Statistik II - Exercise session 1
15.10.2014 & 22.10.2014

Info

- Classroom: SPA1 203
- Time: Wednesdays, 16:15 - 17:45
- in English
- Assignments on webpage (lvb>staff>PB)

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Schedule:

Date	Week	Exercises
15.10.14	E1	4-11, 4-18, 4-19
22.10.14	E1	4-11, 4-18, 4-19
29.10.14	E2	TBA
5.11.14	E2	TBA
12.11.14	E3	TBA
19.11.14	E3	TBA
26.11.14	E4	TBA
03.12.14	E4	TBA
10.12.14	E5	TBA
17.12.14	E5	TBA
07.01.15	E6	TBA
14.01.15	E6	TBA
21.01.15	E7	TBA
28.01.15	–	–
04.02.15	E8	TBA
11.02.15	E8	TBA

Review

- week 1 & week 2
- Slides: random variables(1-45)

Conditional Probability $P(A|B) = \dfrac{P(A \cap B)}{P(B)}\ , \quad P(B) > 0$

Multiplication Rule

$$\begin{aligned} P(A \cap B) &= P(A) \cdot P(B|A) = P(B) \cdot P(A|B) \\ P(A_1 \cap A_2 \cap A_3) &= P(A_1) \cdot P(A_2|A_1) \cdot P(A_3|A_1 \cap A_2) \end{aligned}$$

Total Probability

$$\begin{aligned} P(B) &= P(A_1 \cap B) + P(A_2 \cap B) + \ldots + P(A_n \cap B) = \sum_{i=1}^{n} P(A_i \cap B) \\ &= P(B|A_1) \cdot P(A_1) + P(B|A_2) \cdot P(A_2) + \ldots + P(B|A_n) \cdot P(A_n) \\ &= \sum_{i=1}^{n} P(B|A_i) \cdot P(A_i) \end{aligned}$$

Distribution of discrete random variable

Probability mass function for discrete r.v.

$$\begin{aligned} P(X = x_i) &= f(x_i) \qquad (i = 1, 2, \ldots) \\ P(a \leq X \leq b) &= \sum_{a \leq x_i \leq b} P(X = x_i) = \sum_{a \leq x_i \leq b} f(x_i) \end{aligned}$$

Cumulative distr. function for discrete r.v.

$$F(x) = P(X \leq x) = \sum_{x_i \leq x} f(x_i)$$

Distribution of continuous random variable

Probability mass function for continuous r.v.

$$P(a < X \leq b) = \int_a^b f(x)\,dx \text{ for all} a, b \text{ such that } a \leq b$$

Cumulative distr. function for continuous r.v.

$$F(x) = P(-\infty < X \leq x) = \int_{-\infty}^{x} f(t)\,dt$$

Parameters of random variable

Expected value

discrete variable	$E(X) = \mu_X = \sum\limits_{i=1}^{k} x_i \cdot f(x_i)$
continuous variable	$E(X) = \mu_X = \int\limits_{-\infty}^{+\infty} x \cdot f(x) dx$

Properties of expected value:

$$E(a + b \cdot X) = a + b \cdot E(X) \text{ (}a, b\text{ constant)}$$
$$E(X \pm Y) = E(X) \pm E(Y)$$

Variance

discrete variable $Var(X) = \sigma_X^2 = \sum_{i=1}^{k}(x_i - \mu_X)^2 \cdot f(x_i) = \sum_{i=1}^{k} x_i^2 \cdot f(x_i) - \mu_X^2$

continuous variable $Var(X) = \sigma_X^2 = \int_{-\infty}^{+\infty}(x - \mu_X)^2 \cdot f(x)\,dx = \int_{-\infty}^{+\infty} x^2 \cdot f(x)\,dx - \mu_X^2$

Properties of variance:

$$Var(X) = E[\{X - E(X)\}^2] = E(X^2) - [E(X)]^2$$
$$Var(a + b \cdot X) = b^2 \cdot Var(X) \text{ (}a, b\text{ konstant)}$$
$$Var(X \pm Y) = Var(X) + Var(Y) \pm 2 \cdot Cov(X, Y)$$

Linear combination of random variable

Linear combination	$Z_1 = a \cdot X + b \cdot Y$	$Z_2 = a \cdot X - b \cdot Y$
Expected value	$E(Z_1) = a \cdot E(X) + b \cdot E(Y)$	$E(Z_2) = a \cdot E(X) - b \cdot E(Y)$
Variance	$Var(Z_1) = a^2 \cdot Var(X) + b^2 \cdot Var(Y) + 2 \cdot a \cdot b \cdot Cov(X, Y)$	
	$Var(Z_2) = a^2 \cdot Var(X) + b^2 \cdot Var(Y) - 2 \cdot a \cdot b \cdot Cov(X, Y)$	

Exercises

Exercise 4-11 - Production of Commodity

The company produces only one commodity and offers it for price of 6 EUR per piece. The quantity of sales per month is a random variable with $\mathrm{E}(X) = 1000$ pieces and $\mathrm{var}(X) = 500$ pieces^2. Costs are given by equation $Y = 250 + 3X$. Revenue as $Z = 6X$.

a) State the expected value and variance for monthly revenue.

b) State the expected value and variance for monthly costs.

c) State the expected value and variance for monthly profit.

Exercise 4-18 - Density

There is a function

$$f(x) = \begin{cases} ax^2(1-x) & 0 \le x \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

Determine the constant a that f is a density function of continuous random variable.

Exercise 4-19 - TV show

At TV-show "Who makes fool of himself" the candidate has to choose an answers to a question in every round. 5 possible answers are given, however, only one of them is correct. Questions in particular rounds do not depend on each other. When candidate gives a wrong answer, game is over and he obtains price corresponding to the last question, which was answered correctly. Profits are following:

Round 1: 100 EUR   Round 2: 200 EUR   Round 3: 300 EUR   Round 4: 400 EUR.

(i.e.: answers for 1st question is wrong, he obtains nothing)

What is the expected price (exp. avg. price) when he randomly chooses the answers in each round?