

Statistical Analysis of Neuroeconomic Data

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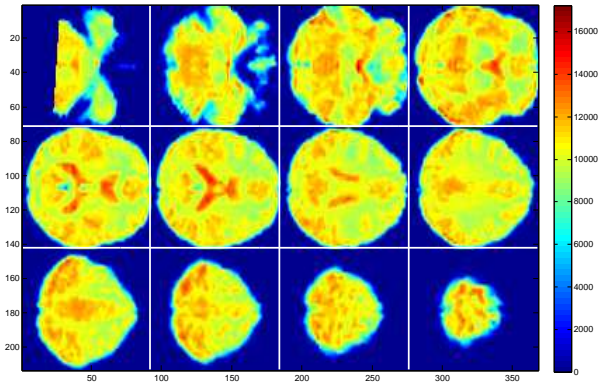


Motivation

- Which part of our brain is activated during *risky decisions* ?
- Can statistical analysis help to detect this area?



Different brain visualization



Motivation

- search for neuro-physiological analogue to the specification of risk type
- massive data set from experiments
 - ▶ statistical analysis necessary
 - ▶ dimension reduction keeping the data structure
 - ▶ **time** consideration (DSFM)



Outline

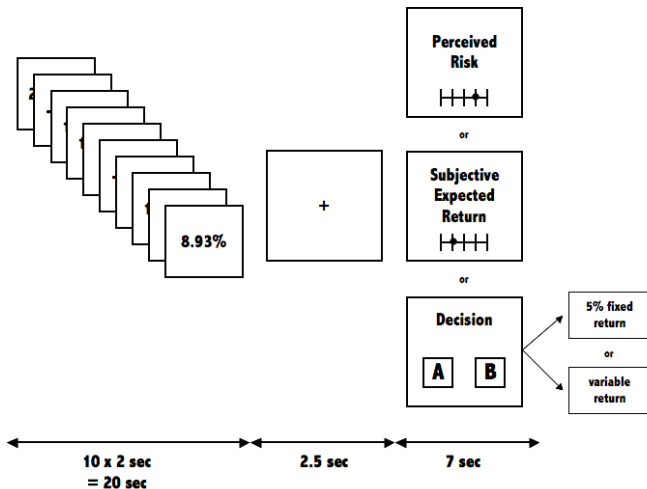
1. Motivation ✓
2. Experiment
3. Statistical Model
4. Results
5. Future Perspectives



Experiment participants

- ▣ 20 volunteers (age 18-35 years)
- ▣ 11 females, 9 males
- ▣ native German speakers, right-handed (according to the Edinburgh Handedness Inventory)
- ▣ no history of neurological or psychiatric diseases
- ▣ flat payment (10 EUR) \pm outcome resulting from the participant's decision
- ▣ 2 participants excluded due to extensive head motion and modeling problems





Risk Perception and Investment Decision (RPID)



RPID task in each trial

1. presentation of a return stream (rs)
 - ▶ ten returns from an investment (each for 2 sec)
 - ▶ each rs independent of the others
 - ▶ 9 different combinations of means (6%, 9%, 12%) and standard deviations (1%, 5%, 9%)



RPID task

2. decision *or* subjective judgment task (chosen randomly)
- ▶ choice between an investment with 5% fixed return (safe investment) and the investment represented by the r_s (risky investment)
 - ▶ subjective expected return judge with range: -5% – 15%
 - ▶ perceived risk judge on scale: 0 (no risk) – 100 (maximum risk)

Altogether: 81 trials (3 tasks 27 times) in 57 mins.



fMRI Acquisition

- fMRI = functional Magnetic Resonance Imaging
- noninvasive technique of recording brain's signals
- BOLD (blood oxygenation level dependent)-sensitive imaging
- 1.5 T Magnetom Sonata MRI system (Siemens)
- 26 axial slices of 4mm thickness



Data Set

Series of 3-dim images

- each scan transformed on the resolution $2 \times 2 \times 2mm^3$
- 91 slices
- observed every 2.5 seconds
- data set: series of 1360 images with $91 \times 109 \times 91$ voxels

High-dimensional, high frequency data.



Dynamic Semiparametric Factor Model (DSFM)

$$Y_{t,j} = m_0(X_{t,j}) + \sum_{l=1}^L Z_{t,l} m_l(X_{t,j}) + \varepsilon_{t,j}, \quad 1 \leq j \leq J.$$

$$X_{t,j} = (X_{t,1}, \dots, X_{t,J})^\top$$

observable covariates defined on \mathbb{R}^d

$$Y_{t,j} = (Y_{t,1}, \dots, Y_{t,J})^\top$$

observable random vector

$$Z_{t,j} = (Z_{t,1}, \dots, Z_{t,L})^\top$$

unobservable L -dimensional process

$$(m_0, \dots, m_L)$$

unknown real-valued functions defined on a subset of \mathbb{R}^d

$$\varepsilon_{t,j} \sim (0, \sigma_{t,j}^2)$$

errors with $\sigma_{t,j}^2 < \infty$



Fitting fMRI Data

- cut off parts of images without brain scan
- reduction of the original data by taking every second slice in each direction
- voxel's index (i_1, i_2, i_3) as covariate $X_{t,j}$
- BOLD signal as $Y_{t,j}$
- then $J_t = 36 \times 46 \times 46$ and $T = 1360$



Estimation of DSFM

- choose $K = 7 \times 8 \times 8 = 448$ parabolic tensor B-splines to estimate \hat{m}
- set $L = 3$

$$1 - RV(L) = \frac{\sum_t^T \sum_j^{J_t} \{Y_{t,j} - \hat{m}_0(X_{t,j}) - \sum_{l=1}^L \hat{Z}_{t,l} \hat{m}_l(X_{t,j})\}^2}{\sum_t^T \sum_j^{J_t} (Y_{t,j} - \bar{Y})^2}$$

No.of factors	$L = 1$	$L = 2$	$L = 3$	$L = 4$	$L = 5$
$1 - RV(L)$	0.8863	0.8864	0.9049	0.8868	0.8869



Fixed effect model

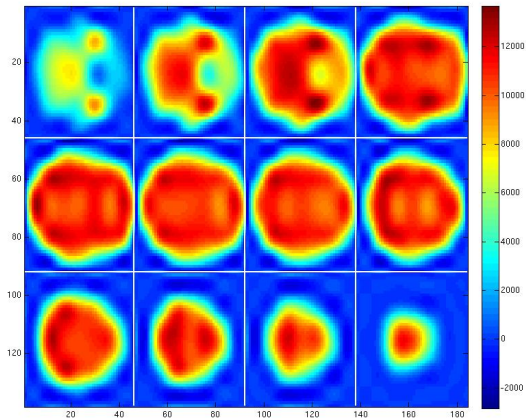
- to obtain a general basis function
- assume fixed effects μ_i for individual i with $\sum_{i=1}^n \mu_i := 0$
- for individual i is then:

$$Y_{t,j}^i = m_0(X_{t,j}) + \sum_{l=1}^L Z_{t,l}^i m_l(X_{t,j}) + \mu_i + \varepsilon_{t,j}^i$$

- and for the average individual:

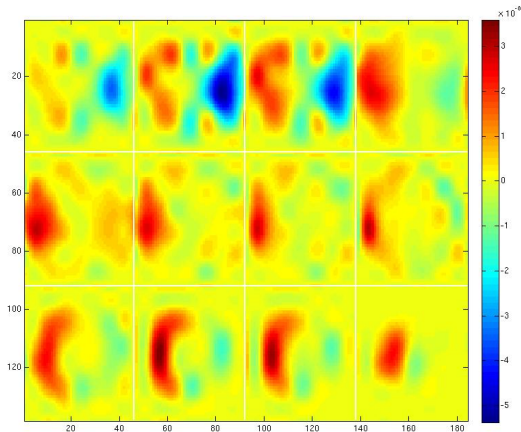
$$\bar{Y}_{t,j} = m_0(X_{t,j}) + \sum_{l=1}^L \bar{Z}_{t,l} m_l(X_{t,j}) + \varepsilon_{t,j}$$





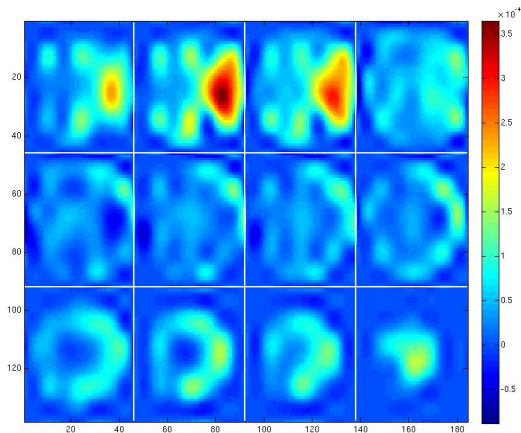
Estimated factor loading \hat{m}_0 with $L = 2$





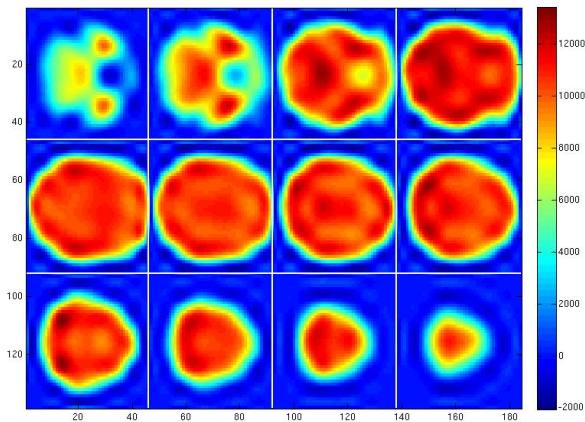
Estimated factor loading \hat{m}_1 with $L = 2$





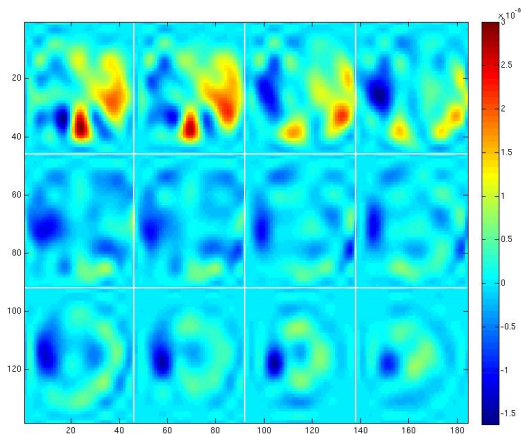
Estimated factor loading \hat{m}_2 with $L = 2$





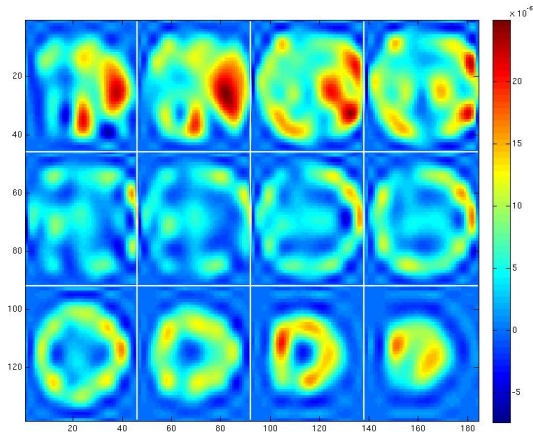
Estimated factor loading \hat{m}_0 with $L = 3$





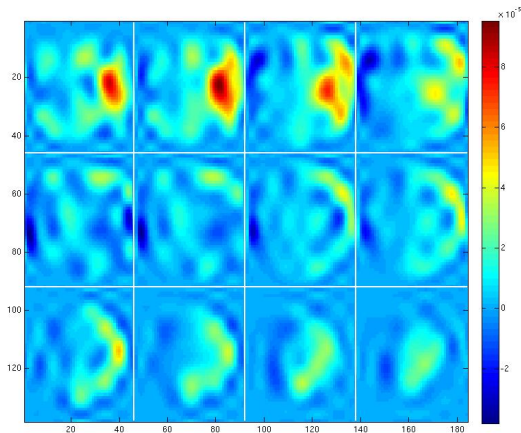
Estimated factor loading \hat{m}_1 with $L = 3$





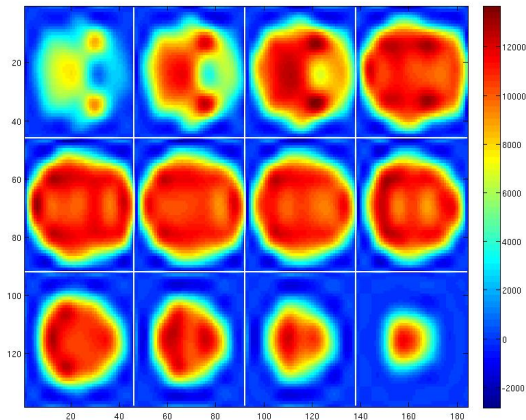
Estimated factor loading \hat{m}_2 with $L = 3$





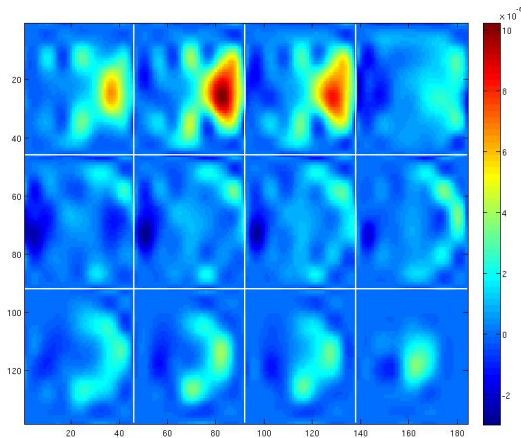
Estimated factor loading \hat{m}_3 with $L = 3$





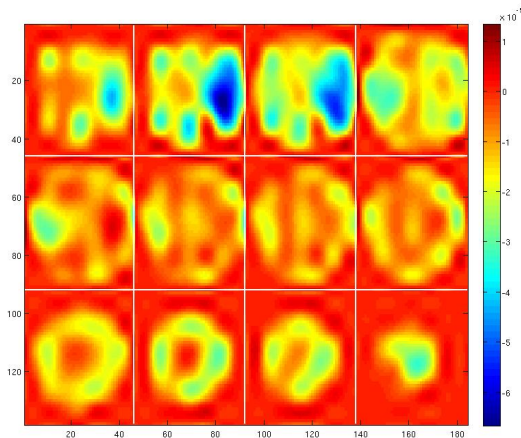
Estimated factor loading \hat{m}_0 with $L = 4$





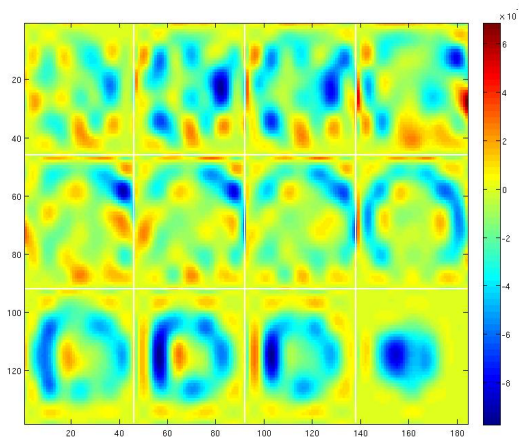
Estimated factor loading \hat{m}_1 with $L = 4$





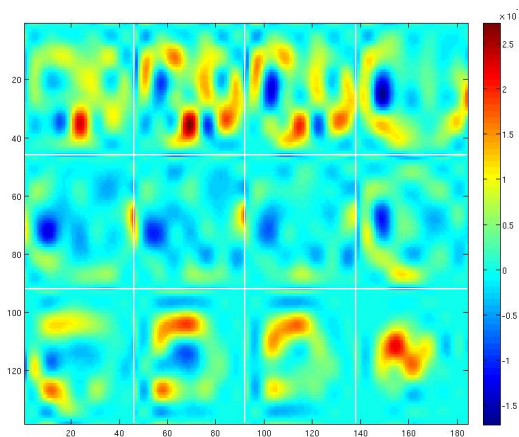
Estimated factor loading \hat{m}_2 with $L = 4$





Estimated factor loading \hat{m}_3 with $L = 4$

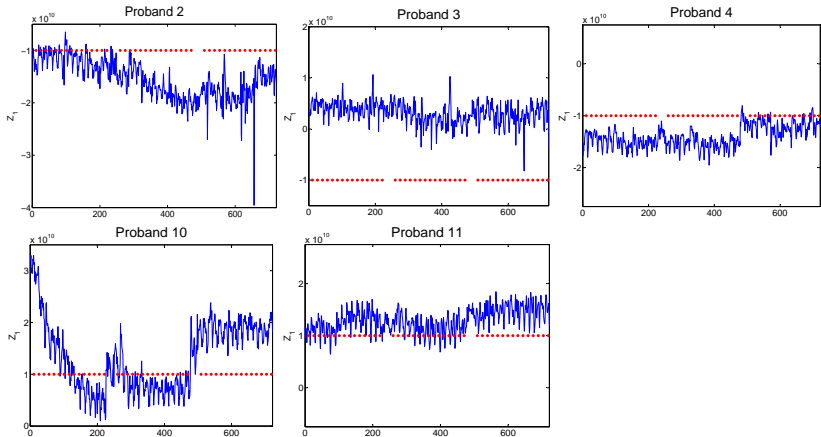




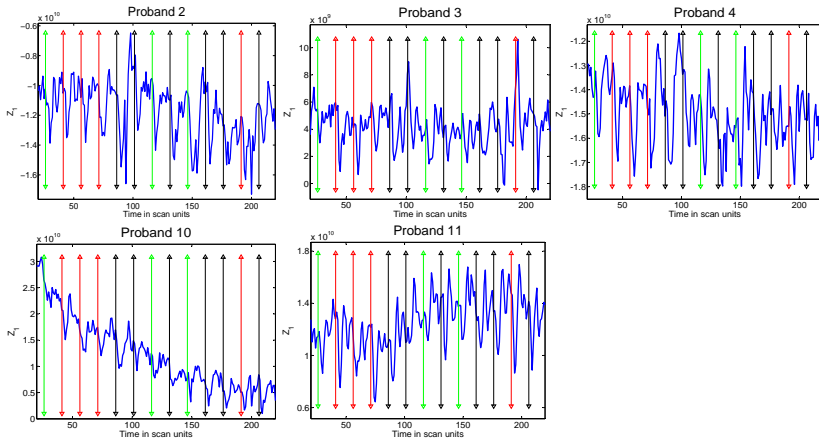
Estimated factor loading \hat{m}_4 with $L = 4$



Factor \hat{Z}_1

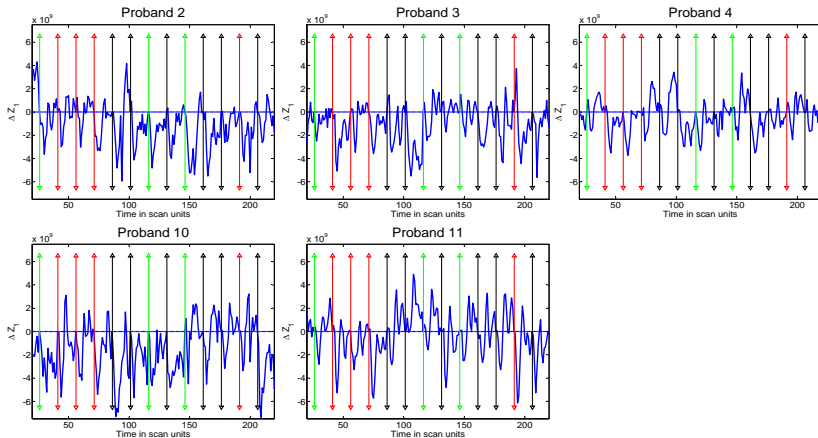


Factor \hat{Z}_1



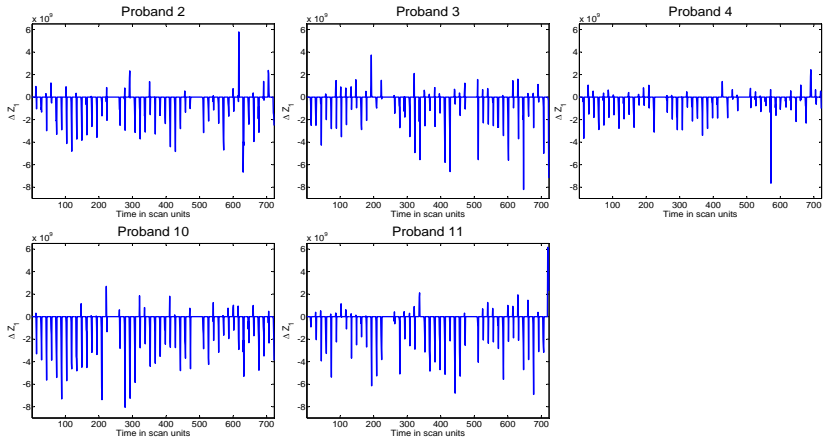
Lines correspond to the time points of judgement tasks: **decision**, **return**, **risk**.

Reaction to stimuli in factor \hat{Z}_1

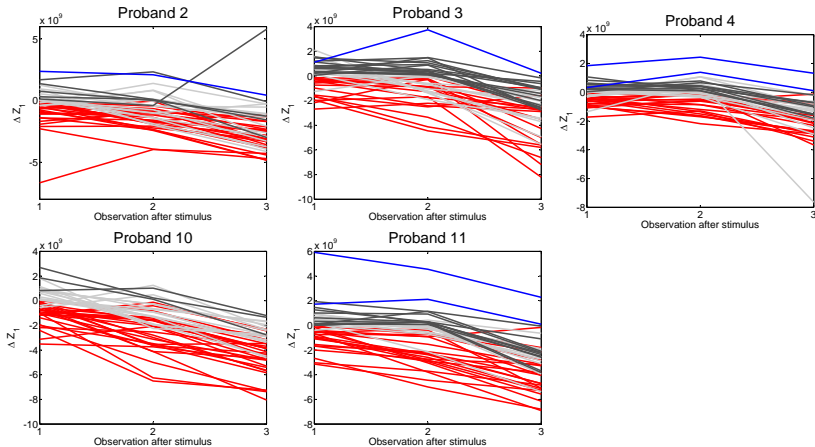


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Reaction to stimuli in factor \hat{Z}_1

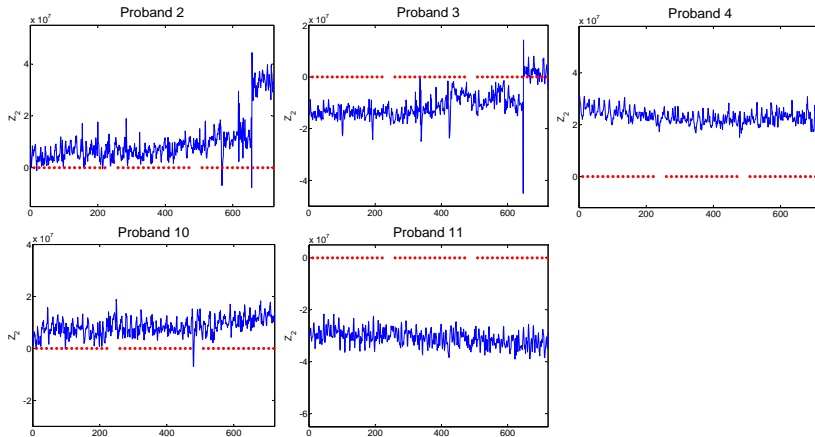


Reaction to stimuli in factor \hat{Z}_1

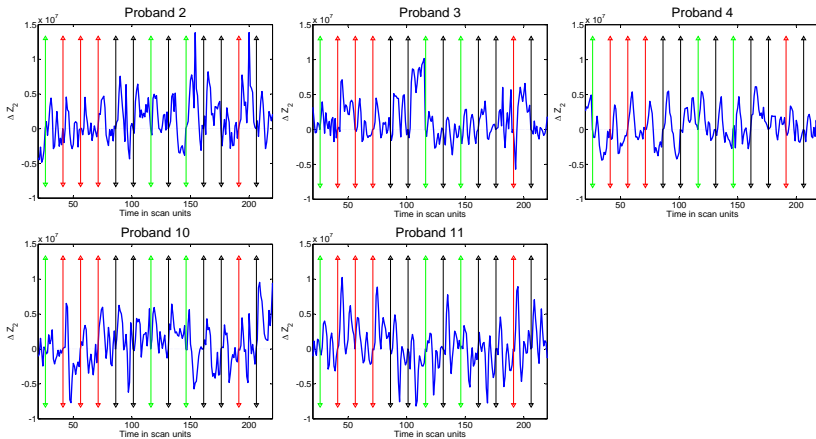


blue: all 3 points > 0 , light gray: 2 points > 0 , gray: 1 point > 0 , red:
all 3 points ≤ 0 .

Factor \hat{Z}_2

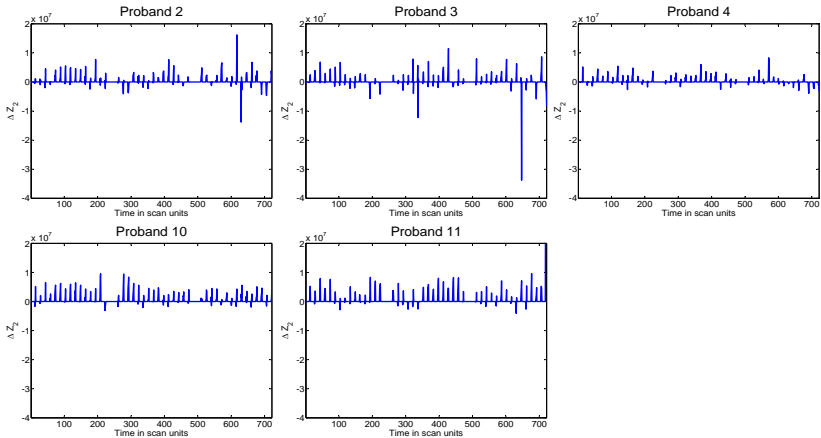


Reaction to stimuli in factor \hat{Z}_2

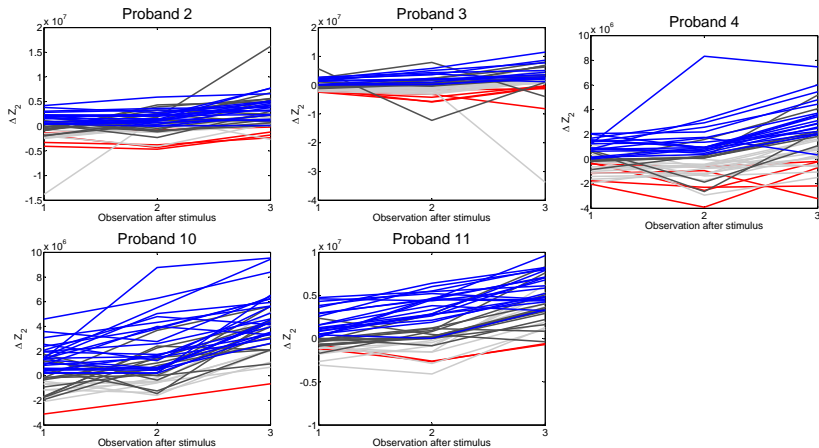


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Reaction to stimuli in factor \hat{Z}_2

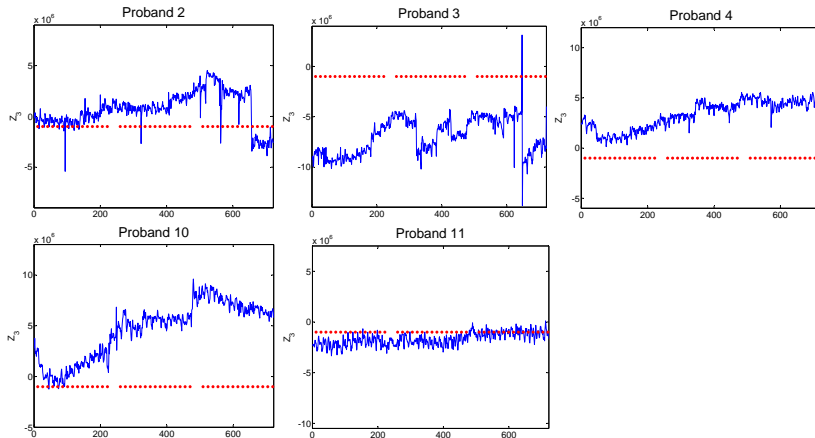


Reaction to stimuli in factor \hat{Z}_2

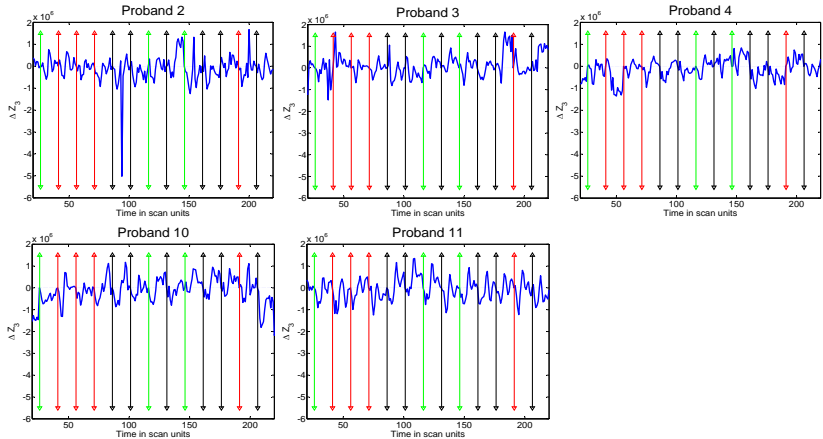


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Factor \hat{Z}_3

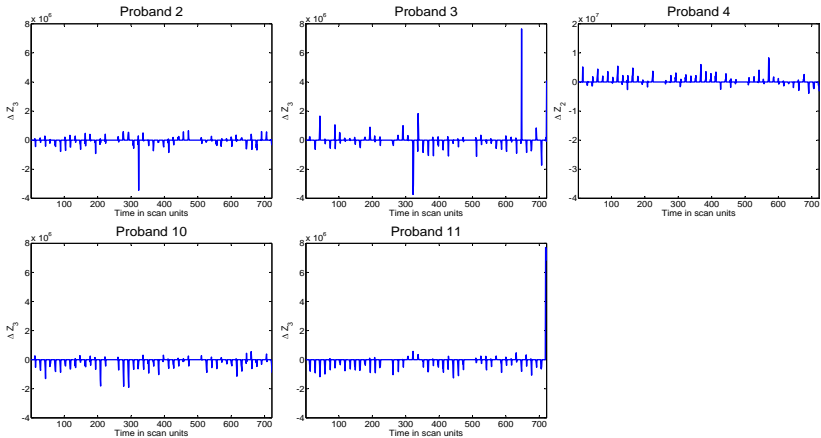


Reaction to stimuli in factor \hat{Z}_3

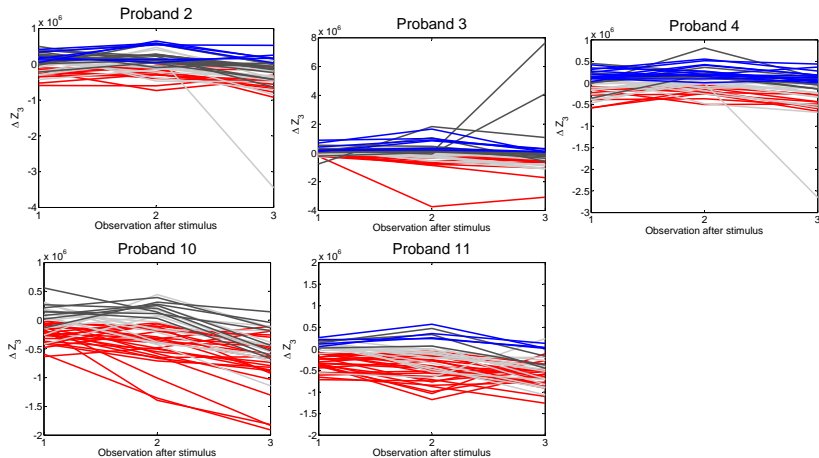


Lines correspond to the time points of judgement tasks: **decision**, **return**, **risk**.

Reaction to stimuli in factor \hat{Z}_3

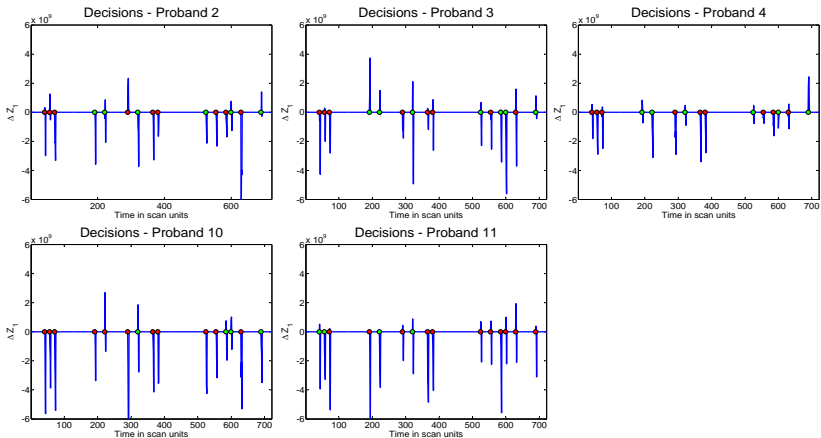


Reaction to stimuli in factor \hat{Z}_3



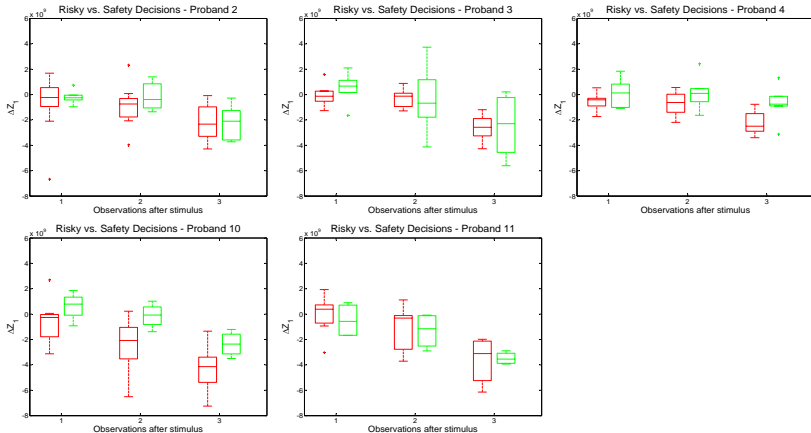
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Reaction after decision tasks in factor \hat{Z}_1



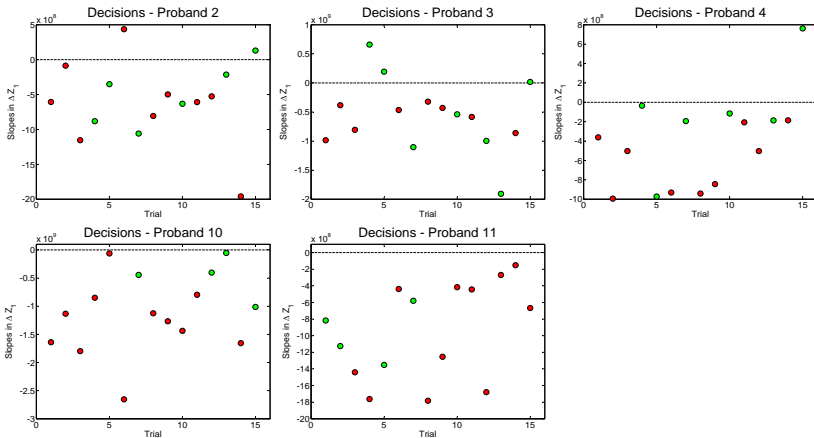
Red points correspond to **risky decisions**, green points to **safety decisions**.

Reaction after decision tasks in factor \hat{Z}_1



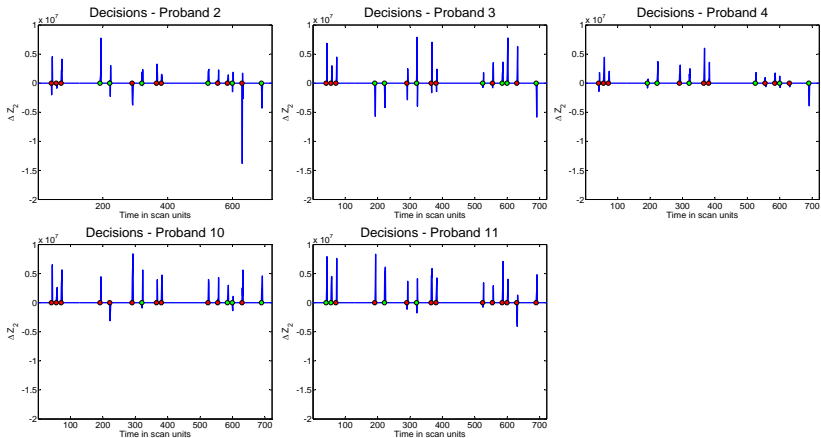
Red boxplots correspond to **risky decisions**, green boxplots to **safety decisions**.

Estimated slopes after decision tasks in factor \hat{Z}_1



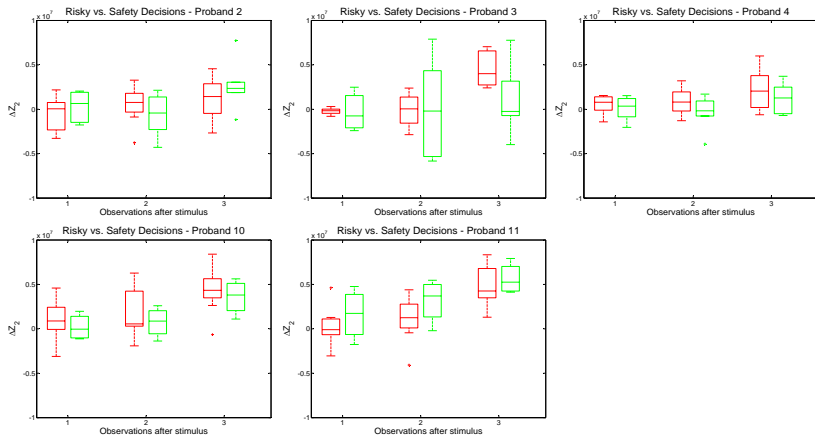
Red points correspond to **risky decisions**, green points to **safety decisions**.

Reaction after decision tasks in factor \hat{Z}_2



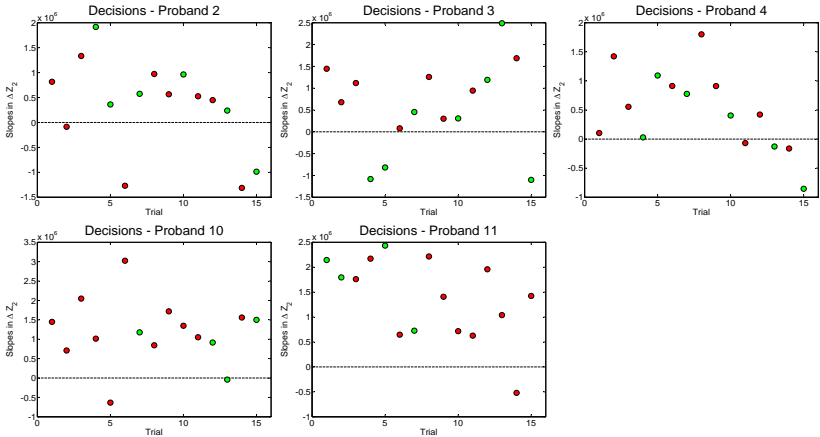
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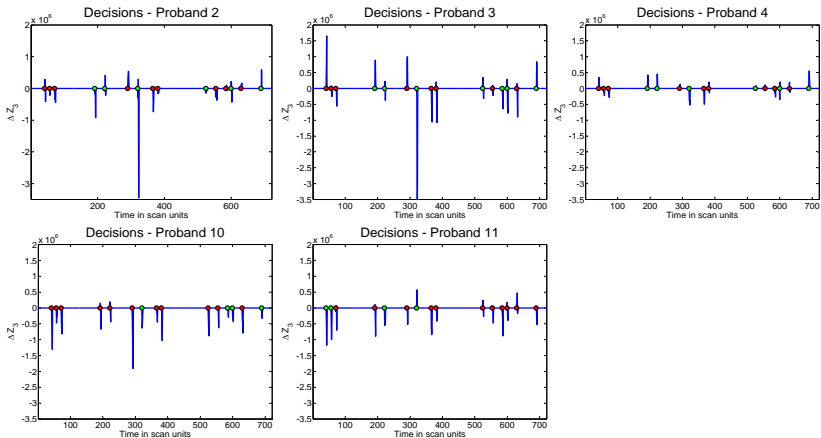


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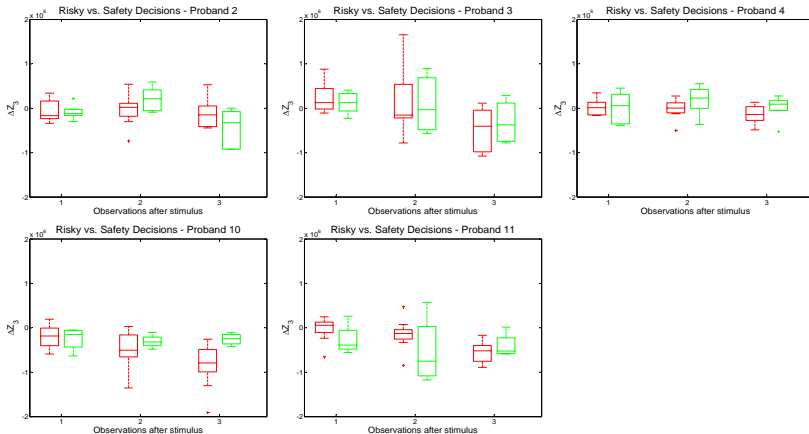


Reaction after decision tasks in factor \hat{Z}_3



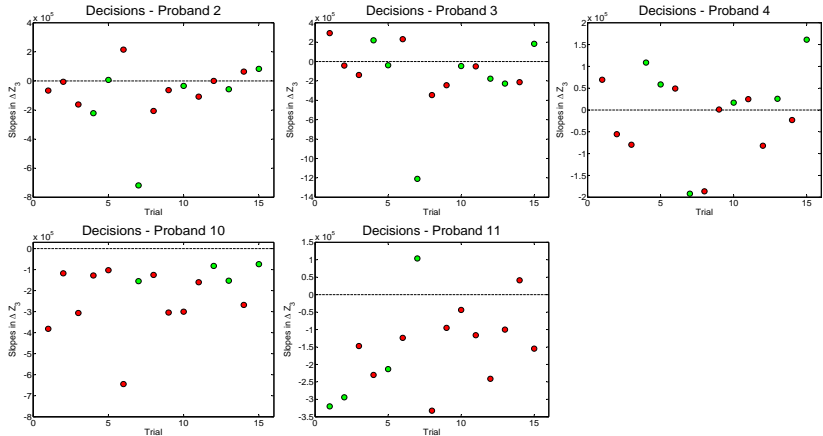
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Reaction after decision tasks in factor \hat{Z}_3



Red boxplots correspond to **risky decisions**, green boxplots to **safety decisions**.

Estimated slopes after decision tasks in factor \hat{Z}_3



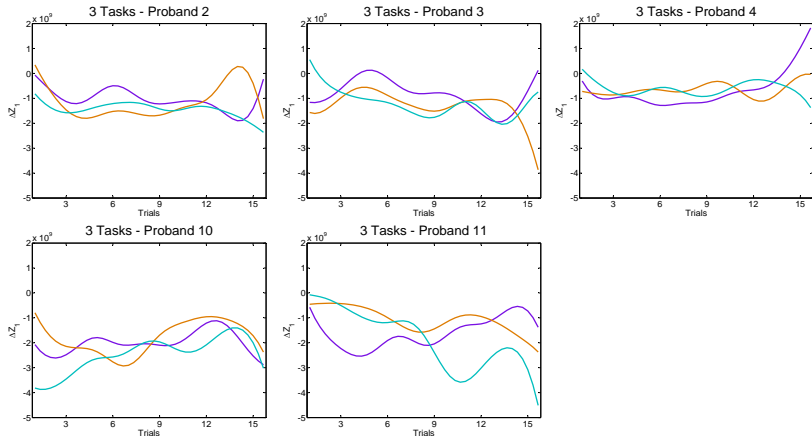
Red points correspond to **risky decisions**, green points to **safety decisions**.

Analysis of the reactions to 3 different stimuli

- only 3 observation after each stimulus taken
- local linear smoother for reactions after each stimulus
- equivalent bandwidth chosen (Silverman's rule of thumb)

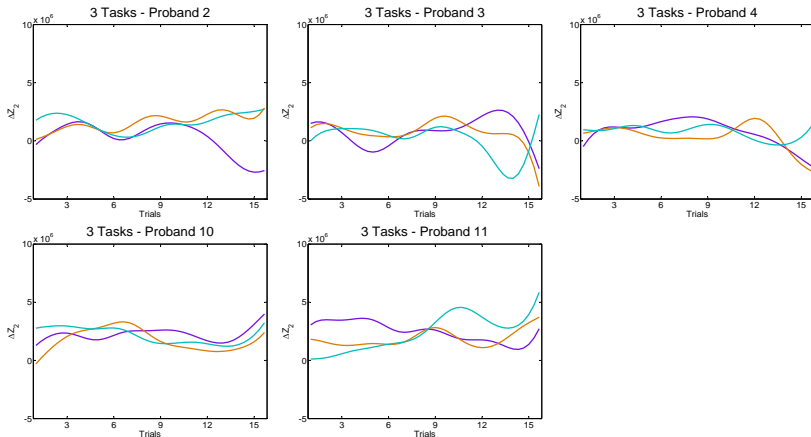


Local linear smoother for reactions in factor \hat{Z}_1



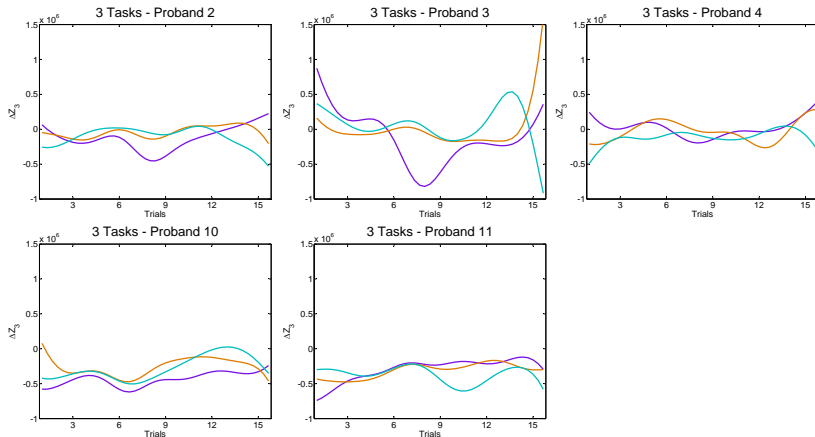
Violet line correspond to decision tasks, dark orange line to expected return tasks, blue line to the perceived risk.

Local linear smoother for reactions in factor \hat{Z}_2



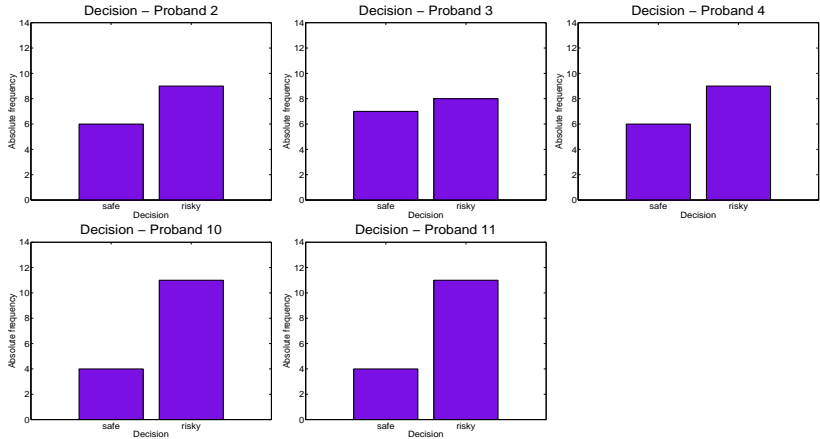
Violet line correspond to decision tasks, dark orange line to expected return tasks, blue line to the perceived risk.

Local linear smoother for reactions in factor \hat{Z}_3



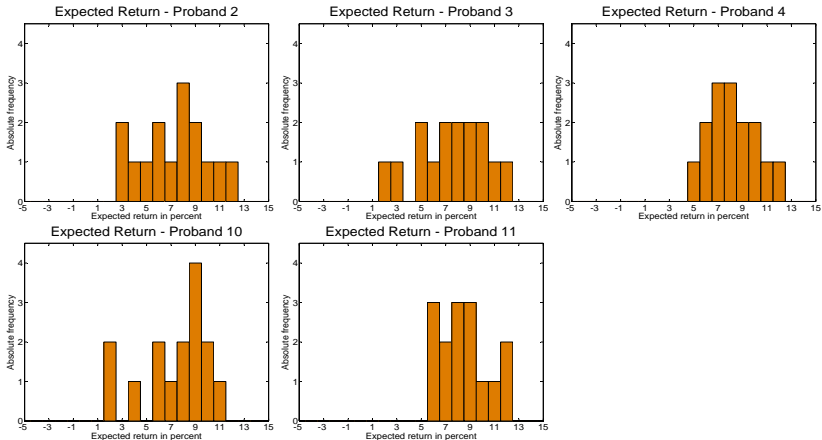
Violet line correspond to decision tasks, dark orange line to expected return tasks, blue line to the perceived risk.

Decision



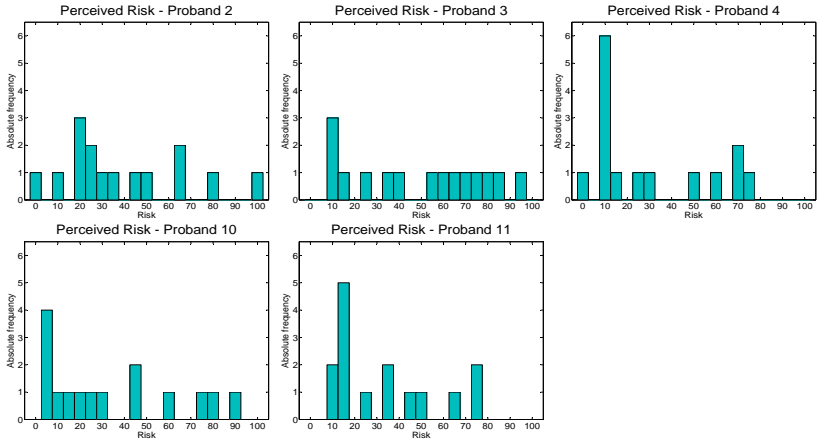
Choice between an investment with 5% fixed return (safe investment) and the investment represented by the return stream (risky investment)

Expected Return



Subjective expected return judge in range (-5% – 15%) of the investment represented by the return stream

Perceived Risk



Perceived risk judge on scale: 0 (no risk) – 100 (maximum risk)