Statistik II - Exercise session 4
26.11.2014 & 03.12.2014

Info

- Classroom: SPA1 203
- Time: Wednesdays, 16:15 - 17:45
- in English
- Assignments on webpage (lvb>staff>PB)

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Schedule:

Date	Week	Exercises
15.10.14	E1	4-11, 4-18, 4-19
22.10.14	E1	4-11, 4-18, 4-19
29.10.14	E2	5-6, 5-10, 5-11, 5-14 ,5-16
5.11.14	E2	5-6, 5-10, 5-11, 5-14 ,5-16
12.11.14	E3	5-16, 5-21, 5-23, 6-1
19.11.14	E3	5-16, 5-21, 5-23, 6-1
26.11.14	E4	6-3, 6-9, 6-13, 7-1
03.12.14	E4	6-3, 6-9, 6-13, 7-1
10.12.14	E5	7-3, 7-5, 7-26, 7-45
17.12.14	E5	7-3, 7-5, 7-26, 7-45
07.01.15	E6	TBA
14.01.15	E6	TBA
21.01.15	E7	TBA
28.01.15	–	–
04.02.15	E8	Review for exam
11.02.15	E8	Review for exam

Review

- week 7 & week 8
- Slides:
  - Distributions (04_Verteilungsmodele)
  - Theory of sampling (05_Stichprobentheorie)

χ^2_n - distribution

If $X_1, \ldots, X_n$ are independent, standard normal random variables, then the sum of their squares $\sum_{i=1}^n X_i^2$ has χ^2 - distribution with n degrees of freedom $\sim \chi^2_n$.

Standard normal distribution

$$X \sim \mathcal{N}(\mu, \sigma) \quad \text{then } Z = \frac{X-\mu}{\sigma} \sim \mathcal{N}(0,1)$$

$$P(X \leq x) = P\left(\frac{X-\mu}{\sigma} \leq \frac{x-\mu}{\sigma}\right) = P\left(Z \leq \frac{x-\mu}{\sigma}\right) = \Phi\left(\frac{x-\mu}{\sigma}\right) = \Phi(z) = P(Z \leq z)$$

Central limit theorem

$X_1, X_2, \ldots, X_n$ are independent, identically distributed random variables with $\mathrm{E}(X_i) = \mu \neq \pm\infty$ and $\mathrm{Var}(X_i) = \sigma^2 < \infty$ (for $i = 1, \ldots, n$). Then random variable $S_n = \Sigma_i X_i$ has mean $E(S_n) = n\mu$ and variance $Var(S_n) = n\sigma^2$. Distribution of standardized random variable

$$Z_n = \frac{S_n - E(S_n)}{\sqrt{Var(S_n)}} = \frac{\sum\limits_{i=1}^n X_i - n \cdot \mu}{\sqrt{n \cdot \sigma^2}} = \frac{1}{\sqrt{n}} \sum_{i=1}^n \frac{X_i - \mu}{\sigma}$$

converges with increasing n to standard normal distribution:

$$\lim_{n \to \infty} P(Z_n \leq z) = \Phi(z).$$

Distributions of sample mean and variance

Sample variables	$E(X_i) = \mu, Var(X_i) = \sigma^2 \quad (i = 1, \ldots, n)$
Sample mean (function)	$\overline{X} = \frac{1}{n} \cdot \sum\limits_{i=1}^n X_i,\ E(\overline{X}) = \mu$

Assume: $X_i \sim N(\mu; \sigma)$ for $i = 1, \ldots, n$

If σ is known, $Z = \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} \sim \mathcal{N}(0,1)$

μ	Sample function	Exp.value	Distribution
known	$S^{*2} = \frac{1}{n} \cdot \sum\limits_{i=1}^n (X_i - \mu)^2$	$E(S^{*2}) = \sigma^2$	$\frac{n \cdot S^{*2}}{\sigma^2} \sim \chi^2_n$
unknown	$S^2 = \frac{1}{n-1} \cdot \sum\limits_{i=1}^n (X_i - \overline{X})^2$	$E(S^2) = \sigma^2$	$\frac{(n-1) \cdot S^2}{\sigma^2} \sim \chi^2_{n-1}$

Exercises

Exercise 6-3 - Pills for Headache

In pills for headache, the amount of the active ingredient is normally distributed. Since the lower amount of active ingredient does not work, but high amount causes side effects, the production must to be monitored. With the help of simple random sample, the average amount of active ingredient μ (in mg) is estimated. What is the probability that function $\bar{X}$ takes values, that are 0,5 mg more then the mean μ:

i) if $\sigma = 1$ mg and $n = 16$?

ii) if $\sigma = 1$ mg and $n = 64$?

iii) if $\sigma = 2$ mg and $n = 64$?

Exercise 6-9 - Tennis instructor An instructor offers 8 hours for training every day in a month (30 days). He has found out that one student per one lecture hits the ball over the court fence (and it will never return) with probability 0,1 for 2 or 7 balls and with probability 30% for 1 or 6 balls. None of his students hits 3,4, less than 1 or more than 7 balls over the fence.

a) What is the approximate probability that 900-1000 balls are hit over the fence in one month?

b) What is the approximate probability that mote then 1500 balls are hit over the fence in one month?

c) Calculate the (symmetric) interval for expected value so, that there will be 99% probability that ball-loss lies in boundaries.

d) Verify, that requirements of applied statistic given in this task.

Exercise 6-13 - Pills for Headache (part II.)

Based on Exercise 6-3, the variance σ^2 of the active ingredient is estimated. What is the probability that sample variance function is more than twice as large as true variance :

i) if the mean is known and $n = 7$?

ii) if the mean is known and $n = 16$?

iii) if the mean is unknown and $n = 16$?

Exercise 7-1 - Unbiasedness

Population has the mean μ and the variance σ^2. Let (X_1, X_2, X_3) be a simple (theoretical) random sample from this population. The following three estimators of μ are given:

$$\hat{\theta}_1 = \frac{1}{3}(X_1 + X_2 + X_3)$$

$$\hat{\theta}_2 = \frac{1}{4}(2X_1 + 2X_3)$$

$$\hat{\theta}_3 = \frac{1}{3}(2X_1 + X_3)$$

i) Which of these estimators are unbiased ?

ii) Which of them would you prefer acc. to criterium of effectiveness? Reason!