Cross Country Evidence for the EPK Paradox

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EPK Paradox: Option Markets

Figure 1: EPK's across moneyness κ and maturity τ for DAX from 20010101 – 20011231, Giacomini and Härdle (2008)

EPK Paradox: Stock Markets

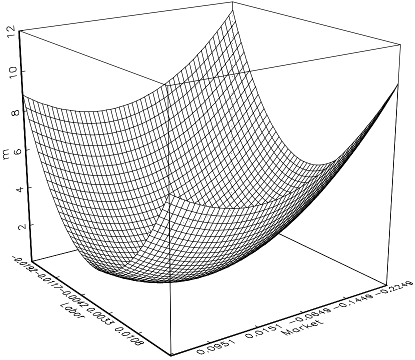


Figure 2: EPK for the US stock market. Data: returns of 20 industry-sorted portfolios from 19630731 to 19951231 with human capital (lagged labor income), Dittmar (2002)

Objectives

(i) Pricing Kernel (PK) estimation
- State-dependent utility
- Generalized Method of Moments (GMM)

(ii) EPK paradox across stock markets
- Time-varying EPK
- Statistical significance

Research Questions

- How well does the proposed methodology explain the EPK paradox (i.e., locally increasing EPK)?
- How strong is the variation of the EPK (parameters) over time and across countries?
- Are the results statistically significant?

Outline

1. Motivation ✓
2. Pricing Kernel (PK)
3. Generalized Method of Moments (GMM)
4. Empirical Results
5. Conclusion

Consumption Based Model

- Representative agent with exogenous income ω_t
- Budget constraints

$$c_t = \omega_t - q_t^\top S_t \tag{1}$$

$$c_{t+1} = \omega_{t+1} + q_t^\top S_{t+1} \tag{2}$$

Consumption c_t, k assets (stocks)

Asset holdings $q_t = (q_{1,t}, \dots, q_{k,t})^\top$, prices $S_t = (S_{1,t}, \dots, S_{k,t})^\top$

Utility

- State-independent utility

$$u(c_t, c_{t+1}) = u(c_t) + \beta \mathsf{E}_t \left[u(c_{t+1}) \right]$$

Preference parameter β, $\mathsf{E}_t[\bullet] = \mathsf{E}[\bullet \mid \mathcal{F}_t]$, $\mathcal{F}_t$ - information set up to t

- State-dependent utility

$$\begin{aligned} u(c_t, c_{t+1}) = {} & u(c_t) + \beta_1 \mathsf{E}_t \left[u(c_{t+1}) \right] \mathbf{I}\{c_t \in [0, x_0)\} \\ & + \beta_2 \mathsf{E}_t \left[u(c_{t+1}) \right] \mathbf{I}\{c_t \in [x_0, \infty)\} \end{aligned}$$

Preference parameters β_1 and β_2, reference point x_0

Utility Maximization

$$\max_{c_t, c_{t+1}} u(c_t, c_{t+1}), \text{under constraints (1) and (2)}$$

$$S_t = \mathsf{E}_t \left[\mathcal{K}_\theta(c_t, c_{t+1}) S_{t+1}\right] \tag{3}$$

with state-dependent PK given parameter $\theta = (x_0, \beta_1, \beta_2)^\top$

$$\mathcal{K}_\theta(c_t, c_{t+1}) = \beta_1 \frac{u'(c_{t+1})}{u'(c_t)} \mathbf{I}\{c_t \in [0, x_0)\} + \\ + \beta_2 \frac{u'(c_{t+1})}{u'(c_t)} \mathbf{I}\{c_t \in [x_0, \infty)\}$$

Pricing Kernel

- Log utility $u(x) = \log x$, $\dfrac{u'(c_{t+1})}{u'(c_t)} = \left(\dfrac{c_{t+1}}{c_t}\right)^{-1}$
- Power utility, $u(x) = \frac{x^\gamma}{\gamma}$, $\dfrac{u'(c_{t+1})}{u'(c_t)} = \left(\dfrac{c_{t+1}}{c_t}\right)^{\gamma-1}$
- Consumption growth is linear in the market portfolio gross return $r_{m,t+1} = S_{m,t+1}/S_{m,t}$, Cochrane (2001)

Pricing Kernel

- State-dependent PK in the market portfolio gross return

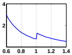
$$\mathcal{K}_\theta\left(r_{m,t+1}\right) = \beta_1 r_{m,t+1}^{-1} \mathsf{I}\left\{r_{m,t+1} \in [0, x_0)\right\} + \\ + \beta_2 r_{m,t+1}^{-1} \mathsf{I}\left\{r_{m,t+1} \in [x_0, \infty)\right\}$$

$$S_t = \mathsf{E}_t\left[\mathcal{K}_\theta\left(r_{m,t+1}\right) S_{t+1}\right]$$

- Rewrite

$$\mathcal{K}_\theta(x) = \left[\frac{x}{\{1 - F(x)\}\beta_1 + F(x)\beta_2}\right]^{-1}$$

$F(x) = 0$ if $x \leq x_0$, $F(x) = 1$ if $x > x_0$



Generalized Method of Moments

- Hansen (1982), expectation of k moment conditions

$$\mathsf{E}_t \left[\mathcal{K}_\theta \left(r_{m,t+1} \right) R_{t+1} - 1_k \right] = 0_k$$

Stock returns $R_{t+1} = \left(S_{1,t+1}/S_{1,t}, \ldots, S_{k,t+1}/S_{k,t} \right)^\top$

- Define $g\left(\theta\right) = \mathcal{K}_\theta \left(r_{m,t+1} \right) R_{t+1} - 1_k, \quad \mathsf{E}_t \left[g\left(\theta\right) \right] = 0_k$
- Sample: $g_n\left(\theta\right) = n^{-1} \sum_{t=0}^{n-1} \left\{ \mathcal{K}_\theta \left(r_{m,t+1} \right) R_{t+1} - 1_k \right\}$

GMM Estimation

1. Iterated GMM

   Hansen and Singleton (1982), Ferson and Foerster (1994)

   $$\widetilde{\theta}_n \stackrel{\text{def}}{=} \arg\min_{\theta} \left\{ g_n^{\top}(\theta)\, g_n(\theta) \right\}, \quad \widetilde{W}_n = n^{-1} \sum_{t=0}^{n-1} g(\widetilde{\theta}_n) g(\widetilde{\theta}_n)^{\top}$$

   $$\widehat{\theta}_n \stackrel{\text{def}}{=} \arg\min_{\theta} \left\{ g_n^{\top}(\theta)\, \widetilde{W}_n^{-1} g_n(\theta) \right\}, \quad \widehat{W}_n = n^{-1} \sum_{t=0}^{n-1} g(\widehat{\theta}_n) g(\widehat{\theta}_n)^{\top}$$

2. GMM with Hansen-Jagannathan (HJ) weighting matrix
   Jagannathan and Wang (1996), Hansen and Jagannathan (1997), $\widetilde{W}_n = n^{-1} \sum_{t=0}^{n-1} R_t R_t^{\top}$

GMM Hypothesis Testing

- Newey and West (1987) - "D-test"
- Test statistic

$$D = n g_n^\top(\widetilde{\theta}_n)\widetilde{W}_n^{-1} g_n(\widetilde{\theta}_n) - n g_n^\top(\check{\theta}_n)\check{W}_n^{-1} g_n(\check{\theta}_n) \xrightarrow{\mathcal{L}} \chi_j^2$$

number of parameter restrictions j, estimates $\widetilde{\theta}_n$ and $\check{\theta}_n$, weighting matrices $\widetilde{W}$ and $\check{W}$

Data

- Markets: Australian Securities Exchange (AUS), Deutsche Börse (GER), Tokyo Stock Exchange (JPN), SIX Swiss Exchange (SUI), LSE (UK), NYSE (US)
- Span: 1 January 1990 - 31 May 2012 (daily data)
- Series: stock market indices, prices of 20 largest blue chips per market
- Windows: $n \in \{250 \text{ (1 year)}, 500 \text{ (2 years)}, 1250 \text{ (5 years)}\}$

PK Estimation

$$\mathcal{K}_\theta(x) = \left[\frac{x}{\{1 - F(x)\}\,\beta_1 + F(x)\,\beta_2} \right]^{-1}$$

- Scenarios

  Case 1. $\beta_1, \beta_2 > 0$ - state-dependent, unconstrained

  Case 2. $\beta_2 > \beta_1 > 0$ - state-dependent, constrained

  Case 3. $\beta_1 = \beta_2 = \beta > 0$ - state-independent

Parameter Dynamics

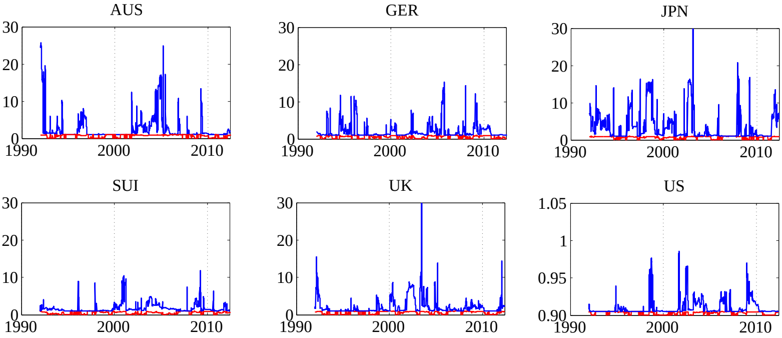


Figure 3: Time series of the estimated parameters β_1 and β_2 across six worldwide largest stock markets for case 2 ($\beta_2 > \beta_1 > 0$). We employ the iterated GMM estimation technique with $n = 500$ (2 years).

Reference Point Analysis

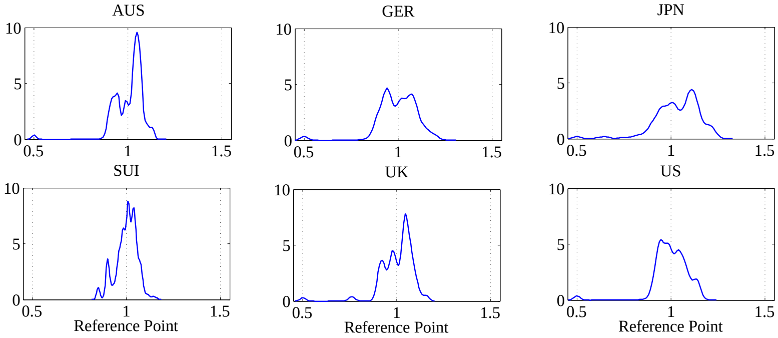


Figure 4: Kernel density plots (Gaussian kernel with optimal bandwidth) of optimal reference point x_0 for case 2 ($\beta_2 > \beta_1 > 0$). We employ the iterated GMM estimation technique with $n = 500$ (2 years).

Empirical Pricing Kernels

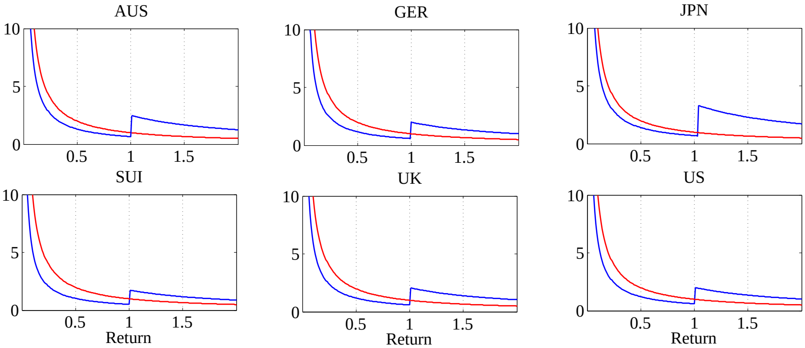


Figure 5: Empirical pricing kernels across six worldwide largest stock markets (for average parameter values): case 1, $\beta_1, \beta_2 > 0$ and case 2, $\beta_1 = \beta_2 = \beta$.

Hypothesis Testing

	Iterated GMM			GMM with HJ matrix		
	1 year	2 years	5 years	1 year	2 years	5 years
AUS	76.32	79.49	67.76	68.64	69.88	70.21
GER	89.94	88.99	81.76	81.55	84.27	86.30
JPN	84.22	83.02	83.15	83.60	84.67	76.93
SUI	92.06	88.47	87.14	85.21	79.77	80.62
UK	82.13	86.43	79.26	86.20	73.61	81.32
US	78.16	75.92	74.85	70.44	52.64	54.81

Table 1: Percentage of rejections of the null hypothesis of the D-test ($H_0 : \beta_1 = \beta_2 = \beta$) as indicator for the existence of the EPK paradox across the worldwide largest six stock markets.

Germany: EPK Dynamics

Figure 6: EPK on the German stock market in 2005.

Conclusion

(i) Pricing Kernel (PK) estimation

- State-dependent utility admits PK nonmonotonicity
- GMM successfully used for estimation and hypothesis testing

(ii) EPK paradox across stock markets

- Time-varying preferences
- Optimal reference point slightly above 1
- Statistically significant results

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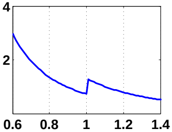

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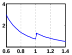

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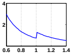

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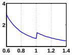

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