

Theory and Application of Dynamic Quantile Models of Rational Behavior  
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A large body of theoretical and empirical work in economics relies on dynamic models based on the expected utility. These models are critical to learning about economic behavior and effects, and to design policy analysis. The goal of this short course is to study new alternative methods, in particular we consider the quantile preferences in a dynamic economic setting by providing a comprehensive analysis of a dynamic rational quantile model. Quantile preferences have useful advantages, such as dynamic consistency, monotonicity, allowing separation between risk aversion and elasticity of intertemporal substitution (EIS), and capturing potential heterogeneity by varying the quantiles.

In the first part of the course, we overview the axiomatization the recursive quantile preferences. The preferences induce an additively separable quantile utility model with standard discounting, that is, the recursive equation is characterized by the sum of the current period utility function and the discounted value of the certainty equivalent, which is a quantile function. In addition, we introduce dynamic programming for intertemporal decisions whereby the economic agent maximizes the present discounted value of the stream of future  $\tau$ -quantile utilities by choosing a decision variable in a feasible set. Although quantiles do not share some of the helpful properties of expectations, such as linearity and the law of iterated expectations, it has been established all the standard results in dynamic models, namely, quantile preferences are dynamically consistent, the corresponding dynamic problem yields a value function, via a fixed point argument, and that concavity, differentiability, and the principle of optimality hold. Additionally, we will show the corresponding Euler equation, which is well suited for using well-known quantile regression methods for estimating and testing the economic model.

In the second part of the course, we review the existing econometric techniques to estimate the quantile Euler equation. We study a smooth generalized method of moments with instrumental variables for estimating nonlinear conditional quantile models. In addition, we illustrate the usefulness of the quantile dynamic model with an empirical model of intertemporal consumption and estimate the implied EIS. With these new methods, empirical researchers can estimate and make inference on the parameters that describe the economy, and also compare and contrast the results for the quantile utility model with the expected utility.

The scope for future research is broad. Aggregation of the quantile preferences (existence of a representative agent) and developing nonlinear fixed effects panel data quantile regression econometric techniques being in the immediate research agenda.

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