# Estimating Inflation Expectation Co-movement Across Countries

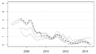
Shi Chen
Wolfgang K. Härdle
Weining Wang

Ladislaus von Bortkiewicz Chair of Statistics
C.A.S.E.-Center for Applied Statistics and Economics
Humboldt–Universität zu Berlin
King's College London
http://lvb.wiwi.hu-berlin.de

# Inflation Expectation

- Important to financial investors and policy makers
- Inflation surprise can make real impact
- Important to estimate underlying state of the economy
- . . .





# General Goal

- Joint modeling of inflation expectation cross regions (states/countries)
  - Providing informative estimates of inflation expectations
- Accounting for the non-linear dependency among countries
  - A GeoCopula Model
- Cross sectional forecast and forecast of inflation expectation

# Measure of Inflation

- Break-even inflation rate (BEIR) with maturity $\tau$,

$$BEIR_t(\tau) = y_t^N(\tau) - y_t^R(\tau)$$

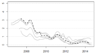
  - $y_t^N(\tau)$ - nominal yield
  - $y_t^R(\tau)$ - real yield

- Decompose BEIR,

$$BEIR_t(\tau) = \pi_t^e(\tau) + else$$

  - $\pi_t^e(\tau)$ is expected inflation
  - else includes risk premium, convexity effects





# BEIR of European Countries

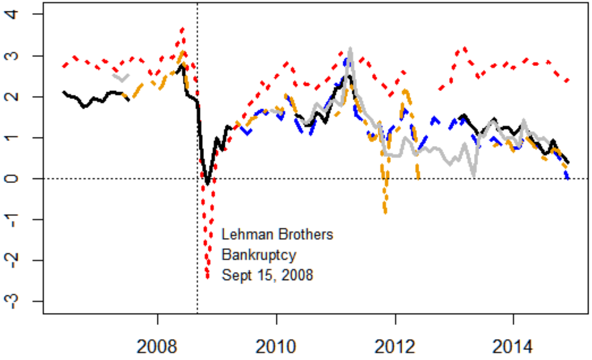



Figure 1: Observed BEI rates (percent) of UK, France, Italy, Sweden and Germany, maturity five years.

MTS_BEIR

- Co-movement exists between countries
- Data from different countries are available at different maturities and month

## Challenges

- Joint yield curve modeling across multiple maturities
- BEIR decomposition
- Panel model of inflation expectations
- Non linear dependency among countries
- New estimation and forecast of inflation expectation

# Outline

1. Motivation ✓
2. Yield Curve Modeling
3. BEIR decomposition
4. Dynamics of Inflation Expectation
5. Empirical Results
6. Conclusion

# Model Approach– Single Country

- Nelson and Siegel (1987): Classical NS model

$$y(\tau) = \beta_0 + \beta_1 \left( \frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) + \beta_2 \left( \frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right)$$

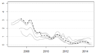
- Diebold and Li (2006): Dynamic NS (DNS) model

$$y_t(\tau) = L_t + S_t \left( \frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) + C_t \left( \frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right)$$

- Christensen et.al (2011): Arbitrage-free DNS (AFNS) model
  The closest match to DNS yield is

$$y_t(\tau) = L_t + S_t \left( \frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) + C_t \left( \frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right) - \frac{A(\tau)}{\tau}$$

  - state variable $X_t^\top = (L_t, S_t, C_t)$

# Joint AFNS model

- The separate AFNS models of nominal and inflation-indexed type for a specific country $i$,

$$y_{it}^N(\tau) = L_{it}^N + S_{it}^N \left( \frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) + C_{it}^N \left( \frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right) - \frac{A_i^N(\tau)}{\tau}$$

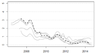
$$y_{it}^R(\tau) = L_{it}^R + S_{it}^R \left( \frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) + C_{it}^R \left( \frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right) - \frac{A_i^R(\tau)}{\tau}$$

- We assume,

$$S_{it}^R = \alpha_i^S S_{it}^N$$

$$C_{it}^R = \alpha_i^C C_{it}^N$$





# Joint AFNS model

- The joint AFNS yield curve for country $i$ with maturity $\tau$ is

$$\begin{pmatrix} y_{it}^N(\tau) \\ y_{it}^R(\tau) \end{pmatrix} = \begin{pmatrix} 1 & \frac{1-e^{-\lambda_i \tau}}{\lambda_i \tau} & \frac{1-e^{-\lambda_i \tau}}{\lambda_i \tau} - e^{-\lambda_i \tau} & 0 \\ 0 & \alpha_i^S \frac{1-e^{-\lambda_i \tau}}{\lambda_i \tau} & \alpha_i^C \left(\frac{1-e^{-\lambda_i \tau}}{\lambda_i \tau} - e^{-\lambda_i \tau}\right) & 1 \end{pmatrix} \begin{pmatrix} L_{it}^N \\ S_{it}^N \\ C_{it}^N \\ L_{it}^R \end{pmatrix} + \begin{pmatrix} \varepsilon_{it}^N(\tau) \\ \varepsilon_{it}^R(\tau) \end{pmatrix} - \begin{pmatrix} \frac{A_i^N(\tau)}{\tau} \\ \frac{A_i^R(\tau)}{\tau} \end{pmatrix}$$

  - state variable $X_{it}^\top = (L_{it}^N, S_{it}^N, C_{it}^N, L_{it}^R)$ ▸ Dynamics of state variable
  - $\frac{A_i(\tau)}{\tau}$ is a yield-adjustment term

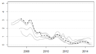
# Yield Curve Modeling

- For country $i$ with maturity $\tau = (\tau_1, \tau_2, \ldots, \tau_n)$ is

$$\begin{pmatrix} y_{it}^N(\tau_1) \\ y_{it}^R(\tau_2) \\ \vdots \\ y_{it}^R(\tau_n) \end{pmatrix} = \begin{pmatrix} 1 & \frac{1-e^{-\lambda_i \tau_1}}{\lambda_i \tau_1} & \frac{1-e^{-\lambda_i \tau_1}}{\lambda_i \tau_1} - e^{-\lambda_i \tau_1} & 0 \\ 0 & \alpha_i^S \frac{1-e^{-\lambda_i \tau_2}}{\lambda_i \tau_2} & \alpha_i^C \left( \frac{1-e^{-\lambda_i \tau_2}}{\lambda_i \tau_2} - e^{-\lambda_i \tau_2} \right) & 1 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \alpha_i^S \frac{1-e^{-\lambda_i \tau_n}}{\lambda_i \tau_n} & \alpha_i^C \left( \frac{1-e^{-\lambda_i \tau_n}}{\lambda_i \tau_n} - e^{-\lambda_i \tau_n} \right) & 1 \end{pmatrix}$$

$$\begin{pmatrix} L_{it}^N \\ S_{it} \\ C_{it} \\ L_{it}^R \end{pmatrix} + \begin{pmatrix} \varepsilon_{it}^N(\tau_1) \\ \varepsilon_{it}^R(\tau_2) \\ \vdots \\ \varepsilon_{it}^R(\tau_n) \end{pmatrix} - \begin{pmatrix} \frac{A_i^N(\tau_1)}{\tau_1} \\ \vdots \\ \frac{A_i^R(\tau_n)}{\tau_n} \end{pmatrix}$$





# Outline

1. Motivation ✓
2. Yield Curve Modeling ✓
3. BEIR decomposition
4. Dynamics of Inflation Expectation
5. Empirical Results
6. Conclusion

# BEIR decomposition

- That is,

$$BEIR_t(\tau) = y_t^N(\tau) - y_t^R(\tau) = \pi_t^e(\tau) + \eta_t(\tau) + \phi_t(\tau)$$

  - $\eta_t(\tau)$ is convexity effect
  - $\phi_t(\tau)$ is Inflation Risk Premium

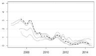
Go to details

-

$$\pi_t^e(\tau) = -\frac{1}{\tau} \log \mathrm{E}_t^P \left[ \exp \left\{ - \int_t^{t+\tau} (r_s^N - r_s^R) ds \right\} \right]$$

  - $r_{it}$ is the instantaneous risk-free rate
  - real type: $r_{it}^R = L_{it}^R + \alpha_i^S S_{it}^N$, nominal type: $r_{it}^N = L_{it}^N + S_{it}^N$





# Inflation Expectation Estimates– Single Country

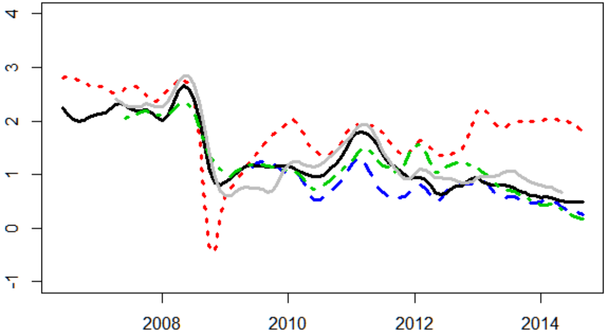



Figure 2: Model-implied inflation expectation for different countries - UK, France, Italy, Sweden and Germany.

# Outline

1. Motivation ✓
2. Yield Curve Modeling ✓
3. BEIR decomposition ✓
4. Dynamics of Inflation Expectation
5. Empirical Results
6. Conclusion

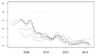
# IE Dynamics with Credit Risk Factor (with a fixed $\tau$)

- $$\hat{\pi}^e_{it} = m_i + n_i \Pi_t + l_i d_{it} + \mu_{it}$$

- The dynamics of the common factor,

$$\Pi_t = p + q\Pi_{t-1} + \nu_t$$

  - where $m_i$, $n_i$, $p$ and $q$ are unknown parameters
  - the errors $\nu_t$s are i.i.d white noises and $\mu_{it}$ follows a GeoCopula model
  - $d_{it}$ is the CDS varying over time

# Copulae

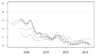
A continuous function $C : [0,1]^d \rightarrow [0,1]$,

$$C(u_1, \dots, u_d) = F\{F_1^{-1}(u_1), \dots, F_d^{-1}(u_d)\}, \quad u_1, \dots, u_d \in [0,1],$$

where $F_1^{-1}(\cdot), \dots, F_d^{-1}(\cdot)$ the quantile functions.

- Separate dependency and marginal distributions
- Represent general dependency





# A GeoCopulae Model

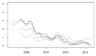
$$\mu_{it} = \alpha_{it} + \xi_{it}, \tag{1}$$

$\alpha_{it}$ the spatial temporal variation, $\xi_{it}$ the i.i.d. noise with mean 0 and variance $\sigma_\xi$, see Bai et. al. 2014.

$$F_{it}(\alpha) = \Phi_{NT}\{\Phi^{-1}(F_{11}(\alpha_{11})), \cdots, \Phi^{-1}(F_{N,T}(\alpha_{NT}))|\Sigma\}, \tag{2}$$

$\Phi_{NT}(\cdot)$ the cumulative distribution function (c.d.f.) of a multivariate Gaussian distribution with a variance covariance matrix $\Sigma$, which models the spatiotemporal dependence. Details





# A Spatialtemporal Variagram

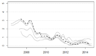
To understand the spatial temporal correlation, define

$$\Gamma(t_1 - t_2, n_1 - n_2) \stackrel{\text{def}}{=} \frac{1}{2} E(\alpha_{t_1 n_1} - \alpha_{t_2 n_2})^2. \tag{3}$$

Closely related to covariance function. For stationary process with $\sigma_2$ are the variance,

$$\Gamma(t_1 - t_2, n_1 - n_2) \stackrel{\text{def}}{=} \sigma_2 - Cov(\alpha_{t_1 n_1}, \alpha_{t_2 n_2}). \tag{4}$$





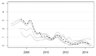
| France | Germany | Italy | Sweden | UK |
|---|---|---|---|---|
| 2.2 | 13.3 | 12.3 | 18.0 | 0.09 |
| 48.5 | 52.3 | 41.5 | 59.1 | 51.3 |

Table 1: The Coordinates of countries, in Degree.

Empirical variagram is defined as

$$\hat{\Gamma}(d_1, d_2) \stackrel{\text{def}}{=} \frac{1}{N_{d_1,d_2}} \sum_{t_1,t_2,n_1,n_2:\|t_1-t_2\|\leq d_1,\|n_1-n_2\|\leq d_2} (\alpha_{t_1 n_1} - \alpha_{t_2 n_2})^2, \tag{5}$$

where $N_{d_1,d_2}$ is the number of pairs, which has the spatial and temporal distance in range.

# Outline

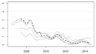
1. Motivation ✓
2. Yield Curve Modeling ✓
3. BEIR decomposition ✓
4. Dynamics of Inflation Expectation ✓
5. Empirical Results
6. Conclusion

# Data

- Bloomberg: monthly zero-coupon government bond yield.
- Type: nominal $y^N$, inflation-indexed $y^R$.

| *Data* | Span | Maturity |
|---|---|---|
| UK | 30.06.2006-31.12.2014 | 3,4,5 years |
| France | 30.06.2006-31.12.2014 | 3,5,10 years |
| Italy | 29.06.2007-31.12.2014 | 3,5,10 years |
| Sweden | 30.04.2007-29.08.2014 | 3,5,10 years |
| Germany | 30.06.2009-31.12.2014 | 5,7,10 years |

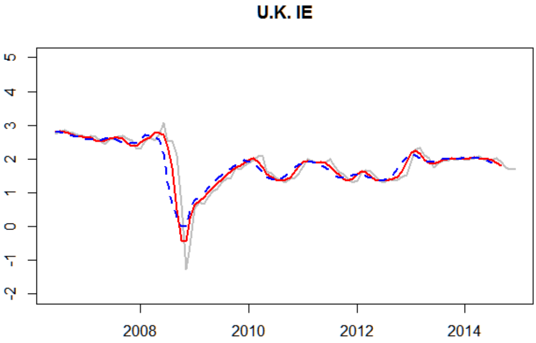




# Outline

1. Motivation ✓
2. Yield Curve Modeling ✓
3. BEIR decomposition ✓
4. Dynamics of Inflation Expectation ✓
5. Empirical Results
    - Single Country
    - Multiple Country
    - With GeoCopulae
6. Conclusion

# Estimated IE– Single Country




MTS_expinf

# 3-year IE forecast– Single Country

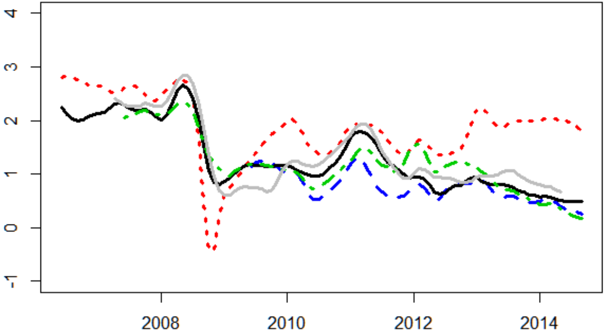
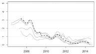



Figure 3: Model-implied inflation expectation for different countries - UK, France, Italy, Sweden and Germany. MTS_expinf





# Outline

1. Motivation ✓
2. Yield Curve Modeling ✓
3. BEIR decomposition ✓
4. Dynamics of Inflation Expectation ✓
5. Empirical Results
   - Single Country ✓
   - Multiple Country
   - With GeoCopulae
6. Conclusion

# Common effect with $d_{it}$

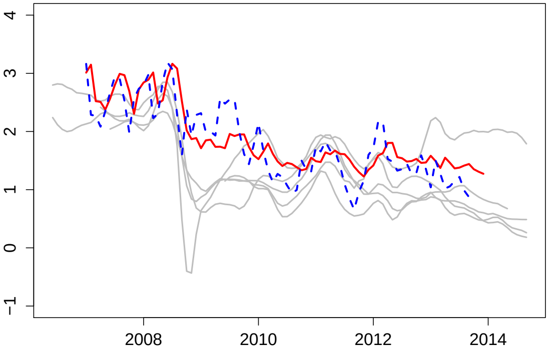



Figure 4: Common inflation factor - predicted $\Pi_t$, filtered $\Pi_t$.

MTS_comexpinf_cds

# Residuals of common effect

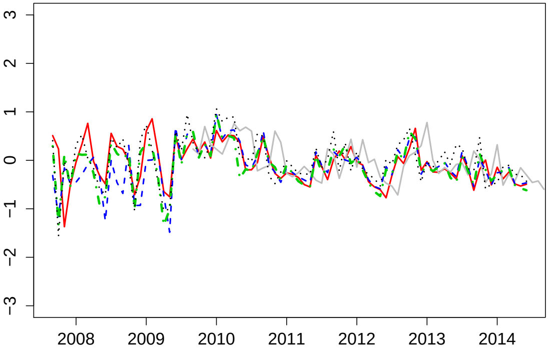
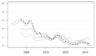



Figure 5: Model residual for IE dynamics without macroeconomic factor - UK, France, Italy, Sweden and Germany. MTS_comexpinf_cds



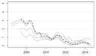


# Estimates with $d_{it}$

| Country-specific equations | | |
|---|---|---|
| UK | $\pi^e_{1t}(\tau) =$ | $-0.358 d_{it} + 0.798 \Pi_t$ |
| France | $\pi^e_{2t}(\tau) =$ | $0.085 d_{it} + 0.714 \Pi_t$ |
| Italy | $\pi^e_{3t}(\tau) =$ | $1.078 d_{it} + 0.531 \Pi_t$ |
| Sweden | $\pi^e_{4t}(\tau) =$ | $-0.621 d_{it} + 0.805 \Pi_t$ |
| Germany | $\pi^e_{5t}(\tau) =$ | $0.045 d_{it} + 0.700 \Pi_t$ |
| Common Effect equation | | |
| | $\Pi_t =$ | $0.382 + 0.976 \Pi_{t-1}$ |

Table 2: Estimates for the dynamics of IE.



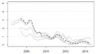


# Variance decomposition

- According to the joint model of IE dynamics, decompose the variation of IE - $\pi^e_{it}$ into parts driven by,
  - common effect variation
  - country-specific variation
  - sovereign risk variation

$$\mathrm{Var}(\pi^e_{it}) = n_i^2 \mathrm{Var}\left(\Pi_t\right) + l_i^2 \mathrm{Var}\left(d_{it}\right) + \mathrm{Var}\left(\mu_{it}\right)$$



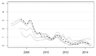


# Joint IE dynamics with $d_{it}$

| | U.K. | France | Italy | Sweden | Germany |
|---|---|---|---|---|---|
| Common effect | 36.08 | 33.59 | 11.54 | 31.87 | 32.84 |
| Country-specific effect | 56.66 | 65.88 | 40.92 | 49.17 | 67.02 |
| Sovereign risk effect | 7.26 | 0.53 | 47.55 | 18.96 | 0.14 |

Table 3: Variations explained in percentage





# Outline

1. Motivation ✓
2. Yield Curve Modeling ✓
3. BEIR decomposition ✓
4. Dynamics of Inflation Expectation ✓
5. Empirical Results
   - Single Country ✓
   - Multiple Country ✓
   - With GeoCopulae
6. Conclusion

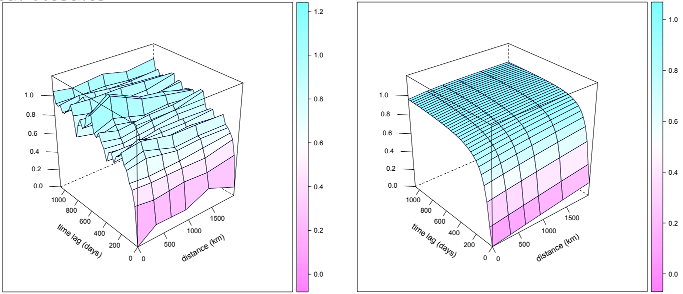



Figure 6: The empirical fitted variogram (left) and the parametrically fitted variogram (right).

$\hat{\eta} = 0.5$, given $\hat{\beta} = 0.1028$, $\hat{a} = 0.001795$ means that the marginal temporal correlation decreases by around 8% with 1 month increase in time, and $\hat{b} = 0.000298$ indicates that the marginal space correlation decays by around 9% with a 100-km increase.

# Forecast

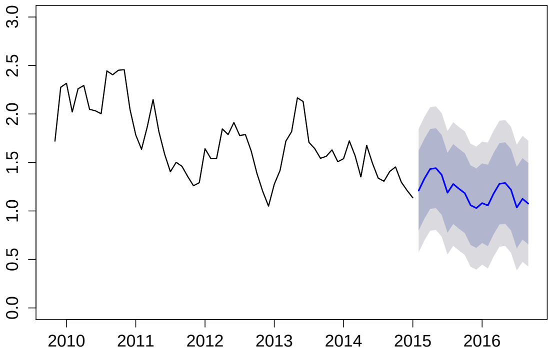



Figure 7: The forecast of common inflation factor derived from the joint model of IE dynamics with CDS.

# Forecast with Geocopula

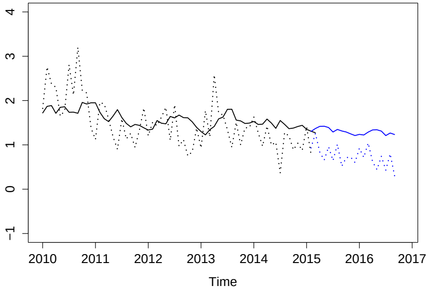



Figure 8: The comparision of two forecasts with (dotted) and without(solid) Geocopula, the estimation results are derived from the joint model of IE dynamics with default factor.

# Comparison

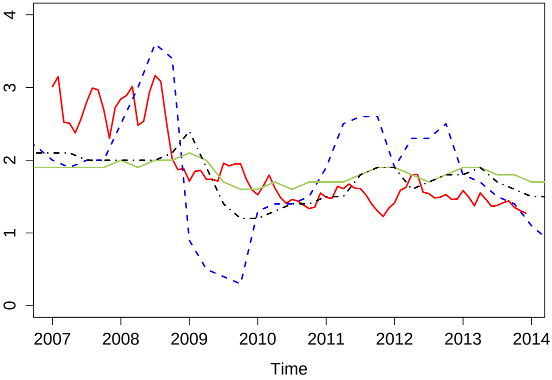
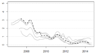



Figure 9: Comparison of model-implied level, the observed inflation level, 1-year SPF forecast and 2-year SPF forecast of inflation (Survey Professional Forecast).





# Forecast inflation and cross sectional forecast

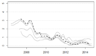
| | U.K. | France | Italy | Sweden | Germany |
|---|---|---|---|---|---|
| Without GeoCopula | 0.413 | 0.381 | 0.463 | 0.386 | 0.298 |
| With GeoCopula | 0.116 | 0.056 | 0.145 | 0.122 | 0.142 |
| Cross sectional forecast | 0.345 | 0.312 | 0.487 | 0.344 | 0.201 |

Table 4: Averaged one year ahead forecast starting from 201401(squared mean squared error)

# Conclusion

- Common inflation factor $\Pi_t$ is an important drivers of country-specific inflation expectations.
- The model extracts informative estimates of inflation expectations
- Will provide good implications for monetary policies





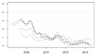
# References


Nelson, C.R., Siegel, A.F.
*Parsimonious modeling of yield curves.*
*Journal of Bussiness*, 60:473-489, 1987.

Francis X Diebold and Li, C.
*Forecasting the term structure of government bond yields.*
*Journal of Econometrics*, 130: 337-364, 2006.

Cochrane, J.H.
*Asset Pricing.*
*Princeton university press*, 2005.

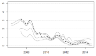


# References

Jens HE Christensen, Francis X Diebold and Glenn D Rudebusch
*The affine arbitrage-free class of Nelson-Siegel term structure models.*
*Journal of Econometrics*, 164(1):4-20, 2011.

Francis X Diebold, Canlin Li and Vivian Z Yue
*Global yield curve dynamics and interactions: a dynamic Nelson-Siegel approach.*
*Journal of Econometrics*, 146(2):351-363, 2008.

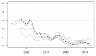



# References


Francis X Diebold, Glenn D Rudebusch and S Boragan Aruoba
*The macroeconomy and the yield curve: a dynamic latent factor approach.*
*Journal of Econometrics*, 115(1):32-39, 2012.

Jens H.E. Christensen, Jose A. Lopez and Glenn D. Rudebusch
*Inflation expectations and risk premiums in an Arbitrage-Free model of nominal and real bond yields.*
*Journal of Money, Credit and Banking*, 11:143-178, 2010.



# Dynamics of state variable

- Derived from affine AF model of Duffie&Kan(2002), the real world P-dynamics is

$$dX_{it} = K_i^P(t)[\theta_i^P(t) - X_{it}]dt + \Sigma_i(t)dW_{it}^P$$

  - $K_i^P(t)$, $\theta_i^P(t)$ can vary freely.
  - $\Sigma_i(t)$ is diagonal volatility matrix.
- Transition equation,

$$X_{it} = \Phi^0_{i,\Delta t} + \Phi^1_{i,\Delta t} X_{i,t-1} + \eta_{it}$$

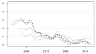
with

$$\Phi^0_{i,\Delta t} = I - \exp(-K_i^P \Delta t)\theta_i^P$$

$$\Phi^1_{i,\Delta t} = \exp(-K_i^P \Delta t)$$

Return





$$F(\alpha_{t_1,n_1},\alpha_{t_2,n_2}) = \Phi_2(\Phi^{-1}(F_{t_1,n_1}(\alpha_{t_1,n_1})),\Phi^{-1}(F_{t_2,n_2}(\alpha_{t_2,n_2}))|\Sigma_{t_1,n_1,t_2,n_2}), \quad (6)$$

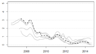
where $\Sigma_{t_1,n_1,t_2,n_2}$ is a submatrix of $\Sigma$.

$$\sigma(n_2 - n_1, t_2 - t_1)$$

$$= \sigma(v,u) \stackrel{\text{def}}{=} \begin{cases} \frac{2\sigma^2\beta}{(a^2u^2+1)^\eta(a^2u^2+\beta)\gamma(\eta)}\left(\frac{b}{2}\left(\frac{a^2u^2+1}{a^2u^2+\beta}\right)^{1/2}v\right)^\eta K_\eta\left(b\left(\frac{a^2u^2+1}{a^2u^2+\beta}\right)^{1/2}v\right) & \text{if } v > 0, \\ \frac{\sigma^2\beta}{(a^2u^2+1)^\eta(a^2u^2+\beta)\gamma(\eta)} & \text{if } v = 0, \end{cases}$$

where $a, b, \beta, \eta$ are parameters, $\gamma(\eta)$ is the gamma function and $K_\eta(\cdot)$ is the Bessel function of the second kind. Go Back





$$l(\theta, d_1, d_2) = \sum_{t_1, t_2, n_1, n_2 : \|t_1 - t_2\| \leq d_1, \|n_1 - n_2\| \leq d_2} \log f_{\alpha_{t_1 n_1}, \alpha_{t_2 n_2}}, \qquad (7)$$

where $\theta \stackrel{\text{def}}{=} (a, b, \beta, \eta)^\top$ .

$$f_{\alpha_{t_1,n_1}, \alpha_{t_2,n_2}} \stackrel{\text{def}}{=} c_\Phi\{F(\alpha_{t_1,n_1}), F(\alpha_{t_2,n_2})\} f(\alpha_{t_1,n_1}) f(\alpha_{t_2,n_2})$$

with

$$c_\Phi\{F(\alpha_{t_1,n_1}), F(\alpha_{t_2,n_2})\} = |\Sigma_{t_1,n_1,t_2,n_2}|^{-1/2} \exp\{q^\top (I_2 - \Sigma^{-1}_{t_1,n_1,t_2,n_2}) q\},$$

$$q \stackrel{\text{def}}{=} (q_{t_1,n_1}, q_{t_2,n_2})$$

and

$$q_{t_i,n_i} = \Phi^{-1}\{\hat{F}(x_{t_i,n_i})\}.$$

▸ Go Back

# BEIR decomposition

- Cochrane (2005), the price of the zero-coupon bond that pay one unit of consumption basket at time $t$,

$$P_t^N(\tau) = \mathsf{E}_t\left(M_{t+1}^N M_{t+2}^N \cdots M_{t+\tau}^N\right)$$

$$P_t^R(\tau) = \mathsf{E}_t\left(M_{t+1}^R M_{t+2}^R \cdots M_{t+\tau}^R\right)$$

  - $M_t^N$ and $M_t^R$ are SDFs.
- Under assumption of no arbitrage,

$$\frac{M_t^N}{M_t^R} = \frac{Q_{t-1}}{Q_t}$$

  - $Q_t$ is the overall price level of consumption basket.

# BEIR decomposition

- Converting equations,

$$y_t(\tau) = -\frac{1}{\tau}\log P_t(\tau)$$

$$\pi_{t+1} = \log\frac{Q_{t+1}}{Q_t}$$

- The BEIR can be defined as follows,

$$\begin{aligned} y_t^N(\tau) - y_t^R(\tau) &= \frac{1}{\tau}\mathsf{E}_t\left(\log\frac{M_{t+1}^N}{M_{t+1}^R}\cdots\frac{M_{t+\tau}^N}{M_{t+\tau}^R}\right) - \frac{1}{2\tau}\mathsf{Var}_t\left(\log\frac{M_{t+1}^N}{M_{t+1}^R}\cdots\frac{M_{t+\tau}^N}{M_{t+\tau}^R}\right) \\ &+ \frac{1}{\tau}\mathsf{Cov}_t\left(\log\frac{M_{t+1}^N}{M_{t+1}^R}\cdots\frac{M_{t+\tau}^N}{M_{t+\tau}^R}, \log M_{t+1}^R\cdots M_{t+\tau}^R\right) \end{aligned}$$

Go Back