Flexible copula-based vector MEM

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Liquidity risk

Example 1

Problem: Let Q be an huge order influencing the market price; thus Q cannot be executed at the actual market price.

Solution: Split Q into smaller amounts $\delta_i \widehat{V}_i$, such that $Q = \sum_{i=1}^{n} \delta_i \widehat{V}_i$, with $\delta_i \in (0,1)$, where $\widehat{V}_i$ is the forecasted trading volumes, $i = 1, \ldots, n$.

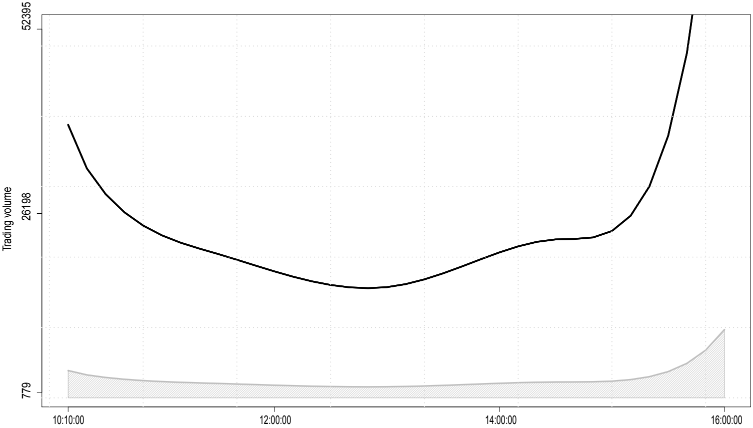

Figure 1: Trading volume of Skyworks Solutions Inc. (SWKS) at the 2009-08-08. Forecasted market volume; limited trade sizes with $\delta_i = 0.05$.

Multiplicative error model (MEM)

Engle (2002) proposed the MEM, i.e.

$$x_i = \mu_i \, \varepsilon_i ,$$
$$\mu_i \stackrel{\text{def}}{=} \mathrm{E}\left(x_i | \mathcal{F}_{i-1}; \xi\right),$$

with

- $\{x_i\}_{i=1}^n$ is a positive valued stochastic process,
- ε_i is iid with $\mathrm{E}\left(\varepsilon_i\right) = 1$ and density $f : \mathbb{R}^+ \to \mathbb{R}^+$,
- Vector of parameters ξ,
- μ_i measurable w.r.t. $\mathcal{F}_{i-1}$.

Research questions

- Joint modeling of volumes, price variations and trading intensities
- Basic ingredients are positive: *no* natural distribution on $\mathbb{R}_+^d$
- More accurate forecasts due to large information set
- Challenges:
  - Non-Gaussian dependencies
  - Two step estimation instead of computationally demanding full-ML estimation

Vector MEM (VMEM)

Cipollini and Gallo (2010) formalized the VMEM as

$$x_i = \mu_i \odot \varepsilon_i,$$

$$\mu_i \stackrel{\text{def}}{=} \mathsf{E}\left(x_i|\mathcal{F}_{i-1};\xi\right) = \underbrace{\omega}_{(d\times 1)} + \underbrace{A}_{(d\times d)} x_{i-1} + \underbrace{B}_{(d\times d)} \mu_{i-1},$$

where "$\odot$" is the componentwise Hadamard product, $\varepsilon_{ij} \geq 0 \ \forall i$, $j = 1, \ldots, d$, $\xi = (\omega, A, B)$ and $\varepsilon_i|\mathcal{F}_{i-1} \sim D\left(1_d, \Sigma\right)$.

Example 2

$\varepsilon_{ij} \sim Exp\left(1\right) = F$ and $D = C\left\{F\left(x_{i1}\right), \ldots, F\left(x_{id}\right); \theta\right\}$

Outline

1. Motivation ✓
2. Estimation
3. HAC
4. Data
5. Preliminary estimation results
6. Summary

Full ML estimation

From VMEM has x_i a conditional density

$$
\begin{aligned}
f_x\left(x_{i1},\ldots,x_{id}|\mathcal{F}_{i-1};\theta,\xi,\alpha\right) &= c\left\{F_1(\varepsilon_{i1};\alpha_1),\ldots,F_d(\varepsilon_{id};\alpha_d);\theta\right\} \\
&\quad \times \prod_{j=1}^{d} \frac{f_j\left(\varepsilon_{ij};\alpha_j\right)}{\mu_{ij}\left(\xi_j\right)},
\end{aligned}
$$

for $\varepsilon_{ij} \stackrel{\text{def}}{=} \varepsilon_{ij}(\xi_j)$, $i = 1,\ldots,n$ and copula density c. Then, the log-likelihood is given through

$$
\begin{aligned}
\ell\left(\theta,\xi,\alpha|\mathcal{F}_{i-1}\right) &= \sum_{i=1}^{n}\sum_{j=1}^{d}\left[\log\left\{\varepsilon_{ij} f_j\left(\varepsilon_{ij};\alpha_j\right)\right\} - \log x_{ij}\right] \\
&\quad + \sum_{i=1}^{n} \log c\left\{F_1(\varepsilon_{i1};\alpha_1),\ldots,F_d(\varepsilon_{id};\alpha_d);\theta\right\}.
\end{aligned}
$$

Advantage:

- Efficient and asymptotically unbiased estimates

Disadvantages:

- If C is flexible structured copula, ℓ has to be optimized for each possible structure
- Curse of dimensionality
  - $2d^2 + 4d - 1$ parameters to estimate (assuming 2 parameters for each margin and $d - 1$ for the copula)
- Parameter restrictions to ensure stationarity and positivity
- Computationally intensive
- ...

Multistage procedure

Similar to Chen and Fan (2006)

1. Estimate $\mu_i(\xi) = \{\mu_{i1}(\xi_1), \dots, \mu_{id}(\xi_d)\}^\top$, $i = 1, \dots, n$
2. Extract the residuals $\hat{\varepsilon}_{i1}, \dots, \hat{\varepsilon}_{id}$ and determine the edfs $\widehat{F}_j$
3. $\hat{\theta} = \arg\max_{\theta} \sum_{i=1}^{n} \log c \left\{ \widehat{F}_1(\hat{\varepsilon}_{i1}), \dots, \widehat{F}_d(\hat{\varepsilon}_{id}); \theta \right\}$

Advantages:

- Separate estimation of $\mu_i(\xi)$ and $C(\cdot;\theta)$
- Applicable for high dimensional time series, conditional on the copula

Disadvantages:

- Matrices A and B should be diagonal
- Estimation equation by equation is inefficient otherwise

Consequence: no real VMEM and no superior forecasting properties

Maximization by parts

Set $\delta = (\xi, \alpha)^\top$ and rewrite the log-likelihood

$$\ell\left(\theta, \delta | \mathcal{F}_{i-1}\right) = \overbrace{\sum_{i=1}^{n}\sum_{j=1}^{d}\left[\log\left\{\varepsilon_{ij} f_j\left(\varepsilon_{ij};\alpha_j\right)\right\} - \log x_{ij}\right]}^{working}$$
$$\underbrace{+\sum_{i=1}^{n}\log c\left\{F_1(\varepsilon_{i1};\alpha_1),\ldots,F_d(\varepsilon_{id};\alpha_d);\theta\right\}}_{error}$$

in two parts

$$\ell\left(\delta, \theta\right) = \ell_w\left(\delta\right) + \ell_e\left(\delta, \theta\right).$$

Shortcuts: $\ell_w^\delta = \partial \ell_w / \partial \delta$, $\ell_e^\delta = \partial \ell_e / \partial \delta$ and $\ell_e^\theta = \partial \ell_e / \partial \theta$

Algorithm (Song et al. (2005))

For $k = 1$:

1. *Solve* $\ell_w^\delta(\delta) = \mathbf{0}$ *for* $\hat{\delta}^1$
2. *Solve* $\ell_e^\theta(\hat{\delta}^1, \theta) = \mathbf{0}$ *for* $\hat{\theta}^1$

For $k = 2, 3, \ldots$:

1. *Solve* $\ell_w^\delta(\delta) = -\ell_e^\delta(\hat{\delta}^{k-1}, \hat{\theta}^{k-1})$ *for* $\hat{\delta}^k$
2. *Solve* $\ell_e^\theta(\hat{\delta}^{k-1}, \theta) = \mathbf{0}$ *for* $\hat{\theta}^k$

HAC

Example 3

3-dimensional fully nested HAC with generators ϕ and ψ:

$$C(u_1, u_2, u_3) = \phi\left\{\phi^{-1} \circ \psi\left\{\psi^{-1}(u_1) + \psi^{-1}(u_2)\right\} + \phi^{-1}(u_3)\right\}$$

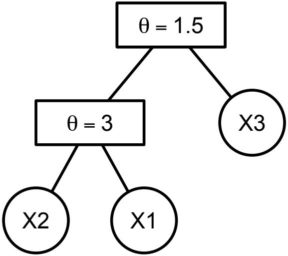

Figure 2: Structure of 3-dim HAC

Estimation

Okhrin et al. (2011) proposed a multi-stage QML-estimation, i.e.

$$\left(\frac{\partial \ell_1}{\partial \theta_1}, \dots, \frac{\partial \ell_{d-1}}{\partial \theta_{d-1}} \right)^\top = \mathbf{0}_{d-1}$$

with quasi log-likelihood

$$\ell(\theta|\mathbf{X}) = n^{-1} \sum_{i=1}^{n} \log \left[c \left\{ \widehat{F}_1(x_{i1}), \dots, \widehat{F}_d(x_{id}), \theta \right\} \right].$$

Data

- Datasource: NASDAQ (LOBSTER)
- Stock: SWKS
- 2nd of November 2009 to 31th of December 2009
- Series is cleaned and adjusted for deterministic intraday patterns
- Aggregation level:
    - 10 min
    - 1331 observations

Variables

High-frequency data sets provide information about

$$\{(p_j, s_j, t_j)\}_{j=1}^{n^*},$$

with

- Price series p_j,
- Amount of traded shares s_j,
- Time stamps t_j,
- Number of observation within a day n^*.

Construct the lower frequent equi-distant series

$$\{(\text{HL}_i, \text{Vol}_i, \text{NT}_i, t_i)\}_{i=1}^{n},$$

with

- $n < n^*$
- $\text{HL}_i = \max\{p_j | t_j \in (t_{i-1}, t_i]\} - \min\{p_j | t_j \in (t_{i-1}, t_i]\}$,
- $\text{NT}_i = \#\{t_j | t_j \in (t_{i-1}, t_i]\}$ and
- $\text{Vol}_i = \text{NT}_i^{-1} \sum_{i_j \in (t_{i-1}, t_i]} s_i$.

Multistage estimation

$$\mu_i = c + a x_{i-1} + b\mu_{i-1}$$

	HL	NT	Vol
c	0.02	0.02	0.18
a	0.11	0.20	0.28
b	0.87	0.77	0.56
α_1	-	0.94	0.50
α_2	-	5.00	11.74
p_{LB10}	0.03	0.01	0.10
p_{LB20}	0.09	0.17	0.40
p_{LB30}	0.15	0.06	0.55

Table 1: Estimated parameters and p-values of the Ljung-Box (LB) test; sample size: 1331. Assumptions: $\varepsilon_{\text{HL}} \sim Exp(1)$, $\varepsilon_{\text{NT}} \sim Ga(\alpha_1, \alpha_2)$ and $\varepsilon_{\text{Vol}} \sim Ga(\alpha_1, \alpha_2)$.

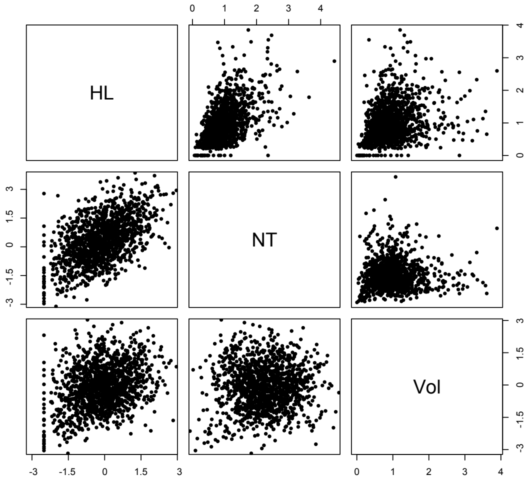

Figure 3: Scatterplot of the residuals.

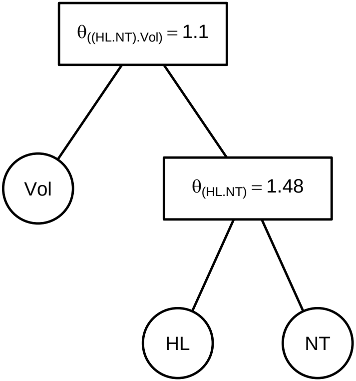

Figure 4: Estimated HAC for the residuals of the univariate processes.

Full ML

$$\mu_i = \begin{pmatrix} 0.22 \\ 0.22 \\ 0.24 \end{pmatrix} + \begin{pmatrix} 0.29 & 0.08 & 0.07 \\ 0.19 & 0.08 & 0.07 \\ 0.09 & 0.24 & 0.22 \end{pmatrix} x_{i-1} + \begin{pmatrix} 0.33 & 0.16 & 0.07 \\ 0.01 & 0.04 & 0.01 \\ 0.00 & 0.00 & 0.15 \end{pmatrix} \mu_{i-1}$$

Stucture	log-lik
$((\mathrm{HL.NT})_{1.85}\mathrm{Vol})_{1.27}$	-2740
$((\mathrm{HL.Vol})_{1.49}\mathrm{NT})_{1.43}$	-2828
$((\mathrm{NT.Vol})_{1.44}\mathrm{HL})_{1.44}$	-2838

With $\alpha_1 = (1.50, 2.73, 3.68)^\top$, $\alpha_2 = (2.98, 2.07, 1.04)^\top$ for $(\mathrm{HL}, \mathrm{NT}, \mathrm{Vol})^\top$.

Conclusion

- Aim at forecasting economic variables
- Two step estimation for VMEM
- Benefit from well developed univariate models and modern copulas

To do:

- Maximization by parts
- Time varying relationships
- Penalization of the parameter space's dimensionality

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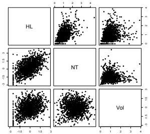

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