

**Statistics II in English**  
**Exercises from *Übungsaufgaben und***  
***Lösungen zu Statistik I und II***  
**Part 3**

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**Exercise 8-1:**

A company produces special refrigerators to conserve certain goods. The wished temperature for that type of refrigerators is  $-25^{\circ}C$ . When goods are insufficiently cooled, they go bad very easily, and since the client base of the company is not big, a defective products would cause the worst case-the ruin of the company. That is why the cooling performance of 100 randomly chosen produced refrigerators shall be tested on a significance level of 2.275% in order to decide whether the production can be carried on or if constructional changes on the refrigerators need to be made. Experience shows that the cooling temperature is normally distributed with standard deviation  $2^{\circ}C$ .

1. What are the hypotheses for this test? Justify your choice and argue with risk.
2. Formulate the underlying sample function formally and verbally as well as its distribution under  $H_0$ .
3. What is the testing function and what is its distribution under  $H_0$ ?
4. Determine the rejecting region (Verwerfungsbereich) of this test.
5. Determine the value of the power function (Gütefunktion) if the true mean of cooling temperature is:  $-24.8^{\circ}C$ ,  $-25.8^{\circ}C$ ,  $-29.0^{\circ}C$ .
6. Sketch the power function.
7. Random sampling yielded a mean of cooling temperature of  $-26^{\circ}C$  and a standard deviation of  $1.5^{\circ}C$ .
  - (a) What is the test decision?
  - (b) Interpret the test result in an exact way statistically as well as from a context point of view.

8. Random sampling yielded a mean of cooling temperature of  $-25.3^{\circ}C$ .
  - (a) What is the test decision?
  - (b) Interpret the test result in an exact way statistically as well as from a context point of view.
  - (c) Which mistake may have been made by taking this test decision?
  - (d) What is the probability that this mistake has really been made?
  - (e) How big is the probability to make this mistake when using this testing procedure and the real  $\mu$  is  $-29^{\circ}C$ ?
9. Why is it sufficient for unilateral test to consider under the null hypothesis only the case  $\mu = \mu_0$ ?

**Exercise 8-3:**

Ecologists are criticizing the pollution of lakes by the household's wastewater, particularly by the amount of phosphate in washing agents. That is why they attack a certain company, as they suspect this company's product to exceed the allowed mean value of  $18g$  per package. The company denies this massively and promises to take the product from the market if the exceeding of the mean value can be shown statistically.

The company is thus offering a test with an error probability of  $0.001$  since a false result shall be avoided with high probability. The ecologist accept this offer since they assume the argumentation to be completely clear. The variance of the amount of phosphate per package of  $36g^2$  is assumed to be known.

A random sample of  $36$  packages yielded the average amount of phosphate of  $20g$ .

1. Formulate the hypothesis.
2. Provide the underlying sampling function formally and verbally. What is its distribution under  $H_0$ ?
3. What is the testing function and what is its distribution under  $H_0$ ?
4. Determine the rejection region for this test.
5. What is the test decision?

The company releases the following press statement:

*With the help of statistical testing our company could prove that the average amount of phosphate per package of cleaning agent does not exceed  $18g$ .  $36$  packages have been chosen randomly. We chose a test setting that yields false decisions with a probability of only  $0.1\%$  only.*

6. Comment on this press release.

7. Determine the probability of false decisions if the average amount of phosphate per package has the true value 21.09g.
8. Is the track of the power function of this test dependent on the sample value or on the sample size?

**Exercise 8-7:**

The two boxers Jim Knockout and Bill Uppercut are both the world's best boxer, based on computer ranking lists. A plaster company wants to offer the world's best boxer an advertisement contract worth 1 Million Euro. The Chef of this company believes that Jim Knockout, who won lots of fight k.o., is the better boxer. To proof this hypothesis statistically, the chef organizes 11 fights between both boxers. In each fight there will be a winner. Draws are excluded. ( $\alpha = 0.05$ )

1. Formulate the hypotheses for this test.
2. Define the test function for this test.
3. How is the test function under  $H_0$  distributed?
4. Determine accepting and rejecting regions for this test.
5. How do you decide this test, if Jim Knockout loses 3 fights?
6. Could you have made an error in you test decision? If yes, which one?
7. The company plans to publish the test result in a short and comprehensive form. Formulate this press release.

**Exercise 8-8:**

In a ski resort every year there is a huge ski race for all guests. This year's slalom shall happen on a newly available slope. A group of ski teacher is charged with constructing a slalom that is drivable for all guests. On average, more than 90% of the guests shall healthily arrive at the finish line. The level of difficulty of the slop needs further assessments. A test on the significancelevel of 10% shall be made with 22 randomly chosen guest that took the slalom.

1. Formulate the hypotheses of this test.
2. What is the test function's distribution under  $H_0$ ?
3. Determine the rejection region of this test.
4. What is the exact probability of incorrect choice of  $H_A$  for this test?
5. One of the 22 test skiers did not reach the finish line. What is the test result?

6. Justify this decision.
7. Give the value of the power function  $g(p)$  for  $p = 0$ ,  $p = 0.1$ , and  $p = 0.2$ .
8. Sketch the graph of the power function and use the values determined in point 7.