Localized Conditional Autoregressive Expectile Model

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Motivation

- Localizing parametric models
  - Adaptively forecasting the Chinese macroeconomy in transition
  - Local adaptive multiplicative error models for high-frequency forecasts - Härdle et al. (2014)
- Time-varying CARE parameters ▸ CARE ▸ Parameter Dynamics

Objectives

(i) Localizing CARE Models

- Local parametric approach (LPA), Spokoiny (1998)
- Balance between modelling bias and parameter variability

(ii) Forecasting Tail Risk

- Estimation windows with potentially varying lengths
- Time-varying expectile parameters

Outline

1. Motivation $\checkmark$
2. Conditional Autoregressive Expectile (CARE)
3. Local Parametric Approach (LPA)
4. Empirical Results
5. Conclusions

Conditional Autoregressive Expectile ▸ Motivation

- Taylor (2008), Kuan et al. (2009)
- CARE specification

$$y_t = e_{t,\tau} + \varepsilon_{t,\tau} \qquad \mathsf{E}\left[\varepsilon_\tau\right] = 0, \mathsf{Var}\left(\varepsilon_\tau\right) = \sigma^2_{\varepsilon,\tau}$$

▸ AND

$$e_{t,\tau} = \alpha_{0,\tau} + \alpha_{1,\tau} y_{t-1} + \alpha_{2,\tau}\left(y^+_{t-1}\right)^2 + \alpha_{3,\tau}\left(y^-_{t-1}\right)^2$$

- Expectile $e_{t,\tau}$ at $\tau \in (0,1)$, $\theta_\tau = \left\{\alpha_{0,\tau}, \alpha_{1,\tau}, \alpha_{2,\tau}, \alpha_{3,\tau}, \sigma^2_{\varepsilon,\tau}\right\}^\top$
- Returns: $y^+_{t-1} = \max\left\{y_{t-1}, 0\right\}$, $y^-_{t-1} = \min\left\{y_{t-1}, 0\right\}$

Parameter Estimation

- Data calibration with time-varying intervals
- Observed returns $y = \{y_t\}_{t=1}^{n}$
- Quasi maximum likelihood estimate (QMLE)

$$\widetilde{\theta}_{I,\tau} = \arg\max_{\theta_\tau \in \Theta} \ell_I\left(y; \theta_\tau\right)$$

- $I = [t_0 - m, t_0]$ - interval of $(m+1)$ observations at t_0
- $\ell_I\left(\cdot\right)$ - quasi log likelihood

Local Parametric Approach (LPA)

- LPA, Spokoiny (1998, 2009)
  - Time series parameters can be locally approximated
  - Finding the (longest) *interval of homogeneity*
  - Balance between modelling bias and parameter variability

- Time series literature
  - GARCH$(1,1)$ models - Čížek et al. (2009)
  - Realized volatility - Chen et al. (2010)
  - Multiplicative Error Models - Härdle et al. (2014)

Interval Selection

- $(K+1)$ nested intervals with length $n_k = |I_k|$

$$\begin{array}{ccccccc} I_0 & \subset & I_1 & \subset \cdots \subset & I_k & \subset \cdots \subset & I_K \\ \widetilde{\boldsymbol{\theta}}_0 & & \widetilde{\boldsymbol{\theta}}_1 & & \widetilde{\boldsymbol{\theta}}_k & & \widetilde{\boldsymbol{\theta}}_K \end{array}$$

Example: Daily index returns

Fix i_0, $I_k = [i_0 - n_k, i_0]$, $n_k = \left[n_0 c^k\right]$, $c > 1$

$\{n_k\}_{k=0}^{11} = \{20 \text{ days}, 25 \text{ days}, \ldots, 250 \text{ days } \}$, $c = 1.25$

Data

- Series
  - DAX and S&P500 returns, 19900102-20120531 (5849 days)
  - Research Data Center (RDC) - Bloomberg
- Setup
  - Expectile levels: $\tau = 0.05$ and $\tau = 0.01$
  - Interval lengths: 20, 60, 125 and 250 trading days

Parameter Dynamics ▸ Motivation

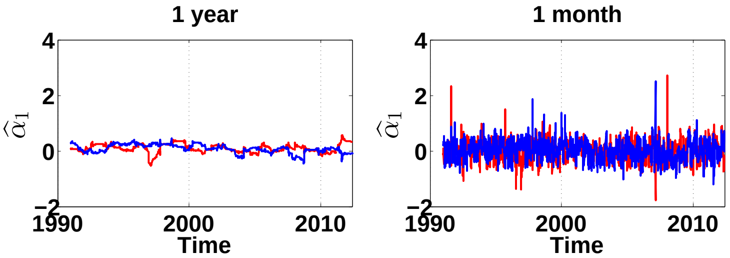

Figure 1: Estimated $\alpha_{1,0.05}$ for DAX and S&P500 using 20 (1 month) or 250 (1 year) observations

Parameter Dynamics ‣ Motivation

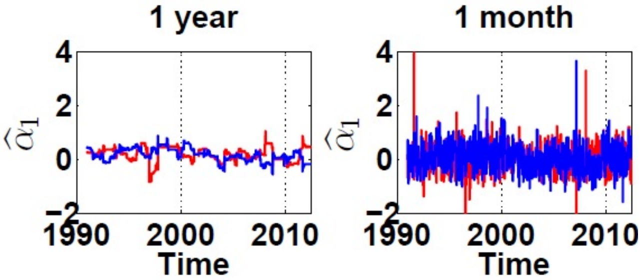

Figure 2: Estimated $\alpha_{1,0.01}$ for DAX and S&P500 using 20 (1 month) or 250 (1 year) observations

Parameter Distributions

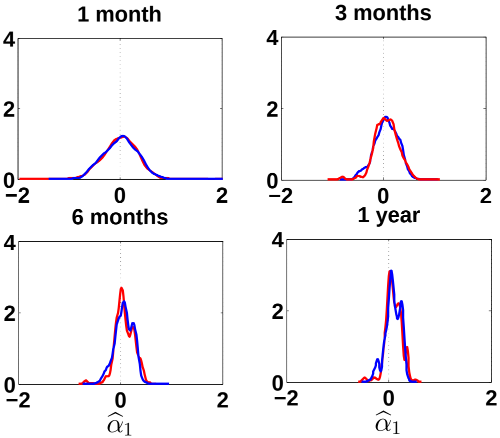

Figure 3: Kernel density estimates of $\alpha_{1,0.05}$ for DAX and S&P500 using 20, 60, 125 or 250 observations

Parameter Distributions

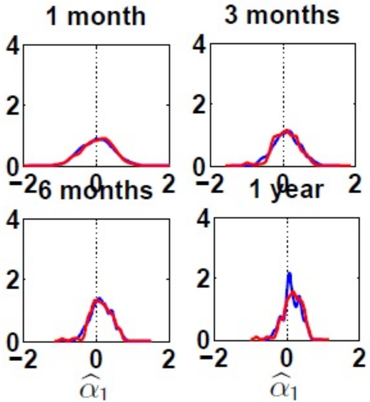

Figure 4: Kernel density estimates of $\alpha_{1,0.01}$ for DAX and S&P500 using 20, 60, 125 or 250 observations

Conclusions

(i) Localizing CARE Models

- Balance between modelling bias and parameter variability
- Parameter dynamics

(ii) Forecasting Tail Risk

- Varying distributional characteristics
- Expectile levels $\tau = 0.05$ and $\tau = 0.01$

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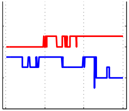

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Asymmetric Normal Distribution ▸ CARE

- If $\varepsilon_\tau \sim \mathrm{AND}\left(\mu, \omega_\varepsilon^2, \tau\right)$ with pdf

$$f(u) = \frac{1}{\omega_\varepsilon}\sqrt{\frac{2}{\pi}} \exp\left\{-\frac{(u-\mu)^2}{2\omega_\varepsilon^2}\right\} \varphi\left(\tau \frac{u-\mu}{\omega_\varepsilon}\right)$$

  - $\mathsf{E}[\varepsilon_\tau] = 0$ when $\mu = -\frac{\tau\omega_\varepsilon}{\sqrt{1+\tau^2}}\sqrt{\frac{2}{\pi}}$
  - $\mathsf{Var}(\varepsilon_\tau) = \sigma_{\varepsilon,\tau}^2 = \omega_\varepsilon^2\left\{1 - \frac{2}{\pi}\left(\frac{\tau}{\sqrt{1+\tau^2}}\right)^2\right\}$