Cross Country Evidence for the EPK Puzzle

Maria Grith
Wolfgang Karl Härdle
Andrija Mihoci

Ladislaus von Bortkiewicz Chair of Statistics
C.A.S.E. – Center for Applied Statistics
and Economics
Humboldt–Universität zu Berlin
http://lvb.wiwi.hu-berlin.de
http://www.case.hu-berlin.de

Motivation

- Pricing kernel (PK)
  - Consumption based models
    - marginal rate of consumption substitution
  - Arbitrage free models
    - Radon-Nikodym derivative of the physical measure w.r.t. the risk neutral measure ▸ Risk Neutral Valuation ▸ PK - Black-Scholes

- Empirical pricing kernel (EPK)
  - $\widehat{\mathcal{K}}$ any estimate of the PK
  - EPK paradox - locally increasing EPK

PK Estimation

- Indirect estimation of the PK

$$\widehat{\mathcal{K}} = \frac{\widehat{q}}{\widehat{p}}$$

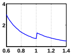
  - q risk neutral density; p physical density;
  - European options and stock index data
  - EPK puzzle emerges
    Ait-Sahalia and Lo (2000), Brown and Jackwerth (2004)

PK Estimation

- Direct estimation of the PK

$$\widehat{\mathcal{K}} = G_{\hat{\theta}}$$

  - $PK \stackrel{\text{def}}{=} G_\theta \propto U'$, U aggregated utility
  - cross-sectional equity returns data
  - mixed evidence for the EPK puzzle
    Dittmar (2002), Schweri (2011)



EPK Paradox: European option market

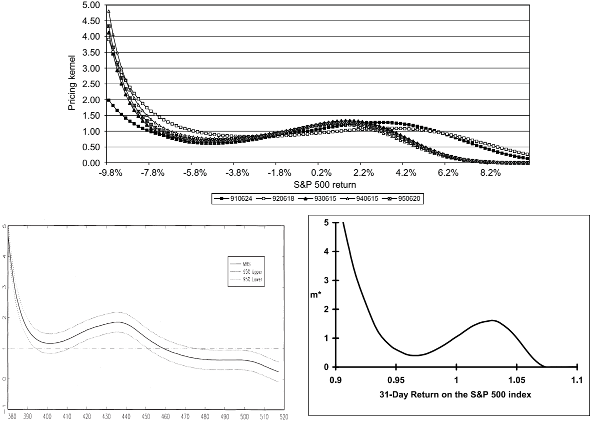
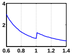


Figure 1: EPK's: Engle and Rosenberg (2002), Ait-Sahalia and Lo (2000), Brown and Jackwerth (2004)



EPK Paradox: European option market

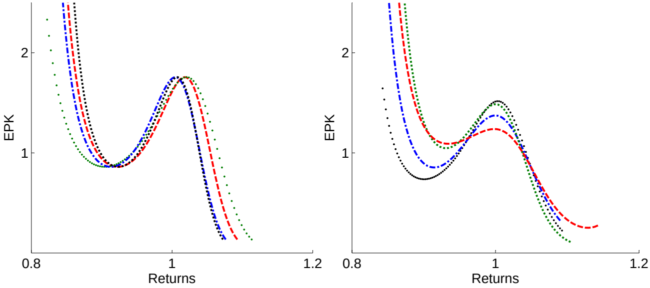


Figure 2: EPK's for various maturities (left) and different estimation dates for fixed maturity 1M (right), Grith et al. (2010)

EPK Paradox: European option market

Figure 3: EPK's across moneyness κ and maturity τ for DAX from 20010101 – 20011231, Giacomini and Härdle (2008)

EPK Paradox

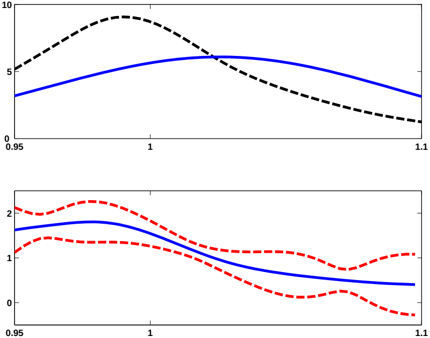


Figure 4: Upper panel: estimated risk neutral density $\widehat{q}$ and historical density $\widehat{p}$. Lower panel: EPK and 95% uniform confidence bands on 20060228, Härdle et al. (2010)

Research Questions

- Parametrization of the PK that admits nonmonotonicity
- Dynamic estimation of the EPK parameters
- Test the significance of the 'bump' in the EPK
- Cross-country variation of the EPK in equity returns

Outline

1. Motivation ✓
2. Pricing Kernel (PK)
3. Generalized Method of Moments (GMM)
4. Empirical Results
5. Conclusion

Modeling Framework

- Neoclassical economy, representative agent
  - Exogenous income ω_t
  - Consumption c_t and financial portfolio of k assets

$$\omega_t = c_t + q_t^\top S_t$$

Asset holdings $q_t = (q_{1,t}, \ldots, q_{k,t})^\top$, prices $S_t = (S_{1,t}, \ldots, S_{k,t})^\top$

  - c_{t+1} contains the future income and all asset payoffs

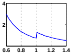
$$c_{t+1} = \omega_{t+1} + q_t^\top S_{t+1}$$

Preferences

- Expected time separable and state-dependent utility

$$
\begin{aligned}
u(c_t, c_{t+1}) = & u(c_t) + \beta_1 \mathsf{E}_t\left[u(c_{t+1})\right] \mathbf{I}\{c_t \in [0, x)\} \\
& + \beta_2 \mathsf{E}_t\left[u(c_{t+1})\right] \mathbf{I}\{c_t \in [x, \infty)\}
\end{aligned}
$$

  - Reference point x, preference parameters β_1 and β_2
  - $\mathsf{E}_t[\bullet] = \mathsf{E}[\bullet \mid \mathcal{F}_t]$



Optimal Portfolio Holding

$$\max_{c_t, c_{t+1}} u(c_t, c_{t+1}) = \max_{q_t} [\, u\left(\omega_t - S_t^\top q_t\right)$$
$$+ \beta_1 \mathrm{E}_t \left[u\left(\omega_{t+1} + q_t^\top S_{t+1}\right)\right] \mathbf{I}\left\{\left(\omega_{t+1} + q_t^\top S_{t+1}\right) \in [0, x)\right\}$$
$$+ \beta_2 \mathrm{E}_t \left[u\left(\omega_{t+1} + q_t^\top S_{t+1}\right)\right] \mathbf{I}\left\{\left(\omega_{t+1} + q_t^\top S_{t+1}\right) \in [x, \infty)\right\} \,]$$

- Consumption based asset pricing

$$S_t = \mathrm{E}_t \left[\left\{\beta_1 \frac{u'(c_{t+1})}{u'(c_t)} \mathbf{I}\{c_t \in [0, x)\} \; + \beta_2 \frac{u'(c_{t+1})}{u'(c_t)} \mathbf{I}\{c_t \in [x, \infty)\}\right\} S_{t+1}\right] \quad (1)$$

Preferences

- Power utility $u(x) = x^{1-\gamma}/(1-\gamma)$

$$\frac{u'(c_{t+1})}{u'(c_t)} = \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}.$$

constant relative risk aversion coefficient (CRRA) $\gamma > 0$

Pricing Kernel

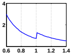
Assumption (Cochrane, 1996)

$$c_{t+1} = r_{m,t+1} = S_{m,t+1}/S_{m,t}$$

- State dependent pricing kernel from (??)

$$\mathcal{K}_{\theta}\left(r_{m,t+1}\right) = \beta_1 r_{m,t+1}^{-\gamma} \mathbf{I}\left\{r_{m,t+1} \in [0, x)\right\} + \beta_2 r_{m,t+1}^{-\gamma} \mathbf{I}\left\{r_{m,t+1} \in [x, \infty)\right\}$$

with $\theta = (\beta_1, \beta_2, \gamma)^{\top}$



Generalized Method of Moments

- Interpret (**??**) as the expectation of k moment conditions

$$\mathsf{E}_t\left[\mathcal{K}_\theta\left(r_{m,t+1}\right)R_{t+1} - 1_k\right] = 0_k, \tag{2}$$

where $R_{t+1} = \left(S_{1,t+1}/S_{1,t}, \ldots, S_{k,t+1}/S_{k,t}\right)^\top$. Then for

$$g\left(\theta\right) = \mathcal{K}_\theta\left(r_{m,t+1}\right)R_{t+1} - 1_k, \quad \mathsf{E}_t\left[g\left(\theta\right)\right] = 0_k$$

the sample analogue of (**??**)

$$g_n\left(\theta\right) = n^{-1}\sum_{t=0}^{n-1}\left\{\mathcal{K}_\theta\left(r_{m,t+1}\right)R_{t+1} - 1_k\right\} \tag{3}$$

over the data sample of size n.

Two-step GMM

- 1^{st} step: weighting matrix I_k

$$\widetilde{\theta}_n \stackrel{\text{def}}{=} \arg\min_{\theta} \left\{ g_n^\top(\theta)\, g_n(\theta) \right\}.$$

$$\widetilde{W}_n = n^{-1} \sum_{t=0}^{n-1} g(\widetilde{\theta}_n) g(\widetilde{\theta}_n)^\top.$$

- 2^{nd} step: weighting matrix $\widetilde{W}_n$

$$\widehat{\theta}_n \stackrel{\text{def}}{=} \arg\min_{\theta} \left\{ g_n^\top(\theta)\, \widetilde{W}_n^{-1} g_n(\theta) \right\}$$

Data

- Cross country analysis
  - Germany and UK, 1998–2007 (daily data)
  - Overlapping monthly returns
  - Rolling window (5y)

- Stock markets
  - Index returns (DAX, FTSE 100)
  - Returns of the largest 20 constituents of each market
  - Reference point: zero simple net market return ($x = 1$); 5y average market return

EPK Dynamics

Figure 5: EPK on the German stock market in 2005. Reference point: zero simple net market return.

Parameter Dynamics

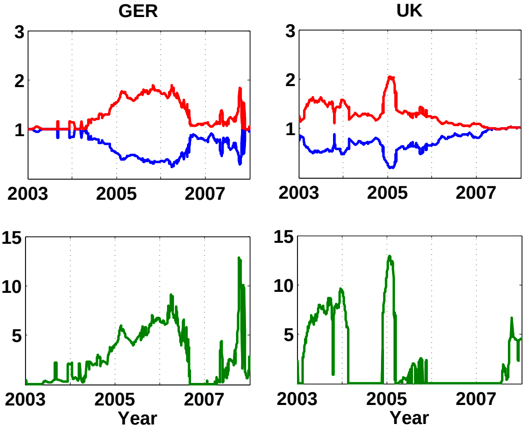
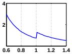


Figure 6: Estimated parameters $\widehat{\beta}_1$, $\widehat{\beta}_2$ and $\widehat{\gamma}$ on the German and the British stock market. Reference point: zero simple net market return.



Parameter Dynamics

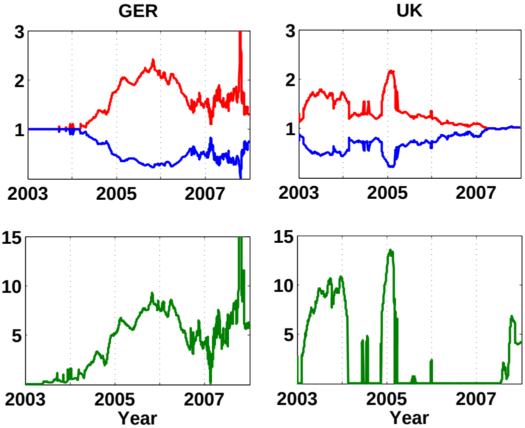
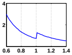


Figure 7: Estimated parameters $\widehat{\beta}_1$, $\widehat{\beta}_2$ and $\widehat{\gamma}$ on the German and the British stock market. Reference point: 5y mean market return.



EPK Puzzle - Stock Markets

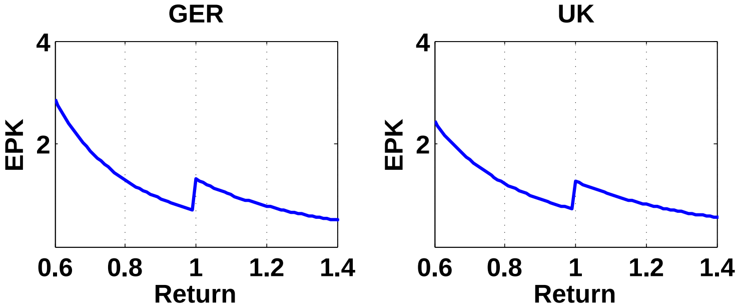
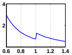


Figure 8: EPK given average estimated parameters from 2003-2007 on the German $\widehat{\theta} = (0.69, 1.31, 2.78)^\top$ and the British stock market $\widehat{\theta} = (0.72, 1.27, 2.39)^\top$. Reference point: zero simple net market return.



EPK Puzzle - Stock Markets

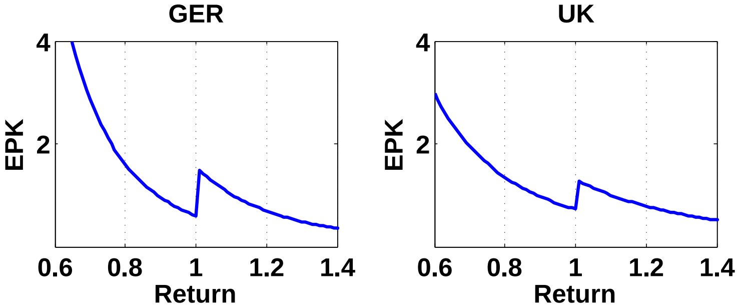
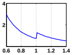


Figure 9: EPK given average estimated parameters from 2003-2007 on the German $\widehat{\theta} = (0.60, 1.54, 4.37)^\top$ and the British stock market $\widehat{\theta} = (0.73, 1.31, 2.76)^\top$. Reference point: 5y mean market return.



Conclusion

(i) Estimating pricing kernels (PK)

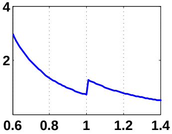
- State-dependent preferences - 'jump' in the PK
- Estimated 'jump' is time-persistent with different intensities

(ii) Cross country study

- Evidence for the existence of the 'jump' in both countries
- Positive comovements between EPKs' parameters

Cross Country Evidence for the EPK Puzzle

Maria Grith
Wolfgang Karl Härdle
Andrija Mihoci



Ladislaus von Bortkiewicz Chair of Statistics
C.A.S.E. – Center for Applied Statistics
and Economics
Humboldt–Universität zu Berlin
http://lvb.wiwi.hu-berlin.de
http://www.case.hu-berlin.de

References

Breeden, D., Gibbons, M. and Litzenberger, R.
Empirical tests of the consumption-oriented CAPM
Journal of Finance **44**: 231–262, 1989

Cochrane, J.
A cross-sectional test of an investment-based asset pricing model
Journal of Political Economy **104**: 572–621, 1996

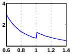
Dittmar, R.F.
Nonlinear Pricing Kernels, Kurtosis Preference, and Evidence from the Cross Section of Equity Returns
Journal of Finance **62**(1): 369–403, 2002

References

Hansen, L.P.
Large sample properties of generalized method of moments estimators
Econometrica **50**: 1029–1054, 1982

Hansen, L.P. and Jagannathan, R.
Assessing specification errors in stochastic discount factor models
Journal of Finance **52**: 557–590, 1997

Schweri, U.
Is the pricing kernel u-shaped?
NCCR Finrisk Working Paper 732, University of Zurich, 2011



Risk Neutral Valuation

Motivation

- Present value of the payoffs $\psi\left(S_T\right)$

$$P_0 = \mathsf{E}_Q\left[e^{-Tr}\psi(s_T)\right] = \int_0^\infty e^{-Tr}\psi(s_T)\ \mathcal{K}(s_T)p(s_T)\ ds_T$$

r risk free interest rate, $\{S_t\}_{t\in[0,T]}$ stock price process,
p pdf of S_T, Q risk neutral measure, $\mathcal{K}\left(\cdot\right)$ pricing kernel

PK under the Black-Scholes Model

▸ Motivation

- Geometric Brownian motion for S_t

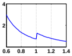
$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t$$

μ mean, σ volatility, W_t Wiener process

- Physical density p is log-normal, $\tau = T - t$

$$p_t(S_T) = \frac{1}{S_T\sqrt{2\pi\sigma^2\tau}} \exp\left[-\frac{1}{2}\left\{\frac{\log(S_T/S_t) - \left(\mu - \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}\right\}^2\right]$$

- Risk neutral density q is log-normal: replace μ by r



PK under the Black-Scholes Model ▸ Motivation

- PK is a decreasing function in S_T for fixed S_t

$$\mathcal{K}(S_t, S_T) = \left(\frac{S_T}{S_t}\right)^{-\frac{\mu - r}{\sigma^2}} \exp\left\{\frac{(\mu - r)\left(\mu + r - \sigma^2\right)\tau}{2\sigma^2}\right\}$$
$$= b\left(\frac{S_T}{S_t}\right)^{-\delta}$$

$b = \exp\left\{\frac{(\mu - r)\left(\mu + r - \sigma^2\right)\tau}{2\sigma^2}\right\}$ and $\delta = \frac{\mu - r}{\sigma^2} \geq 0$ constant relative risk aversion (CRRA) coefficient