Portfolio optimization strategies in the cryptocurrency market

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Correlations between fiat assets and cryptos

	BTC	ETH	XRP	LTC	DASH
USD/EUR	−0.05	−0.04	0.04	−0.06	−0.01
JPY/USD	0.02	−0.04	−0.03	−0.04	0.09
USD/GBP	−0.06	−0.09	0.04	−0.09	−0.01
Gold	0.05	0.04	0.04	0.05	−0.01
SP500	0.00	−0.05	0.05	−0.05	0.02
XWD	0.01	−0.03	0.02	−0.07	0.03
EEM	0.00	−0.09	0.04	−0.09	0.00
REIT	0.03	−0.09	0.04	0.05	0.00
DTB3	0.02	0.09	0.00	0.02	0.03
DGS10	−0.02	−0.08	0.00	−0.02	0.01

Table 1: Correlations between cryptos and conventional financial assets: 3 exchange rates, gold, 3 stock indices, real estate and the US Treasury Bills Rates.

Benefit of crypto investment: higher return in European markets

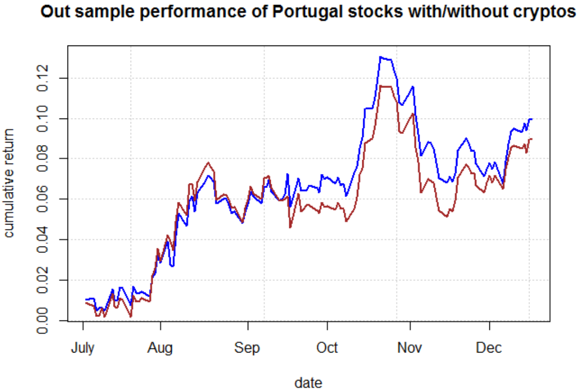

Figure 1: Markowitz portfolios with cryptos and without cryptos

Challenge of crypto investment: lower return in American markets

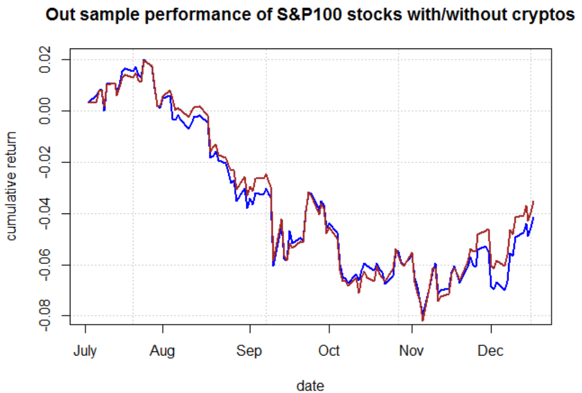

Figure 2: Markowitz portfolios with cryptos and without cryptos

Is Markowitz rule appropriate?

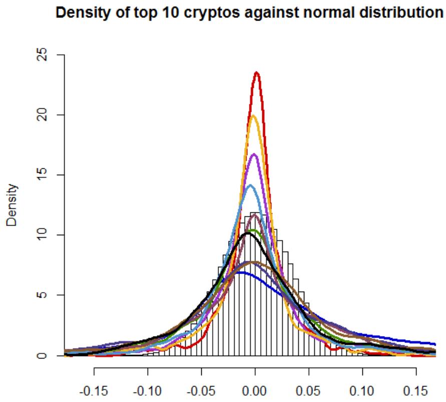

Figure 3: The observation period is 2014-03-30 to 2016-07-24.

CCSHistReturnsDensity

Challenge of crypto investment: high risk

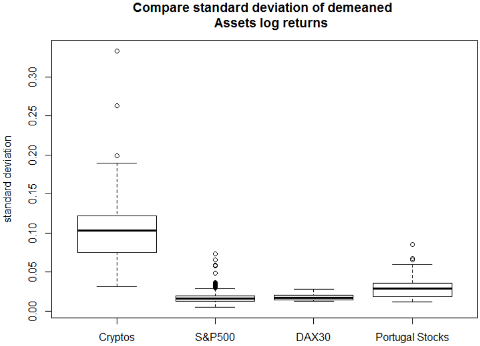

Figure 4: Crypto currencies have higher volatilities than stocks, highlighting the importance of risk management when investing on them LIBRObox1

Challenge of crypto investment: low trading volume

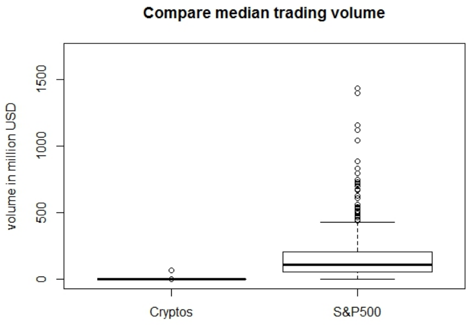

Figure 5: Crypto currencies have much lower trading volume compared to traditional assets LIBRObox2

Cryptos from an investment viewpoint

- Elendner et al. (2016) & Yermack (2014): Cryptos show low correlation with traditional assets
- Eisl et al. (2015), Briere (2015): Bitcoin improves the risk-return trade-off of portfolios.
- Härdle and Trimborn (2015) & Trimborn and Härdle (2016): Constructing market index for cryptos (CRIX)
- Chen et al. (2016): Analyzing dynamics of CRIX
- Li and Trimborn (2017): Liquidity constrained Markowitz portfolios in crypto markets

Objectives

- Risk-Based portfolio strategies in crypto-currency market
  - Out-of-sample performance analysis – is there best asset allocation model?
  - Do weights of Bitcoin and altcoins depend on the optimization procedure used?
  - Effect of liquidity constrains for the performance

Outline

1. Motivation ✓
2. Methodology
3. Empirical results
4. Data
5. Conclusion

Investment strategies

Model	Reference	Abbreviation
Mean – Var – max Sharpe	Jagannathan and Ma (2003)	MV – Sharpe
Mean – Var – max return	Markowitz (1952)	MV – max ret
Mean – Var – min var	Merton (1980)	Min Var
Equally weighted	DeMiguel et al. (2009)	EQ
Equal Risk Contribution	Roncalli et al. (2010)	ERC
Mean – CVaR – max return	Rockafellar and Uryasev (2000)	MCVaR – max ret
Mean – CVaR – min risk	Rockafellar and Uryasev (2000)	MCVaR – min risk

Table 2: List of asset allocation models

MV ERC Mean-CVaR LIBRO

Data Information

- Core assets
  - MSCI World Commodity
  - MSCI EM
  - FTSE100
  - FTSE EPRA/NAREIT
- Satellites: 75 crypto-currencies
- Time span 2015-03-20 to 2017-05-12 (560 trading days)
  - 400 days – in-sample period
  - 160 days – out-of-sample period

Performance of allocation strategies'

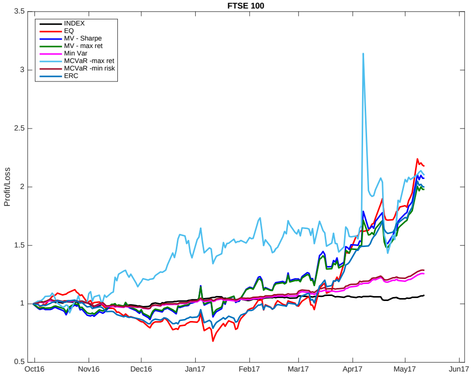

Figure 6: Performance of allocation strategies for the FTSE100 benchmark index without liquidity constraints

Performance of allocation strategies'

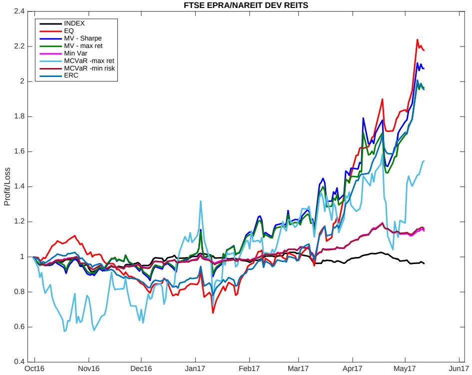

Figure 7: Performance of allocation strategies for the FTSE EPRA/NAREIT DEV REITS without liquidity constraints

Performance of allocation strategies'

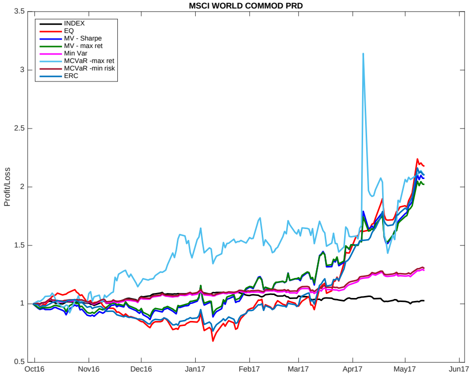

Figure 8: Performance of allocation strategies for the MSCI WORLD COMMOD PR benchmark index without liquidity constraints

Performance of allocation strategies'

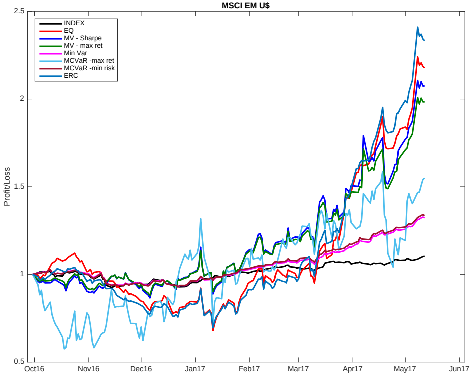

Figure 9: Performance of allocation strategies for the MSCI EM benchmark index without liquidity constraints

Performance of allocation strategies' – LIBRO

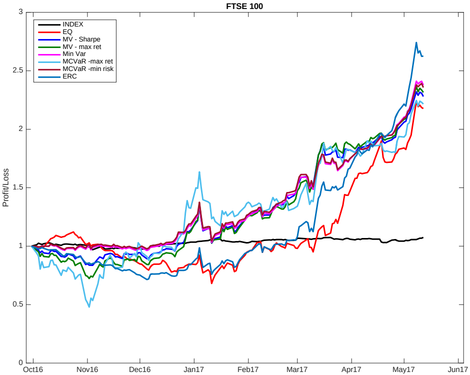

Figure 10: Performance of allocation strategies for the FTSE100 benchmark index with invested sum 1 mln US$

Performance of allocation strategies' – LIBRO

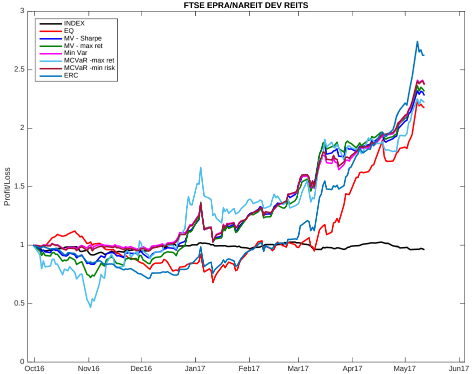

Figure 11: Performance of allocation strategies for the FTSE EPRA/NAREIT DEV REITS with invested sum 1 mln US$

Performance of allocation strategies' – LIBRO

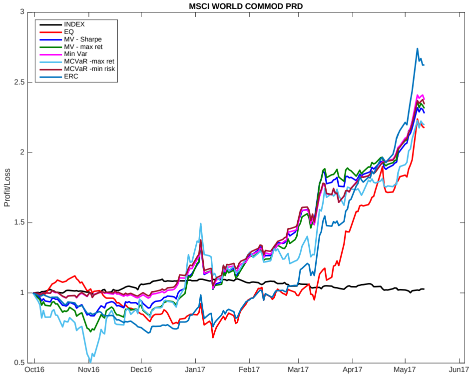

Figure 12: Performance of allocation strategies for the MSCI WORLD COMMOD PR benchmark index with invested sum 1 mln US$

Performance of allocation strategies' – LIBRO

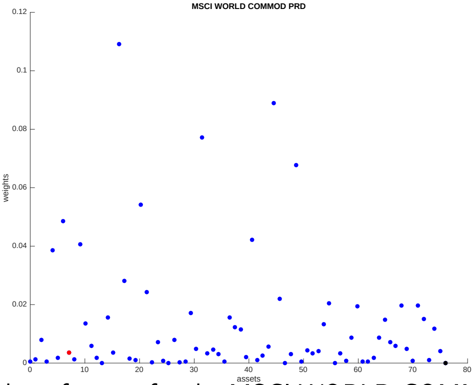

Figure 13: Weights of assets for the MSCI WORLD COMMOD PR benchmark index with invested sum 1 mln US$ (ERC)

Performance of allocation strategies' – LIBRO

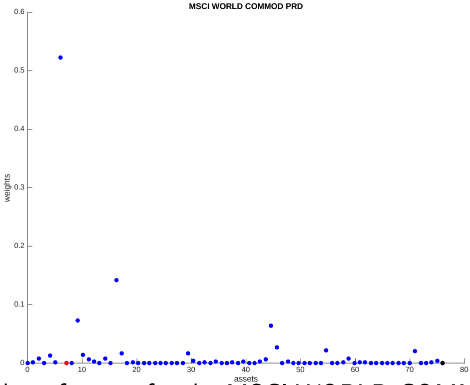

Figure 14: Weights of assets for the MSCI WORLD COMMOD PR benchmark index with invested sum 1 mln US$ (MV-min)

Performance of allocation strategies' – LIBRO

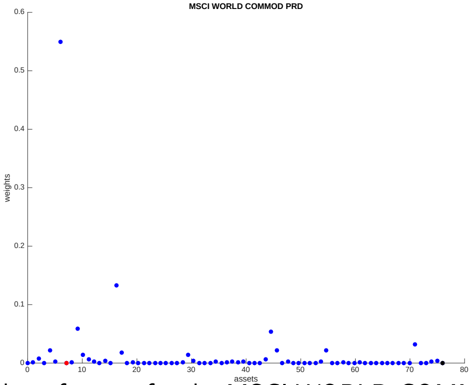

Figure 15: Weights of assets for the MSCI WORLD COMMOD PR benchmark index with invested sum 1 mln US$ (CVAR-min)

Performance of allocation strategies' – LIBRO

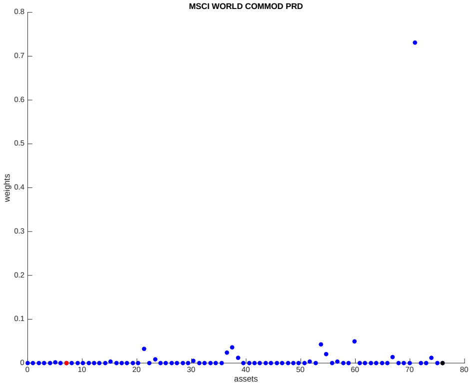

Figure 16: Weights of assets for the MSCI WORLD COMMOD PR benchmark index with invested sum 1 mln US$ (CVAR-max)

Performance of allocation strategies' – LIBRO

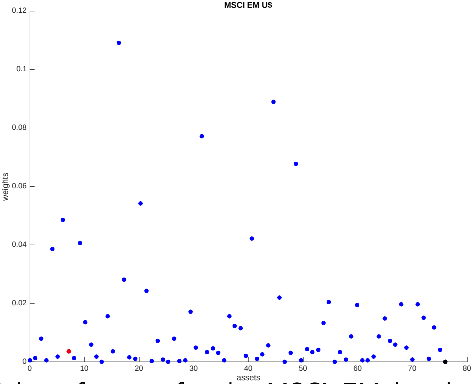

Figure 17: Weights of assets for the MSCI EM benchmark index with invested sum 1 mln US$(ERC)

Performance of allocation strategies' – LIBRO

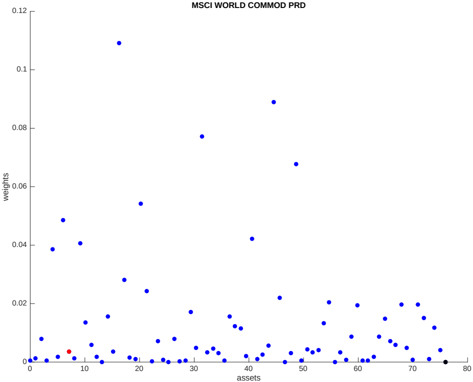

Figure 18: Weights of assets for the MSCI WORLD COMMOD PR benchmark index with invested sum 1 mln US$(ERC)

Performance of allocation strategies' – LIBRO

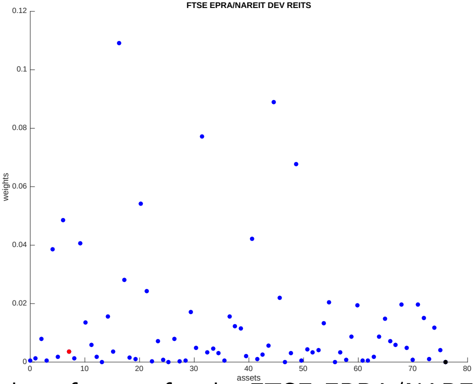

Figure 19: Weights of assets for the FTSE EPRA/NAREIT DEV REITS benchmark index with invested sum 1 mln US$(ERC)

Portfolios' performance

Model	No constraints		10 mln	
	Cumulative return	Sharpe ratio	Cumulative return	Sharpe ratio
INDEX	1.07	0.01	1.07	0.01
EQ	2.18	0.16	2.18	0.16
MV-max Sharpe	2.07	0.13	2.28	0.2
Min Var	1.26	0.21	2.38	0.24
MV-max return	1.98	0.13	2.32	0.18
ERC	2.00	0.20	2.63	0.22
MCVaR -min risk	1.29	0.22	2.36	0.24
MCVaR- max ret	2.10	0.04	2.22	0.12

Table 3: Performance of allocation strategies for the FTSE 100 benchmark index

Portfolios' performance

Model	No constraints		10 mln	
	Cumulative return	Sharpe ratio	Cumulative return	Sharpe ratio
INDEX	0.96	-0.01	0.96	-0.01
EQ	2.18	0.16	2.18	0.16
MV-max Sharpe	2.07	0.13	2.28	0.2
Min Var	1.15	0.10	2.38	0.24
MV-max return	1.97	0.13	2.32	0.18
ERC	1.95	0.19	2.63	0.22
MCVaR -min risk	1.16	0.11	2.30	0.18
MCVaR- max ret	1.55	0.05	2.37	0.24

Table 4: Performance of allocation strategies for the FTSE EPRA/NAREIT benchmark index

Portfolios' performance

Model	No constraints		10 mln	
	Cumulative return	Sharpe ratio	Cumulative return	Sharpe ratio
INDEX	1.03	-0.01	1.03	-0.01
EQ	2.18	0.16	2.18	0.16
MV-max Sharpe	2.07	0.13	2.07	0.13
Min Var	1.28	0.17	1.28	0.17
MV-max return	2.02	0.14	2.02	0.14
ERC	2.10	0.19	2.10	0.19
MCVaR -min risk	1.30	0.19	3.09	0.10
MCVaR- max ret	2.10	0.04	2.10	0.04

Table 5: Performance of allocation strategies for the MSCI WORLD COMMOD PR benchmark index

Portfolios' performance

Model	No constraints		10 mln	
	Cumulative return	Sharpe ratio	Cumulative return	Sharpe ratio
INDEX	1.10	0.01	1.10	0.01
EQ	2.11	0.16	2.18	0.16
MV-max Sharpe	2.07	0.13	2.28	0.2
Min Var	1.32	0.23	2.38	0.24
MV-max return	1.98	0.13	2.32	0.18
ERC	2.33	0.19	2.63	0.22
MCVaR -min risk	1.34	0.23	2.30	0.18
MCVaR- max ret	1.55	0.05	2.15	0.11

Table 6: Performance of allocation strategies for the MSCI EM benchmark index

Cryptos as satellites

- Out-of-sample performance:
  - Including cryptos can provide better risk-return trade off with various traditional cores
  - Higher performance of portfolio optimization based on higher moments
- Weights of Bitcoin and alt coins are not robust and depend on the optimization procedure
- Liquidity constraints on investing in cryptos improves performance

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Risk Parity (Equal risk contribution)

Let $\sigma(w) = \sqrt{w^\top \Sigma w}$. Euler decomposition:

$$\sigma(w) \stackrel{\text{def}}{=} \sum_{i=1}^{n} \sigma_i(w) = \sum_{i=1}^{n} w_i \frac{\sigma(w)}{\partial w_i}$$

where $\frac{\sigma(w)}{\partial w_i}$ is the marginal risk contribution and $\sigma_i(w) = w_i \frac{\sigma(w)}{\partial w_i}$ the risk contribution of i-th asset. The idea of ERC strategy is to find risk balanced porfolio, such that:

$$\sigma_i(w) = \sigma_j(w)$$

i.e. the risk contribution is the same for all assets of the portfolio

Back to "Methodology

Mean-Variance Asset Allocation

Log returns $X_t \in \mathbb{R}^p$:

$$\begin{aligned} \min_{w_t \in \mathbb{R}^p} \quad & \sigma^2_{P,t}(w_t) \stackrel{\text{def}}{=} w_t^\top \Sigma_t w_t \\ \text{s.t.} \quad & \mu_{P,t}(w_t) = r_T, \\ & w_t^\top 1_p = 1, \\ & w_{i,t} \geq 0 \end{aligned} \tag{1}$$

where r_T "target" return, $\Sigma_t \stackrel{\text{def}}{=} E_{t-1}\{(X_t - \mu)(X_t - \mu)^\top\}$

Back to "Methodology

Conditional CFVaR optimization

Given $\alpha > 0.5$ confidence level,

$$\min_{w_t \in \mathbb{R}^p} \mathrm{CVaR}_\alpha(w_t), \quad \text{s.t. } \mu_{P,t}(w_t) = r_T, w_t^\top 1_p = 1, w_{i,t} \geq 0, \tag{2}$$

$$\mathrm{CVaR}_\alpha(w_t) = -\frac{1}{1-\alpha} q_\alpha^*(w_t) \sigma_{P,t}(w_t), \tag{3}$$

Proof

where (via Cornish-Fisher (CF) expansion):

$$q_\alpha^*(w_t) = \left\{ 1 + \frac{S_{P,t}(w_t)}{6} z_\alpha + \frac{K_{P,t}(w_t)}{24}(z_\alpha^2 - 1) - \frac{S_{P,t}^2(w_t)}{36}(2z_\alpha^2 - 1) \right\} \varphi(z_\alpha), \tag{4}$$

here $S_p(w_t)$ skewness of the portfolio, $K_p(w_t)$ excess kurtosis of the portfolio, z_α is $\mathrm{N}(0,1)$ α-quantile. If $S_p(w_t)$, $K_p(w_t)$ are zero, then the problem reduces to the Markowitz case.

Back to "Methodology

Solve the optimization problem

Target optimization problem:

$$\min\ w^\top \widehat{\Sigma} w$$
$$\text{s.t. } \mathbf{1}_p^\top w = 1,\ \mu^\top w \geq \bar{r},\ ||w||_1 \leq c,\ w \leq a$$

- $\widehat{\Sigma}$: estimated covariance matrix
- $w = (w_1, w_2, \cdots, w_p)^\top$: weight on assets
- a are some upper bounds concerning liquidity
- Difficulty:
    - $||w||_1 \leq c$ alone has 2^p constraints
    - Problematic when p large

Back to "Methodology

Solve the optimization problem

How to simplify:

- Cryptocurrencies can not be shortselled
- Limited liquidity contradicts shortselling
- Then results $||w||_1 = 1$.

Which gives:

$$\begin{aligned} &\min\ w^\top \widehat{\Sigma} w \\ &\text{s.t. } \mathbf{1}_p^\top w = 1,\ \mu^\top w \geq \bar{r},\ 0 \leq w \leq a \end{aligned}$$

Back to "Methodology

Constrained Portfolios

Define

$$R(w) = w^\top \Sigma w \quad R_n(w) = w^\top \widehat{\Sigma} w$$

Let

$$w_{opt,a} = \underset{w^\top 1=1, \mu^\top w \geq \bar{r}, ||w||_1 \leq c, w \leq a}{\arg\min} R(w)$$

$$\widehat{w}_{opt,\hat{a}} = \underset{w^\top 1=1, \mu^\top w \geq \bar{r}, ||w||_1 \leq c, w \leq \hat{a}}{\arg\min} R_n(w)$$

$$b_n = ||\widehat{\Sigma} - \Sigma||_\infty$$

$$\widehat{a} = a + \varepsilon$$

Details | Back to "Methodology

Implications for application

- Range of ε bounded by c
- Proper choice of c sets upper bound
- Divergence in volatility risk increases, but liquidity risk controlled for
- Trade-off between two risk sources increases volatility risk of portfolio

Constrained Portfolios I

Fan et al. (2012):

$$|R(w_{opt,a}) - R_n(\widehat{w}_{opt,a})| \leq b_n c^2$$

Define:

$$w_{opt,a} \leq a$$
$$w_{opt,\widehat{a}} \leq \widehat{a}$$
$$a = w_{opt,a} + \delta_1, \quad \delta_1 \text{ slack variable}$$
$$\widehat{a} = w_{opt,\widehat{a}} + \delta_2, \quad \delta_2 \text{ slack variable}$$
$$w_{opt,\widehat{a}} = w_{opt,a} + \delta_1 - \delta_2 + \varepsilon$$

Back

Constrained Portfolios II

Consider $\delta_2 = 0$:

- All estimated weights are at boundary

$$
\begin{aligned}
|R_n(\widehat{w}_{opt,a}) - R_n(\widehat{w}_{opt,\hat{a}})| &= |\widehat{w}_{opt,\widehat{a}}^\top \widehat{\Sigma} \varepsilon| \\
|R(w_{opt,a}) - R_n(\widehat{w}_{opt,\hat{a}})| &\leq |\widehat{w}_{opt,\widehat{a}}^\top \widehat{\Sigma} \varepsilon| + b_n c^2 \\
A &= |\widehat{w}_{opt,\widehat{a}}^\top \widehat{\Sigma} \varepsilon|
\end{aligned}
$$

- A bigger when $\varepsilon > 0$ than if $\varepsilon < 0$
- Overestimation of a causes smaller upper bound for error
- ε bounded by the choice of c

Constrained Portfolios III

$$
\begin{aligned}
&|R_n(\widehat{w}_{opt,a}) - R_n(\widehat{w}_{opt,\hat{a}})| \\
&= |\widehat{w}_{opt,a}^\top \widehat{\Sigma} \widehat{w}_{opt,a} - \widehat{w}_{opt,\hat{a}}^\top \widehat{\Sigma} \widehat{w}_{opt,\hat{a}}| \\
&= |(\widehat{w}_{opt,a} + \delta_1)^\top \widehat{\Sigma} (\widehat{w}_{opt,a} + \delta_1) \\
&\quad - (\widehat{w}_{opt,a} + \delta_1 - \delta_2 + \varepsilon)^\top \widehat{\Sigma} (\widehat{w}_{opt,a} + \delta_1 - \delta_2 + \varepsilon)| \\
&= |(\widehat{w}_{opt,a} + \delta_1)^\top \widehat{\Sigma} (\delta_2 - \varepsilon) - \delta_2^\top \widehat{\Sigma} (\delta_2 - \varepsilon) + \varepsilon^\top \widehat{\Sigma} (\delta_2 - \varepsilon)|
\end{aligned}
$$

Situation of interest:

- Any $\varepsilon_i > 0$ and any $\delta_{2,i} = 0$
- Then a weight is at a non optimal point

Back

Constrained Portfolios IV

Consider $\delta_2 = 0$:

$$
\begin{aligned}
& |-(\widehat{w}_{opt,a} + \delta_1)^\top \widehat{\Sigma} \varepsilon - \varepsilon^\top \widehat{\Sigma} \varepsilon| \\
=& |-(a + \varepsilon)^\top \widehat{\Sigma} \varepsilon| \\
=& |-\widehat{w}_{opt,\widehat{a}}^\top \widehat{\Sigma} \varepsilon|
\end{aligned}
$$

Back

Constrained Portfolios V

$$
\begin{aligned}
&|R(w_{opt,a}) - R_n(\widehat{w}_{opt,\widehat{a}})| \\
=&|R(w_{opt,a}) - R_n(\widehat{w}_{opt,a}) + R_n(\widehat{w}_{opt,a}) - R_n(\widehat{w}_{opt,\widehat{a}})| \\
\leq&|R(w_{opt,a}) - R_n(\widehat{w}_{opt,a})| + |R_n(\widehat{w}_{opt,a}) - R_n(\widehat{w}_{opt,\widehat{a}})| \\
\leq& b_n c^2 + |(\widehat{w}_{opt,a} + \delta_1)^\top \widehat{\Sigma}(\delta_2 - \varepsilon) - \delta_2^\top \widehat{\Sigma}(\delta_2 - \varepsilon) + \varepsilon^\top \widehat{\Sigma}(\delta_2 - \varepsilon)|
\end{aligned}
$$

Back