Non-constant Price Reset Hazards and Inflation Dynamics

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Abstract

This paper demonstrates that tractability gained from the Calvo pricing assumption is costly in terms of aggregate dynamics. I derive a generalized New Keynesian Phillips curve featuring a generalized hazard function and real rigidity. Analytically, I find that important dynamics in the NKPC are canceled out due to the restrictive Calvo assumption. The richer dynamic structure resulted from the non-constant hazards is quantitatively important for inflation dynamics and monetary policy. With plausible parameter values, the increasing hazard model generates hump-shaped impulse responses of inflation to the monetary shock, and the real effects of monetary shocks are 2-3 times higher than those in the Calvo model.

JEL classification: E12; E31

Key words: Hazard function, Nominal rigidity, Real rigidity, New Keynesian Phillips curve

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1 Introduction

The Calvo pricing assumption (Calvo, 1983) has become predominant in the world of applied monetary analysis under nominal rigidity. The main argument for using this approach, however, is solely based on its tractability. In recent years, detailed micro-level data sets have become available for researchers. Empirical work using these data sets[1] generally reach the consensus that, instead of having economy-wide uniform price stickiness, the frequency of price adjustments differs substantially within the economy. In addition, the Calvo assumption also implies a constant hazard function of price setting, meaning that the probability of adjusting prices is independent of the length of the time since last adjustment. Unfortunately, constant hazard functions are also largely rejected by empirical evidence from the micro level data. Cecchetti (1986) used newsstand prices of magazines in the U.S. and Goette et al. (2005) apply Swiss restaurant prices. Both studies find strong support for increasing hazard functions. By contrast, recent studies using more comprehensive micro data find that hazard functions are first downward sloping and then mostly flat, interrupted periodically by spikes (See, e.g.: Campbell and Eden, 2005, Alvarez, 2007 and Nakamura and Steinsson, 2008).

Given this conflict between theory and empirical evidence, it is important to understand the consequences of a non-constant price reset hazard function for inflation dynamics and implications of monetary policy.

To tackle these questions, I construct a generalized time-dependent pricing model and derive the New Keynesian Phillips curve (NKPC) featuring a general hazard function and real rigidity. The resulting NKPC incorporates components, such as lagged inflation, future and lagged expectations of inflation and real marginal costs. This version of the Phillips curve nests the Calvo case in the sense that, under a constant hazard function, effects of lagged inflation exactly cancel those of lagged expectations, so that, as in the Calvo NKPC, only current real marginal cost and expected future inflation remain in the expression. In the general case, however, expectations of future variables, lagged expectations and lagged inflation all should be presented in the dynamic structure of the Phillips curve.

The economic reason why those lagged dynamic components should appear in the GNKPC but miss in the Calvo model is because: first, due to nominal rigidity, some fraction of past reset prices continue to affect the current aggregate price $\hat{p}_t$. Lagged expectational terms reflect influences of past optimal prices on the current aggregate price and hence on current inflation. The higher past optimal price is, the higher the current aggregate price and hence current inflation. Second, the inclusion of past inflations reflects the influence of past reset prices on the lagged aggregate price $\hat{p}_{t-1}$. The higher the past inflations prevail, higher the lagged aggregate price would be, and thereby it deters current inflation to be high. Putting them together, these two opposing effects change current inflation through $\hat{p}_t$ and $\hat{p}_{t-1}$ respectively, and the magnitudes of those two effects depend on the shape of the price reset hazard function. In the general case, the extent of past optimal prices affecting current aggregate price $\hat{p}_t$ should be different to those affecting lagged aggregate price $\hat{p}_{t-1}$. As a result, lagged expectations and lagged inflations should appear in the generalized NKPC. Conversely, in the Calvo case, the constant hazard function leads old reset prices to exert the exactly same amount of impact on both current

[1] See: e.g. Bils and Klenow (2004), Alvarez et al. (2006), Midrigan (2007), Nakamura and Steinsson (2008) among others.

aggregate price and lagged aggregate price, and thereby it causes them to be cancelled out. Results presented above have important implications for monetary policy. It reveals that the dependence of inflation on its own lags should be influenced by the monetary policy through expectations, and therefore models[2] that treat this dependence as a fixed primitive coefficient should be subject to the Lucas critique, and thereby can not be used in the monetary policy analysis.

In the numerical experiments, I simulate the general equilibrium model, combining the generalized NKPC with a standard IS curve and the nominal money growth rule. The simulation results show that, even without real rigidity, the increasing hazard function helps to increase both persistence of inflation and output gap. When introducing some degree of real rigidity, the generalized NKPC gives rise to substantially different inflation dynamics, namely, the impulse response of inflation to a nominal money growth shock becomes hump-shaped. The economic intuition behind these results is that, on the one hand, increasing hazard function postpones the timing of the price adjustment. On the other hand, strategic complementary makes earlier adjusting firms choose a small size for the adjustment, while the later adjusting firms make a larger price adjustment. In another words, the increasing-hazard pricing together with some degree of real rigidity not only affect the timing of the price adjustment, but also the average magnitude of firms' adjustments, leading to a hump-shaped response. Last but not least, when the real effects of monetary policy shocks are measured by the accumulative impulse responses of the real output gap, models with an increasing hazard function generate real effects of monetary policy which are 2-3 times larger than those in the corresponding Calvo model.

In the literature, the general-hazard-pricing model has been studied in different contexts. Wolman (1999) raised the issue that inflation dynamics should be sensitive to the hazard function underlying different pricing rules and thereby implications of the constant-hazard Calvo model is not robust to the shape of the hazard function. He showed this result in a partial equilibrium analysis. Kiley (2002) compared the Calvo and Taylor staggered-pricing models and showed the dynamics of output following monetary shocks are both quantitatively and qualitatively quite different across the two pricing specifications unless one assumes a substantial level of real rigidity in the economy. Mash (2003) constructed a general pricing model that nests both the Calvo and Taylor cases. He found that implications for optimal monetary policy based on those limiting cases are not robust to the change in the hazard function. Sheedy (2007) focused on the relationship between the shape of hazard functions and inflation persistence. He parameterized the hazard function in such a way that the resulting NKPC has a positive coefficient on lagged inflation given that the hazard function is upward sloping. This result, however, is only valid under his hazard function specification. The most closely related work is Whelan (2007), who derived the generalized NKPC under a general hazard function, but rejected this model based on the observation from the reduced-form Phillips curve regression that inflation is positively dependent on its lags. However, this argument is vulnerable in light of the evidence presented by Dotsey (2002), who shows that the positive reduced-form coefficients themselves could be spurious due to omitted variables in a misspecified regression model. This argument is supported by Cogley and Sbordone (2006), who find that when correctly accounting for the time-varing trend of inflation, the purely forward-looking model explains the persistence

[2]See: e.g. Gali and Gertler (1999) and Christiano et al. (2005).

of the inflation deviation from its trend quite well.

The remainder of the paper is organized as follows: in section 2, I present the model with the generalized time-dependent pricing and derive the New Keynesian Phillips curve; section 3 shows analytical results regarding new insights gained from relaxing the constant hazard function underlying the Calvo assumption; in section 4, I simulate the complete DSGE model with some commonly used parameter values in the literature and then present the simulation results; section 5 contains some concluding remarks.

2 The Model

In this section, I present a DSGE model of sticky prices based on both nominal and real rigidities. The scheme of nominal rigidity in the model allows for a general shape of the hazard function. A hazard function of price setting is defined as the probabilities of price adjustment conditional on the spell of the time elapsed since the price was last set. Real rigidity is introduced similarly as in Sbordone (2002), who incorporates upward-sloping marginal cost as a source of strategic complementarity.

2.1 Representative Household

A representative, infinitely-lived household derives utility from the composite consumption good C_t, its labor supply and the real money holding M_t^d/P_t, and it maximizes a discounted sum of utilities of the form:

$$\max_{\{C_t, M_t^d, L_t, B_{t+1}\}} \quad E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \chi_H \frac{L_t^{1+\phi}}{1+\phi} + \chi_M \log\left(\frac{M_t^d}{P_t}\right) \right) \right]$$

Here C_t denotes an index of the household's consumption of each of the individual goods $C_t(i)$ following a constant-elastisity-of-substitution aggregator (Dixit and Stiglitz, 1977).

$$C_t \equiv \left[\int_0^1 C_t(i)^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}, \tag{1}$$

where $\eta > 1$, and it follows that the corresponding cost-minimizing demand for $C_t(i)$ and the welfare based price index P_t are given by

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\eta} C_t \tag{2}$$

$$P_t = \left[\int_0^1 P_t(i)^{1-\eta} di \right]^{\frac{1}{1-\eta}} \tag{3}$$

For simplicity, I assume that household supplies homogeneous labor units (L_t) in an enocomy-wide competitive labor market.

The flow budget constraint of the household at the beginning of period t is

$$P_t C_t + M_t^d + \frac{B_t}{R_t} \leq M_{t-1}^d + W_t L_t + B_{t-1} + \int_0^1 \pi_t(i) di. \tag{4}$$

Where B_t is a one-period nominal bond and R_t denotes the gross nominal return on the bond. $\pi_t(i)$ represents the nominal profits of a firm that sells good i. I assume that each household owns an equal share of all firms. Finally this sequence of period budget constraints is supplemented with a transversality condition of the form $\lim_{T\to\infty} E_t\left[\frac{B_T}{\Pi_{s=1}^T R_s}\right] \geqslant 0$.

The solution to the household's optimization problem can be expressed in three first order necessary conditions. First, optimal labor supply is related to the real wage:

$$\chi_H L_t^{\phi} C_t^{\sigma} = \frac{W_t}{P_t}, \tag{5}$$

Second, the Euler equation gives the relationship between the optimal consumption path and asset prices:

$$1 = \beta E_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma} \frac{R_t P_t}{P_{t+1}}\right], \tag{6}$$

Finally, the demand of real money balance is determined by weighting between the benefits and costs of holding money.

$$\chi_M \frac{M_t}{P_t} = \frac{C_t^{\sigma}}{1 - R_t^{-1}}, \tag{7}$$

2.2 Firms in the Economy

In the economy, there is a continuum of monopolistic competitive firms, who use labor as the single input to produce good i.

$$Y_t(i) = Z_t L_t(i)^{1-a} \tag{8}$$

where Z_t denotes an aggregate productivity shock. Log deviations of the shock $\hat{z}_t$ follow an exogenous AR(1) process $\hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon_{z,t}$, where $\varepsilon_{z,t}$ is white noises and $\rho_z \in [0, 1)$. $L_t(i)$ is the demand of labor by firm i. Following equation (2), demand for intermediate goods is given by:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\eta} Y_t \tag{9}$$

2.2.1 Pricing Decisions under Real Rigidity

In Appendix (A), I derive the economy-wide optimal relative price, which is the ratio between the average optimal price chosen by the adjusting firms and aggregate price index. Note that even through the individual optimal prices are not the same due to the fact that marginal costs generally depend on the amount produced, we can still derive the aggregate optimal relative-price ratio at period t from the average marginal cost in the economy.

$$\frac{P_t^*}{P_t} = \left(\frac{\eta}{\eta - 1}\frac{1}{1-a}\right)^{\frac{1-a}{1-a+\eta a}} Y_t^{\frac{\phi+\sigma(1-a)+a}{1-a+\eta a}} Z_t^{-\frac{1+\phi}{1-a+\eta a}} \tag{10}$$

To show how real rigidity affects price setting in this model, I log-linearize the relative price equation (10). Define $\hat{x}_t = logX_t - log\bar{X}$ as the log deviation from the steady state, up to a log

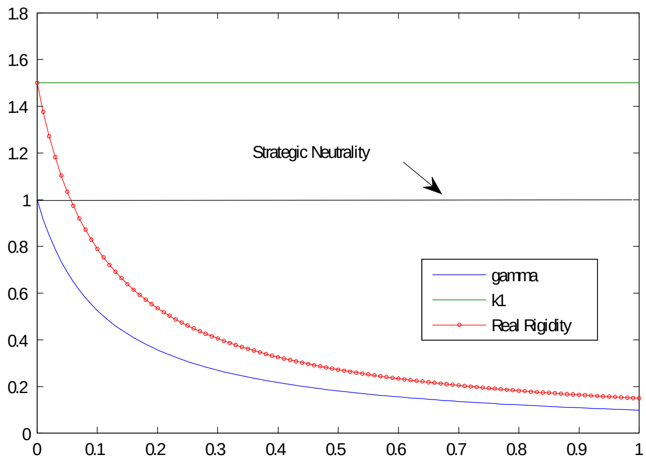


Figure 1: Real Rigidity, when $\sigma = 1, \phi = 0.5$ and $\eta = 10$

linearization approximation, one can show that the log deviation of the relative price is equal to the log deviation of the economy-wide marginal cost, which in turn is a linear function of log deviations of output gap and the technology shock.

$$
\begin{aligned}
\widehat{rp}_t &= \widehat{mc}_t = \gamma\left(\kappa_1 \hat{y}_t - \kappa_2 \hat{z}_t\right) \\
where &: \\
\gamma &= \frac{1}{1-a+a\eta} \\
\kappa_1 &= a+\phi+\sigma(1-a) \\
\kappa_2 &= 1+\phi
\end{aligned}
$$

Parameters γ and κ_1 have the economic interpretation as the measure of real rigidity. γ is the elasticity of relative prices to the change in real marginal cost, while κ_1 measures the sensitivity of real marginal cost to the change in the output gap. Following Woodford (2003), price-setting decisions are called strategic complementarity when $\gamma\kappa_1 < 1$. When we assume that the monetary authority controls the growth rate of the nominal aggregate demand $\hat{d}_t$, then at equilibrium we have $\hat{y}_t = \hat{d}_t - \hat{p}_t$. In this case, price adjustments are "sticky" even under a flexible price setting, because relative price reacts less than one-to-one to a monetary shock. On the other hand, price setting decisions can be dubbed strategic substitutes when $\gamma\kappa_1 > 1$, so that relative price reacts strongly to monetary policy shocks.

Now we can discuss how changes in the labor share a affect the magnitude of real rigidity of price setting in the model. When setting a equal to zero, creating a linear production technology, then $\gamma = 1$ and $\kappa_1 = \delta + \phi$. Under the standard calibration values in the RBC literature ($\delta = 1$ and $\phi = 0.5$), the real rigidity parameter $\gamma\kappa_1$ is equal to 1.5 and price decisions are strategic substitutes. When the value of a rises, the real rigidity parameter becomes smaller, and price decisions turn into strategic complementarity.

In Figure (1), I plot values of γ and κ_1 against values of a, while setting $\sigma = 1, \phi = 0.5$ and $\eta = 10$. In this special case, the sensitivity of real marginal cost to the change in the output gap κ_1 is not affected by the labor share, while γ decreases fairly quickly as a becomes larger. This means that, given the parameter values, real rigidity is mainly driven by the sensitivity of the relative price to changes in real marginal cost, and the degree of real rigidity is decreasing in a. Only with a modest value of the labor share (around 0.1), real rigidity drops below the strategic neutrality threshold.

2.2.2 Pricing Decisions under Nominal Rigidity

In this section, I introduce a general form of nominal rigidity, which is characterized by an arbitrary hazard function. Many well known price setting models in the literature can be shown to have the incorporation of a hazard function of one form or another. The hazard function in this price setting is defined as the probability of price adjustment conditional on the spell of time elapsed since the price was last set. I assume that monopolistic competitive firms cannot adjust their price whenever they want. Instead, opportunities for re-optimizing prices depend on the hazard function h_j, where j denotes the time-since-last-adjustment and $j \in \{0, J\}$. J is the maximum number of periods in which a firm's price can be fixed. To keep the model general, I do not parameterize the hazard function, so that the relative magnitudes of hazard rates are totally free. As a result, this model is able to nest a wide range of staggered pricing New Keynesian models.

Dynamics of the vintage distribution In the economy, firms' prices are heterogeneous with respect to the time since their last price adjustment. I call them price vintages, while the vintage label j indicates the age of each price group. Table (1) summarizes key notations concerning the dynamics of vintages.

Vintage	Hazard Rate	Non-adj. Rate	Survival Rate	Distribution
j	h_j	α_j	S_j	$\theta(j)$
0	0	1	1	$\theta(0)$
1	h_1	$\alpha_1 = 1 - h_1$	$S_1 = \alpha_1$	$\theta(1)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
j	h_j	$\alpha_j = 1 - h_j$	$S_j = \prod_{i=0}^{j} \alpha_i$	$\theta(j)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
J	$h_J = 1$	$\alpha_J = 0$	$S_J = 0$	$\theta(J)$

Table 1: Notations of the dynamics of price-vintage-distribution.

Using the notation defined in Table (1), and also denoting the distribution of price durations at the beginning of each period by $\Theta_t = \{\theta_t(0), \theta_t(1) \cdots \theta_t(J)\}$, we can derive the ex post distribution of firms after price adjustments ($\tilde{\Theta}_t$)

$$\tilde{\theta}_t(j) = \begin{cases} \sum_{i=1}^{J} h_j \theta_t(i) \text{ , when } j = 0 \\ \alpha_j \theta_t(j) \text{ , when } j = 1 \cdots J \end{cases} \tag{11}$$

Intuitively, those firms that reoptimize their prices in period t are labeled as 'vintage 0', and the proportion of those firms is given by hazard rates from all vintages multiplied by their corresponding densities. The firm left in each vintage are the firms that do not adjust their prices. When period t is over, this ex post distribution $\tilde{\Theta}_t$ becomes the ex ante distribution for the new period Θ_{t+1}. All price vintages move to the next one, because all prices age by one period.

As long as the hazard rates lie between zero and one, dynamics of the price-duration distribution can be viewed as a Markov process with an invariant distribution, Θ, and is obtained by solving $\theta_t(j) = \theta_{t+1}(j)$. It yields the stationary price-duration distribution $\theta(j)$ as follows:

$$\theta(j) = \frac{S_j}{\sum_{j=0}^{J-1} S_j} \text{ , for } j = 0, 1 \cdots J-1. \tag{12}$$

Here, I give a simple example. When $J = 3$, then the transition matrix of the price-duration-group Markov chain is illustrated as follows:

j	0	1	2	3
0	0	1	0	0
1	h_1	0	α_1	0
2	h_2	0	0	α_2
3	1	0	0	0

According to equation (12), this Markov chain eventually converges to the stationary price-duration distribution $\Theta = \left\{ \frac{1}{1+\alpha_1+\alpha_1\alpha_2}, \frac{\alpha_1}{1+\alpha_1+\alpha_1\alpha_2}, \frac{\alpha_1\alpha_2}{1+\alpha_1+\alpha_1\alpha_2} \right\}$.

Let's assume the economy converges to this invariant distribution quickly, so that regardless of the initial price-duration distribution, I only consider the economy with the invariant distribution of price durations.

The Optimal Pricing under Nominal Rigidity In a given period when a firm is allowed to reoptimize its price, the optimal price chosen should reflect the possibility that it will not be re-adjusted in the near future. Consequently, adjusting firms choose optimal prices that maximize the discounted sum of real profits over the time horizon during which the new price is expected to be fixed. The probability that the new price is fixed is given by the survival function, S_j, defined in Table (1).

Here I setup the maximization problem of an average adjustor as follows:

$$\max_{P(i)_t^*} E_t \sum_{j=0}^{J-1} S_j Q_{t,t+j} \left[Y_{t+j|t}^d \frac{P(i)_t^*}{P_{t+j}} - \frac{TC(i)_{t+j}}{P_{t+j}} \right]$$

Where E_t denotes the conditional expectation based on the information set in period t, and $Q_{t,t+j}$ is the stochastic discount factor appropriate for discounting real profits from t to $t+j$. Note that here $P(i)_t^*$ is defined as the average optimal price chosen by the average adjusting firm.

Therefore $TC(i)_t$ denotes the average total costs of producing output $Y(i)^d_t$. The representative adjusting Firm maximizes profits subject to demand for intermediate goods in period $t+j$ given that the firm resets the price in period t, $(Y(i)^d_{t+j|t})$.

$$Y(i)^d_{t+j|t} = \left(\frac{P(i)^*_t}{P_{t+j}}\right)^{-\eta} Y_{t+j},$$

It yields the following first order necessary condition for the optimal price:

$$P(i)^*_t = \frac{\eta}{\eta-1} \frac{\sum\limits_{j=0}^{J-1} S_j E_t[Q_{t,t+j} Y_{t+j} P^{\eta-1}_{t+j} MC(i)_{t+j}]}{\sum\limits_{j=0}^{J-1} S_j E_t[Q_{t,t+j} Y_{t+j} P^{\eta-1}_{t+j}]} \tag{13}$$

MC_t denotes the average nominal marginal costs of adjusting firms. The optimal price is equal to the markup multiplied by a weighted sum of future marginal costs, where weights depend on the survival rates. In the Calvo case, where $S_j = \alpha^j$, this equation reduces to the Calvo optimal pricing condition.

Finally, given the stationary distribution $\theta(j)$, aggregate price can be written as a distributed sum of all vintage prices. I define the aggregate optimal price which was set j periods ago as P^*_{t-j}. Following the aggregate price index equation (3), the aggregate price is then obtained by:

$$P_t = \left(\sum_{j=0}^{J-1} \theta(j) P^{*1-\eta}_{t-j}\right)^{\frac{1}{1-\eta}} \tag{14}$$

2.3 New Keynesian Phillips Curve

In this section, I derive the New Keynesian Phillips curve for this generalized model. To do that, I first log-linearize equation (13) around the constant price steady state. The log-linearized optimal price equations are obtained by

$$\begin{aligned} \hat{p}^*_t &= E_t\left[\sum_{j=0}^{J-1} \frac{\beta^j S(j)}{\Omega} \left(\widehat{mc}_{t+j} + \hat{p}_{t+j}\right)\right], \qquad (15) \\ where &: \\ \Omega &= \sum_{j=0}^{J-1} \beta^j S(j) \text{ and } \widehat{mc}_t = \frac{a+\phi+\sigma(1-a)}{1-a+a\eta}\hat{y}_t - \frac{1+\phi}{1-a+a\eta}\hat{z}_t. \end{aligned}$$

In a similar fashion, I derive the log deviation of the aggregate price by log linearizing equation (14).

$$\hat{p}_t = \sum_{k=0}^{J-1} \theta(k)\ \hat{p}^*_{t-k}. \tag{16}$$

After some algebraic manipulations on equations (15) and (**??**), I obtain the New Keynesian Phillips curve as follows[3]

[3] The detailed derivation of the NKPC can be found in the technical Appendix (B).

$$
\begin{aligned}
\hat{\pi}_t \quad = \quad & \sum_{k=0}^{J-1} \frac{\theta(k)}{1-\theta(0)} E_{t-k} \left(\sum_{j=0}^{J-1} \frac{\beta^j S(j)}{\Psi} \widehat{mc}_{t+j-k} + \sum_{i=1}^{J-1} \sum_{j=i}^{J-1} \frac{\beta^j S(j)}{\Psi} \hat{\pi}_{t+i-k} \right) \\
& - \sum_{k=2}^{J-1} \Phi(k) \hat{\pi}_{t-k+1}, \quad \text{where } \Phi(k) = \frac{\sum_{j=k}^{J-1} S(j)}{\sum_{j=1}^{J-1} S(j)}, \quad \Psi = \sum_{k=0}^{J-1} \beta^j S(j).
\end{aligned} \tag{17}
$$

At the first glance, this Phillips curve is quite different from the one in the Calvo model. It involves not only lagged inflation but also lagged expectations that were built into pricing decisions in the past. All coefficients in the NKPC are derived from structural parameters which are either the hazard function parameters or the preference parameters. When $J = 3$, for example, then the NKPC is of the following form

$$
\begin{aligned}
\hat{\pi}_t \quad = \quad & \frac{1}{(\alpha_1 + \alpha_1\alpha_2)\Psi} \widehat{mc}_t + \frac{\alpha_1}{(\alpha_1 + \alpha_1\alpha_2)\Psi} \widehat{mc}_{t-1} + \frac{\alpha_1\alpha_2}{(\alpha_1 + \alpha_1\alpha_2)\Psi} \widehat{mc}_{t-2} \\
& + \frac{1}{\alpha_1 + \alpha_1\alpha_2} E_t \left(\frac{\beta\alpha_1}{\Psi} \widehat{mc}_{t+1} + \frac{\beta^2\alpha_1\alpha_2}{\Psi} \widehat{mc}_{t+2} + \frac{\beta\alpha_1 + \beta^2\alpha_1\alpha_2}{\Psi} \hat{\pi}_{t+1} + \frac{\beta^2\alpha_1\alpha_2}{\Psi} \hat{\pi}_{t+2} \right) \\
& + \frac{\alpha_1}{\alpha_1 + \alpha_1\alpha_2} E_{t-1} \left(\frac{\beta\alpha_1}{\Psi} \widehat{mc}_{t} + \frac{\beta^2\alpha_1\alpha_2}{\Psi} \widehat{mc}_{t+1} + \frac{\beta\alpha_1 + \beta^2\alpha_1\alpha_2}{\Psi} \hat{\pi}_{t} + \frac{\beta^2\alpha_1\alpha_2}{\Psi} \hat{\pi}_{t+1} \right) \\
& + \frac{\alpha_1\alpha_2}{\alpha_1 + \alpha_1\alpha_2} E_{t-2} \left(\frac{\beta\alpha_1}{\Psi} \widehat{mc}_{t-1} + \frac{\beta^2\alpha_1\alpha_2}{\Psi} \widehat{mc}_{t} + \frac{\beta\alpha_1 + \beta^2\alpha_1\alpha_2}{\Psi} \hat{\pi}_{t-1} + \frac{\beta^2\alpha_1\alpha_2}{\Psi} \hat{\pi}_{t} \right) \\
& - \frac{\alpha_1\alpha_2}{\alpha_1 + \alpha_1\alpha_2} \hat{\pi}_{t-1},
\end{aligned} \tag{18}
$$

$$
\textit{where} \quad : \quad \Psi = 1 + \beta\alpha_1 + \beta^2\alpha_1\alpha_2.
$$

In this example, we see more clearly how current inflation depends on marginal costs, lagged inflation and a complex weighted sum of lagged expectations. All coefficients are expressed in terms of hazard rates ($\alpha_j = 1 - h_j$) and a preference parameter β.

3 Analytical Result

In this section, I explore the dynamic structure of the generalized NKPC (17) to show which new insights we can learn from relaxing the constant hazard function underlying the Calvo assumption.

3.1 Economic Intuition behind the Generalized NKPC

Proposition 1 : *When assuming the hazard function is constant over the infinite horizon, the generalized NKPC (17) reduces to the standard Calvo NKPC:*

$$
\hat{\pi}_t = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} mc_t + \beta E_t \hat{\pi}_{t+1} \tag{19}
$$

Proof : *see Appendix (C).*

By iterating equation (19) backwards, the following equations hold

$$\begin{aligned}
\hat{\pi}_{t-1} &= E_{t-1}\left((1-\alpha)(1-\alpha\beta)\sum_{i=0}^{\infty}\alpha^i\beta^i mc_{t+i-1} + (1-\alpha)\sum_{i=0}^{\infty}\alpha^i\beta^i\hat{\pi}_{t+i-1}\right)\\
\hat{\pi}_{t-2} &= E_{t-2}\left((1-\alpha)(1-\alpha\beta)\sum_{i=0}^{\infty}\alpha^i\beta^i mc_{t+i-1} + (1-\alpha)\sum_{i=0}^{\infty}\alpha^i\beta^i\hat{\pi}_{t+i-1}\right)\\
&\vdots
\end{aligned}$$

In light of these analytical results, we learn that the generalized NKPC nests the Calvo Phillips curve in the sense that, given the constant hazard function, the effects of lagged inflation terms exactly equal the effects of lagged expectations. Moreover, lagged inflation and lagged expectations are not extrinsic to the time-dependent nominal rigidity model. They are missing in the Calvo setup only because the constant hazard assumption causes them to be canceled out.

To understand the economic intuition of the generalized NKPC (GNKPC here after), we need to decompose inflation into its parts and study the influence of each dynamic component of the GNKPC on the parts of inflation. By definition, inflation is equal to the log difference between two consecutive aggregate prices and the aggregate price in the period t can be further written as the distributed sum of current and past optimal prices.

$$\begin{aligned}
\hat{\pi}_t &= \quad \hat{p}_t \quad - \quad \hat{p}_{t-1}\\
& \qquad \Downarrow\\
&= \underline{\left[\theta(0)\hat{p}^*_t + \theta(1)\hat{p}^*_{t-1} + \cdots + \theta(J)\hat{p}^*_{t-J}\right]} - \hat{p}_{t-1}
\end{aligned}$$

Next, I summarize the GNKPC into the following three components: 1) all forward-looking and current terms (first two rows of the example NKPC 18), 2) Lagged expectations (third and fourth rows) and 3) lagged inflations (last row). In the following expression, I represent these three components of the GNKPC with short-hand notations $E_t(.)$, $E_{t-j}(.)$ and $\hat{\pi}_{t-k}$ respectively and $W_x(h_j)$.represents coefficients attached to those terms, which depend on the hazard function h_j.

$$\begin{array}{rcccccc}
\hat{\pi}_t &=& W_1(h_j)E_t(.)\,+ & & W_2(h_j)E_{t-j}(.)\,- & & W_3(h_j)\hat{\pi}_{t-k}\\
& & \Downarrow & & \Downarrow & & \Downarrow\\
\hat{\pi}_t &=& \underline{\theta(0)\hat{p}^*_t} & + & \underline{\left[\theta(1)\hat{p}^*_{t-1} + \cdots + \theta(J)\hat{p}^*_{t-J}\right]} & - & \underline{\hat{p}_{t-1}}
\end{array}$$

The economic reason why those three components should appear in the GNKPC is: first, because the optimal price decision in this model is based on the sum of all current and future real marginal costs over the spell of time during which the reset price is fixed, the current and forward-looking terms reflect the influence of the current reset price on the current aggregate price. This is the channel highlighted in the Calvo model. Second, due to nominal rigidity, some fraction of past reset prices continue to affect the current aggregate price. Lagged expectational terms represent those influences of past reset prices on the current aggregate price and hence on current inflation. The higher past reset price is, the higher the current aggregate price is and hence the current inflation. Last, past inflations reflect the influence of past price decisions on the lagged aggregate price $\hat{p}_{t-1}$. The higher the past inflations prevail, higher the lagged aggregate price would be, and thereby it deters current inflation to be high. Put them together, the last two counteracting channels change current inflation through $\hat{p}_t$ and $\hat{p}_{t-1}$ respectively. The magnitudes of these effects depend on the price reset hazard function. In the general case, the effect of past optimal prices on current aggregate price $\hat{p}_t$ should be different to those affecting lagged aggregate price $\hat{p}_{t-1}$. As a result, lagged expectations and lagged inflations should appear in the generalized NKPC. Conversely, in the Calvo case, the constant hazard function leads relevant reset prices to exert the exactly same amount of impact on both $\hat{p}_t$ and $\hat{p}_{t-1}$, and thereby it causes them to be cancelled out. This insight can be also seen in the derivation of the Calvo NKPC,

$$\begin{aligned}
\hat{p}_t &= (1-\alpha)\sum_{j=0}^{\infty}\alpha^j \hat{p}^*_{t-j} \\
&= (1-\alpha)\left[\hat{p}^*_t + \alpha\hat{p}^*_{t-1} + \alpha^2\hat{p}^*_{t-2} + \cdots\right] \\
&= (1-\alpha)\hat{p}^*_t + \underbrace{(1-\alpha)\left[\alpha\hat{p}^*_{t-1} + \alpha^2\hat{p}^*_{t-2} + \cdots\right]}_{=\alpha\hat{p}_{t-1}} \\
\hat{p}_t &= (1-\alpha)\hat{p}^*_t + \alpha\hat{p}_{t-1} \\
&\vdots \\
\hat{\pi}_t &= \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\widehat{mc}_t + \beta E_t(\hat{\pi}_{t+1}).
\end{aligned}$$

The crucial substitution from line (3) to line (4) is only possible, when the distribution of price durations takes the power function form under the Calvo assumption. Therefore the dynamic relationship between inflation and real marginal cost is over-simplified by the constant hazard function assumed in the Calvo sticky price model.

4 Numerical Results

4.1 The General Equilibrium Model

In the numerical experiment, I study the behavior of inflation dynamics in a general equilibrium setup. For this purpose, I close the model by adding a nominal money stock growth rule. The log-linearized equilibrium equations are summarized here:

$$
\begin{aligned}
\hat{\pi}_t &= \sum_{k=0}^{J-1} W_1(k) E_{t-k} \left(\sum_{j=0}^{J-1} W_2(j) \widehat{mc}_{t+j-k} + \sum_{i=1}^{J-1} W_3(i) \hat{\pi}_{t+i-k} \right) - \sum_{k=2}^{J-1} W_4(k) \hat{\pi}_{t-k+1} \\
\widehat{mc}_t &= \frac{\phi+\sigma+a}{1+\eta\phi+\eta a} \hat{y}_t - \frac{1+\phi}{1+\eta\phi+\eta a} \hat{z}_t \\
\sigma E_t \left[\hat{y}_{t+1} \right] &= \sigma \hat{y}_t + \left(\hat{\imath}_t - E_t \left[\hat{\pi}_{t+1} \right] \right) \\
\hat{m}_t &= \sigma \hat{y}_t - \frac{\beta}{1-\beta} \hat{\imath}_t \\
\hat{m}_t &= \hat{m}_{t-1} - \hat{\pi}_t + g_t \quad where \;\; g_t \backsim N(0, 0.0025^2) \\
\hat{z}_t &= \rho_z * \hat{z}_{t-1} + \epsilon_t \quad where \;\; \epsilon_t \backsim N(0, 0.007^2)
\end{aligned}
$$

Where all variable are expressed in terms of log deviations from the non-stochastic steady state. The weights (W_1, W_2, W_3, W_4) in the NKPC are defined in the equation (17). $\hat{m}_t$ is the real money balance, and g_t denotes the growth rate of the nominal money stock, which consists of a constant g and a white-noise shock u_t, representing the regular and irregular parts of the standing monetary policy.

4.2 Calibration

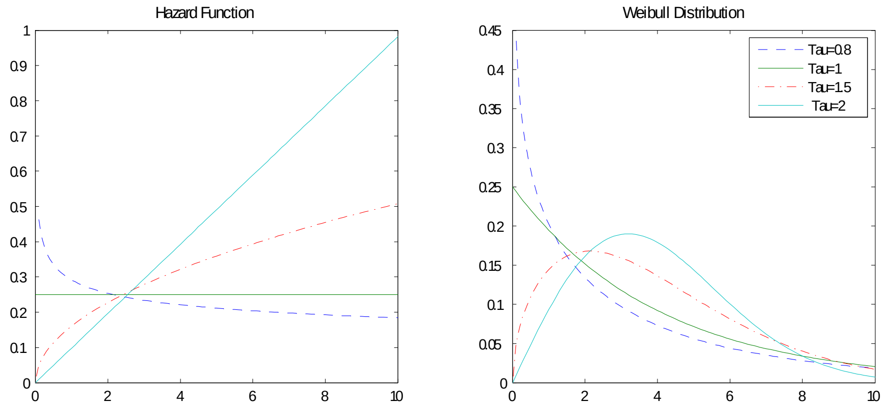
In the calibration, instead of referring to any microeconometric evidence on the hazard function, I parameterize the hazard function a parsimonious way. The reason is that, until now, there is not yet consensus on the shape of hazard functions in the empirical literature. As discussed in the introduction, it is evident that the shape of hazard functions is changing over time with the underlying economic conditions. Since the main purpose of the paper is to demonstrate the impact of varying hazard rates on the inflation dynamics, I choose to calibrate it based on the statistical theory of duration analysis. In particular, the functional form I apply is the hazard function of the Weibull distribution, which has two parameters:

$$
h(j) = \frac{\tau}{\lambda} \left(\frac{j}{\lambda} \right)^{\tau-1} \tag{20}
$$

λ is the scale parameter, which controls the average duration of the price adjustment, while τ is the shape parameter to determine the monotonic property of the hazard function. It enables the incorporation of a wide range of hazard functions by using various values for the shape parameter. In fact, any value of the shape parameter that is greater than one corresponds to an increasing hazard function, while values ranging between zero and one lead to a decreasing hazard function. By setting the shape parameter to one, we can retrieve the Poisson process from the Weibull distribution.

In this numerical experiment, I choose λ, such that it implies an average price duration of 3 quarters, which is largely consistent with the median price durations of 7 - 9 months documented by Nakamura and Steinsson (2008). The shape parameter is set in the interval between one and three, which covers a wide range of shapes of the hazard function[4]. As for the rest of the

[4] This range only covers increasing hazard functions because it makes the maximum number of price duration J well defined.



structural parameters, I use some common values in the literature to facilitate comparison the results. In the calibration of the preference parameters, I assume $\beta = 0.9902$, which implies a steady state real return on financial assets of about four percent per annum. I also assume the intertemporal elasticity of substitution $\sigma = 1$, implying log utility of consumption. I choose the Frisch elasticity of the labor supply to equal 0.5, a value that is motivated by using balanced-growth-path considerations in the macro literature. As for the technology parameters, I set labor's share $(1 - a)$ to be either 1 or 0.64 to show the effect of real rigidity. The elasticity of substitution between intermediate goods $\eta = 10$, which implies the desired markup over marginal cost should be about 11%. Finally, I set the standard deviation of the innovation to the nominal money growth rate to be 25 basic points per quarter. For the aggregate technology shock, I choose $\rho_z = 0.95$ and the standard deviation of 0.007, in line with commonly used values in the RBC literature, for example King and Rebelo (2000).

4.3 Simulation Results

To evaluate the quantitative implication for the aggregate dynamics, I apply the standard algorithm to solve for the log-linearized rational expectation model.

4.3.1 Effects of Increasing Hazard Functions

In the first experiment, I study the effects of varying the shape parameter on the equilibrium dynamics without any real rigidity and the trend inflation. In Table (2), I report second moments generated by the theoretical models, which are different with respect to the shape of the hazard function. Because I use the Weibull hazard function to calibrate the model, I can change the shape of the hazard function by varying the value of the shape parameter τ. In this experiment, I focus on the comparison between the baseline Calvo case, with a corresponding shape parameter of $\tau = 1$, and the increasing hazard models, where τ falls in the range between 1.6 and 3. In all cases, the moments are for a Hodrick-Prescott filtered time series. For each of these hazard functions, two sets of statistics are reported: first, the first-order autocorrelation coefficient of deviations on inflation, real marginal cost and output; and second, contemporaneous correlation coefficients between inflation and real marginal cost. In all models, I use a persistent technology shock and a transitory monetary shock, whose stochastic properties are specified above.

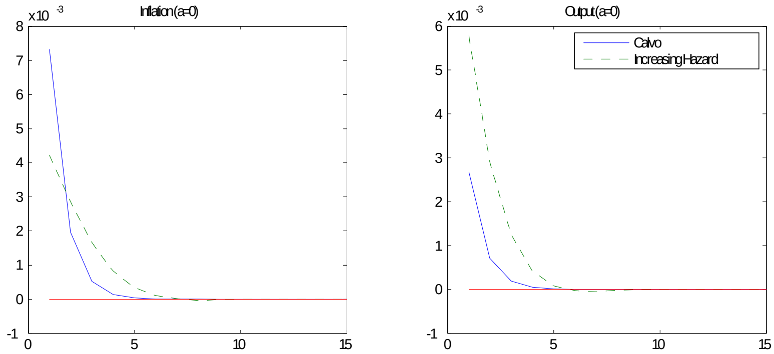


Figure 2: Comparing impulse responses functions

	Calvo Model	Increasing Hazard Models				
τ	1	1.6	1.8	2	2.5	3
$AR(1)\ \hat{\pi}$	0.166	0.524	0.537	0.549	0.567	0.576
$AR(1)\ \hat{y}$	0.811	0.876	0.874	0.873	0.870	0.868
$AR(1)\ \widehat{mc}$	0.169	0.362	0.338	0.318	0.280	0.264
$Corr(\hat{\pi}, \widehat{mc})$	0.998	0.977	0.965	0.950	0.915	0.891

Table 2: Second moments of the simulated data (HP filtered, lambda=1600)

The first noteworthy result from the table is that models with increasing hazard rates generate much higher persistence in inflation than in the Calvo model, *ceteris paribus*. Secondly, increases in the shape parameter reduces the persistence of real marginal cost and output. In the Calvo case, because inflation persistence is solely determined by the dynamics of real marginal cost, inflation persistence cannot exceed persistence of real marginal cost. In the increasing hazard model, however, the autoregressive terms of real marginal cost are brought into the Phillips curve through lagged expectations, and thus, in comparison to the Calvo model, this new transmission mechanism propagates more inflation persistence. Fuhrer (2006) presented empirical evidence showing that it is difficult to have a sizable coefficient on the driving process in the Calvo NKPC and that a reduced form shock in the NKPC explains a significant portion of the inflation persistence. We can understand this evidence through the lens of the generalized NKPC. The problem of the conventional NKPC is essentially caused by ignoring terms like lagged inflations and lagged expectations. As I show in the analytical result, this is not the case in the more general time-dependent pricing model. The misspecified Phillips curve fails to explain inflation persistence with its limited structure. Consequently, we either need to introduce the ad hoc backward-looking behavior or a persistent reduced-form shock to achieve a good fit to the data. Last but not least, as shown in the final row of the table, the increasing-hazard pricing model also helps to reduce the correlation between inflation and current real marginal cost, a rather robust feature of the data (See: e.g. Hornstein, 2007).

Figure 2 shows the impulse responses of the Calvo model compared to the increasing-hazard model with the shape parameter of 2. The left panel depicts the impulse responses of inflation

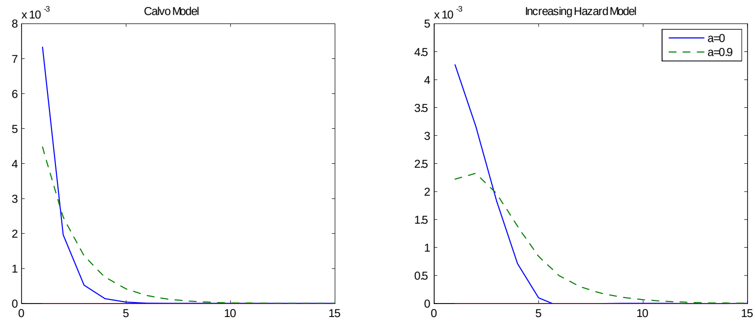


Figure 3: Impulse responses of inflation with real rigidity

while the right panel shows those of the output gap to a 1% increase in the annual nominal money growth rate. Without real rigidity and trend inflation, we observe that, even though the impulse response function of the increasing-hazard model is somewhat more persistent, the general pattern of the impulse responses are the same in both cases, namely, they drop monotonically back to the steady state.

4.3.2 Effects of Real Rigidity

As influentially argued in Woodford (2003), real price rigidity plays an important role in inflation dynamics in addition to nominal rigidity. In this model I introduce real rigidity in a parsimonious way, following Sbordone (2002). I now set the labor share parameter $(1-a)$ equal to 0.64. Combining this with other parameter values in the model, it implies that the real rigidity parameter $(\gamma\kappa_1 = \frac{a+\phi+\sigma(1-a)}{1-a+a\eta})$ equals 0.35, representing a modest level of strategic complementarity.

In Figure (3), I compare the impulse responses of inflation to a transitory money growth shock with and without real rigidity. The left panel shows the comparison in the Calvo model. Incorporation of real rigidity makes the impulse responses more long-lasting, but still monotonic. By contrast, in the right panel, impulse responses of inflation in the increasing hazard model change substantially with real rigidity. One can see that not only the persistence of the impulse response function gets improved, but, more importantly, the shape of it as well. In this case, the IRF becomes hump-shaped with a peak at around the second quarter.

The economic intuition behind this result is that, on the one hand, increasing hazard function postpones the timing of the price adjustment, i.e. only a few firms adjust their prices immediately after a shock, and more and more adjust later on. On the other hand, real rigidity helps to amplify this postponing effect even further. Because price decisions are strategic complementary, when fewer firms adjust their prices at the beginning phase of the IRF, even the adjusting firms choose a small size of the adjustment. Afterwards, however, when more firms reset their prices, the size of the price adjustment becomes also larger. In another words, the increasing-hazard pricing together with some degree of real rigidity not only affect the timing of the price adjustment, but also the average magnitude of firms' adjustments, leading to a hump-shaped response.

4.3.3 Real Effects of the Monetary Shock

In the previous sections, I have informally shown that the real effects of the monetary shock is larger in the increasing hazard model than in the Calvo case. Here I introduce a quantitative measure of the real effects of money. In Table (3), I report the accumulative IRF of the real output gap to a transitory 1% increase in the annual nominal money growth rate. The accumulative IRF is the area below the impulse response function over the whole horizon, and it is in the unit of percentage of the steady state level of real output.

Real Effects	Calvo Model		Increasing Hazard Model ($\tau = 2$)		
	a=0	a=0.36	a=0, g=1	a=0.36, g=1	a=0.36, g=1.02
$Acc.IRF$ (%)	0.09	0.26	0.22	0.48	0.56

Table 3: Real Effects of A Transitory Monetary Shock) with varying trend inflation

In the Calvo model without any real rigidity, the real effect of money is only about 0.09% of real output in the steady state, while this figure rises by a factor of 3 when a modest level of real rigidity is present. On the other hand, the increasing hazard model can generate this level of real effects of the monetary shock even without any helping features. When adding real rigidity into the increasing hazard model, however, real effects rise to 0.48% of steady state real output, and presenting trend inflation reinforces real effects even further. All in all, the increasing hazard model implies 2-3 times more real effects of the monetary shock than the constant-hazard Calvo model.

5 Conclusion

The central theme of this paper is to study consequences of a non-constant price reset hazard function for inflation dynamics and implications of monetary policy. I derive a general New Keynesian Phillips curve under a general hazard function and real rigidity. My main analytical results show that the dynamic relationship between inflation and real marginal cost is over-simplified by the constant hazard function assumed in the Calvo sticky price model. Results presented above have important implications for monetary policy. It reveals that the dependence of inflation on its own lags should be influenced by the monetary policy through expectations. The expectation channel plays a central role in the propagation mechanism of the monetary shock. Even though the Calvo sticky price model also delivers this key message through the forward-looking expectations, here I show that this expectation channel has a long-lasting effect on inflation due to the presence of lagged expectations in the generalized NKPC. Furthermore models that treat the dependence between current inflation and lagged inflation as a fixed primitive coefficient should be subject to the Lucas critique, and thereby can not be used in the monetary policy analysis.

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A Deviation of the Marginal Cost

I assume that there is an economy-wide competitive labor market, and hence intermediate firms are price takers in this market. In each period, firms choose optimal demands for labor inputs to maximize their real profits given wage and the production technology (8).

$$\max_{L_t(i)} \Pi_t(i) = \frac{P_t(i)}{P_t} Y_t(i) - \frac{W_t}{P_t} L_t(i) \tag{21}$$

Real marginal cost can be derived from this maximization problem in the form:

$$mc_t(i) = \frac{W_t/P_t}{(1-a)\, Z_t L_t(i)^{-a}}$$

Using the production function (8), output demand equation (9), the labor supply condition (5) and the fact that at the equilibrium $C_t = Y_t$, we obtain the real marginal cost as follows:

$$mc_t(i) = \frac{1}{1-a} Y_t^{\frac{\phi+\sigma(1-a)+a}{1-a}} Z_t^{-\frac{1+\phi}{1-a}} \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\eta a}{1-a}} \tag{22}$$

Because marginal costs depend on the demand of the individual good, the price set by the firm also affects the marginal costs of the firm. Next, firms determine their optimal prices given marginal costs and the market demand for their goods (9)

$$\max_{P_t(i)} \Pi_t(i) = Y_t(i) \left(\frac{P_t(i)}{P_t} - mc_t(i)\right)$$

The first order condition for $P_t(i)$ yields:

$$\frac{P_t^*(i)}{P_t} = \frac{\eta}{\eta-1} mc_t(i)$$

The optimal relative price is equal to the markup multiplied by real marginal cost. By substituting the real marginal cost with equation (22), we get the economy-wide average relative price in the form:

$$\frac{P_t^*}{P_t} = \left(\frac{\eta}{\eta-1}\frac{1}{1-a}\right)^{\frac{1-a}{1-a+\eta a}} Y_t^{\frac{\phi+\sigma(1-a)+a}{1-a+\eta a}} Z_t^{-\frac{1+\phi}{1-a+\eta a}} \tag{23}$$

B Deviation of the New Keynesian Phillips Curve

Here I derive the NKPC for $g = 1$, Starting from 15

$$\begin{aligned}
\hat{p}_t^* &= E_t\left[\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\left(\widehat{mc}_{t+j}+\hat{p}_{t+j}\right)\right] && (24)\\
&= E_t\left[\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\widehat{mc}_{t+j}\right]+E_t\left[\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{p}_{t+j}\right] && (25)
\end{aligned}$$

The last term can be further expressed in terms of future rates of inflation

$$\begin{aligned}
\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{p}_{t+j} &= \frac{1}{\Psi}\hat{p}_t+\frac{\beta S_1}{\Psi}\hat{p}_{t+1}+\cdots+\frac{\beta^{J-1}S_{J-1}}{\Psi}\hat{p}_{t+J-1}\\
&= \frac{1}{\Psi}\hat{p}_t+\frac{\beta S_1}{\Psi}\hat{p}_t+\frac{\beta S_1}{\Psi}\left(\hat{p}_{t+1}-\hat{p}_t\right)+\cdots+\frac{\beta^{J-1}S_{J-1}}{\Psi}\hat{p}_{t+J-1}\\
&= \left(\frac{1}{\Psi}+\frac{\beta S_1}{\Psi}\right)\hat{p}_t+\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+j}+\frac{\beta^2 S_2}{\Psi}\hat{p}_{t+1}+\cdots+\frac{\beta^{J-1}S_{J-1}}{\Psi}\hat{p}_{t+J-2}\\
&= \left(\frac{1}{\Psi}+\frac{\beta S_1}{\Psi}+\frac{\beta^2 S_2}{\Psi}\right)\hat{p}_t+\sum_{j=1}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+j}+\sum_{j=2}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+j-1}\\
&\quad +\frac{\beta^3 S_3}{\Psi}\hat{p}_{t+1}+\cdots+\frac{\beta^{J-1}S_{J-1}}{\Psi}\hat{p}_{t+J-2}\\
&\quad \vdots\\
&= \left(\frac{1}{\Psi}+\frac{\beta S_1}{\Psi}+\cdots+\frac{\beta^{J-1}S_{J-1}}{\Psi}\right)\hat{p}_t+\sum_{j=1}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+j}\\
&\quad +\sum_{j=2}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+j-1}+\ldots+\frac{\beta^{J-1}S_{J-1}}{\Psi}\hat{\pi}_{t+1}\\
&= \hat{p}_t+\sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+i}
\end{aligned}$$

The optimal price can be expressed in terms of inflation rates, real marginal cost and aggregate prices.

$$\hat{p}_t^* = \hat{p}_t+E_t\left[\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\widehat{mc}_{t+j}\right]+E_t\left[\sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+i}\right] \quad (26)$$

Next, I derive the aggregate price equation as the sum of past optimal prices. I lag equation

26 and substitute it for each $\hat{p}^*_{t-j}$ into equation 16

$$
\begin{aligned}
\hat{p}_t &= \theta(0)\ \hat{p}^*_t + \theta(1)\ \hat{p}^*_{t-1}\ + \cdots + \theta(J-1)\hat{p}^*_{t-J+1} \\
&= \theta(0)\left[\hat{p}_t + E_t\left(\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\widehat{mc}_{t+j}\right) + E_t\left(\sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+i}\right)\right] \\
&+ \theta(1)\left[\hat{p}_{t-1} + E_{t-1}\left(\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\widehat{mc}_{t+j-1}\right) + E_{t-1}\left(\sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+i-1}\right)\right] \\
&\vdots \\
&+ \theta(J-1)\left[\hat{p}_{t-J+1} + E_{t-J+1}\left(\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\widehat{mc}_{t+j-J+1}\right) + E_{t-J+1}\left(\sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+i-J+1}\right)\right] \\
\hat{p}_t &= \sum_{k=0}^{J-1}\theta(k)\left[\hat{p}_{t-k} + \underbrace{E_{t-k}\left(\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\widehat{mc}_{t+j-k} + \sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+i-k}\right)}_{F_{t-k}}\right] \qquad (27)
\end{aligned}
$$

Where F_t summarizes all current and lagged expectations formed at period t.
Finally, we derive the New Keynesian Phillips curve from equation 27.

$$
\begin{aligned}
\hat{p}_t &= \sum_{k=0}^{J-1}\theta(k)\ \hat{p}_{t-k} + \underbrace{\sum_{k=0}^{J-1}\theta(k)F_{t-k}}_{Q_t} \\
\hat{\pi}_t &= \sum_{k=0}^{J-1}\theta(k)\ \hat{p}_{t-k} - \hat{p}_{t-1} + Q_t \\
&= \theta(0)\ (\hat{p}_t - \hat{p}_{t-1}) + \theta(0)\hat{p}_{t-1} + \theta(1)\hat{p}_{t-1} + \cdots + \theta(J-1)\hat{p}_{t-J+1} - \hat{p}_{t-1} + Q_t \\
&= \theta(0)\ (\hat{p}_t - \hat{p}_{t-1}) + (\theta(0) + \theta(1))\,\hat{p}_{t-1} + \theta(2)\hat{p}_{t-2} + \cdots + \theta(J-1)\hat{p}_{t-J+1} - \hat{p}_{t-1} + Q_t \\
&= \underbrace{\theta(0)}_{W(0)}\hat{\pi}_t + \underbrace{(\theta(0)+\theta(1))}_{W(1)}\hat{\pi}_{t-1} + (\theta(0)+\theta(1)+\theta(2))\,\hat{p}_{t-2}\cdots + \theta(J-1)\hat{p}_{t-J+1} - \hat{p}_{t-1} + Q_t \\
&\vdots \\
&= W(0)\ \hat{\pi}_t + W(1)\hat{\pi}_{t-1} + \cdots + W(J-2)\hat{\pi}_{t-J+2} + \underbrace{W(J-1)}_{=1}\hat{p}_{t-J+1} - \hat{p}_{t-1} + Q_t \\
&= W(0)\ \hat{\pi}_t + \cdots + W(J-2)\hat{\pi}_{t-J+2} + \underbrace{\hat{p}_{t-J+1} - \hat{p}_{t-J+2}}_{-\hat{\pi}_{t-J+2}} + \hat{p}_{t-J+2} - \cdots + \underbrace{\hat{p}_{t-2} - \hat{p}_{t-1}}_{-\hat{\pi}_{t-1}} + Q_t \\
(1 - W(0))\hat{\pi}_t &= -(1 - W(2))\hat{\pi}_{t-1} - \cdots - (1 - W(J-1))\hat{\pi}_{t-J+2} + Q_t \\
\hat{\pi}_t &= -\sum_{k=2}^{J-1}\frac{1 - W(k)}{1 - \theta(0)}\hat{\pi}_{t-k+1} + \sum_{k=0}^{J-1}\frac{\theta(k)}{1 - \theta(0)}F_{t-k}
\end{aligned}
$$

The generalized New Keynesian Phillips curve is:

$$
\begin{aligned}
\hat{\pi}_t &= \sum_{k=0}^{J-1} \frac{\theta(k)}{1-\theta(0)} E_{t-k} \left(\sum_{j=0}^{J-1} \frac{\beta^j S_j}{\Psi} \widehat{mc}_{t+j-k} + \sum_{i=1}^{J-1} \sum_{j=i}^{J-1} \frac{\beta^j S_j}{\Psi} \hat{\pi}_{t+i-k} \right) \\
&\quad - \sum_{k=2}^{J-1} \Phi(k) \hat{\pi}_{t-k+1}, \qquad where\ \Phi(k) = \frac{\sum\limits_{j=k}^{J-1} S_j}{\sum\limits_{j=1}^{J-1} S_j}, \quad \Psi = \sum_{j=0}^{J-1} \beta^j S_j
\end{aligned} \tag{28}
$$

C Proof for Proposition 1

In the Calvo pricing case, all hazards are equal to a constant between zero and one. Denote the constant hazard as $h = 1 - \alpha$, and substitute it into the NKPC (17):

$$
\begin{aligned}
\hat{\pi}_t + \sum_{k=1}^{\infty} \alpha^k \hat{\pi}_{t-k} &= (1-\alpha) \sum_{k=0}^{\infty} \alpha^k E_{t-k} \left((1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-k} + \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-k} \right) \\
\hat{\pi}_t + \alpha \hat{\pi}_{t-1} + \alpha^2 \hat{\pi}_{t-2} + \cdots &= E_t \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right) \\
&+ \alpha E_{t-1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-1} \right) \\
&+ \alpha^2 E_{t-2} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-2} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-2} \right) \\
&\vdots
\end{aligned} \tag{29}
$$

Iterate this equation one period forward, I obtain

$$
\begin{aligned}
\hat{\pi}_{t+1} + \alpha \hat{\pi}_t + \alpha^2 \hat{\pi}_{t-1} + \alpha^3 \hat{\pi}_{t-2} \cdots &= E_{t+1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i+1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i+1} \right) \\
&+ \alpha E_t \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right) \\
&+ \alpha^2 E_{t-1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-1} \right) \\
&\vdots
\end{aligned}
$$

Use equation (29) to substitute terms in the left hand side of the equation ($\hat{\pi}_t, \hat{\pi}_{t-1}, \hat{\pi}_{t-2} \cdots$), I get

$$
\begin{aligned}
&\hat{\pi}_{t+1} + \alpha E_t \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right) \\
&+\alpha^2 E_{t-1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-1} \right) \\
&+\alpha^3 E_{t-2} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-2} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-2} \right) \\
&\vdots \\
&= E_{t+1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i+1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i+1} \right) \\
&+\alpha E_t \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right) \\
&+\alpha^2 E_{t-1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-1} \right). \\
&\vdots
\end{aligned}
$$

After canceling out equal terms from both sides of the equation, It yields the following equation:

$$
\hat{\pi}_{t+1} = E_{t+1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i+1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i+1} \right)
$$

Lag this equation and rearrange it, I obtain the NKPC of the Calvo model.

$$
\begin{aligned}
\hat{\pi}_t &= E_t \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right) \qquad (30) \\
\hat{\pi}_t &= (1-\alpha)(1-\alpha\beta) mc_t + (1-\alpha)\hat{\pi}_t + \alpha\beta E_t \left(\hat{\pi}_{t+1} \right) \\
\hat{\pi}_t &= \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} mc_t + \beta E_t \left(\hat{\pi}_{t+1} \right)
\end{aligned}
$$

Proof done