Hilda Geiringer Lecture 2016

Asset Pricing

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"Asset pricing theory all stems from one simple concept, presented in the first page of the first chapter of this book: **price equals expected discounted payoff**. The rest is elaboration, special cases, and a closet full of tricks that make the central equation useful for one or another application." (Cochrane (2005), p. xiii)

Motivation

- **Price equals expected discounted payoff**
  - what is the appropriate risk-free rate?
  - what is the appropriate compensation for risk?
    - how to explain the equity risk premium?
    - how to explain return differences of stocks?
    - how to explain the prices of all the other assets, as e.g. options, commodities, housing, foreign exchange, . . . ?

- Central questions: **which risks matter?**
  - aggregate consumption
    (high return for assets that do poor when consumption is low)
  - $\rightarrow$ equity premium puzzle
  - $\rightarrow$ there are risks beyond just "normal consumption risk"

First generation models

- link prices to investor preferences and fundamentals (aggregate consumption)
- fail to explain level of risk-free rate and equity risk premium

Second generation models

- more sophisticated preferences and fundamentals
- solve equity premium puzzle and risk-free rate puzzle
- focus: "all the other problems"

Consumption based asset pricing

Individual optimality

- each investor maximizes life-time utility by deciding on consumption, savings, and investments
  $\rightarrow$ asset allocation
- investor preferences
  - CRRA-utility
  - recursive preferences
  - habit formation

Market clearing

- demand equals supply
- demand follows from individual optimization problems (in many cases: representative investor)
- supply: exogenously given aggregate consumption (endowment economies)

First generation: (Simple) **Lucas tree economy**

Investor

- representative investor has CRRA preferences
- life-time utility

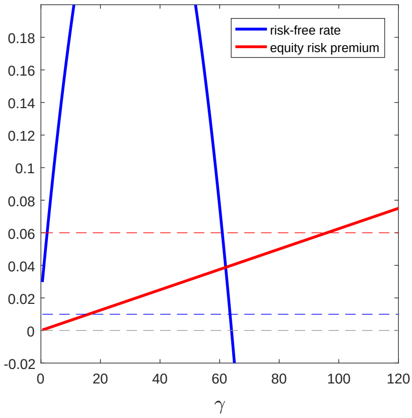
$$\sum_{t=0}^{\infty} e^{-\delta t} E\left[\frac{C_t^{1-\gamma}}{1-\gamma}\right]$$

Fundamentals

- investor is endowed with one tree
- fruits of the tree
  $=$ aggregate consumption
  (every year, the investor just consumes the fruits)
- dynamics of aggregate consumption are given exogenously

Lucas Tree Economy: Numbers

- Investor preferences
  - relative risk aversion: 2-10
  - time preference rate: 2%
- Aggregate consumption
  - average growth rate: 2%
  - volatility: 2.5%
- Puzzles
  - equity premium puzzle
  - risk-free rate puzzle



The world is more complicated...

...and there are risks beyond "normal consumption risk"

	Investor preferences	Fundamentals
Disaster risk	time additive CRRA preferences	consumption is subject to rare but severe disasters rare: one or twice a century severe: average drop 30-40%
Habit formation	utility depends on consumption and habit level	
Long-run risk	recursive utility	consumption dynamics depend on (persistent) state variables (capture state of economy)

(and there are combinations of these approaches)

Long Run Risk Models

(Bansal, Yaron (2004))

Investor

- representative investor has recursive preferences
- implication: state variables are priced

Fundamentals

- consumption dynamics depend on persistent state variables
- state variables capture state of economy, e.g. expected growth, uncertainty, disaster risk, . . .

- solves equity premium puzzle and risk-free rate puzzle
- one of the main work-horses of modern asset pricing

Fundamentals are Driven by State Variables

- Dynamics of consumption C and dividends D

$$
\begin{aligned}
\ln C_{t+1} - \ln C_t &= (\mu_c + x_t) + \sigma_c \sqrt{V_t}\epsilon^c_{t+1} \\
\ln D_{t+1} - \ln D_t &= (\mu_d + \phi_d x_t) + \sigma_d \sqrt{V_t}\epsilon^d_{t+1} \\
x_{t+1} &= \kappa_x x_t + \sigma_x \sqrt{V_t}\epsilon^x_{t+1} \\
V_{t+1} &= (1-\kappa_v)\bar{V} + \kappa_v V_t + \sigma_v \sqrt{V_t}\epsilon^v_{t+1}
\end{aligned}
$$

- State variables describe overall state of the economy
  - long-run growth rate x
  - variance risk V
- Important in the following: high persistence
  - changes in state variables and in particular in x have long-lasting impact
  - therefore: long-run risk model

- **CRRA preferences** (in discrete time)

$$\frac{\mathcal{U}_t^{1-\gamma}}{1-\gamma} = \left(1-e^{-\delta}\right)\frac{C_t^{1-\gamma}}{1-\gamma} + e^{-\delta}\frac{E_t\left[\mathcal{U}_{t+1}^{1-\gamma}\right]}{1-\gamma}$$

  - C: consumption at t
  - $\mathcal{U}$: certainty equivalent of future consumption
  - relative risk aversion: γ (variation across states and time)

- **Recursive preferences** (in discrete time)

$$\frac{\mathcal{U}_t^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}} = \left(1-e^{-\delta}\right)\frac{C_t^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}} + e^{-\delta}\frac{\left(E_t\left[\mathcal{U}_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}\right)^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}}$$

  - relative risk aversion: γ (variation across states)
  - intertemporal elasticity of substitution: ψ (variation over time)

Recursive Preferences cont'd

- Stochastic discount factor

$$M_{t,t+1} \quad = \quad \underbrace{e^{-\delta}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}}_{\substack{\text{CRRA term} \\ \text{consumption risk}}} \underbrace{\left(e^{-\delta}\frac{W_{t+1}/C_{t+1}}{W_t/C_t-1}\right)^{\frac{\gamma-\frac{1}{\psi}}{1-\frac{1}{\psi}}}}_{\substack{\text{additional risk} \\ \text{beyond consumption risk}}}$$

- There are premiums for consumption risk...
- ...and for variation in the wealth-consumption ratio
  - which captures future variation in continuation utility
  - which is driven by variation in state variables

Model Implications

- Equity risk premium: $\surd$

$$\underbrace{\gamma\, Cov_t(\ln D_{t+1}, \ln C_{t+1})}_{\substack{\text{compensation for}\\ \text{consumption risk}}} + \underbrace{\frac{\gamma - \frac{1}{\psi}}{1 - \frac{1}{\psi}} Cov_t\left(\ln D_{t+1}, \ln \frac{W_{t+1}}{C_{t+1}}\right)}_{\substack{\text{compensation for}\\ \text{state variables}}}$$

1. compensation for consumption risk: again small
2. compensation for state variables: large
    - increases in persistence of state variables
    - mainly due to growth rate risk

- Risk-free rate: $\surd$

- Two puzzles solved...
- ...and still a lot of facts to explain!

Classics: **predictability by price-dividend ratio**

- Predicability of **excess returns** by price-dividend ratio

$$R_{t+1} - R_{f,t} = \underbrace{ERP_0 + ERP_v V_t}_{\text{predictable}} + \underbrace{\sigma_R' \epsilon_{t+1}}_{\text{unpredictable}}$$

price-dividend ratio depends on V_t ...
...and can thus predict future excess returns

- Predicability of **dividend growth** by price-dividend ratio

$$\ln D_{t+1} - \ln D_t = \underbrace{\mu_d + \phi_d x_t}_{\text{predictable}} + \underbrace{\sigma_d \sqrt{V_t} \epsilon_{t+1}^d}_{\text{unpredictable}}$$

price-dividend ratio depends on x_t ...
...and can thus predict future dividend growth

Where Do we Stand Right Now

We can make the model more complicated...
...if it can then explain more empirical findings

Model Specification

- larger set of risk factors, in particular uncertainty risk
  (stochastic central tendency, stochastic vol-of-vol, stochastic jump intensity)
- jump risk
- interaction
  (uncertainty or inflation has an impact on expected growth)

Model Implications

- predictability
- variance risk premium
  (level, predictive power)
- option prices
  (level and slope of volatility smile)
- exchange rates
- cross-sectional asset pricing
- further assets
  (commodities, housing, ...)

Question 1: Specification of Risk Factors

Add more risk factors...

...and give the reasons why we really need them

The Dynamics of Crises and the Equity Premium

(Branger, Kraft, Meinerding (2016))

Model Specification

- consumption is subject to moderate downward jumps
  (similar to disaster risk models)
- two regimes: low jump intensity in good state, high jump intensity in bad states
- regime switch into bad state simultaneously with jump in consumption

Model Implications

- better fit to consumption dynamics
  (several small jumps instead of one large jump)
- joint regime switches and consumption jumps
  → higher equity risk premium
  (than for separate jumps)

Starting Point: Disaster Risk Models

Starting point: **Disaster risk models**

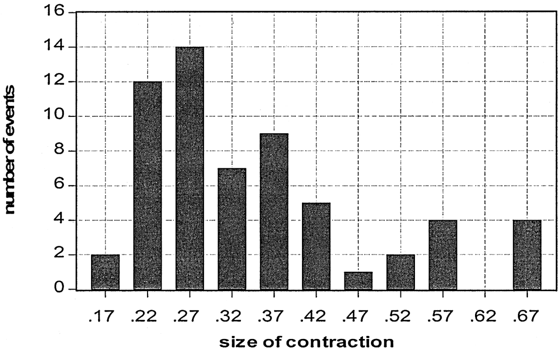
- Consumption dynamics

$$d \ln C_t = \mu \, dt + \sigma \, dW_t + L \, dN_t$$

- Consumption is subject to rare but severe disasters
  - rare: once or twice a century
  - severe: average jump size of -30% or -40%

- Model is able to explain the equity risk premium...
- ...but how realistic are the assumptions on disasters?

Consumption disasters can really be large:



Histogram of consumption disasters of 35 countries during the 20th century (adjusted for trend growth, see Barro (2006))

However: Peak-to-Trough Calibration

Consumption disasters can really be large

. . . but it is crucial how we measure disasters

(Constantinides (2008))

- Calibration of Barro: disaster size is measured from peak (in year t) to trough (in year $t + h$)
- Critique of Constantinides (2008)
  - peak-to-trough disasters often last for several years
  - thus: not one large disaster, but several small disasters
- Does it matter?
  - assumption is crucial to explain the equity premium
    (equity risk premium for intensity = 0.01/size = -0.4 is much larger than risk premium for intensity = 0.02/size = -0.2)

  $\Rightarrow$ the model is in trouble

Solution: Clustering of Jumps

- Our idea: **jump clustering** can solve the problem
- Two states
  - good state with low jump intensity

$$d \ln C_t \quad = \quad \mu \, dt + \sigma \, dW_t + L \, dN_t^{g,g} + L^{g,b} \, dN_t^{g,b}$$

  - bad state with high jump intensity

$$d \ln C_t \quad = \quad \mu \, dt + \sigma \, dW_t + L \, dN_t^{b,b} + 0 \, dN_t^{b,g}$$

  - regime switch from good to bad state
    - joint with downward jump in consumption: $L^{g,b} = L$
    - separate from downward jump in consumption: $L^{g,b} = 0$

Solution: Clustering of Jumps

Joint jumps: $L^{g,b} = L$

- jump clustering: $\surd$
- high equity risk premium

- main point: combination of two bad events (downward jump in consumption, regime switch into bad state) gives really bad event

Separate jumps: $L^{g,b} = 0$

- jump clustering: $\surd$
- low equity risk premium
- there is a premium for regime changes, but it is not large enough
- simply adding a bad regime in which jumps cluster does not do the job

Level and Slope of Volatility Smiles in Long-Run Risk Models

(Branger, Rodrigues, Schlag (WP 2016))

Model Specification

- state variables are subject to jumps
- uncertainty risk factors
  - two factors for stochastic variance: variance and mean-reversion level of variance
  - stochastic jump intensity

Model Implications

- focus on option pricing
- model with additional jump intensity factor can match negative correlation of level and slope of volatility smile

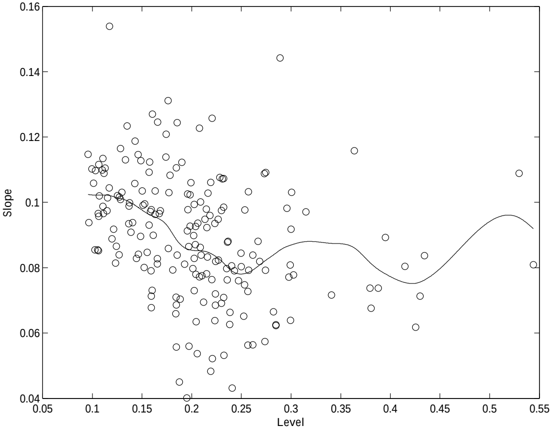
Model Setup

$$
\begin{aligned}
d\ln C_t &= (\mu_c + x_t)\,dt + \sigma_c\sqrt{\omega_c V_t + 1 - \omega_c}\,dW_{c,t} \\
d\ln D_t &= (\mu_\delta + \phi x_t)\,dt + \sqrt{\omega_\delta V_t + 1 - \omega_\delta}\,(\sigma_{\delta,c}\,dW_{c,t} + \sigma_{\delta,\delta}\,dW_{\delta,t}) \\
dx_t &= -\kappa_x x_t\,dt + \sigma_x\sqrt{\omega_x V_t + 1 - \omega_x}\,dW_{x,t} + \xi_{x,t}\,dN_{x,t} \\
dV_t &= (k_v\bar{\sigma}_t^2 - \kappa_v V_t)\,dt + \sigma_v\sqrt{V_t}\,dW_{v,t} + \xi_{v,t}\,dN_{v,t} \\
d\alpha_t &= (k_\alpha\bar{\sigma}_t^2 - \kappa_\alpha \alpha_t)\,dt + \sigma_\alpha\sqrt{\alpha_t}\,dW_{\alpha,t} + \xi_{\alpha,t}\,dN_{\alpha,t} \\
d\bar{\sigma}_t^2 &= \kappa_{\bar{\sigma}}(\omega - \bar{\sigma}_t^2)\,dt + \sigma_{\bar{\sigma}}\sqrt{\bar{\sigma}_t^2}\,dW_{\bar{\sigma},t}
\end{aligned}
$$

- our model: jump intensity is proportional to α
  - $\rightarrow$ jump and diffusion risk can change independently
- simplification: jump intensity is proportional to V_t
  - $\rightarrow$ jump and diffusion risk move in lockstep

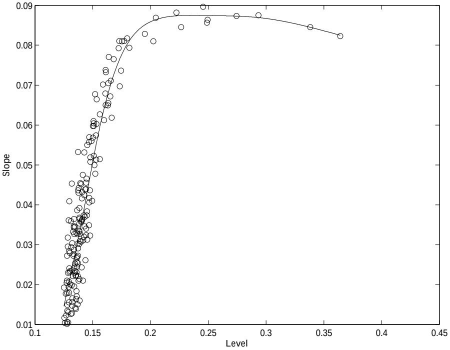
Model Implications

- Our focus: **level and slope of volatility smile**
  - level: implied volatility of ATM options
  - slope: difference between implied volas of OTM and ATM put
- Empirical evidence on joint behavior of level and slope

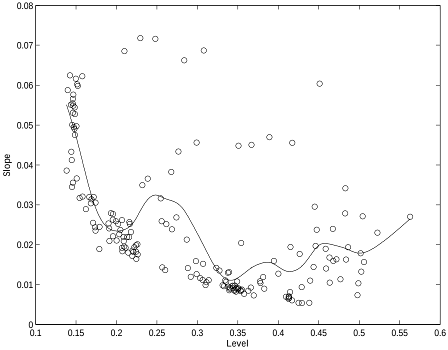


Model Implications: Evidence

jump intensity $\propto$ diffusion variance



jump intensity $\neq$ diffusion variance



Question 2: Modeling the Cross-Section

Risk premia differ in the cross section of stocks. . .

. . . and we want know why they do so

Equilibrium Asset Pricing in Directed Networks...

(Branger, Konermann, Meinerding, Schlag (WP 2015/16))

Model Specification

- cross section of stocks is modeled via joint dividend dynamics
- dividends (and consumption) are subject to downward jumps
- dependence of jump intensities is modeled via directed network

  (jump in asset i has an impact on jump intensity of asset j)

Model Implications

- measure for connectedness: shock-propagation capacity (spc)
- return volatility

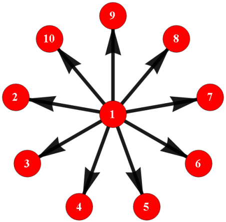
  $\rightarrow$ decreases in spc
- market price of risk

  $\rightarrow$ increases in spc
- expected excess return

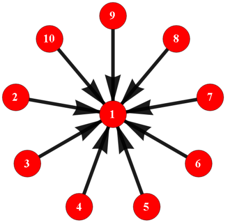
  $\rightarrow$ it depends

Motivation

- Two important features of a **financial network**
  - → shocks can be ‘passed on’ from one firm to another (potentially with a time lag)
  - → links between firms can have a direction
- Does the **direction of links** matter for the cross-section of excess returns, return volatilities, and market prices of risk?



star network



reverse star network

Contributions

- **Links between firms**
  - mutually exciting jump processes in dividends
    (jump in asset i has an impact on jump intensity for asset j)
  - risk is passed on through the network
  - links have a direction
- Characterize network via simple measures
  $\rightarrow$ shock-propagation capacity
- Model replicates empirical findings:
  - return volatility: decreasing in spc
  - market prices of (jump) risk: increasing in spc
  - expected excess returns: depends ...
- **Summary: direction of links matters for asset pricing**

Question 3: Long-Run Survival

Investors in the market differ from each other...

...who survives when they trade with each other?

That is not pure asset pricing...

...but again, recursive preferences can change everything

Survival

- Heterogeneous investors
  - consumption sharing rule (equilibrium outcome)
    - implemented via consumption-savings decision...
    - ...and via investment decision
  - consumption shares change over time
- **Survival** = non-vanishing consumption share of investor

- Why do we care about survival?
  - if we explain prices, volatilities, trading by investor heterogeneity...
    ...then we want both investors to survive in the long run
  - curiosity: how far can we get without e.g. using an OLG model?

Preference Heterogeneity and the Long-Term Evolution of Consumption Shares

(Branger, Dumitrescu, Ivanova, Schlag (WP 2016))

Model Specification

- investors have recursive preferences
- heterogeneity w.r.t. risk aversion and intertemporal elasticity of substitution
- two models for consumption
  - i.i.d.
  - stochastic growth rate

Model Implications

- recursive preferences give parameter region for joint survival

  (one investor has low risk aversion, other investor has large IES)
- region is larger with stochastic growth rate than for i.i.d. consumption

Recursive preferences change the rules of the game

CRRA preferences

- speculate his way out of extinction
  - investor with lower RRA
- save his way out of extinction
  - investor with larger IES
  - larger IES $\widehat{=}$ lower RRA

only investor with lower RRA survives

Recursive Utiltiy

- speculate his way out of extinction
  - investor with lower RRA
- save his way out of extinction
  - investor with larger IES

both investor with lower RRA and investor with larger IES can survive

Long-run survival meets long-run risk

- Region of joint survival
  - combination of preference parameters for which both investors survive
  - subset of: (low RRA, low IES) - (high RRA, high IES)
- Size of the region depends on fundamentals
  - i.i.d. consumption growth: rather small
  - long-run growth risk: much larger

Long-Run Survival

Who survives with long-run growth risk?

$\downarrow \psi_2 \mid \psi_1 \rightarrow$	0.40	0.50	0.60	0.70	0.80	0.90	1.10	1.20	1.30	1.40	1.50	1.60
0.40	1	1	1	1	1	1	1	1	1	1	1	1
0.50	1/2	1	1	1	1	1	1	1	1	1	1	1
0.60	1/2	1/2	1	1	1	1	1	1	1	1	1	1
0.70	1/2	1/2	1/2	1	1	1	1	1	1	1	1	1
0.80	1/2	1/2	1/2	1	1	1	1	1	1	1	1	1
0.90	1/2	1/2	1/2	1/2	1	1	1	1	1	1	1	1
1.10	2	1/2	1/2	1/2	1/2	1/2	1	1	1	1	1	1
1.20	2	1/2	1/2	1/2	1/2	1/2	1	1	1	1	1	1
1.30	2	1/2	1/2	1/2	1/2	1/2	1	1	1	1	1	1
1.40	2	1/2	1/2	1/2	1/2	1/2	1/2	1	1	1	1	1
1.50	2	1/2	1/2	1/2	1/2	1/2	1/2	1/2	1	1	1	1
1.60	2	1/2	1/2	1/2	1/2	1/2	1/2	1/2	1	1	1	1

Who survives with i.i.d. consumption growth?

$\downarrow \psi_2 \mid \psi_1 \rightarrow$	0.40	0.50	0.60	0.70	0.80	0.90	1.10	1.20	1.30	1.40	1.50	1.60
0.40	1	1	1	1	1	1	1	1	1	1	1	1
0.50	1/2	1	1	1	1	1	1	1	1	1	1	1
0.60	2	1/2	1	1	1	1	1	1	1	1	1	1
0.70	2	2	1/2	1	1	1	1	1	1	1	1	1
0.80	2	2	1/2	1/2	1	1	1	1	1	1	1	1
0.90	2	2	2	1/2	1/2	1	1	1	1	1	1	1
1.10	2	2	2	2	1/2	1/2	1	1	1	1	1	1
1.20	2	2	2	2	2	1/2	1/2	1	1	1	1	1
1.30	2	2	2	2	2	1/2	1/2	1	1	1	1	1
1.40	2	2	2	2	2	2	1/2	1/2	1	1	1	1
1.50	2	2	2	2	2	2	1/2	1/2	1/2	1	1	1
1.60	2	2	2	2	2	2	1/2	1/2	1/2	1/2	1	1

Conclusion

- **Asset pricing**
  - risk-free rate and equity risk premium: $\surd$
  - predictability, option prices, commodities, . . . : **current topics**
- **Long-run risk models** are one of the main work-horses
  - we need more risk factors (here uncertainty risk factors). . .

    . . . to explain options
  - we need jumps and jump clustering. . .

    . . . to "rescue" diaster risk models
  - we need to model the cross section of dividends. . .

    . . . to explain the cross section of stocks
- **And there are still a lot of open questions!**