Cross Country Evidence for the EPK Paradox

Maria Grith
Wolfgang Karl Härdle
Andrija Mihoci

Ladislaus von Bortkiewicz Chair of Statistics
C.A.S.E. – Center for Applied Statistics
and Economics
Humboldt–Universität zu Berlin
http://lvb.wiwi.hu-berlin.de
http://www.case.hu-berlin.de

Motivation

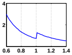
- Pricing kernel (PK), $\mathcal{K}$
  - Preference based models
    - Marginal rate of substitution (MRS)
  - Arbitrage free models
    - Radon-Nikodym derivative of the physical measure w.r.t. the risk neutral measure

- Empirical pricing kernel (EPK)
  - $\widehat{\mathcal{K}}$ any estimate of the PK
  - EPK paradox - locally increasing EPK

PK Estimation

- Indirect estimation, $\widehat{\mathcal{K}} = \dfrac{\widehat{q}}{\widehat{p}}$
  - q risk neutral density; p physical density
  - Aït-Sahalia and Lo (2000), Engle and Rosenberg (2002), Brown and Jackwerth (2004), Giacomini and Härdle (2008), Härdle et al. (2010), Grith et al. (2010)

- Direct estimation, $\widehat{\mathcal{K}} = G_{\widehat{\theta}}$
  - Parameter θ and given function G
  - Dittmar (2002), Schweri (2011)



EPK Paradox: Option Markets

Figure 1: EPK's across moneyness κ and maturity τ for DAX from 20010101 – 20011231, Giacomini and Härdle (2008)

EPK Paradox: Stock Markets

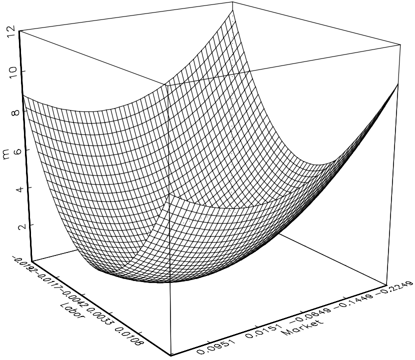


Figure 2: EPK for the US stock market. Data: returns of 20 industry-sorted portfolios from 19630731 to 19951231 with human capital (lagged labor income), Dittmar (2002)

Objectives

- Pricing Kernel (PK) estimation
  - State-dependent utility
  - Generalized Method of Moments (GMM)

- EPK paradox across equity markets
  - Time-varying EPK
  - Statistical properties

Research Questions

- Does empirical evidence from equity data suggest a locally increasing EPK?
- Are the estimation results significant?
- How do PK estimates vary across countries and over time?

Outline

1. Motivation ✓
2. Pricing Kernel (PK)
3. Generalized Method of Moments (GMM)
4. Empirical Results
5. Conclusion

Preference Based Model

- Representative agent with exogenous income ω_t
- Budget constraints

$$c_t = \omega_t - q_t^\top S_t \tag{1}$$

$$c_{t+1} = \omega_{t+1} + q_t^\top S_{t+1} \tag{2}$$

with consumption c_t, k assets, prices $S_t = (S_{1,t}, \dots, S_{k,t})^\top$, asset holdings $q_t = (q_{1,t}, \dots, q_{k,t})^\top$

- Returns $R_{t+1} = (S_{1,t+1}/S_{1,t}, \dots, S_{k,t+1}/S_{k,t})^\top$

State-Independent Preferences

- Utility maximization

$$\max_{c_{t+1}} \mathsf{E}_t \left[u \left(c_{t+1} \right) \right], \text{under constraints (1) and (2)}$$

with utility function of the representative agent $u(\cdot)$,
$\mathsf{E}_t[\cdot] = \mathsf{E}[\cdot \mid \mathcal{F}_t]$, $\mathcal{F}_t$ - information set up to t

State-Independent Preferences

- State-independent PK

$$\mathcal{K}_t\left(c_{t+1}\right) = \beta \frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)}$$

with discount factor β

State-Dependent Preferences

- Reference dependent utility function

  $$u\left(c_{t+1}, x\right) = u\left(c_{t+1}\right) \mathbf{I}\left\{x \in [0, x_0)\right\} + b u\left(c_{t+1}\right) \mathbf{I}\left\{x \in [x_0, \infty)\right\},$$

  with state variable x, reference point x_0 and parameter b

- Utility maximization

  $$\max_{c_{t+1}} \mathrm{E}_t\left[u\left(c_{t+1}, x\right)\right],$$ under constraints (1) and (2)

State-Dependent Preferences

- State-dependent PK

$$\mathcal{K}_t\left(c_t, c_{t+1}\right) = \beta_1 \frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)} \mathbf{I}\left\{x \in \left[0, x_0\right)\right\} + \\ + \beta_2 \frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)} \mathbf{I}\left\{x \in \left[x_0, \infty\right)\right\}$$

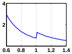
$\beta_1 = \beta$, $\beta_2 = \beta b$

Pricing Kernel

- Consumption growth is linear in the market portfolio gross return, Cochrane (2001)

  Define $c_{t+1} \stackrel{\text{def}}{=} r_{m,t+1} = S_{m,t+1}/S_{m,t}$
- State variable x is $r_{m,t+1}$

- Parameter $\theta = (\beta_1, \beta_2, x_0)^\top$
- Log utility, $u(y) = \log y$, $u'(y) = 1/y$



Pricing Kernel

- Pricing Kernel

$$
\begin{aligned}
\mathcal{K}_{\theta,t}\left(r_{m,t+1}\right) = \beta_1 r_{m,t+1}^{-1} \mathrm{I}\left\{r_{m,t+1} \in \left[0, x_0\right)\right\} + \\
+ \beta_2 r_{m,t+1}^{-1} \mathrm{I}\left\{r_{m,t+1} \in \left[x_0, \infty\right)\right\}
\end{aligned}
$$

$$
\begin{aligned}
S_t &= \mathsf{E}_t\left[\mathcal{K}_{\theta,t}\left(r_{m,t+1}\right) S_{t+1}\right] \\
1_k &= \mathsf{E}_t\left[\mathcal{K}_{\theta,t}\left(r_{m,t+1}\right) R_{t+1}\right]
\end{aligned}
$$

Generalized Method of Moments

- Hansen (1982), expectation of k moment conditions

$$\mathsf{E}_t\left[\mathcal{K}_{\theta,t}\left(r_{m,t+1}\right) R_{t+1} - 1_k\right] = 0_k$$

- Define $g\left(\theta\right) = \mathcal{K}_{\theta,t}\left(r_{m,t+1}\right) R_{t+1} - 1_k, \quad \mathsf{E}_t\left[g\left(\theta\right)\right] = 0_k$
- Sample: $g_n\left(\theta\right) = n^{-1}\sum_{t=0}^{n-1}\left\{\mathcal{K}_{\theta,t}\left(r_{m,t+1}\right) R_{t+1} - 1_k\right\}$

GMM Estimation

1. Iterated GMM

   Hansen and Singleton (1982), Ferson and Foerster (1994)

   $$\widetilde{\theta}_n \stackrel{\text{def}}{=} \arg\min_{\theta} \left\{ g_n^{\top}(\theta)\, g_n(\theta) \right\}, \quad \widetilde{W}_n = n^{-1} \sum_{t=0}^{n-1} g(\widetilde{\theta}_n) g(\widetilde{\theta}_n)^{\top}$$

   $$\widehat{\theta}_n \stackrel{\text{def}}{=} \arg\min_{\theta} \left\{ g_n^{\top}(\theta)\, \widetilde{W}_n^{-1} g_n(\theta) \right\}, \quad \widehat{W}_n = n^{-1} \sum_{t=0}^{n-1} g(\widehat{\theta}_n) g(\widehat{\theta}_n)^{\top}$$

2. GMM with Hansen-Jagannathan (HJ) weighting matrix
   Jagannathan and Wang (1996), Hansen and Jagannathan (1997), $\widetilde{W}_n = n^{-1} \sum_{t=0}^{n-1} R_t R_t^{\top}$

GMM Hypothesis Testing

- Newey and West (1987) - "D-test"
- Test statistic

$$D = n g_n^\top(\widetilde{\theta}_n)\widetilde{W}_n^{-1} g_n(\widetilde{\theta}_n) - n g_n^\top(\check{\theta}_n)\check{W}_n^{-1} g_n(\check{\theta}_n) \xrightarrow{\mathcal{L}} \chi^2_j,$$

with j parameter restrictions, two estimates $\widetilde{\theta}_n$ and $\check{\theta}_n$ with weighting matrices $\widetilde{W}$ and $\check{W}$, respectively

Data

- Markets: Australian Securities Exchange (AUS), Deutsche Börse (GER), Tokyo Stock Exchange (JPN), SIX Swiss Exchange (SUI), LSE (UK), NYSE (US)
- Span: 1 January 1990 - 31 May 2012 (daily data)
- Series: stock market indices, prices of 20 largest blue chips per market
- Windows: $n \in \{250 \text{ (1 year)}, 500 \text{ (2 years)}, 1250 \text{ (5 years)}\}$

PK Estimation

- Scenarios

  Case 1. $\beta_1, \beta_2 > 0$ - state-dependent, unconstrained

  Case 2. $\beta_2 > \beta_1 > 0$ - state-dependent, constrained

  Case 3. $\beta_1 = \beta_2 = \beta > 0$ - state-independent

Parameter Dynamics

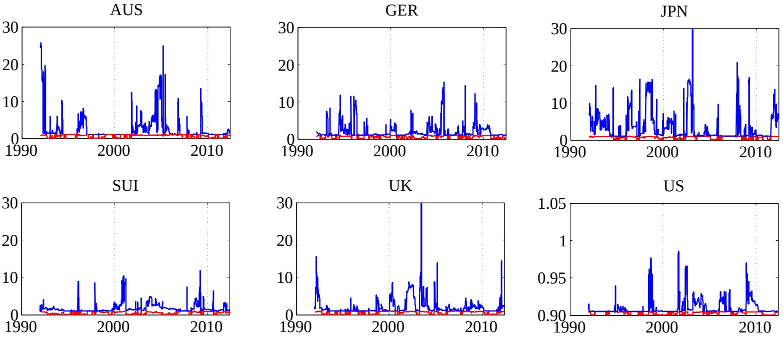


Figure 3: Time series of the estimated parameters β_1 and β_2 across six worldwide largest stock markets for case 2 ($\beta_2 > \beta_1 > 0$). We employ the iterated GMM estimation technique with $n = 500$ (2 years).

Reference Point Analysis

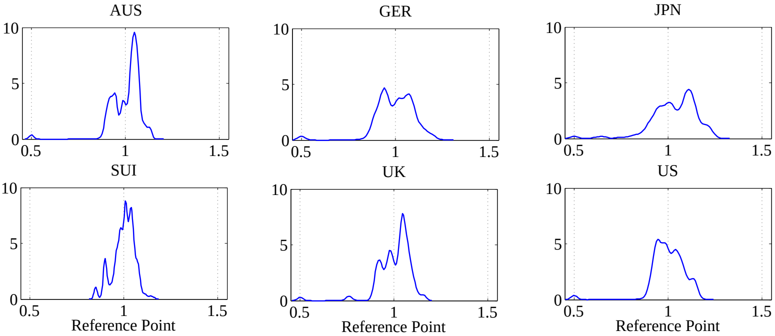
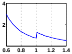


Figure 4: Kernel density plots (Gaussian kernel with optimal bandwidth) of optimal reference point x_0 for case 2 ($\beta_2 > \beta_1 > 0$). We employ the iterated GMM estimation technique with $n = 500$ (2 years).



Empirical Pricing Kernels

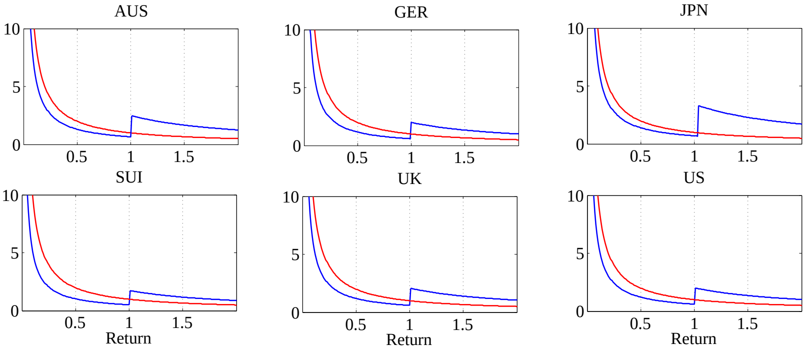
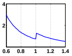


Figure 5: Empirical pricing kernels across six worldwide largest stock markets (for average parameter values): case 1, $\beta_1, \beta_2 > 0$ and case 2, $\beta_1 = \beta_2 = \beta$.



Hypothesis Testing

	Iterated GMM			GMM with HJ matrix		
	1 year	2 years	5 years	1 year	2 years	5 years
AUS	76.32	79.49	67.76	68.64	69.88	70.21
GER	89.94	88.99	81.76	81.55	84.27	86.30
JPN	84.22	83.02	83.15	83.60	84.67	76.93
SUI	92.06	88.47	87.14	85.21	79.77	80.62
UK	82.13	86.43	79.26	86.20	73.61	81.32
US	78.16	75.92	74.85	70.44	52.64	54.81

Table 1: Percentage of rejections of the null hypothesis of the D-test ($H_0 : \beta_1 = \beta_2 = \beta$) as indicator for the existence of the EPK paradox across the worldwide largest six stock markets.

Germany: EPK Dynamics

Figure 6: EPK on the German stock market in 2005.

Conclusion

- Pricing Kernel (PK) estimation
  - State-dependent utility admits PK nonmonotonicity
  - GMM successfully used for estimation and hypothesis testing

- EPK paradox across equity markets
  - Time-varying preferences
  - Optimal reference point slightly above 1
  - Statistically significant results

Future Research

- Distribution of reference points
- Cross-country estimation
- Statistical arbitrage
- Option and equity data

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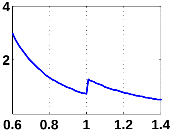

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References

Cochrane, J.H.
Asset Pricing
Princeton University Press, 2001

Dittmar, R.F.
Nonlinear Pricing Kernels, Kurtosis Preference, and Evidence from the Cross Section of Equity Returns
Journal of Finance **62**(1): 369–403, 2002

Ferson, W.E. and Foerster, S.R.
Finite sample properties of the Generalized Method of Moments in tests of conditional asset pricing models
Journal of Financial Economics **36**: 29–55, 1994

References

Giacomini, E. and Härdle, W. K.
Dynamic Semiparametric Factor Models in Pricing Kernel Estimation
in Functional and Operational Statistics, Dabo-Niang, S. and Ferraty, F. (Eds), Contributions to Statistics, Springer Verlag ISBN 978-3-7908-2061-4. 181–187, 2008

Hansen, L.P.
Large sample properties of generalized method of moments estimators
Econometrica **50**: 1029–1054, 1982

References

Hansen, L.P. and Jagannathan, R.
Assessing specification errors in stochastic discount factor models
Journal of Finance **52**: 557–590, 1997

Hansen, L.P. and Singleton, K.J.
Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models
Econometrica **50**: 1269–1286, 1982

Jagannathan, R. and Wang, Z.
The Conditional CAPM and the Cross-Section of Expected Returns
Journal of Finance **51**(1): 3–53, 1996

References

Newey, W.K. and West, K.D.
Hypothesis Testing with Efficient Method of Moments Estimation
International Economic Review **28**(3): 777–787, 1987

Risk Neutral Valuation

▸ Motivation

- Present value of the payoffs $\psi(S_T)$

$$P_0 = \mathsf{E}_Q\left[e^{-Tr}\psi(s_T)\right] = \int_0^\infty e^{-Tr}\psi(s_T)\,\mathcal{K}(s_T)p(s_T)\,ds_T$$

r risk free interest rate, $\{S_t\}_{t\in[0,T]}$ stock price process,
p pdf of S_T, Q risk neutral measure, $\mathcal{K}(\cdot)$ pricing kernel

PK under the Black-Scholes Model

▸ Motivation

- Geometric Brownian motion for S_t

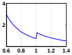
$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t$$

μ mean, σ volatility, W_t Wiener process

- Physical density p is log-normal, $\tau = T - t$

$$p_t(S_T) = \frac{1}{S_T\sqrt{2\pi\sigma^2\tau}} \exp\left[-\frac{1}{2}\left\{\frac{\log(S_T/S_t) - \left(\mu - \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}\right\}^2\right]$$

- Risk neutral density q is log-normal: replace μ by r



PK under the Black-Scholes Model

▸ Motivation

- PK is a decreasing function in S_T for fixed S_t

$$\mathcal{K}(S_t, S_T) = \left(\frac{S_T}{S_t}\right)^{-\frac{\mu - r}{\sigma^2}} \exp\left\{\frac{(\mu - r)\left(\mu + r - \sigma^2\right)\tau}{2\sigma^2}\right\}$$

$$= b\left(\frac{S_T}{S_t}\right)^{-\delta}$$

$b = \exp\left\{\frac{(\mu - r)\left(\mu + r - \sigma^2\right)\tau}{2\sigma^2}\right\}$ and $\delta = \frac{\mu - r}{\sigma^2} \geq 0$ constant relative risk aversion (CRRA) coefficient