case, we argue that from the point of view of society, and despite the fact that the allocation is (loosely speaking, since we do not consider the demand side) Pareto efficient, the market is not functioning optimally because individuals are not investing $\overline{I}$. Obviously, even in the case where it is possible to identify this problem, there may be no simple remedy for it ■

Remark 8 *Idiosyncratic shocks*

Note that for the variant of the model we used in Remark 7, if we interpret I as private savings (for own consumption) there is potentially a need for social security. Suppose for simplicity that the return on the investment is being consumed rather than consumed (set p equal to 1). We assume that every individual agent believes that for all *other* agents, independently of their belief, $(\overline{P}_L, \overline{P}_H)$ is the distribution of returns, r. Then according to every agent, under a laissez fare policy, the aggregate social welfare (using a kind of law of large numbers) is

$$Q\left\{\overline{P}_L u[W_1 - I^0, I^0 r_L] + \overline{P}_H u[W_1 - I^0, I^0 r_H]\right\} + (1-Q)\left\{\overline{P}_L u[W_1 - I^1, I^1 r_L] + \overline{P}_H u[W_1 - I^1, I^1 r_H]\right\}$$

This aggregate social welfare is strictly smaller than

$$\overline{P}_L u[W_1 - \overline{I}, \overline{I} r_L] + \overline{P}_H u[W_1 - \overline{I}, \overline{I} r_H]$$

which is the aggregate social welfare ensuing from forcing everyone to invest $\overline{I}$. Thus there is universal agreement about the consequences of enforcing a uniform investment level $\overline{I}$ for everyone and the resulting allocation is objectively optimal. This holds, period by period, irrespectively of whether agents use (long run) rational beliefs or not, i.e. the condition $QP^0 + (1-Q)P^1 = \overline{P}$ need for instance not hold ■

Remark 9 *Overconfidence and Undersavings*

There is disagreement among economists about whether or not undersavings is a serious problem in high-income societies. The reasons for this disagreement is twofold: firstly it is not clear how to define adequacy of savings, secondly there are data issues. According to calibrations by Scholz et al.(2006) only about 15 % of Americans in the $1931-41$ cohort save too little while more save too much. These authors do not provide figures on for how many households the minimum bound on savings imposed by social security rules (and possibly other mandatory savings programs) is effectively binding. Engen et al.(2004) report adequate or even oversavings for most of the population in the same cohort as used by Scholz et. al (2006), but that the bottom 25 % are undersaving and rely heavily on social security. Bernheim and Rangel(2005) cite evidence suggesting the prevalence of

[20] Thus, we use "pecuniary externality" not in the traditional sense (see f.i. Loong and Zeckhauser, 1982), but relate it to an ex-post concept of efficiency.

undersavings, in particular that consumption tends to fall around the time of retirement, indicating that many people have been unsuccessful in achieving consumption smoothing over their life time. In the model we just studied, without some minimum required savings, some agents will save too much, some too little. Whether this will lead to over- or undersavings in the aggregate depends on the parameters of the model. On the other hand, with an ex-post optimal minimum savings level imposed there is aggregate oversavings. Furthermore, as in Engen et al.(2004) we see some agents saving close to the imposed minimum, while others are saving, from an objective point of view, too much. This feature of the model is however subject to qualifications. We mentioned in Remark 6 how it may come about that only a subset of agents who are undersaving find the minimum savings requirement binding. If we add to this the possibility that, without any minimum savings requirement, most agents are undersaving, we may arrive at a situation where even with a minimum savings requirement there is undersavings by most agents.

In the context of rational belief models, there are several ways of arriving at a situation where most agents undersave. Suppose that the individual agent solves a problem of the following sort:

$$\max_{S_1,S_2,\ldots,S_N} Eu(W_1 - \sum_{n=1}^{N} S_n, \sum_{n=1}^{N} S_n R_n) \text{ s.t. } S_n \geq 0,\ n = 1,2,\ldots,N$$

where $\sum_n S_n$ is total savings and S_n is placed in an investment projects with stochastic return R_n, $n = 1,2,\ldots,N$. These returns each have a support (r_L, r_H), $r_L < r_H$ and empirical probabilities $\overline{P} = (\overline{P}_L, \overline{P}_H)$. Let us assume that agents' beliefs are in the form of the extreme probabilities $P^0 = (1,0)$ and $P^1 = (0,1)$ with frequency Q and $1-Q$ respectively. The rationality condition that $QP^0 + (1-Q)P^1 = \overline{P}$ then implies that $Q = \overline{P}_L$. Suppose again the distribution of beliefs across agents is constant, so that the fraction of agents that hold the belief $(P^{i_1}, P^{i_2}, \ldots, P^{i_N})$ with $i_n \in \{0,1\}$ is

$$Q^{(N-\sum_{n=1}^{N} i_n)}(1-Q)^{(\sum_{n=1}^{N} i_n)}$$

This means that, except for a fraction $Q^N = \overline{P}_0^N$ of agents, all agents believe that there is at least one investment which delivers the return r_H with probability 1. If high returns lead to low savings there will be widespread undersavings and also undersavings in the aggregate.
We do not need such extreme beliefs to get this result. What is important is that there is, according to most savers' beliefs, a portfolio that with high probability delivers a return above $\overline{P}_L r_L + \overline{P}_H r_H$ ■

4 Portfolio Choice

So far we have considered a rationalization of mandatory savings program. A remarkable aspect of the recent debate about Social Security in the USA (during the first year of the second term of president Bush, jr.) is that both of the main camps (those who wanted to fix it by making more funds available to it and those who wanted to reform it) agreed that there should be some limits to choices. Firstly, there were the, at least as stated, fundamental agreement that there should be some sort of mandatory savings for retirement. Secondly, there were also an agreement that there should be restrictions on how these savings are placed (to avoid what the president called "frivolous investments"). As the program works now, the running surplus is placed in government bonds, while reformers have suggested that individuals should have the option to place a certain fraction of their contributions for retirement in a limited array of portfolios of relatively "safe" assets, the income from these individual accounts then being used to pay out pensions on retirement.

A majority of Americans think, according to a recent poll by The Wall Street Journal(2004), that there should be something like Social Security. Interestingly, 46% of the individuals polled expressed opposition to letting individuals invest part of their social security savings in stocks and bonds, while only 38% stated that they would *not* take advantage of this opportunity, should it become available.

In the following we shall see that in the context of rational overconfidence such opinions make perfect sense. In particular we shall explain, using ex-post optimality as welfare criterion, why the idea of letting people freely choose how to invest the government imposed savings may not be desirable.

In the previous model, we assumed that there is an *outside* investment for agents, i.e. that production takes place. Here we shall instead assume a stochastic exchange economy, with two trees, A and B in fixed supply of 1 each. The two stochastic sequences $\{\Omega_{At}\}_{t=1}^{\infty}$ and $\{\Omega_{Bt}\}_{t=1}^{\infty}$ with state space $\Re_+$ are the returns of these two trees and we shall assume that they are both known to be independently distributed over time and mutually independent, and furthermore that they are *empirically* known to be mutually independent with the same i.i.d., one-period distribution, $\overline{P}$ with associated expectation $\overline{E}$. The latter assumptions means that it is known, for any set $S \subset \Re_+$, that for almost all realizations $\{\omega_{ct}\}_{t=1}^{\infty}$, the frequency of dates where $\omega_{ct} \in S$ is $\overline{P}(S)$ for $c = A, B$.

We shall assume an overlapping generations structure such that young agents hold the asset and sell it to the then young agents on retirement. To simplify the analysis we shall assume that agents only consume when old. As young they have a certain endowment, e of the single commodity and their only problem then is how to invest this endowment in a portfolio of the two trees.

As before all agents know the empirical distribution of the dividends of the two trees. Assuming risk aversion they would, if the two distributions were believed by all to be the empirical distribution, invest $e/2$ in each of the trees. Since this investment decision results in the consumption $e + (\Omega_{At} + \Omega_{Bt})$ for all old agents (i.e. perfect risk sharing), this is also the investment that the government would like them to make.

Knowledge of the empirical distribution does not rule out that agents believe that the true unknown distribution of $\{\Omega_{At}\}_{t=1}^{\infty}$ and $\{\Omega_{Bt}\}_{t=1}^{\infty}$ differ. We shall assume that each agent may have one of two one-period beliefs (in terms of distributions), P_0 and P_1 about $\Omega_c, c = A, B$ such that there is a $0 < Q < 1$ with $QP_0 + (1-Q)P_1 = \overline{P}$. At every date t there is a measure Q^2 of agents who use the distribution $P_0 \times P_0$ on $\Re_+^2$, a measure $Q(1-Q)$ who use $P_0 \times P_1$, a measure $(1-Q)Q$ who use $P_1 \times P_0$, and a measure $(1-Q)^2$ who use $P_1 \times P_1$.

Let p_{ct} be the price of the commodity in terms of tree $c, c = A, B$. The problem, at date t, of an agent of type (z_A, z_B), $Z_c \in \{0, 1\}$ (meaning that he holds belief (P_{z_A}, P_{z_B}) with associated expectation E^{Z_A, Z_B}) now is:

$$Max_{q\in[0,1]} E^{z_A, z_B} u \left[\left(q p_{At}[\frac{1}{p_{At+1}} + \Omega_{At+1}] + (1-q) p_{Bt}[\frac{1}{p_{Bt+1}} + \Omega_{Bt+1}] \right) e \right] \tag{7}$$

where u is a strictly increasing, strictly concave utility function. Since, in terms of the distribution of beliefs, this economy does not change between dates, it makes sense to concentrate on those equilibria where p_{ct} is constant $= p, c = A, B$, and where consequently the fraction of total real endowment invested in each tree, π is constant and equal to 1/2 (owing to the symmetry of beliefs). In that case the problem of an agent of type (z_A, z_B) becomes

$$Max_{q\in[0,1]} E^{z_A, z_B} u \left[\left(q p[\frac{1}{p} + \Omega_{At+1}] + (1-q) p[\frac{1}{p} + \Omega_{Bt+1}] \right) e \right] \tag{8}$$

and in equilibrium, $\pi e p = 1$ i.e. $p = \frac{2}{e}$ (demand equal to supply on the markets for the trees). It follows that the problem of the agent is:

$$Max_{q\in[0,1]} E^{z_A, z_B} u \left[\left(q[1 + \frac{2\Omega_{At+1}}{e}] + (1-q)[1 + \frac{2\Omega_{Bt+1}}{e}] \right) e \right] \tag{9}$$

with solution π_{z_A,z_B} where $\pi_{0,1} = 1 - \pi_{1,0}$ (due to symmetry), $\pi_{0,0} = \pi_{1,1} = \frac{1}{2}$ (due to risk aversion), consequently $Q^2\pi_{0,0} + (1-Q)^2\pi_{1,1} + Q(1-Q)\pi_{0,1} + (1-Q)Q\pi_{1,0} \equiv \pi = 1/2$ as was postulated before. We consider the case where $\pi_{0,1} \neq \pi_{1,0}$. At every date t the sum (or average) of agents' utilities is:

$$\overline{u}(\Omega_{At}, \Omega_{Bt}) \equiv \left[(1-Q)^2 + Q^2\right] u(e + \Omega_{At} + \Omega_{Bt}) + Q(1-Q)u(e + 2\{\pi_{01}\Omega_{At} + (1-\pi_{01})\Omega_{Bt}\}) \\ +Q(1-Q)u(e + 2\{\pi_{10}\Omega_{At} + (1-\pi_{10})\Omega_{Bt}\}) \quad (10)$$

To find the average (over time) sum (across agents) of realized utility, by stability, we just take expectations in (10) with respect to the stationary distribution $\overline{P}$. We shall use U^D to refer to this value:

$$U^D = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \{\overline{u}(\Omega_{At}, \Omega_{Bt})\} = \overline{E}\{\overline{u}(\Omega_{At}, \Omega_{Bt})\} = \\ \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} E^P\{\overline{u}(\Omega_{At}, \Omega_{Bt})\}$$

(where the second equality holds almost surely) for any rational belief, P about $\{(\Omega_{At}, \Omega_{Bt})\}_{t=1}^{\infty}$, hence expectation E^P. Here the last equality follows from stability and the three expressions have the same interpretations as government criterion functions as in Section 3.

The optimal portfolio, using $\overline{P}$, is $\overline{\pi} = \frac{1}{2}$ with resulting realized sum (across agents) of utility, $u(e + \Omega_{At} + \Omega_{Bt})$ and resulting average (across agents and time) realized utility

$$\overline{U} = \overline{E}u(e + \Omega_{At} + \Omega_{Bt}) \quad (11)$$

which, by strict concavity of u, is greater than U^D.

Note that both U^D and $\overline{U}$ are objective quantities, i.e. (i) these are the average utilities that will be realized in this economy for the two cases of no intervention and intervention respectively and (ii) all agents agree this is so. We summarize in the following proposition:

Proposition 2
With the welfare weights being equal for all agents, the policy to require all agents to invest half of their savings in each tree, $\pi_{z_A,z_B} = \frac{1}{2}$ for all (z_A, z_B) leads to an allocation which is optimal according to any individual rational belief, hence objectively optimal, and about which there is universal agreement about consequences. This allocation is Pareto dominated by the allocation resulting from the laissez faire policy ■

Remark 10 *Distribution of beliefs*
Notice that because this is a stochastic exchange economy, the policy leads to the unique expected ex-post efficient allocation (in the sense of Harris, 1978) independently of the rational belief being used.[21] There is, in other words, a strong case for arguing that, irrespectively of the distribution of beliefs, everyone of the identical (in terms of preferences) and strictly risk averse agent should consume $e + (\Omega_{At} + \Omega_{Bt})$ which is supported by the portfolio $(\frac{1}{2}, \frac{1}{2})$. This observation is akin to Proposition 3.2 in Harris(1978) ■

5 The government's Information

A criticism of Social Security is that not all agents have the same quantitative needs for savings for retirement. A 25-year old who knows that 5 years from now, he will inherit a fortune would not want to save anything from

[21] In fact for any belief that is in accordance with the fundamental facts about the stochastic process.

his current income. When the government, as seems likely, is not able to identify the "objectively" optimal savings level for each individual, there is an important trade-off between the need to attain social efficiency and to accommodate for private information about needs and preferences, i.e. a trade-off between *coherence of expectations* and *responsiveness to private information*. We sketch this problem my means of a variant of the OLG model we studied before. Assume there is only one tree, with return Ω_t. In addition, allow for agents to have utility also of consumption in the first periods of their lives, so utility is now $u(C_1, C_2)$ where u is strictly increasing, strictly concave, twice continuously differentiable on $\Re^2_{++}$, and satisfies the Inada conditions. For simplicity of the exposition of the model, assume that there are only two exogenous states, $\Omega^L < \Omega^H$. Let us finally assume that the population is split evenly up into two types, A and B where agents A and B have different endowments in the first period of their lives, $e_A = e_B + \Delta e$, $\Delta e \geq 0$ but that the government is not necessarily able to identify the two types.

In the rational expectations case, the problem of agent $c, c = A, B$ becomes (with $\overline{P}(s)$ the probability of state Ω^s):

$$\max_{I \in [0, e_c]} \sum_{s \in \{L,H\}} \overline{P}(s) u \left[e_c - I, I \frac{\pi + \Omega^s}{\pi} \right] \tag{12}$$

where π is aggregate investment in the economy. We shall assume that savings are strictly decreasing in π so that the equilibrium is unique. In this REE equilibrium, markets are effectively complete (in the sense that adding any contingent claims will not change the equilibrium consumption allocation), and the resulting investment levels (I_A^*, I_B^*) are Pareto optimal. We let $\hat{\pi}^* = \frac{1}{2}(I_A^* + I_B^*)$.

For the rational beliefs case we assume that at each date half of each type (A and B) are optimists, holding the belief $(P_1(L), P_1(H)) = (\overline{P}(L) - \delta, \overline{P}(H) + \delta)$, $\delta \geq 0$, while the other half are pessimists, holding the belief $(P_2(L), P_2(H)) = (\overline{P}(L) + \delta, \overline{P}(H) - \delta)$. Without government intervention the resulting RBE has the investment levels $(\hat{I}_{A1}, \hat{I}_{A2}, \hat{I}_{B1}, \hat{I}_{B2})$ where $\hat{I}_{cz}$ is the solution to

$$\max_{I \in [0, e_c]} \sum_{s \in \{L,H\}} P_z(s) u \left[e_c - I, I \frac{\hat{\pi} + \Omega^s}{\hat{\pi}} \right] \tag{13}$$

and $\hat{\pi} = \frac{1}{4} \sum_{c \in \{A,B\}} \sum_{z=1}^{2} \hat{I}_{cz}$. Also this equilibrium exists and is unique.

Remark 11 *Equilibrium savings*
If we assume that the utility function is homothetic $\hat{I}_{Az} = \frac{e_A}{e_B} \hat{I}_{Bz}, z = 1, 2$. If savings are also increasing in $P(H)$ (i.e. in δ) we have that $\hat{I}_{c2} < I_c^* < \hat{I}_{c1}, c = A, B$.[22] To see this, suppose $\frac{1}{2}(\hat{I}_{A1} + \hat{I}_{A2}) \leq I_A^*$. Then $\hat{\pi} \leq \hat{\pi}^*$ which implies that $\hat{I}_{A1} = I_{A1}(\hat{\pi}) \geq I_{A1}(\pi^*) > I_A(\pi^*) = I_A^*$, so that $\hat{I}_{A2} < I_A^*$. The same argument gives $\hat{I}_{B2} < I_B^* < \hat{I}_{B1}$. If, on the other hand, $\frac{1}{2}(\hat{I}_{A1} + \hat{I}_{A2}) > I_A^*$, we must have $\hat{\pi} > \pi^*$, so that $\hat{I}_{A2} < I_A^*$ and thus $\hat{I}_{A1} > I_A^*$ and the same relations hold for B ■

The government's objective function is

$$\sum_{c \in \{A,B\}} \sum_{z=1}^{2} w_{cz} \sum_{s \in \{L,H\}} \overline{P}(s) u(C_{z1}^c, C_{z2}^c(s)) \tag{14}$$

where C_{z1}^c is consumption of type (c, z) agent in period 1 , $C_{z2}^c(s))$ is consumption in period 2 of type (c, z) agent in state s, and w_{cz} is the weight attached to type (c, z). We assume that these weights are such that the REE is optimal (redistribution of wealth is not an issue), with associated welfare W^*. We consider two types of policies:

[22] An example of a utility function fulfilling all requirements: $u(C_1, C_2) = C_1^\alpha + C_2^\alpha, \alpha \in (0, 1)$.

(i) No intervention, with resulting welfare $\hat{W}$.
(ii) Everyone is required to invest at least $\overline{I}$ which is chosen to maximize the government's welfare function, leading to the welfare level $\overline{W}$. In equilibrium an agent of type c, z then solves:

$$\max_{I \in [\overline{I}, e_c]} \sum_{s \in \{L,H\}} P_z(s) u \left[e_c - I, I \frac{\hat{\pi}(\overline{I}) + \Omega(s)}{\hat{\pi}(\overline{I})} \right] \tag{15}$$

with solution $\hat{I}_{c,z}(\overline{I})$ and $\hat{\pi}(\overline{I}) = \frac{1}{4} \sum_c \sum_z \hat{I}_{c,z}(\overline{I})$ implying that welfare is

$$\overline{W}(\overline{I}) = \sum_{c \in \{A,B\}} \sum_{z=1}^{2} w_{cz} \sum_{s \in \{L,H\}} \overline{P}(s) u \left[e_c - \hat{I}_{c,z}(\overline{I}), \hat{I}_{c,z}(\overline{I}) \frac{\hat{\pi}(\overline{I}) + \Omega^s}{\hat{\pi}(\overline{I})} \right]$$

The following result illustrates the trade-off the government is facing:

Proposition 3 *(1) Given any $\Delta e > 0$, there is a $\overline{\delta} > 0$ s.t. for $0 \leq \delta \leq \overline{\delta}$, Policy (i) is optimal.*
(2) Given any $\delta > 0$, there is a $\overline{\Delta e} > 0$ s.t for $0 \leq \Delta e \leq \overline{\Delta e}$, Policy (ii) is strictly better than Policy (i).

Proof: (1) For $\delta = 0$, $\hat{W} = W^* > \overline{W}(\overline{I})$ whenever $\overline{I}$ is strictly binding for some agents. By continuity of the equilibrium and the welfare function, the last inequality continues to hold for $\delta > 0$ but small.
(2) For $e_A = e_B$, $I_A^* = I_B^*$ which is also an optimal choice of $\overline{I}$. The resulting welfare is $\overline{W} = W^* > \hat{W}$. By continuity, the last inequality continues to hold for $\Delta e > 0$, but small ■

We assume that the government knows e_B and Δe, but not the value of δ nor individual agents' types. Its beliefs about the former is a distribution over $[0, \delta^M]$ where δ^M is the maximum value that δ can take.[23] If this distribution has most of its weight close to 0, Policy (i) is better. On the other hand for a given distribution over beliefs, if the income inequality is sufficiently small (sufficiently small Δe) Policy (ii) is better. The example illustrates a trade-off between allowing agents to choose portfolios which are adjusted to their objective individual characteristics and avoiding that their overconfidence leads to a choice of suboptimal levels of savings. Such a trade-off was not present in the previous models where there was only one type of agents.

6 On Welfare Economics and Behavioral Welfare Economics

6.1 Welfare Economics?

Traditional economics is not blind to stylished behavioral facts - monotonicity (more is better at last for some commodities) and risk aversion being two prominent examples. What is then the difference between the traditional approach and behavioral economics? Certainly, behavioral economists are more willing to incorporate particular psychological traits into their models. Gul and Pesendorfer(2005) suggest a more fundamental difference: behavioral economics (the authors refer to its branch, neuroeconomics) is "therapeutic" in the sense that it does not equate what the individual wants (i.e. perceived needs and hence choices) with what it needs (i.e. true well being).[24] Part of the task of the behavioral economist is then seen as identifying (true) needs and designing economic institutions that meet those needs.

Whether or not there is a disparity between wants and needs is important if welfare economics is to be taken seriously as a discipline. However Gul and Pesendorfer(2005) seem to think that economics should be

[23] In Nielsen(2003) and Nielsen(2007a) we assume the government to have a distribution over agents' beliefs.
[24] Gul and Pesendorfer(2005) seem to think that the typical therapist would somehow have the authority to *override* the choice preferences of the patient, where more likely the role is to *modify* these preferences.

entirely a positive endeavor in which case only choices (behavior, that is) are relevant and the question of the link between these and well being becomes uninteresting. They state (p. 4) that "Standard welfare economics functions as a part of positive economics", more precisely: "Economic welfare analysis is a tool for analyzing economic institutions and models" (p. 30). To these authors Pareto suboptimal institutions are inherently unstable and we should not expect to observe them in reality.[25] This means that the apparent identification of an institution which is not Pareto optimal simply calls for a further explanation (a better model), in other words, such an apparent identification is merely a step in the positive analysis.

But if, as this view seems to suggest, the underlying, a priori assumption is that any observed institution is constrained Pareto optimal, there is not even a potential role for policy intervention. This de facto emptying of the normative content of welfare economics would have as consequence that economics becomes a purely descriptive exercise with no practical political implications.

We think (and probably many economist will agree with this) that neither the position of Behavioral Economics nor that of Gul and Pesendorfer reflect the essentials of economics. To our mind, the object of economic research is (ultimately) economic institutions and policies and its most important theoretical question is which of these lead, for given constraints (possibly political, i.e. normative), from individual rationality to collective rationality.[26] If it f.i. has been decided politically (maybe as an outcome of a public debate) that there should be restrictions on what drugs may be consumed, economists may help decide how such restrictions can be implemented in an efficient manner.

Traditional economics does not question the preferences or actions of the individual and is concerned with improving society (economic institutions), not the individual. Implicit in this position is the view that choices (preferences) to a large extend do reflect the true interests or well-being (which is not necessarily the same as "happiness") of the individual. We hold that even if this is not so, the determination of what is in the interest of the individual and more generally what is the proper form of preferences is a topic for public debate rather than something to be decided by (neuro-) economists.

6.2 Behavioral Welfare Analysis

From the point of view of economics, "rationality" means that the agent has well defined preferences and acts according to them using the available information. Note that assuming that an agent chooses a for him optimal bundle in particular implies the strong assumption that he makes no logical mistakes (it does however not preclude that he has "wrong" expectations). To rephrase what we just said, the traditional assumption of economics is that the individual is rational but society is not necessarily so, i.e. it may not be efficiently organized. We then suggest as a crucial distinction between the traditional approach and that of behavioral economics the assumption of individual rationality, accepted by the former but not by the latter. Taken literally, the assumption of individual rationality is obviously indefensible which does, however, not preclude it from being a meaningful and useful assumption in economic models.

Bernheim and Rangel(2005, p. 4) state:

[25] Even if inefficient organizations are unstable one would expect adjustments to be gradual so these organizations may survive for a long time. Economics can then be seen as attempting to speed up and improve on the process of innovation of more efficient institutions.

[26] To take some theoretical examples, the prisoner's dilemma game, an economy with private provision of public goods and *The King's Dilemma* with no intervention are "institutions" that lead to collectively irrational outcomes. On the other hand, a coordination game with no Pareto ranked Nash equilibria, a market economy with private goods or savings by agents holding rational expectations are examples where there is no identifiable collective irrationality. In contrast, from an empirical point of view, an important question is if and how actual economic institutions are inefficient.

> "Welfare analysis has two main components. First, one determines how policies affect the well-being of each individual. Second, one aggregates across individuals. As is well-known, the second step involves some thorny issues (e.g. those raised by Arrow's Impossibility Theorem). However, since these are common to both neo-classical and behavioral approaches, we will say no more about them".

In our approach it is not possible to decompose welfare analysis as suggested by the quote. It is f.i. not possible to precisely assess whether a certain policy will affect a given individual positively or negatively. We can only make such assessments about the aggregate, society.

Bernheim and Rangel(2005) continue (on p. 17) by saying that in the neoclassical world "Assuming information is not an issue, there is no role for government in the absence of pre-existing distortions" and "Reasons for government intervention involve market failures, not individual decision making failures".

We disagree. In our simple models there are no market failures, no *identifiable* individual decision making failures (i.e. agents do employ well-defined preferences and use information efficiently) and yet, there is, we argue, a role for government intervention. The misunderstanding, common to both the rational expectation and the behavioral economics approach, is that rational agents cannot have mutually inconsistent expectations.

Bernheim and Rangel(2005) and Bernheim and Rangel(2007) acknowledge that there is a "danger" associated with an approach to economic policy that allows for agents being declared irrational and as a consequence their preferences being (partially) ignored. A paternalistic rationale for government intervention (Odean, 1998 and Russo and Schoemaker, 1992) may very well be abused. The challenge for Behavioral Welfare Economics is then to define in general terms when it is legitimate to override individual preferences. However, Bernheim and Rangel offer no systematic approach (which conceivably would be less prone to manipulations) to this question. Situations where the planner is not supposed to "mimic" the agent's choices (i.e. respect the agent's choice preferences) include (Bernheim and Rangel, 2007, p. 469) the agent using the information available to him incorrectly (forecasting consequences of his choices incorrectly, and learning more slowly from past experience than the objective information would permit). This amounts to a claim that some authority (the planner) is able to make objectively better forecasts than any individual. Such kind of reasoning is however a slippery slope: when some agents are declared irrational or ignorant and hence not to be trusted with decisions about consumption would it then not be natural to hold that they should also not be trusted with voting?

Our approach is different. We do not claim that any authority knows better, only that in some situations the fact that agents' expectations are incongruent reveals an inefficiency that sometimes can be rectified at the social level.

Bernheim and Rangel(2005) consider three non-traditional models of consumer behavior, all related to commitment problems and all seeming to suggest mandatory savings programs as welfare improving: Hyperbolic discounting, hot/cold mode consumers and the temptation model of Gul and Pesendorfer(2001). Hyperbolic discounting models have a long tradition (going back at least to Feldstein(1985)) in the undersavings literature, but there still seems to be no general agreement about how they should be interpreted and about their usefulness in explaining undersavings (see Laibson et al.,1998 including the critique provided by William G. Gale in the discussion, as well as Salanie and Treich, 2006 who suggest that hyperbolic preferences, when properly understood typically lead to oversavings). In the second model an agent may either be in a "cold" or "hot" mode, the latter leading to "binge consumption". Without argument the authors only considers policies that maximize the preferences of the cold mode consumer. We maintain that, in case such two types do exist, which of them to support is not something neuroscience or economics can decide; rather it should be a topic for public debate (and morality). In the third approach, the temptation model, in any period agents suffer from temptation to consume "more" than they actually do. Thus for given consumption, typically the bigger is the present budget set, the worse off is the agent.[27] One salient feature of this model is that preferences are well defined and the agents cannot be said to be irrational.

All three models point to the need for devices for commitment, in fact they seem to suggest that social security could well thrive as a voluntary program. However in the context of savings for retirement, such devices seem rare (maybe because of legal issues, maybe because of a lack of demand). Also, for these models, the definition of the period is crucial.[28] All three models suggest that allowing agents to withdraw funds from their social security programs with one-period delay (and for the hot/cold mode model, conditionally on repeated requests) will lead to welfare improvements. All this is in contrast with our model where some undersave because they are consistently overconfident.

Bernheim and Rangel(2005) also argue that hyperbolic discounting, temptation and decision making malfunction models may shed light on addiction. We shall not argue that the typical drug addict is rational. However, before addictions sets in, decisions may very well be rational, but based on mistaken expectations. Some people seem to get away with experimenting with drugs without becoming addicted to them, some even seem to be able to have fulfilling lives while permanently using hard drugs (maybe because they are physiologically less prone to becoming addicted). People who start experimenting with drugs may confidently think they belong to this group. Seen from the point of view of the collective, it may however be deemed that the average risk of becoming addicted merits a universal ban on experimentation with (usage of) hard drugs. Such reasoning is akin to what was illustrated in *The King's Dilemma*.

Further remarks

Psychological observations and interpretations are often difficult to defend as hard facts and many types of behavior differ across cultures, ages and situations.[29] By ignoring a myriad of psychological models and descriptions, traditional economic theory may miss some particular points. On the other hand it gains in generality and escapes the danger of becoming topical, involved in the type of endless discussions that seem to haunt f.i. cultural studies and management science.

If certain institutions lead to undesirable ("irrational") outcomes in models where all agents are assumed rational, in many cases this will also be so in the context of bounded rationality. Furthermore, policy conclusions derived from models with rational agents are not based on a claim (that can always be challenged) of supposedly superior (as compared to "ordinary" people's) insights and are therefore more robust. Here, we argued that the prevalence of social security or mandatory savings for retirement may be explained by a "symmetric" model with rational agents. Such institutions may also make sense in "asymmetric" models of boundedly rational agents, however at least to this author an explanation based on rational agents finds itself on a more solid ground.

6.3 Welfare analysis and rational overconfidence

The reader may have met or heard of people who as a rationale for not quitting smoking use that grandfather (or Churchill!) who was a heavy smoker reached 90 with no effort and that he or she expects to repeat that feat. The factual statement that smoking on average decreases life expectancy by X years, is about the population

[27] We remark that there is a literature (see among others Sen, 1989, Pattanaik and Xu, 1998) that views the size of the choice set in a very different light, holding that increasing it means an increased degree of freedom and hence well-being.

[28] Laibson et al. (1998), p. 145 suggest that the effective period of the hyperbolic discounting model is one week.

[29] For an example, see Henrich et al.(2001).

and does not have as logical implication that a corresponding (conditionally probabilistic) decrease must hold for every single individual. Each an every smoker may rationally declare himself to be an exception *and* accept the population wide tendency: in that case the only thing we can conclude is that taken as a group smokers are "irrational". The same can be said about people who decide to drive without a safety belt, to experiment with addictive drugs or not to use a hard hat on a construction site (see also Remark 8).

In all these instances, it may be decided (possibly behind a "veal of ignorance") to impose restrictions that make the behavior of the collective rational. The exact circumstances under which this would (and should) happen is an obvious topic for economists as well as moral philosophers. At least three aspects of any situation must be emphasized: (i) Does the restrictions put on choices have undesired side effects? We discussed such side effects in the context of portfolio choice. Likewise, safety belts may impose additional dangers on some drivers in some situations, etc. (ii) Does the expected benefits from restricting choices justify the negative feelings associated with encroachment on individual liberties? If in *The King's Dilemma* the rules are changed such that the unsuccessful prince is simply sent back rather than having to face the dragon, the Pareto optimal decision may well be considered acceptable. (iii) Is there any consensus on the probabilistic properties related to the situation considered and is this consensus sufficient to support a collective policy decision? Here a theory of aggregation of probabilities may be needed. A natural requirement is that if everyone thinks event A is more likely than event B then the aggregated probability must exhibit this relation as well.[30] In Remark 10 we discussed how the distribution of beliefs may affect policy decisions.

When we say that decisions to reach social rationality may be made behind a veil of ignorance, we think of the possibility that a general principle of social rationality be instituted before the particular cases to which it will be applied (or individual beliefs) are known. Like in Rawls' (1999) original contribution, this "initial position" implies a greater degree of symmetry in the sense that we do not know who will be effected by possible restrictions on individual actions (for instance, who will be required to save more than they would like to).

Paternalism has traditionally been identified with the decision maker (or social welfare function) not adhering to the Pareto principle. In principle decisions regarding the collective could be made by a benevolent dictator like in *The King's Dilemma*. However, since they are not based on someone being better informed or more rational than any other individual, it is also possible to think about them as being made by universal consent.[31] In this case, the word "paternalism" is not appropriate. This view is supported by the fact that the policies we consider are objectively improving (for the given welfare weights), i.e. each agent agrees that they make society as a whole better off.

To summarize, the approach to welfare economics, we present here, is not paternalistic, since it is based on individual rationality and symmetry. Is is also non-libertarian, since it imposes restrictions on individual behavior *even* when such behavior is not harmful to others.

Finally, it should be pointed out that the principle of using consistent expectations based on objective information in theory also is applicable to models with bounded rationality. The fundamental basis for this principle is after all symmetry. Thus, we may agree that each of us is boundedly rational, but disagree about who are less or more rational, and yet we may decide on rules that treat everyone symmetrically, acknowledging that there is no unanimity about who is wrong. A policy built on such a principle would not be paternalistic

[30]Note that two different rational, beliefs, μ_1 and μ_2, of the type studied here, are always singular, i.e. there are two nonoverlapping sets, A and B, of realized infinite sequences s.t. $\mu_1(A) = 1$ and $\mu_2(B) = 1$.

[31]If the social welfare functions reflects that individual agents care about the general welfare of society this should in principle also show up in individual preferences. We follow what seems to be the general procedure, namely to only model individual preferences over private consumption. This is consistent with the way Rawls constructed his social welfare function, in that he supposed that in the initial position agents are "mutually disinterested".

and there may even be no difference in the policy implications between the case of common rationality and the case of symmetric bounded rationality

Remark 12 *Analysis of The King's Dilemma*

The unknown state of the world is the preferences of the princess (by assumption they play no role in welfare considerations). They could be either of the six constellations: $W \succ L \succ C$, $L \succ W \succ C$, $C \succ L \succ W$, $L \succ C \succ W$, $C \succ W \succ L$ or $W \succ C \succ L$ (excluding ties). Beliefs are then of the form $p = (p_1, \ldots, p_6)$. If only one prince shows up, with probability 1 he is chosen by the princess. We assume that for any prince disutility from death is high - so high that if the lottery "probability 1/2 of getting married to the princess, probability 1/2 of meeting the dragon" was offered, it would be rejected by any prince.

Consider first the case from the introduction where Wit's subjective probability is, say $(1/2, 0, 0, 0, 0, 1/2)$. Suppose that Charm has been allowed to go. If also Wit is allowed to go, he is definitely better off (since he is sure he will be chosen by the princess). Charm is not negatively affected by this, since he is sure to be selected, no matter who else shows up. Thus sending only one prince is Pareto dominated by sending two princes and the same kind of reasoning leads to the conclusion that this decision is in turn Pareto dominated by sending three princes, which is then the unique Pareto optimal decision.

Consider next the case where everyone holds the same belief about the preferences of the princes $(p_1, p_2, \ldots, p_6)$. In that case, sending at least two princes to try their luck is Pareto dominated: For at least one prince the probability of being chosen is less than or equal to $1/2$, i.e. he would prefer not to go. In conclusion, the only Pareto optimal decision is to allow only one prince to go. This is then also the only expected ex-post efficient solution (the two solution concepts coincide, when all agents hold the same beliefs) and likewise the only ex-post welfare optimal solution.

If disutility from death is sufficiently low, there are other expected ex-post efficient decisions. However, for any level of disutility from death the only universally ex-post efficient decision is to send only one prince. Thus the two concepts differ, even when they are both applicable, contrary to what seems to be the thinking of Harris(1978).

To elucidate this difference between universal and expected ex-post efficiency points, consider the situation where the lottery "probability 1/3 of getting married, probability 2/3 of meeting the dragon" is strictly preferred by all princes over being bachelor with probability 1. Suppose that the common belief about the preferences of the princess is uniform $(1/6, 1/6, \ldots, 1/6)$. In this case sending all princes is Pareto optimal as well as expected ex-post efficient (as is sending two princes or only one). Even with subjective beliefs slightly different from $(1/6, 1/6, \ldots, 1/6)$ this is so. However it seems objectively "wasteful" to send all princes to the contest in this situation and certainly, such a decision is not universally ex-post efficient.

One solution that might be available is to design a lottery where only the winner is allowed to go and with the same probabilities as the commonly agreed probabilities of being the most preferred by the princess (i.e. the probability of Wit winning would be $P_W = p_1 + p_6$ and so on). Every prince should prefer such a lottery over the decision to let all of them go. Another potential way to "convexify" the discrete choice set (and hence avoid the dilemma) might be available: introducing private property rights over permissions to go. This would only work though, if the princes have sufficient wealth to trade. None of these remedies would solve the original dilemma where beliefs are diverse ■

7 Conclusion

One of our conclusions may be paraphrased as follows: Any individual agent (or investment fund!) is unlikely to experience a higher return than the historical average. Thus, mandating that all agents hold the market portfolio and save up what is, on average, required to have a desirable distribution of consumption over time, ensures that most agents are better off. If the government does not intervene, we may see low savings rates by overconfident agents and placement of savings in too risky (subjectively superior) portfolios. Most agents will not benefit from this freedom (but some may).

In our model everyone agrees on the average performance of the economy. This is not so in reality, however one would expect that, based on the historical experience, there is some consensus about how the economy is likely to perform in the future. For instance, historical evidence is often the reference point when economists state that stocks are "overvalued" or "undervalued".[32] In this note we have argued that such a consensus should be the basis for a decision about how much individuals are supposed to save up for retirement and how these savings should be invested.

Mandating individuals to save up a certain amount for retirement means not only overruling their subjective beliefs but also ignoring their private information about preferences, wealth and future income. An inventor who would like to place all his income in developing a new product with a high return in the future may be harmed if he is forced to place some of his income in lower earning financial instruments (and if he his unable or unwilling to obtain seed money from other sources). There is then a trade-off between ensuring that society as a whole acts in a consistent way and using privately available information in an efficient way. If one accepts the premise of our study it becomes an important question how to deal with this trade-off. We hope to address this question in more detail in future research.

Our formulation of this trade-off may also shed some new light on the classical debate about the merits of central planning versus decentralized markets. We agree with Hayek(1945) that each individual knows something of relevance for the optimal planning problem and that this information may (for many different reasons) not be (easily) communicated to a hypothetical central planning entity. However, the individual decisions of agents may involve incoherent models because of, as Hayek(1945) states (on p. 519), "all the separate individuals frequently contradictory knowledge" (a problem he ignores in his further arguments). In our model this "contradictory knowledge" is in the form of mutually contradictory statistical models by individual agents. Each of these models contains (is) knowledge since it is consistent with, i.e. embodies, past statistical observations, but none the less departs from any other model in its interpretation of the available information.

We have mostly considered the case where the government is fully informed about agents' beliefs, an assumption that was quite innocuous in our set-up. In less simple models than the ones studied here, the distribution of (rational) beliefs may become important for determining the effects of different economic institutions. In Nielsen(2003) and Nielsen(2007) we have shown how one may model the government's uncertainty about the distribution of rational beliefs in the economy and in particular how to use a notion of "genericity" in the context of such beliefs.

Strategic behavior played no role in our model, however, in a model where agents have more choice variables, it could easily become an issue. From the point of view of public economics it would then become a problem how to design, in an ex-post sense, optimal pension plans when taking into consideration that agents' private

[32] Some commentators and investors, at least until recently, argued that the economy had entered a "new phase" where the historical evidence was no longer a good guide. In effect this means that they held stock returns to be non-stationary, just like agents in our model do.

savings and investment behavior may partially offset the effects of such plans.

Thus our study leaves many open questions to be addressed. Hopefully, some readers have been convinced that the concept of Pareto optimality is problematic not only in the context of stochastic exchange economies (which are after all theoretical constructs) but also for economies with production, where uncertainty becomes of key importance. There are many aspects of economic life where individual choice is restricted by laws and regulations. We have studied one such, namely savings for retirements, but the approach to economic policy, we proposed and used here, may be fruitful in other areas as well, to mention a few: health care (health insurance), traffic laws, and regulation of financial markets.

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