IMPATIENCE AND COORDINATION

First Draft

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This paper introduces impatient players in a population of agents playing a global game. Global games are coordination games with incomplete information and they are used to study a variety of interesting contexts: currency crises, bank runs, debt crises and revolutions against an autocratic regime. In this model the *patient* agents have the option to postpone their decision with the advantage of receiving additional information; the *impatient* agents must make a choice at time zero. The assumption that some agents are impatient seems a plausible one since in situations of distress individuals are forced to make an immediate decision. The model shows that defaults or regime changes are more likely if there are impatient agents in the economy. This paper is composed by three parts. The first part proposes a stylized example on *Revolution Against An Autocratic Regime*, the generalization of this example, a specification of the background and the main contribution of the paper. The second part describes and analyzes the model. The third part concludes.

PART I

EXAMPLE (Revolution Against An Autocratic Regime). Island I has an autocratic regime, its citizens are unhappy and they are considering to attack the regime. To this purpose, a boat is recruiting dissenters in city A. The boat leaves city A in $t = 0$ and the goal is to attack city B in $t = 1$. Dissenters in city A have the option to jump on the boat and participate to the revolution. In city A there are two types of dissenters: *no-swimmers* and *swimmers*. The no-swimmers are a fraction f of the dissenters and the swimmers are a fraction $1 - f$. When the boat leaves city A the no-swimmers must decide if they want to jump on the boat or not. Jumping on the boat is equivalent to attack the regime. The swimmers have the option to postpone their decision. In fact either they attack in $t = 0$ either they swim to city B, at a cost ε, and once they reach the boat in city B they decide if they want to attack or not attack the regime. In city B, the swimmers can observe (with a small noise) how many dissenters are on the boat before making their choice. Hence, the swimmers have the option to postpone their decision and to observe additional information at a cost ε while the no-swimmers don't. The variable θ parameterizes the strength of the regime, which is never common knowledge and dissenters instead have private noisy information about θ.

GENERALIZATION. This example captures the role of coordination in a variety of interesting contexts: models of currency crises, bank runs, debt crises and revolution against an autocratic regime. In all these environments the dichotomy *swimmers/no-swimmers* is substituted by the dichotomy *patient agents/impatient agents*. The patient agents have the option to postpone their decision with the advantage of observing additional information; the impatient agents must make a choice in $t = 0$ and they do not have the option to postpone their choice. The assumption that some agents are impatient seems a plausible one since in situations of distress individuals are forced to make an immediate decision.

BACKGROUND. This example has already been studied under the assumption that $f = 1$: all agents move simultaneously in $t = 0$ because everybody is impatient (or simply because agents have an agnostic view about who moves when). Morris and Shin, in fact, demonstrate the existence of a unique equilibrium when θ is never common knowledge and individuals receive private noisy information about the fundamentals. Denote the model with $f = 1$ as **M&S benchmark**. This model has also been studied under the assumption that $f = 0$: all agents can postpone their choice to $t = 1$ because everybody is patient. Agents that postpone their choice are able to observe a statistic based on the proportion of agents who chose to attack at $t = 0$ (this signal is observed with some idiosyncratic noise). Dasgupta analyzed this situation therefore denote the model with $f = 0$ as **Dasgupta benchmark**.

PAPER CONTRIBUTION. The contribution of this paper is to analyze the example under the assumption that $0 < f < 1$: the economy is aware that some agents are unable to postpone their decision and some agents can.

This analysis will deliver insights that were unexplored by the previous two studies highlighting how the strategic interaction is affected when some of the participants to a coordination game with incomplete information are impatient.

⧫ The impatient agents are forced to reveal, through their aggregate action in $t = 0$, the information that they received. Knowing that the patient agents will observe their choice in $t = 1$ and can potentially coordinate with them, how aggressively will the impatient agents act?

⧫ Will the patient agents be willing to postpone their decision in order to observe the choice of the impatient agents? If so, in which circumstances they will? Does their decision to postpone depend on the cost ε?

⧫ Does the choice or the aggressiveness of the impatient agents in $t = 0$ depend on the cost ε?

⧫ Is this regime more fragile compared with M&S benchmark? Is this regime more fragile compared with Dasgupta benchmark?

⧫ If this model admits a unique equilibrium, is it possible to suggest novel comparative static exercises to understand how to reduce/increase the probability that the regime falls in equilibrium?

PART II

THE MODEL. The situation described in the example can be generalized as follows. There are two possible regimes, the status quo and an alternative. There is a continuum of agents, indexed by $i \in [0, 1]$. Each agent can choose between an action that is favorable to the alternative regime and an action that is favourable to the status quo. Denote these actions, respectively, *attack* and *not attack*. Agents are heterogeneous, a fraction f must act in $t = 0$ and a fraction $(1 - f)$ can either attack in $t = 0$ either postpone the decision to $t = 1$ at a cost ε. If they do so, they are able to observe a statistic based on the proportion of agents who chose to attack at $t = 0$ (this signal is observed with some idiosyncratic noise). The fraction of impatient agents, f, is common knowledge. Denote the regime outcome with $D \in \{0, 1\}$, where $D = 0$ represents survival of the status quo and $D = 1$ represents collapse. The following payoffs are realized at the end of the game:

IMPATIENT	$D = 1$	$D = 0$
Attack	b	$-c$
Not Attack	0	0

PATIENT	$D = 1$	$D = 0$
Attack at $t = 0$	b	$-c$
Attack at $t = 1 \mid$ *Delay at* $t = 0$	$b - \varepsilon$	$-c - \varepsilon$
Not Attack at $t = 1 \mid$ *Delay at* $t = 0$	$-\varepsilon$	$-\varepsilon$

Denote the action of an impatient agent with $a^i_{IMP} \in A^i_{IMP} \equiv \{0, 1\}$, where $a^i_{IMP} = 0$ represents *not attack* and $a^i_{IMP} = 1$ represents *attack*; the action of a patient agent in $t = 0$ with $a^i_{0,P} \in A^i_{0,P} \equiv \{0, 1\}$, where $a^i_{0,P} = 0$ represents *delay* and $a^i_{0,P} = 1$ represents *attack*; the action of a patient agent in $t = 1$ with $a^i_{1,P} \in A^i_{1,P} \equiv \{0, 1\}$, where $a^i_{1,P} = 0$ represents *not attack* and $a^i_{1,P} = 1$

represents *attack*. Hence the impatient types in $t = 0$ maximize the following expected utility:

$$\begin{aligned} t &= 0: \quad \max_{a_i^{IMP}} u_i^{IMP} \\ u_i^{IMP} &= a_i^{IMP}(D_0^{IMP} - r) \end{aligned} \tag{1}$$

where $r = \frac{c}{b+c} \in (0,1)$. Note that D_0^{IMP} represents the probability that the regime falls in $t = 1$ and this probability will be computed by the impatient agents conditional on their information in $t = 0$. The patient agents in $t = 0$ maximize the following expected utility:

$$\begin{aligned} t &= 0: \quad \max u_i^P \\ u_i^P &= \sup \left\{ \max_{a_{0,P}^i} \left[a_{0,P}^i (D_0^P - r) \right]; u^* \right\} \end{aligned} \tag{2}$$

where $u^* = \max_{a_{1,P}^i} \left[a_{1,P}^i (D_1^P - r) - \varepsilon \right]$ is the equilibrium continuation payoff of delaying the decision to $t = 1$. Note that D_0^P represents the probability that the regime falls in $t = 1$ and this probability will be computed by the patient agents conditional on their information in $t = 0$ and conditional on the fact that they act in $t = 0$. D_1^P represents the probability that the regime falls in $t = 1$ and this probability will be computed by the patient agents conditional on their information in $t = 0$ and conditional on the fact that they act optimally in $t = 1$. Note that there is no reason to think that different agents belonging to the same group will have different response function, so the focus is on symmetric equilibria. Finally, the status quo is abandoned ($D = 1$) if and only if $A \geq \theta$ where $A \equiv \int a_i di \in [0,1]$ denotes the mass of agents attacking and $\theta \in$

$\mathbb{R}$ parameterizes the exogenous strength of the status quo (or the economic fundamentals). The aggregate attack $A = A^0 + A^1$ is the sum of the mass of agents attacking in $t = 0$ and the mass attacking in $t = 1$. Note that $A^0 = A^0_{IMP} + A^0_P$, i.e. the size of the attack in $t = 0$, is the sum of the attack by the impatient agents and the attack by the patient agents that do not delay; while $A^1 = A^1_P$, i.e. the size of the attack in $t = 1$, is equal to the attack by the patient agents that delay their decision. The actions of the agents are strategic complements, since it pays for an individual to attack if and only if the status quo collapses and, in turn, the status quo collapses if and only if a sufficiently large fraction of the agents attacks. This coordination problem is the heart of the model. The fundamentals θ are never common knowledge and individuals instead have private noisy information about θ. Private information serves as an anchor for individual's actions that may avoid the indeterminacy of expectations about others actions. Initially agents have a common prior about θ; for simplicity let this prior be (degenerate) uniform over the entire real line. Agent i then observes his private signal $x_i = \theta + \sigma\varepsilon_i$; where the idiosyncratic noise ε_i is $N(0,1)$ and is independent of θ. The signal x_i is thus a sufficient statistic for the private information of an agent. Note that because there is a continuum of agents the information contained by the entire economy, $\{x_i\}_{i\in[0,1]}$ is enough to infer the fundamentals θ. However, this information is dispersed throughout the population. In $t = 1$ the patient types that postponed their decision will also observe an additional signal, a statistic based on the proportion of agents who chose to attack at $t = 0$.

$$y_i = \Phi^{-1}\left(A^0\right) + \tau\eta_i$$

Note that this signal is private and it assumes that agents learn form the aggregate action. A modification of the model will be proposed in which the second period signal is a public indicator function.

$$\begin{aligned} I_{A^0} &= 1 \; if \; A^0 \geq \hat{A} \; (Bad \; News) \\ &= 0 \; \; if \; A^0 < \hat{A} \; (Good \; News) \end{aligned}$$

It is interesting to see how the strategic interaction and the equilibria of the game change due to this modification of the information structure (see Appendix). There are two possible scenarios: **(i)** *No Delay*: the entire population moves in $t = 0$. **(ii)** *Delay*: in $t = 0$ the impatient agents choose between *attack* and *not attack* and the patient agents choose between *attack* and *delay*. Remember that the impatient types do not have the option to postpone. In $t = 1$ the patient types that postponed their choice, a fraction smaller or equal than $(1 - f)$, choose between *attack* and *not attack*. Strategies, beliefs and equilibrium are defined as follows.

♦ Strategies:

$$\begin{aligned} \sigma_{IMP,i} &: \; [X_i] \rightarrow \Delta A^i_{IMP} \; \forall i \in IMP \\ \sigma_{0,P,i} &: \; [X_i] \rightarrow \left\{\Delta A^i_{0,P}\right\} \; \forall i \in P \\ \sigma_{1,P,i} &: \; [X_i] \times [Y_i] \rightarrow \Delta A^i_{1,P} \; \forall i \in P \end{aligned}$$

Note that X_i is the signal's support for agent $i \in \{P, IMP\}$ in $t = 0$; A^i_{IMP} is the action space for an agent $i \in IMP$ in $t = 0$; $A^i_{0,P}$ is the action space for an agent $i \in P$ in $t = 0$; $A^i_{1,P}$ is the action space for an agent $i \in P$ in $t = 1$ and $[Y_i]$ is the support of the signal received by the patient agents that postpone their decision.

♦ Beliefs:

$$
\begin{aligned}
D_0^{IMP} &\equiv \mu_0^{IMP}(\cdot \mid x_i)_{x_i \in X_i} \in [\Delta(\theta)]^{X_i} \\
D_0^{P} &\equiv \mu_0^{P}(\cdot \mid x_i)_{x_i \in X_i} \in [\Delta(\theta)]^{X_i} \\
D_1^{P} &\equiv \mu_1^{P}(\cdot \mid x_i, y_i)_{x_i \in X_i; y_i \in Y_i} \in [\Delta(\theta)]^{X_i \times Y_i}
\end{aligned}
$$

♦ Equilibrium (symmetric therefore drop the i):

$\left\{\sigma^*_{IMP}, \sigma^*_{0,P}, \sigma^*_{1,P}\right\}$ *such that*

(1) σ^*_{IMP} optimal given $(D_0^{IMP}, \sigma^*_{0,P}, \sigma^*_{1,P})$

(2) $\sigma^*_{0,P}$ optimal given $(D_0^{P}, D_1^{P}, \sigma^*_{IMP}, \sigma^*_{1,P})$

(3) $\sigma^*_{1,P}$ optimal given $(D_0^{P}, D_1^{P}, \sigma^*_{IMP}, \sigma^*_{0,P})$

(4) Private Beliefs $(D_0^{IMP}, D_0^{P}, D_1^{P})$ must be consistent given $\left\{\sigma^*_{IMP}\sigma^*_{0,P}, \sigma^*_{1,P}\right\}$.

The objective is to look for equilibria in which: **(i)** The regime falls when fundamentals are weaker than a threshold denoted $\theta^*(f)$. **(ii)** Impatient agents choose monotone strategies with threshold (x^*_{IMP}) and patient agents choose monotone strategies with thresholds $\left(x^*_{0,P}, s^*_{1,P}\right)$. The thresholds are such that: impatient agents attack at $t = 0$ if $x_i \leq x^*_{IMP}$, patient agents attack at $t = 0$ if $x_i > x^*_{0,P}$, if $x_i \leq x^*_{0,P}$ they delay; in $t = 1$ they attack if and only if $s_i \leq s^*_{1,P}$.

	Impatient's Strategy	Patient's Strategy
$t = 0$	$\forall x_i \leq x^*_{IMP} \Longrightarrow a_i^{IMP} = 1$	$\forall x_i \leq x^*_{0,P} \Longrightarrow a^i_{0,P} = 0$
	$\forall x_i > x^*_{IMP} \Longrightarrow a_i^{IMP} = 0$	$\forall x_i > x^*_{0,P} \Longrightarrow a^i_{0,P} = 1$
$t = 1$		$\forall s_i \leq s^*_{1,P} \Longrightarrow a^i_{1,P} = 1$
		$\forall s_i > s^*_{1,P} \Longrightarrow a^i_{1,P} = 0$

THE ANALYSIS

[**M&S Benchmark**] If it is common knowledge that $f = 1$ (everybody must move in $t = 0$) or if there is an equilibrium with No Delay (impatient and patient agents act in $t = 0$) the model is isomorphic to Morris and Shin 1998. The aggregate attack in $t = 0$ is given by:

$$\begin{aligned}
A(\theta) &= f \Pr\left(x \leq x^*_{MS} \mid \theta\right) + (1 - f) \Pr\left(x \leq x^*_{MS} \mid \theta\right) \\
A(\theta) &= f \Phi\left(\frac{x^*_{MS} - \theta}{\sigma}\right) + (1 - f)\, \Phi\left(\frac{x^*_{MS} - \theta}{\sigma}\right) \ \forall \ \textit{given } f \\
A(\theta) &= \Phi\left(\frac{x^*_{MS} - \theta}{\sigma}\right) \ \textit{independent of } f
\end{aligned}$$

The aggregate attack is a decreasing function of θ (same as Morris and Shin). Given that the regime falls whenever $A(\theta) \geq \theta$ and since $A(\theta)$ is decreasing in

θ there will be a θ^*_{MS} such that

$$\Phi\left(\frac{x^*_{MS} - \theta^*_{MS}}{\sigma}\right) = \theta^*_{MS} \tag{3}$$

and therefore $\forall\theta \leq \theta^*_{MS}$ the regime falls and $\forall\theta > \theta^*_{MS}$ it survives. From the indifference condition of the agents, i.e. $\Pr\left(\theta \leq \theta^*_{MS} \mid x^*_{MS}\right) = r$, it is possible to derive another equation in θ^*_{MS} and x^*_{MS}

$$1 - \Phi\left(\frac{x^*_{MS} - \theta^*_{MS}}{\sigma}\right) = r \tag{4}$$

hence given equation (3) and (4), it is possible to pin down two unique thresholds θ^*_{MS} and x^*_{MS} characterizing the unique equilibrium.

Proposition 1 *When $f = 1$ the game is isomorphic to M&S and there is a unique equilibrium, represented by the thresholds $(\theta^*_{MS}, x^*_{MS})$ where θ^*_{MS} is a decreasing function of r: $\theta^*_{MS} = 1 - r$.*

[**Dasgupta Benchmark**] If it is common knowledge that $f = 0$ (everybody can postpone the decision to attack to $t = 1$) the model is isomorphic to Dasgupta 2007. The model has a unique equilibrium. In $t = 0$ agents attack iff $x_i \leq x^*_D$, otherwise they choose to wait. Conditional on reaching $t = 1$ they attack iff $s_i \leq s^*_D$. The regime falls when $\theta \leq \theta^*_D$.

Proposition 2 *When $f = 0$ the game is isomorphic to Dasgupta benchmark. In the limit as $\tau \to 0$ and $\sigma \to 0$, given the existence of a cost of delay, $\theta^*_{MS} \leq \theta^*_D$. Thus, successful coordinated attack becomes more likely when agents can choose both how to act and when to act.*

[**This model**] This paper analyzes the model under the assumption that $f \in (0,1)$: only patient agents, representing a fraction $(1-f)$ of the population, can postpone the decision to $t=1$. The analysis proceeds in four steps. **(i)** Write the aggregate attack A as a function of θ. The aggregate attack is the sum of $A^0(\theta)$, the attack in $t=0$ and $A^1(\theta)$, the attack in $t=1$. **(ii)** Prove that $A(\theta)$ is not monotone in θ and since the regime changes if $A(\theta) \geq \theta$, there might be multiple intersections. If there is only one, the regime falls when the realization of θ is less than θ^*, hence the bigger is this threshold the higher is the probability that the regime falls in equilibrium. If there are many intersections the instability of the regime is indeterminate. The probability that $\theta \leq \theta^*$ (conditional on the available information) represents the belief of the agents on the possibility that the regime changes, thus this probability is essential to pin down the optimality conditions of the patient and impatient agents in $t=0$ and in $t=1$. **(iii)** The optimality conditions plus the regime change conditions give a system of four equations in four unknowns whose solution/s deliver the equilibrium/equilibria. **(iv)** Describe these equations and discuss how the solution/s and therefore the equilibrium/equilibria look like.

What is the attack in $t=0$ when the impatient agents act and some patient agents do not delay?

$$\begin{aligned} A^0(\theta) &= f \Pr\left(x \leq x^*_{IMP} \mid \theta\right) + (1-f)\Pr\left(x > x^*_{0,P} \mid \theta\right) \ \forall \ given\ f \\ A^0(\theta) &= f\Phi\left(\frac{x^*_{IMP}-\theta}{\sigma}\right) + (1-f)\left(1-\Phi\left(\frac{x^*_{0,P}-\theta}{\sigma}\right)\right) \end{aligned} \quad (5)$$

This attack is increasing in f. What is the attack in $t=1$ by the patient agents that delayed their decision at $t=0$?

$$\begin{aligned} A^1(\theta) &= (1-f)\Pr\left(x \leq x^*_{0,P}; s \leq s^*_{1,P} \mid \theta\right) \ \forall \ given \ f \\ A^1(\theta) &= (1-f)\Phi(\cdot) \ \forall \ given \ f \end{aligned} \tag{6}$$

What is the aggregate attack in $t=1$ when some patient agents do not delay?

$$A \equiv A^0(\theta) + A^1(\theta)$$

$$\begin{aligned} A &= f\Pr\left(x \leq x^*_{IMP} \mid \theta\right) \ + (1-f)\Pr\left(x > x^*_{0,P} \mid \theta\right) + \\ &\quad + (1-f)\Pr\left(x \leq x^*_{0,P}; s \leq s^*_{1,P} \mid \theta\right) \end{aligned}$$

$$\begin{aligned} A &= f\Phi\left(\frac{x^*_{IMP}-\theta}{\sigma}\right) \ + (1-f)\left(1-\Phi\left(\frac{x^*_{0,P}-\theta}{\sigma}\right)\right) + \\ &\quad + (1-f)\Phi(\cdot) \end{aligned} \tag{7}$$

When f is common knowledge the aggregate attack A is a function of θ for any given f; when f is a random variable the aggregate attack is a function of two unknowns, θ and f. The regime falls whenever $A(\theta) \geq \theta$, this regime change condition can be rewritten using (7):

$$f\Phi\left(\frac{x^*_{IMP}-\theta}{\sigma}\right) \ + (1-f)\left(1-\Phi\left(\frac{x^*_{0,P}-\theta}{\sigma}\right)\right) + (1-f)\Phi(\cdot) \ \geq \theta$$

Lemma 3 *If $A(\theta)$ is not monotone in θ, for any given f, there are multiple intersection points $\theta^*(f)$ such that*

$$f\Phi\left(\frac{x^*_{IMP} - \theta^*(f)}{\sigma}\right) + (1-f)\left(1 - \Phi\left(\frac{x^*_{0,P} - \theta^*(f)}{\sigma}\right)\right) +$$
$$+ (1-f)\,\Phi(\cdot) = \theta^*(f) \tag{8}$$

hence the probability that the regime falls is indeterminate. Given some restrictions, there is a unique intersection $\theta^*(f)$ such that $\forall\theta \leq \theta^*(f)$ the regime falls and $\forall\theta > \theta^*(f)$ it survives.

Note that $\theta^*(f)$ is a function of f whose analytical form will be pinned down in equilibrium. Note also that when f is common knowledge agents know $\theta^*(f)$ in equilibrium, but when f is a random variable the switching threshold $\theta^*(f)$ is a function of f and therefore it is also a random variable. The optimality conditions plus the regime change condition give a system of four equations in four unknowns whose solution/s deliver the equilibrium/equilibria. The thresholds that characterize each equilibrium are

$$(x^*_{IMP}, x^*_{0,P}, s^*_{1,P}, \theta^*(f))$$

♦ Indifference Condition Patient at $t = 0$ (trade off the expected benefit of attacking early against the expected benefit of waiting and then act optimally)

$$(\Pr\left(\theta \leq \theta^*(f) \mid x^*_{0,P}\right) - r) = (\Pr\left(\theta \leq \theta^*(f), s \leq s^*_{1,P} \mid x^*_{0,P}\right) - r) - \varepsilon \tag{9}$$

♦ Indifference Condition Impatient at $t = 0$

$$\Pr(\theta \leq \theta^*(f) \mid x^*_{IMP}) = r \tag{10}$$

♦ Indifference Condition Patients at $t = 1$

$$\Pr\left(\theta \leq \theta^*(f) \mid s^*_{1,P}\right) = r \tag{11}$$

♦ Critical Mass Condition

$$\begin{aligned} f \Pr\left(x \leq x^*_{IMP} \mid \theta^*(f)\right) \; + (1-f)\Pr\left(x > x^*_{0,P} \mid \theta^*(f)\right) + \\ + (1-f)\Pr\left(x \leq x^*_{0,P}; s \leq s^*_{1,P} \mid \theta^*(f)\right) = \theta^*(f) \end{aligned} \tag{12}$$

This system can be reduced to a simpler one. Rewrite condition (9)

$$\varepsilon = \Pr\left(\theta \leq \theta^*(f), s \leq s^*_{1,P} \mid x^*_{0,P}\right) - \Pr\left(\theta \leq \theta^*(f) \mid x^*_{0,P}\right) \tag{9\^{}}$$

Also condition (10) can be rewritten

$$\begin{aligned} 1 - \Phi_{IMP}\left(\frac{x^*_{IMP} - \theta^*(f)}{\sigma}\right) &= r \\ x^*_{IMP} &= \sigma\Phi^{-1}_{IMP}(1-r) + \theta^*(f) \end{aligned} \tag{10\^{}}$$

and condition (11) can be rewritten

$$\begin{aligned} 1 - \Phi_{0,P}\left(\frac{s^*_{1,P} - \theta^*(f)}{\sigma}\right) &= r \\ s^*_{1,P} &= \sigma\Phi^{-1}_{0,P}(1-r) + \theta^*(f) \end{aligned} \tag{11\^{}}$$

where $M_{IMP} \equiv \sigma\Phi^{-1}_{IMP}(1-r)$ and $M_{0,P} = \sigma\Phi^{-1}_{0,P}(1-r)$. Hence the new system is:

$$\varepsilon = \Pr\left(\theta \leq \theta^*(f), s \leq s^*_{1,P} \mid x^*_{0,P}\right) - \Pr\left(\theta \leq \theta^*(f) \mid x^*_{0,P}\right) \tag{9\^{}}$$

$$x^*_{IMP} = M_{IMP} + \theta^* (f) \qquad (10\hat{})$$

$$s^*_{1,P} = M_{0,P} + \theta^* (f) \qquad (11\hat{})$$

$$f \Pr \left(x \leq x^*_{IMP} \mid \theta^* (f)\right) \;\; + (1-f) \Pr \left(x > x^*_{0,P} \mid \theta^* (f)\right) + \qquad (12\hat{})$$

$$+ (1-f) \Pr \left(x \leq x^*_{0,P}; s \leq s^*_{1,P} \mid \theta^* (f)\right) = \theta^* (f)$$

Condition (9ˆ) says that in equilibrium the delay cost ε must be equal to the benefit, which is measured by the increase in the expected probability that the regime falls conditional on the information that the patient agents receive in $t = 0$ (and given that they act optimally in $t = 1$). The left hand side of this expression (delay benefit) is decreasing in $x_{0,P}$ (see Lemma 4, Appendix) and crosses ε when $x_{0,P} = x^*_{0,P}$. Hence $\forall x_{0,P} \leq x^*_{0,P}$ the delay benefit is bigger than the cost and is optimal to delay; $\forall x_{0,P} > x^*_{0,P}$ the cost is bigger than the benefit and is optimal to attack in $t = 0$. Suppose ε decreases. When ε decreases the switching threshold $x^*_{0,P}$ is bigger, this increases the probability that each patient agent delays in equilibrium and hence the mass of patient agents that delay in $t = 0$. How does it affect the equilibrium strategy of the impatient agents in $t = 0$ and the equilibrium strategy of the patient agents in $t = 1$? It can be shown that when $x^*_{0,P}$ increases and $\tau \to 0$, i.e. the second period signal is very precise, the aggregate attack is bigger for any given θ (see Lemma 5, Appendix) hence $\theta^*(f)$ is bigger (using Lemma 3 under the uniqueness restrictions). According to the equilibrium conditions (10ˆ) and (11ˆ), if $\theta^*(f)$ increases the equilibrium thresholds $\left(x^*_{IMP}, s^*_{1,P}\right)$ for the impatient agents in $t = 0$ and for the patient agents in $t = 1$ also increase. Hence when ε decreases the patient agents in $t = 0$ are more willing to delay and are more willing to attack in $t = 1$, also the impatient in $t = 0$ are more willing to attack. The regime is more fragile when ε decreases. Now suppose ε increases. When ε increases the switching threshold $x^*_{0,P}$ is smaller, reducing the probability that each patient agent delays in equilibrium and hence the mass of patient agents that delay in $t = 0$. By Lemma 5, a decrease of $x^*_{0,P}$ reduces the aggregate attack for any given θ which implies a smaller $\theta^*(f)$ and therefore smaller equilibrium thresholds $\left(x^*_{IMP}, s^*_{1,P}\right)$. Hence when ε increases, the patient agents in $t = 0$ are more willing to attack, but the impatient in $t = 0$ and the patient in $t = 1$ don't. The second effect more than compensate the first one and this explain why, the regime is stronger when ε increases. Lemma 3,4,5 and the equilibrium conditions (9ˆ), (10ˆ), (11ˆ),(12ˆ) lead to the following proposition.

Proposition 4 *When $0 < f < 1$ the game has a unique equilibrium given some restrictions on the precision of the information received by the patient agents. The equilibrium is characterized by the equilibrium thresholds*

$$\left(x^*_{IMP}, x^*_{0,P}, s^*_{1,P}, \theta^*(f)\right)$$

where $\theta^*(f)$ is a decreasing function of r and ε.

PART III

CONCLUSIONS. This model introduces a novel type of friction in coordination games with incomplete information. Impatient agents must make a choice at time zero, patient agents have the option to postpone their decision with the advantage of observing additional information. Delay has a cost, ε. When this cost is small, the patient agents are more willing to postpone their choice in order to coordinate with the impatient agents. In equilibrium the impatient agents anticipate this and use a more aggressive strategy. As a result everybody act more aggressively, and the equilibrium probability of a regime change becomes bigger the smaller is the delay cost.

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