Statistik I - Exercise session 4
16.6.2014

Info

- Classroom: SPA1 220
- Time: Mondays, 16:15 - 17:45
- in English
- Assignments on webpage (lvb>staff>PB)

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Schedule:

Date	Week	Exercises
28.04.14	E1	1-2, 1-3 (even), 1-10
05.05.14	E1	1-2, 1-3 (even), 1-10
12.05.14	E2	1-20, 1-22, 1-32
19.05.14	E2	1-20, 1-22, 1-32
26.05.14	E3	1-80, 1-83, (1-98)
02.06.14	E3	1-80, 1-83, (1-98)
09.06.14	–	–
16.06.14	E4	2-4, 2-14, 3-1, 3-7, (3-11)
23.06.14	E5	3-25, 3-37, 3-55
30.06.14	E5	3-25, 3-37, 3-55
07.07.14	E6	TBA
14.07.14	E6	TBA

Review

- week 7 & week 8
- Slides: Fundamentals of Theory of Probability (cca 1-35)

1 Combinatorics

Combinatorics

	without repetition	with Repetition
Permutation	$P(n) = n!$	$P(n; g_1, \ldots, g_r) = \frac{n!}{g_1! \cdot g_2! \cdot \ldots \cdot g_r!}$
Variation	$V(n,k) = \frac{n!}{(n-k)!}$	$V^R(n,k) = n^k$
Combination	$K(n,k) = \frac{n!}{k! \cdot (n-k)!} = \binom{n}{k}$	$K^R(n,k) = \binom{n+k-1}{k}$

Permutation	any order n Elements
Variation	Selection of k from n with respect to order
Kombination	Selection of k from n without respect to order

2 Theory of Probability

2.1 Events - Notation

$A = S$
$A = \emptyset$
$A \subset B$
$A \equiv B$
$A \cap B = \emptyset$
$A = \bigcup A_i$
$A = \bigcap A_i$

2.2 Laplace Formula

$$P(A) = \frac{|A|}{|S|} = \frac{\text{\# of all events in A}}{\text{\# of all possible events}}$$

Review - notes

- **FACTORIAL** $n! = \prod_{k=1}^{n} k = 1 \cdot 2 \cdot \ldots \cdot n \qquad n \in \mathbb{N}+$
  - ↳ $0! = 1$
  - ↳ usage: → Euler number $e = \sum_{k=0}^{\infty} \frac{1}{n!}$
    → Binomial coef. $\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$

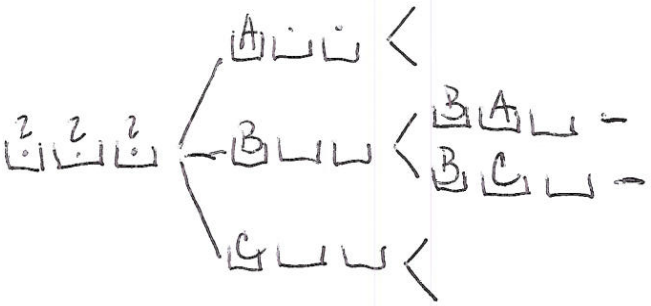
$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

COMBINATORICS:

- **PERMUTATION** $S = \{A, B, C\}$

$P(3) = 3! = 6$

$P(n) = n!$



$3 \times 2 \times 1$

- **PERMUTATION W/ REP.**

(! repetition of event in sample
! space/set
! not in „output")

f.e: 3-letter words from MOM

M_1OM_2 ~~M_2OM_1~~

M_1M_2O ~~M_2M_1O~~

OM_1M_2 ~~OM_2M_1~~

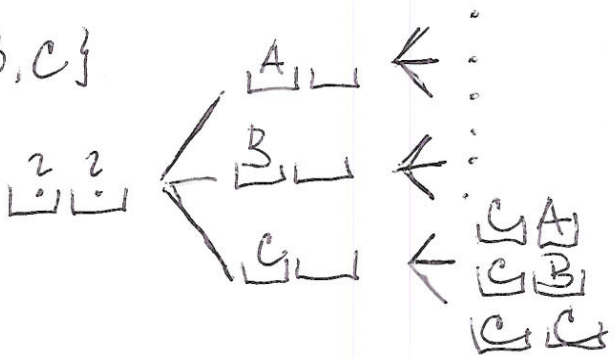
$$P^R(3; 2, 1) = \frac{3!}{2!\,1!} = \frac{\text{all permutations}}{\text{duplications}}$$

- **VARIATION W/ REP.**

$V^R(n, k) = n^k$

$V^R(3, 2) = 3^2$

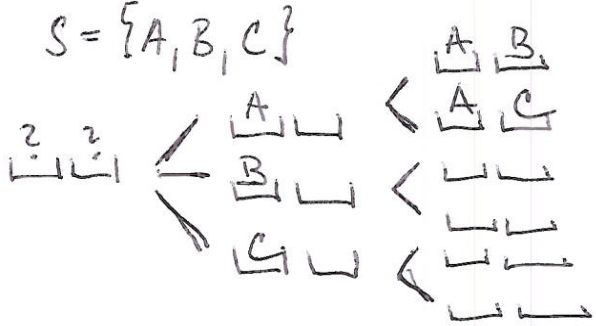
$S = \{A, B, C\}$



3×3

• VARIATION (W/O REP.)

$S = \{A, B, C\}$



$$V(n,k) = n(n-1)\cdot\ldots\cdot(n-k+1)$$

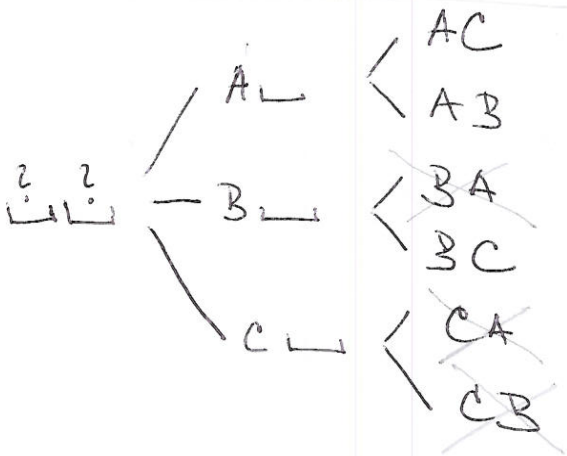
$$= \frac{n(n-1)\ldots(n-k+1)\cdot(n-k)\ldots 1!}{(n-k)\ldots 1!}$$

$$= \frac{n!}{(n-k)!}$$

• COMBINATION (W/O REP) $S = \{A, B, C\}$

$$C(n,k) = \frac{V(n,k)}{P(k)}$$

$$= \frac{n!}{(n-k)!\,k!} = \binom{n}{k}$$



variations / permutations of k

• COMBINATION W. REP

put 3 balls into 4 boxes

1	2	3	4
•••			
	•••		
		•••	
			•••
••	•		
••		•	
••			•

etc...

notation:

•••|||
|•••||
||•••|
|||•••
••|•||
••||•|
••|||•

3x•
3x|
from 6 choose 6 ⇒ permutation with repetition

k balls k x •
n boxes (n-1) x |
$\Rightarrow P(k+n-1, k, n-1)$

$$C^R(n,k) = P(k+n-1; k, n-1)$$

$$= \frac{(n+k-1)!}{k!\,(n-1)!}$$

$$= \binom{n+k-1}{k}$$

Exercises

Exercise 2-80 - Birthday Party

It is your birthday. However, you can only invite 6 of your 12 friends (all similar) to your party.

a) How many choices do you have to select the guests?

b) How many possible seating arrangements are there for your 6 guests at the birthday table ?

c) You have invited 3 male friends and 3 female friends. How many possible seating arrangements do you have when 3 males friends and 3 female friends re considered in each case as the same?

Exercise 1-80 - Trials

Trail are marked with signs consisting of 2 colored lines.

How many colors are required if

a) for 36 trails, where the order is concidered and the repetition is accepted?

b) for 21 trails, the order of in not considered and the repetition is not allowed?

c) for 15 trails, the order of in not considered and the repetition is allowed?

Exercise 3-1-Dice

The random experiment "throw a dice twice" was done.

Define the events: A={ 6 on the first throw } and A={ 6 on the second throw }

a) Set the sample space for this experiment.

b) Calculate the number of elementary events using combinatorics.

c) Define the events in the meaning of elementary events.

d) Determine the union and the intersection of the two events A and B.

e) Set the sample space, events A and B, their union and intersection in Venn-diagram.

f) Give an immpossible event of this random experiment.

g) Give the complementary events for A and B respectively.

h) Does $A \subset B$?

i) Are the events A and B disjoint?

Exercise 3-7- Non-disjoint subset

A, B and C are non-disjoint subsets of sample space S. Only with symbols of union, intersection, difference and complement of the event and also letters A,B, and C, write down the expressions of events:

a) at least one occurs

b) only A occurs

c) A and B occurs but not C

d) all three occur

e) none occurs

f) exactly one occurs

g) most two occur (no more than two)

Exercise 3-11

The random experiment consists of throwing a single ”ideal” dice. What is the probability for:

a) throwing 3

b) throwing 1 or 5

c) throwing an even number

d) What definition of probability did you applied?