

# Spatial Risk Premium on Weather and Hedging Weather Exposure in Electricity

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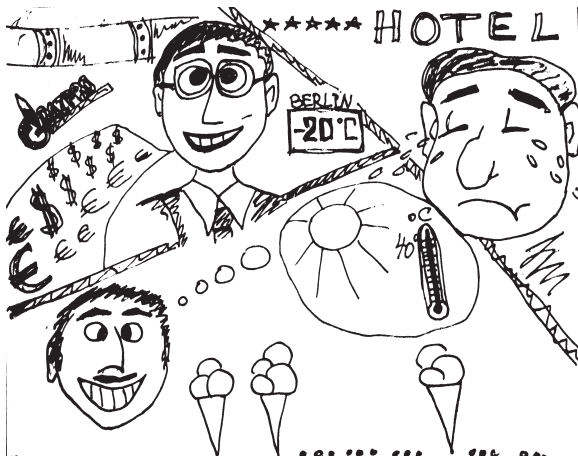
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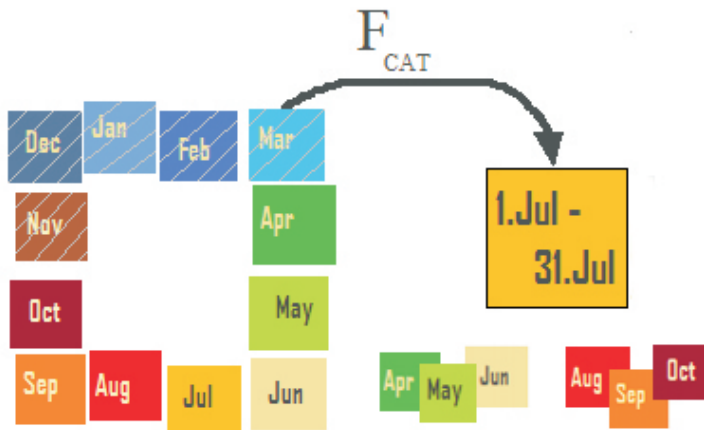
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"Everybody talks about the weather but nobody does anything about it." Mark Twain





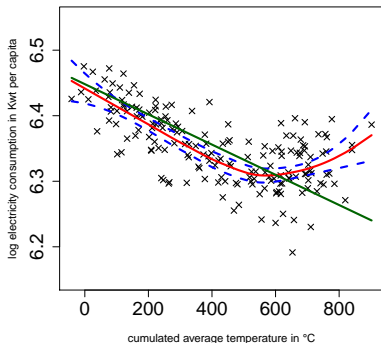


Figure 1: Estimated dependency of log electricity consumption in Germany on cumulative average temperature: 19960101-20100930, effects of prices and income removed (from Akdeniz Duran et al., 2011).



## Future Contracts on Temperature

- transfer weather related risks,
- number of agents limited,
- weather non-tradable, insurance nature,
- geographically separated markets, spatial relationship.





Figure 2: Contracts on 11 cities in Europe are traded on the CME.



## Pricing Model

□ Econometrics of temperature (Benth et al., 2007)

- ▶ seasonal function

$$\Lambda(t) = a + bt + \sum_{j=1}^k \left\{ c_{2j-1} \sin\left(\frac{2j\pi t}{365}\right) + c_{2j} \cos\left(\frac{2j\pi t}{365}\right) \right\}, \quad (1)$$

- ▶  $p$ -dimensional Ornstein-Uhlenbeck process  $\mathbf{X}(t)$ :

$$d\mathbf{X}(t) = A\mathbf{X}(t)dt + \mathbf{e}_p \sigma(t)dB(t),$$

- ▶  $B(t)$  is Brownian motion,  $\mathbf{e}_p$   $p$ th column of  $I_p$ ,  $\sigma(t)$  seasonal variation.



## Pricing Model

Price of a future on **C**umulative **A**verage **T**emperature index (CAT):  
(Benth et al., 2007)

$$\begin{aligned}
 F_{CAT}(t, \tau_1, \tau_2) = & \int_{\tau_1}^{\tau_2} \Lambda(u) du + \mathbf{a}(t) \mathbf{X}(t) & (2) \\
 & + \int_t^{\tau_1} \theta(u) \sigma(u) \mathbf{a}(u) \mathbf{e}_p du \\
 & + \int_{\tau_1}^{\tau_2} \theta(u) \sigma(u) \mathbf{e}_1^\top \mathbf{A}^{-1} [\exp \{ \mathbf{A}(\tau_2 - u) \} - I_p] \mathbf{e}_p du.
 \end{aligned}$$

with  $\mathbf{a}(t) = \mathbf{e}_1^\top \mathbf{A}^{-1} [\exp \{ \mathbf{A}(\tau_2 - t) \} - \exp \{ \mathbf{A}(\tau_1 - t) \}]$

- MPR  $\theta(t)$  unknown for specific locations.





## Pricing Temperature around the Globe

- $\theta(t)$  varies across locations.
- Connect  $\theta(t)$  to a known risk factor!
- Spatial model to interpolate for other locations.

How can FPCA help pricing an arbitrary location?



## Outline

1. Motivation ✓
2. Spatial Model for Risk Premium
  - ▶ Functional Principal Components (FPCA)
  - ▶ Geographically Weighted Regression (GWR)
3. Empirical Risk Premia and Hedging Weather Exposure
4. Outlook



## Risk Premium

Given (2) the RP at  $t = \tau_1$  for  $i$ th location and  $j$ th contract ( $RP_{ij}$ ):

$$\begin{aligned} RP_{ij}(t = \tau_1, \tau_2) &= F_{CAT,ij}(\theta^i, t = \tau_1, \tau_2) - \widehat{F}_{CAT,ij}(0, t = \tau_1, \tau_2) + \varepsilon_{ij}, \\ &= \int_{\tau_1}^{\tau_2} \theta^i(u) \sigma_{ij}(u) \mathbf{e}_1^\top A^{-1} \\ &\quad \times [\exp\{A(\tau_2 - u)\} - I_p] \mathbf{e}_p du + \varepsilon_{ij}. \end{aligned}$$

$\widehat{F}_{CAT,ij}(0, t = \tau_1, \tau_2)$  estimated price for  $j$ th contract in  $i$ th location, zero MPR and  $\varepsilon_{ij}$  noise.



- a **functional regression** set up with scalar response:

$$w^i(t) \stackrel{\text{def}}{=} \mathbf{e}_1^\top A^{-1} [\exp \{A(\tau_2 - t)\} - I_p] \mathbf{e}_p \text{ and} \\ \theta_w^i(t) = \theta(t)w^i(t):$$

$$RP_{ij}(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \theta_w^i(u) \sigma_{ij}(u) du + \varepsilon_{ij}.$$

- Regress RP on PC scores of  $\sigma(t)$  for dimension reduction.
- Note  $\theta_w^i(t)$  contains location dependent parameters, need spatial setting.



## Functional Principal Components

### 1. Decompose

$$\sigma_{ij}(t) = \{\sigma_{ij}(t) - \bar{\sigma}_i(t)\} + \bar{\sigma}_i(t),$$

$\sigma_{ij}(t)$  variation curve for  $i$ th location and  $j$ th month,

$\bar{\sigma}_i(t)$  average curve for  $i$ th location.

$$RP_{ij} = \int_{\tau_1}^{\tau_2} \theta_w^i(u) \bar{\sigma}_i(u) du + \int_{\tau_1}^{\tau_2} \theta_w^i(u) \underbrace{\{\sigma_{ij}(u) - \bar{\sigma}_i(u)\}}_{\text{FPCA for } \sigma_{ij}} du.$$

### 2. Perform FPCA: derive scores for temperature variation



## PC Scores

- PC scores for functions  $\sigma_{ij}(t)$   $i = 1, \dots, 9$  (9 cities),  $j = 1, \dots, 7$  (7 traded months):

$$c_{ijk} = \int_{\tau_1}^{\tau_2} \xi_{ik}(t) \{ \sigma_{ij}(t) - \bar{\sigma}_i(t) \} dt,$$

$c_{ijk}$  scores for  $K$  largest eigenvalues,  $\xi_{ik}(t)$  orthonormal eigenfunctions of  $\text{Cov}\{\sigma(\cdot)\}$  operator.

- Collect scores capturing the variance in the data in matrix  $C$ .
- Parametrize the relationship to RP by geographically weighted regression.



3. Regress the response  $RP_{ij}$  at  $t = \tau_1$  on the PC scores (Ramsay & Silverman, 2008):

$$RP_{ij} = \beta_{i0} + \int_{\tau_1}^{\tau_2} \sum_{k=1}^K \beta_{ik} \xi_{ik}(t) \{ \sigma_{ij}(t) - \bar{\sigma}_i(t) \} dt + \nu_{ij}$$

for  $i$ th location and  $j$ th month,  $K$  truncation parameter for the considered scores chosen by researcher.

- Functional form of MPR:  $\theta_w^i(t) = \sum_k \beta_{ik} \xi_{ik}(t)$  ▶ APPENDIX
- Need spatial model for regression on PC scores.



## Spatial Specification: GWR

Why GWR (Fotheringham et al., 2002)?

- ▣ distance based weights,
- ▣ nonstationarity over space,
- ▣ local nature of spatial dependence.





## GWR: the Model

$$W_i^{\frac{1}{2}} RP = W_i^{\frac{1}{2}} C \beta_i + e_i,$$

$$RP = (RP_{1,1}, RP_{2,1}, \dots, RP_{n,1}, RP_{1,2}, \dots \dots, RP_{n,7})^T,$$

$$C = \begin{pmatrix} c_{1,1,1} & \dots & c_{1,1,K} \\ c_{2,1,1} & \dots & c_{1,1,K} \\ \dots & \dots & \dots \\ c_{n,7,1} & \dots & c_{n,7,K} \end{pmatrix}$$

$n$  – total number of locations

$K$  – number of PC scores



## GWR: the Model

$$W_i = \text{diag}(w_i), \quad i = 1, \dots, n$$

$$w_i = \text{diag} \left[ \exp \left\{ -\frac{1}{2} \left( \frac{d_{i1}}{h^*} \right)^2 \right\}, \dots, \exp \left\{ -\frac{1}{2} \left( \frac{d_{in}}{h^*} \right)^2 \right\} \right],$$

$$h^* = \arg \min_{h \in H} \sum_{m=1}^{7n} \left\{ RP_m - \widehat{RP}_{\neq m}(h) \right\}^2,$$

with  $d_{il}$ ,  $l = 1, \dots, n$  euclidean distances to  $i$ th city,  $\widehat{RP}_{\neq m}(h)$  estimated RP without the  $m$ th value using  $h$ .



## Temperature Data

City	First Date	Last Date	First $F_{CAT}$ Trade
Amsterdam	19730101	20101231	20030401
Berlin	19480101	20101231	20030401
Barcelona	19730101	20101231	20050401
Essen	19700101	20101231	20050401
London	19730101	20101231	20030401
Madrid	19730101	20101231	20050401
Paris	19730101	20101231	20030401
Rome	19730101	20101231	20050401
Stockholm	19730101	20101231	20030401

Table 1: Average Temperatures without 29th February. Source Bloomberg and DWD.



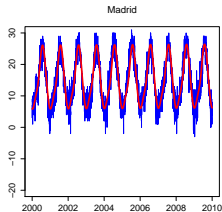
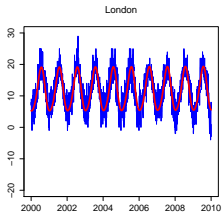
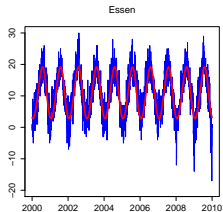
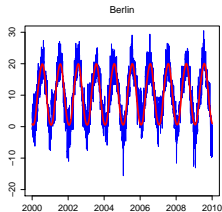
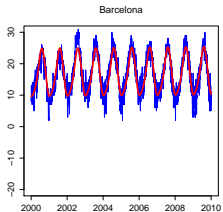
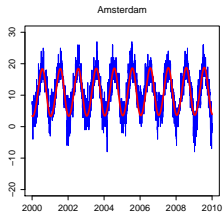
## Volatility Functions $\sigma_i(t)$

Fit to data

- ▣ Seasonality  $\Lambda(t)$ ,
- ▣ AR( $p$ ) process with seasonally heteroscedastic errors,
- ▣ estimate  $RP_{ij}$ ,  $i = 1, \dots, 9$  and  $j = 1, \dots, 7$ ,
- ▣ estimate  $\sigma_i(t)$ ,  $t \in [1, 365]$  using residual standard deviation for each day of year and smooth by Fourier series.



## Seasonality



## Seasonality

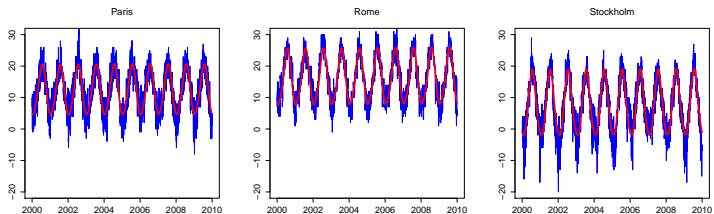


Figure 3: Daily average temperatures  $T(t)$  (blue) and seasonality  $\Lambda(t)$  (red).



	Essen		London		Madrid	
	estimate	t.stat	estimate	t.stat	estimate	t.stat
$a$	10.66	171.30	10.81	217.33	13.97	286.51
$b$	$0.1 \cdot 10^{-4}$	0.66	$0.7 \cdot 10^{-4}$	11.10	$0.9 \cdot 10^{-4}$	14.64
$c_1$	-2.32	-52.79	-2.48	-70.43	-3.30	-95.68
$c_2$	-7.82	-177.64	-6.42	-182.57	-8.93	-259.17
$c_3$	0.49	11.21	0.77	21.85	1.67	48.46
$c_4$	—	—	0.23	6.66	0.25	7.21
$c_5$	—	—	—	—	-0.19	-5.39
$c_6$	—	—	—	—	-0.34	-9.87

Table 2: Estimated Parameters of seasonality (1) for Essen, London, Paris



$AR(p)$ 

	Amsterdam		Barcelona		Berlin	
	estimate	t.stat	estimate	t.stat	estimate	t.stat
$\alpha_1$	0.89	105.05	0.70	83.14	0.92	139.69
$\alpha_2$	-0.19	-16.76	0.03	3.17	-0.20	-23.14
$\alpha_3$	0.09	10.46	0.01	1.29	0.08	11.99
$\alpha_4$	-	-	0.03	3.64	-	-

Table 3: Estimated Parameters of  $AR(p)$  for Amsterdam, Barcelona, Berlin



RP

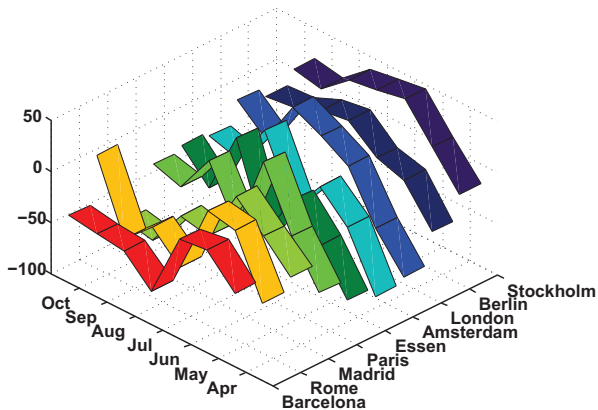
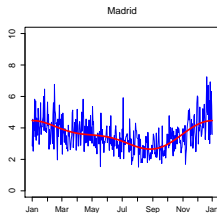
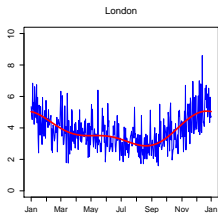
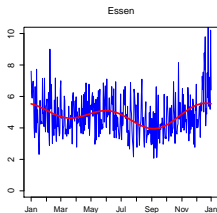
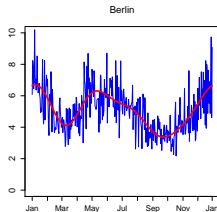
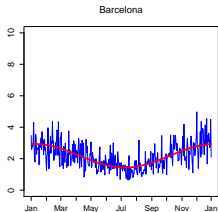
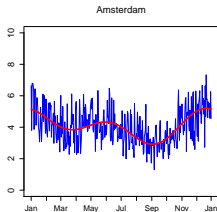


Figure 4: Average RP for traded locations computed according to (2)



## Seasonal Variation



## Seasonal Variation

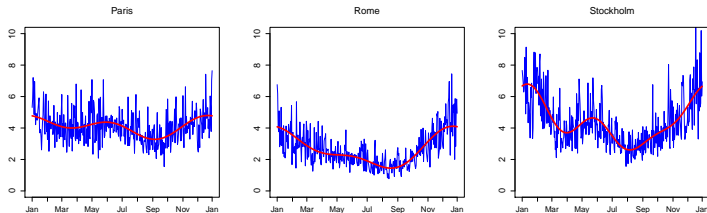
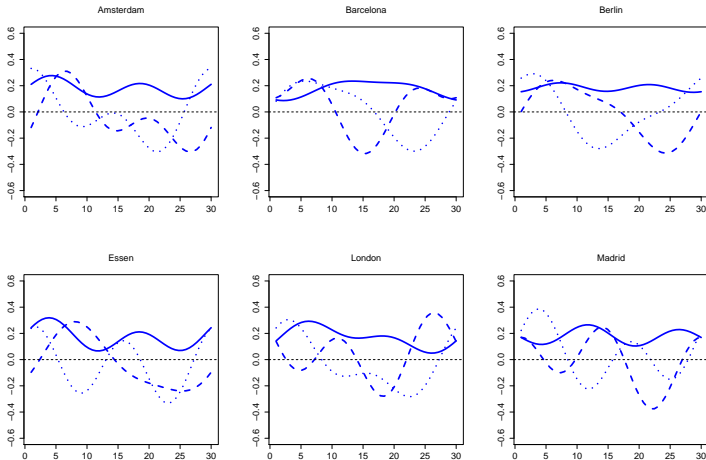


Figure 5: Estimated  $\sigma(t)$  (blue) and smoothed by Fourier series (red).



# Eigenfunctions



## Eigenfunctions

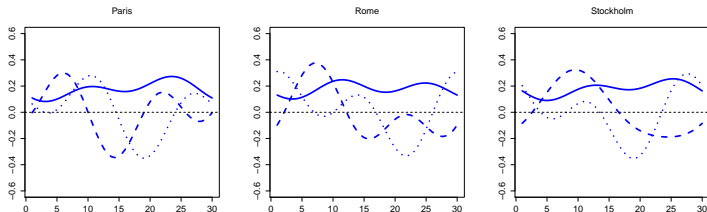
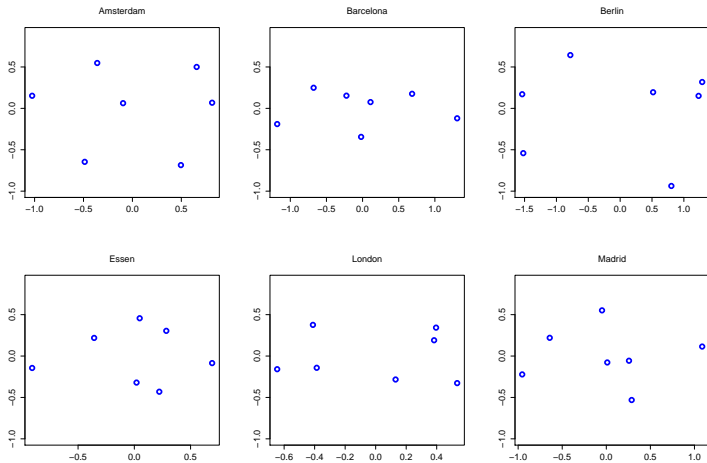


Figure 6: FPCA weight functions: eigenfunction  $\xi_1$  (solid),  $\xi_2$  (dashed),  $\xi_3$  (dotted).



## FPCA Scores



## FPCA Scores

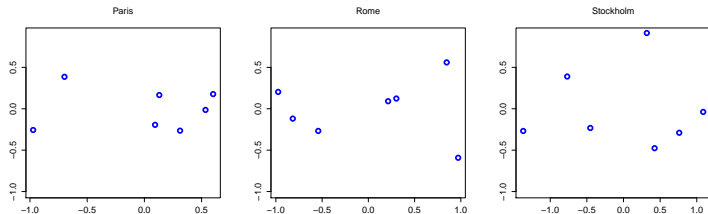
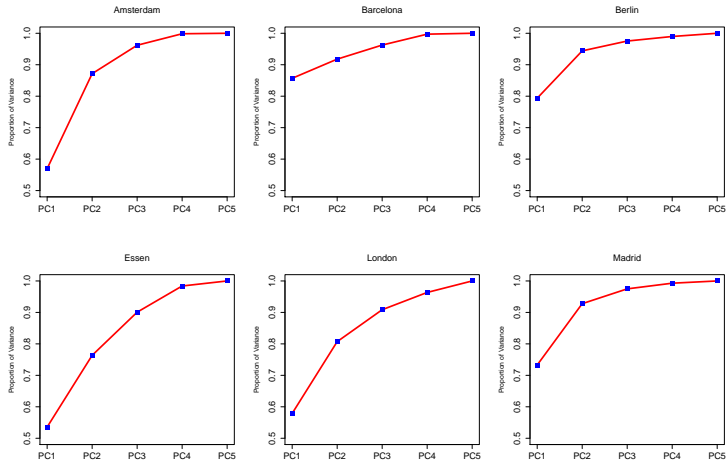


Figure 7: FPCA scores  $c_{ij1}$  and  $c_{ij2}$



## Explained Proportion of Variance





## FPCA Scores

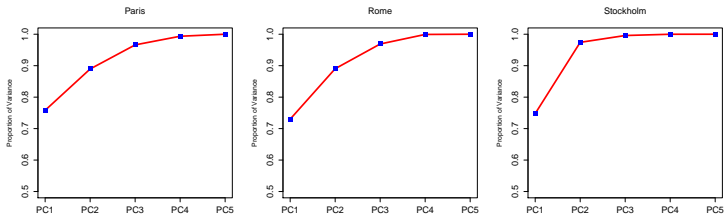
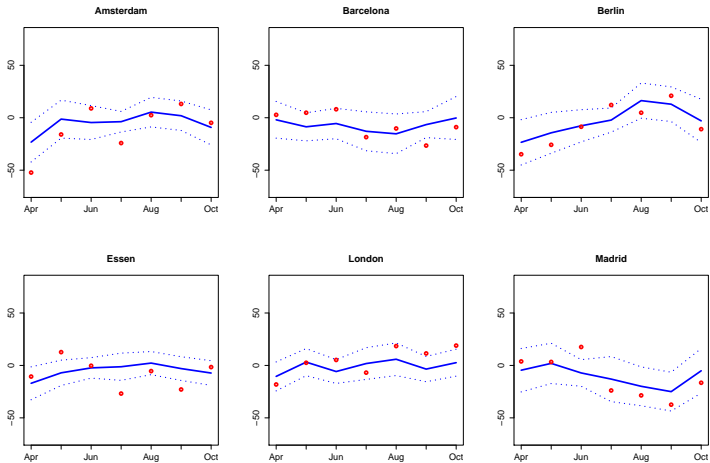


Figure 8: Proportion of Variance explained by the corresponding PC.



## GWR Estimation Results



## GWR Estimation Results

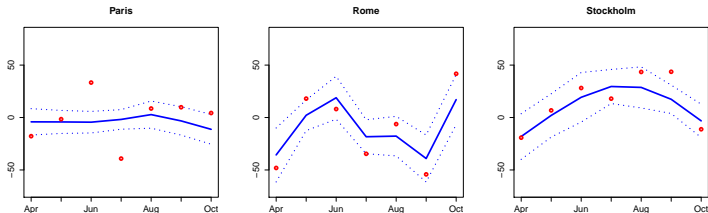


Figure 9: RP (red) and fitted values with 95% CI (blue) returned by GWR.



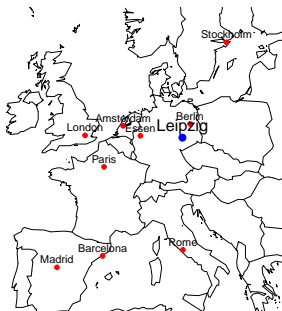
City	$\beta_0$	$\beta_1$	$\beta_2$	$\beta_3$	$R_{loc}^2$
Amsterdam	-4.68**	-5.28	15.31**	15.04**	0.25
Barcelona	-7.35	6.17**	-5.53	8.27	0.34
Berlin	-3.06	-8.07	17.58**	-9.90*	0.61
Essen	-5.10**	-4.86	14.87**	10.85*	0.30
London	-3.27	-5.43	12.89**	18.14**	0.22
Madrid	-10.40	11.17**	-7.98	30.55**	0.42
Paris	-3.75**	-2.85	10.98*	13.38*	0.21
Rome	-5.54 <sup>o</sup>	12.00*	-23.38**	-77.01**	0.73
Stockholm	10.82	-16.29	16.91**	-41.96**	0.72
Leipzig	-4.42	-6.64	16.07	-7.60	-

Table 4: Estimated Parameters of GWR ( $h^*=4.98$ ). \*\* indicate significance on  $\leq 1\%$  level, \* – on 5% and <sup>o</sup> – on 10%. Dummy variables for north and south sea coast cities omitted here. Weights for Leipzig (0.13, 0, 0.42, 0.24, 0.02, 0, 0.05, 0.07, 0.07)<sup>T</sup>.



## Example: Hedging weather risk in electricity demand

- An electricity provider in **Leipzig** transfers risk via CAT futures.
- What **RP** one would pay for  $F_{CAT}$  in August 2010?



## Out-of-Sample Forecast: Leipzig

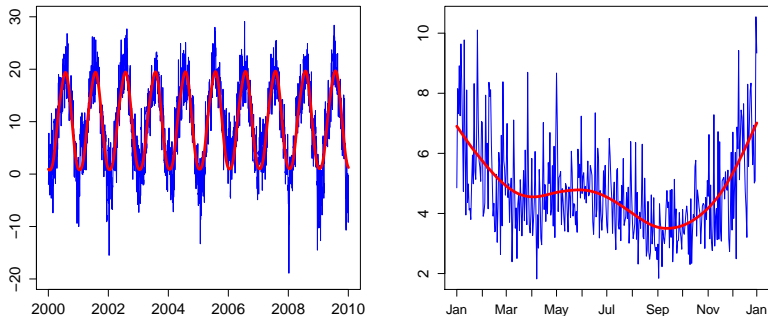


Figure 10:  $\Lambda_t, \sigma_t$  for Leipzig.



## Out-of-Sample Forecast: Leipzig

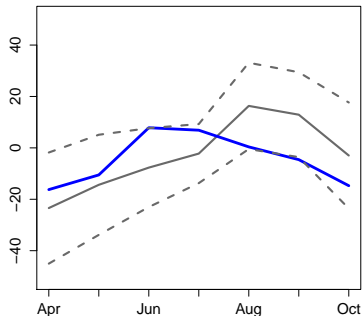
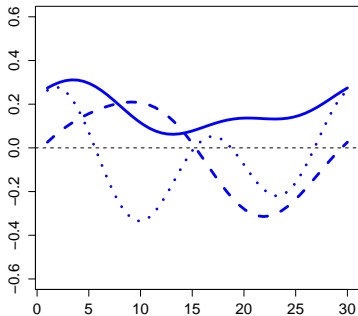
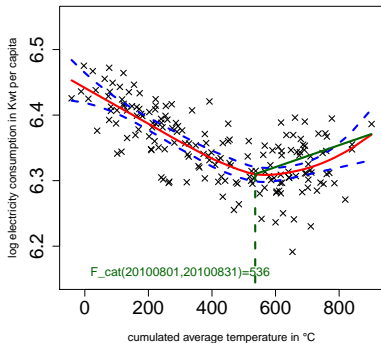


Figure 11:  $\xi$  and the resulting forecast for *RP* for Leipzig vs *RP* of Berlin.



$$\begin{aligned} F_{CAT}(\hat{\theta}, 20100801, 20100831) &= F_{CAT}(0, 20100801, 20100831) + RP \\ &= 534.6 + 1.4 = 536. \end{aligned}$$





## Example: Hedging weather risk in electricity demand

- $c$  – marginal costs of meeting additional log demand of 1% per person
- $b$  – estimated marginal effects of  $1^\circ\text{C}$  CAT on log demand starting from threshold  $F_{CAT}$
- $\alpha$  – number of WD hold,  $p$  – tick value of WD (for traded futures in Europe: 20EUR)

exposure	benefits
$cb(CAT - F_{CAT})$	$\alpha p(CAT - F_{CAT})$

- hedging, s.t.

$$cb = \alpha p$$



## Example: Hedging weather risk in electricity demand

parameter	value	units
$c$	100,000	EUR per 1% log Kwt/pP
$b$	0.0016	1% log Kwt/pP and 1° C CAT.
$p$	20	EUR per 1° C CAT
$\alpha$	8	contracts long

Table 5: An elementary example of a hedging strategy.



## Appendix

$$\begin{aligned}
 RP_{ij} &= \underbrace{\int_{\tau_1}^{\tau_2} \theta_w^i(u) \bar{\sigma}_i(u) du}_{\beta_{i0}} + \int_{\tau_1}^{\tau_2} \underbrace{\theta_w^i(u)}_{\sum_m \beta_{im} \xi_{im}(u)} \underbrace{\{\sigma_{ij}(u) - \bar{\sigma}_i(u)\}}_{\sum_k c_{ijk} \xi_{ik}(u)} du \\
 &= \beta_{i0} + \sum_m \sum_k \beta_{im} c_{ijk} \underbrace{\int_{\tau_1}^{\tau_2} \xi_{ik}(u) \xi_{im}(u) du}_{= \begin{cases} 0, & k \neq m, \\ 1, & k = m \end{cases}} \\
 &= \beta_{i0} + \sum_k \beta_{ik} c_{ijk}.
 \end{aligned}$$

▶ BACK



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