Pricing Temperature around the Globe

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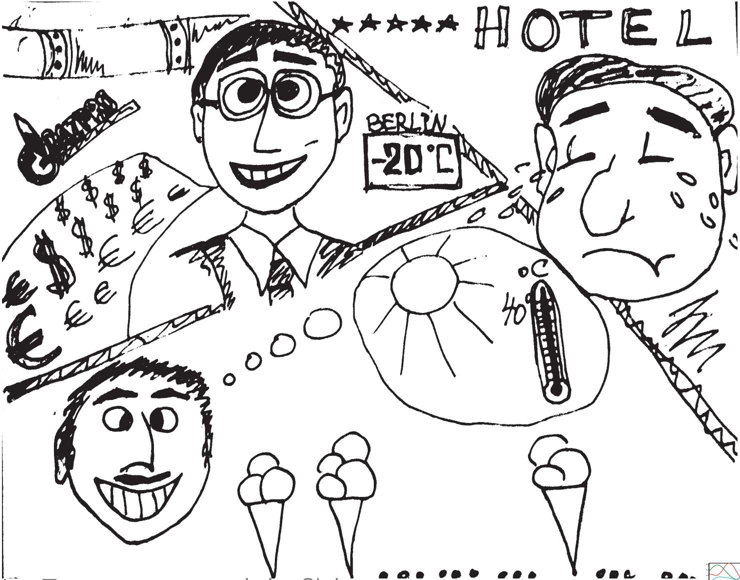
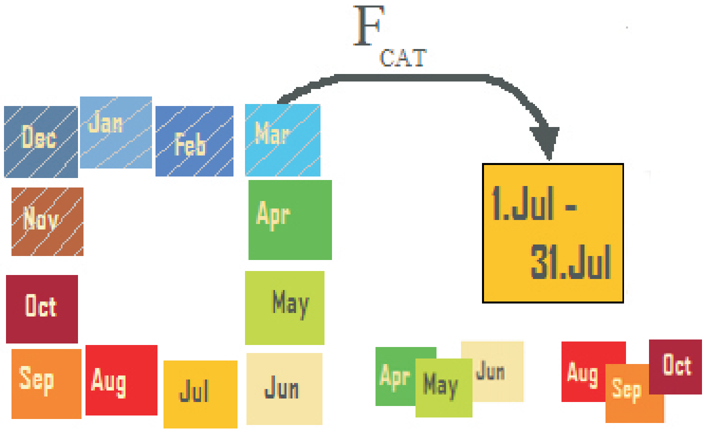

HOTEL
BERLIN
-20°C
°C
40°


F
CAT
Dec
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Aug
Sep
Oct

Future Contracts on Temperature

- transfer weather related risks,
- number of agents limited,
- weather non-tradable, insurance nature,
- geographically separated markets, spatial relationship.

Figure 1: Contracts on 9 cities in Europe are traded on CME

Seasonality

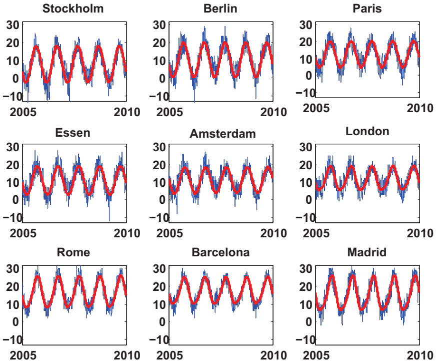


Figure 2: Daily average temperatures $T(t)$ (blue) and seasonality $\Lambda(t)$ (red).

Pricing Model

- Econometrics of temperature
  - seasonal function

$$\Lambda(t) = a + bt + \sum_{j=1}^{k} \left\{ c_j \sin\left(\frac{2j\pi}{365}\right) + d_j \cos\left(\frac{2j\pi}{365}\right) \right\},$$

  - p-dimensional Ornstein-Uhlenbeck process $\mathbf{X}(t)$:

$$d\mathbf{X}(t) = A\mathbf{X}(t)dt + \mathbf{e}_p \sigma(t) dB(t),$$

  - $B(t)$ is Brownian motion, $\mathbf{e}_p$ pth column of I_p, $\sigma(t)$ seasonal variation.

Seasonal Variation

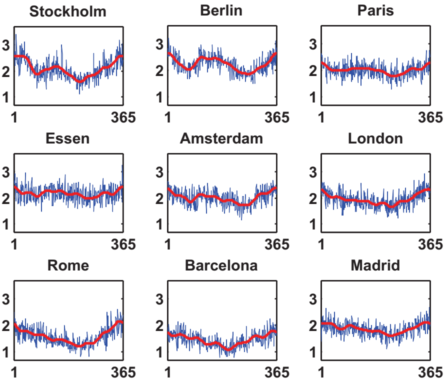


Figure 3: Estimated $\sigma(t)$ (blue) and smoothed by Fourier series (red).

Pricing Model

Price of a future on Cumulative Average Temperature index (CAT):

$$
\begin{aligned}
F_{CAT(t,\tau_1,\tau_2)} &= \int_{\tau_1}^{\tau_2} \Lambda(u)du + \mathbf{a}(t)\mathbf{X}(t) \\
&+ \int_{t}^{\tau_1} \theta(u)\sigma(u)\mathbf{a}(u)\mathbf{e}_p du \\
&+ \int_{\tau_1}^{\tau_2} \theta(u)\sigma(u)\mathbf{e}_1^\top A^{-1}\left[\exp\left\{A(\tau_2 - u)\right\} - I_p\right]\mathbf{e}_p du.
\end{aligned}
\tag{1}
$$

with $\mathbf{a}(t) = \mathbf{e}_1^\top A^{-1}\left[\exp\left\{A(\tau_2 - t)\right\} - \exp\left\{A(\tau_1 - t)\right\}\right]$

- MPR $\theta(t)$ unknown for specific locations.

Pricing Temperature around the Globe

- $\theta(t)$ varies across locations,
- Connect $\theta(t)$ to a known risk factor!
- Spatial model to interpolate for other locations.

How can FPCA help pricing an arbitrary location?

Outline

1. Motivation ✓
2. Functional Principal Components (FPCA)
3. Bayesian Geographically Weighted Regression (BGWR)
4. Empirical Findings
5. Outlook

- RP for $t = \tau_1$, ith location and jth contract month (RP_{ij}) assuming (1),
- a functional regression set up with scalar response: $w(t) \stackrel{def}{=} \mathbf{e}_1^\top A^{-1} \left[\exp\{A(\tau_2 - t)\} - I_p\right] \mathbf{e}_p$ and $\theta_w(t) = \theta(t) w(t)$:

$$RP_{ij}(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \theta_w^{ij}(u) \sigma_{ij}(u) du + \varepsilon_{ij}.$$

- No "registration" problem: domain $t \in [1, 365]$ fixed.
- Regress RP on PC scores of $\sigma(t)$ for dimension reduction.

Functional Principal Components

1. Decompose

$$\sigma_{ij}(t) = \{\sigma_{ij}(t) - \bar{\sigma}_i(t)\} + \bar{\sigma}_i(t),$$

$\sigma_{ij}(t)$ variation curve for ith location and jth month,

$\bar{\sigma}_i(t)$ average curve for ith location.

$$\begin{aligned} RP_{ij} &= \int_{\tau_1}^{\tau_2} \theta_w^{ij}(u)\bar{\sigma}_i(u)du \\ &\quad + \int_{\tau_1}^{\tau_2} \theta_w^{ij}(u) \underbrace{\{\sigma_{ij}(u) - \bar{\sigma}_i(u)\}}_{\text{FPCA for } \sigma_{ij}} du + \varepsilon_{ij}. \end{aligned}$$

2. Perform FPCA: derive scores for temperature variation

FPC Scores

- PC scores for functions $\sigma_{ij}(t)$ $i = 1, \ldots, 9$ (9 cities), $j = 1, \ldots, 7$ (7 traded months):

$$c_{ijk} = \int \xi_{ik}(t) \left\{ \sigma_{ij}(t) - \bar{\sigma}_i(t) \right\} dt,$$

  c_{ijk} scores for K largest eigenvalues, $\xi_{ik}(t)$ orthonormal eigenfunctions of $Cov\{\sigma(t)\}$ operator.

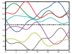
- Collect scores capturing the variance in the data in matrix C.

3. Regress the response RP_{ij} at $t = \tau_1$ on the PC scores Ramsay (2008):

$$RP_{ij} = \beta_0^i \quad + \quad \int \sum_{k=1}^{K} \beta_k^i \xi_{ik}(t)\{\sigma_{ij}(t) - \bar{\sigma}_i(t)\}dt + \nu_{ij}$$

for ith location and jth month,

- Recover functional form of MPR: $\theta_w^{ij}(t) = \sum_k \beta_k^i \xi_{ik}(t)$
- Need spatial model for regression on PC scores.



Spatial Specification: BGWR

Why BGWR (LeSage 1998)?

- distance based weights,
- nonconstant variance over space and regime shifts,
- prior info about spatial dependence,
- valid inference in small samples.

Starting Point: GWR

$$
\begin{aligned}
W_i^{\frac{1}{2}} RP &= W_i^{\frac{1}{2}} C\beta_i + \varepsilon_i, \\
C &- \text{matrix of FPCA scores}, \\
W_i &= diag\left\{\exp\left(\frac{-d_{i1}}{h}\right), \ldots, \exp\left(\frac{-d_{in}}{h}\right)\right\}, \\
d_{il},\ l = 1, \ldots, n &- \text{distances to } i\text{th city}, \\
h &- \text{decay bwth by CV.}
\end{aligned}
$$

- no valid inferences
- sensitive to outliers

BGWR: the Model

$$
\begin{aligned}
W_i^{\frac{1}{2}} RP &= W_i^{\frac{1}{2}} C\beta_i + \varepsilon_i, \\
C &- \text{ matrix of FPCA scores,} \\
W_i &= diag\left\{\exp\left(\frac{-d_{i1}}{h}\right), \ldots, \exp\left(\frac{-d_{in}}{h}\right)\right\}, \\
d_{il},\ l = 1, \ldots, n &- \text{ distances to } i\text{th city,} \\
h &- \text{ decay bwth by CV,} \\
\varepsilon_i &\sim N(0, \sigma_i^2 V_i), \quad V_i = diag(\nu_{1i}, \ldots, \nu_{ni}), \\
\nu_{ij} &\sim IG(r, r),\ r \text{ dispersion in } V_i.
\end{aligned}
$$

Example: Three Cities

Berlin, **A**msterdam and **R**ome

$$D = \begin{pmatrix} d_{BB} & d_{BA} & d_{BR} \\ \dots & d_{AA} & d_{AR} \\ \dots & \dots & d_{RR} \end{pmatrix} = \begin{pmatrix} 0 & 74.0 & 112.8 \\ \dots & \ddots & 167.6 \\ \dots & \dots & 0 \end{pmatrix}$$

$$W_B = diag\left\{1, \exp\left(-\frac{74}{h}\right), \exp\left(-\frac{112.8}{h}\right)\right\}$$

$$W_A = diag\left\{\exp\left(-\frac{74}{h}\right), 1, \exp\left(-\frac{167.6}{h}\right)\right\}$$

$$W_R = diag\left\{\exp\left(-\frac{112.8}{h}\right), \exp\left(-\frac{167.6}{h}\right), 1\right\}$$

BGWR: the Model

$$
\begin{aligned}
\beta_i &= (\omega_{i1} \otimes I_k \ldots \omega_{in} \otimes I_k) \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_n \end{pmatrix} + u_i, \\
\omega_{ij} &= \frac{\exp\left(\frac{-d_{ij}}{h}\right)}{\sum_{j=1}^{n} \exp\left(\frac{-d_{ij}}{h}\right)}, \ \omega_{ii} = 0, \\
u_i &\sim N\left\{0, \sigma_i^2 \left(C W_i^{\frac{1}{2}} V_i^{-1} W_i^{\frac{1}{2}} C\right)^{-1}\right\}, \\
\sigma_i^2 &\sim IG(1,1).
\end{aligned}
$$

BGWR: Posteriors

$$
\begin{aligned}
p(\beta_i|\ldots) &\propto N\left\{\hat{\beta}_i, \sigma_i^2\left(C^\top W_i^{\frac{1}{2}} V_i^{-1} W_i^{\frac{1}{2}} C\right)^{-1}\right\},\\
\hat{\beta}_i &= \left(C^\top W_i^{\frac{1}{2}} V_i^{-1} W_i^{\frac{1}{2}} C\right)^{-1}\left(C^\top W_i^{\frac{1}{2}} V_i^{-1} W_i^{\frac{1}{2}} RP\right),\\
p(\sigma_i^2|\ldots) &\propto IG\left\{\frac{m}{2}+1, \frac{1}{2}\sum_{j=1}^{m}\left(\frac{e_{ij}^2}{\nu_{ij}}\right)+1\right\}, \ m \text{ obs with } \omega \neq 0,\\
p(\nu_{ij}|\ldots) &\propto IG\left\{\frac{1}{2}+r, \frac{1}{2}\frac{e_{ij}^2}{\sigma_i^2}+r\right\},\\
e_{ij} &= W_i^{\frac{1}{2}} RP_j - W_i^{\frac{1}{2}} C_j\beta_i.
\end{aligned}
$$

Estimation via GS

Draw from conditional posteriors of β_i, σ_i, V_i:

- start with $\beta_i^0, \sigma_i^0, V_i^0$,
- for $i = 1, \ldots, n$ obtain:
  - β_i^1 from $p(\beta_i | \sigma_i^0, V_i^0, \beta_{-i}^0)$,
  - V_i^1 from $p(V_i | \sigma_i^0, \beta_{-i}^0, \beta_i^1)$,
- sample σ_i^1 from $p(\sigma_i | V_i^1, \beta_i^1)$,
- start from the beginning with the new values.

After N steps and M burn-in we get $N - M$ samples from the joint density.

Temperature Data

City	First Date	Last Date	First F_{CAT} Trade
Amsterdam	19730101	20090930	20030401
Berlin	19480101	20090930	20030401
Barcelona	19730101	20090930	20050401
Essen	19700101	20090930	20050401
London	19730101	20090930	20030401
Madrid	19730101	20090930	20050401
Paris	19730101	20090930	20030401
Rome	19730101	20090930	20050401
Stockholm	19730101	20090930	20030401

Table 1: Average Temperatures without 29th February. Source Bloomberg and DWD.

Volatility Functions $\sigma_i(t)$

Fit to data

- Seasonality $\Lambda(t)$,
- AR(p) process with seasonally heteroscedastic errors,
- estimate RP_{ij},
- estimate $\sigma_i(t)$, $t \in [1, 365]$ using residual standard deviation for each day of year and smooth by Fourier series.

RP

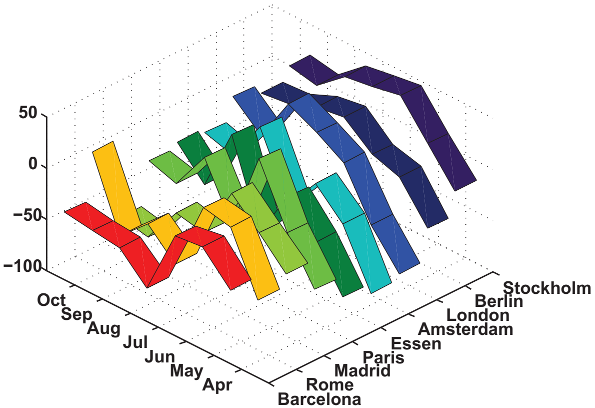


Figure 4: Average RP for Traded Locations computed according to (1)

Volatility Functions $\sigma_i(t)$

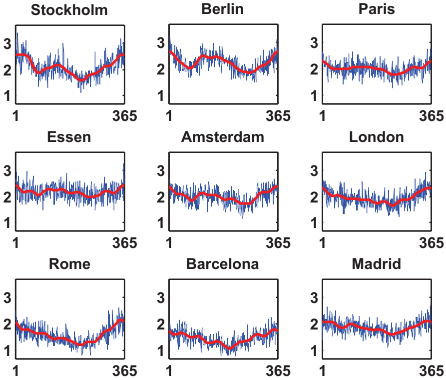


Figure 5: Estimated seasonal volatilities (blue) and smoothed by Fourier series of order 9 (red)

Eigenfunctions

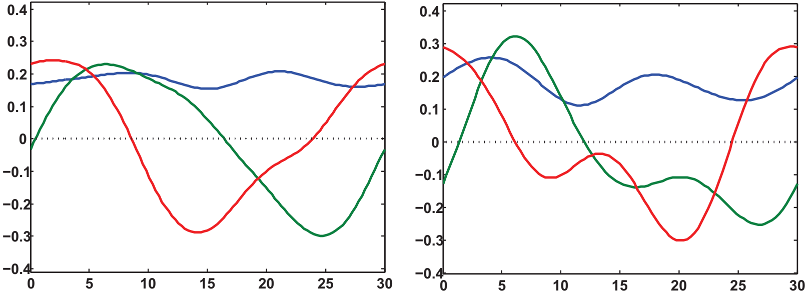
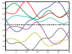


Figure 6: FPCA weight functions for Berlin (left) and Amsterdam (right): eigenfunction ξ_1: 79.8%, 59.6% (blue), ξ_2: 15.3%, 29.4% (green),ξ_3: 3.4%, 7.8%(red)



Eigenfunctions

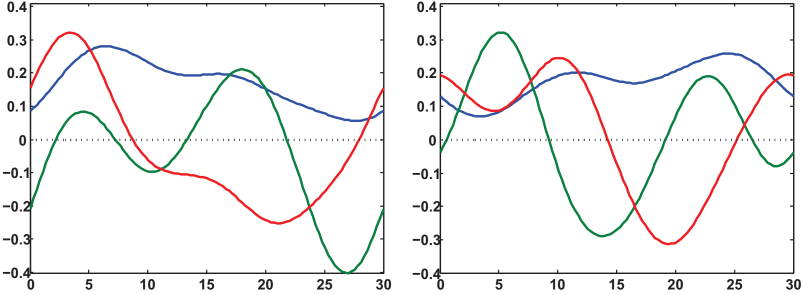


Figure 7: FPCA weight functions for London (left) and Paris (right): eigenfunction ξ_1: 64.3%, 85.4% (blue), ξ_2: 18.0%, 7.3% (green), ξ_3: 10.0%, 4.1% (red)

FPCA Scores

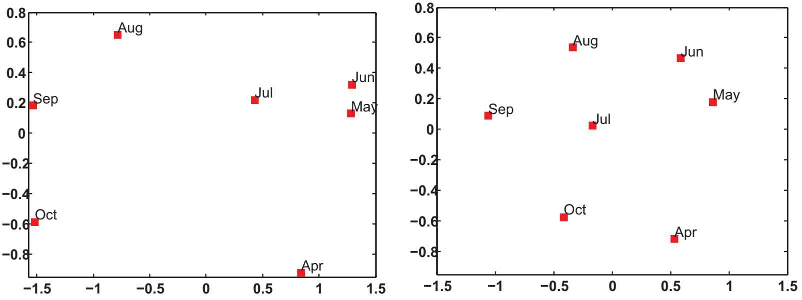
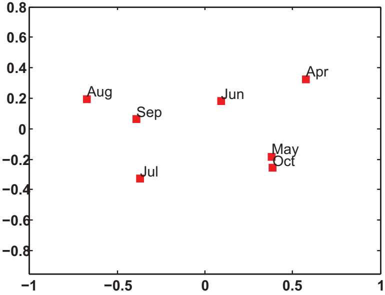


Figure 8: FPCA scores c_{ij1} and c_{ij2} for Berlin (left) and Amsterdam (right)

FPCA Scores



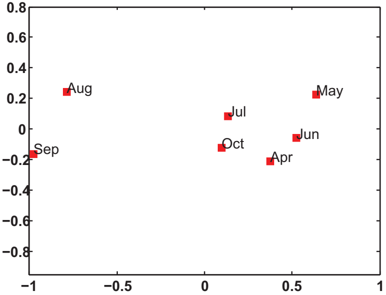


Figure 9: FPCA scores c_{ij1} and c_{ij2} for London (left) and Paris (right)

BGWR Estimation Results

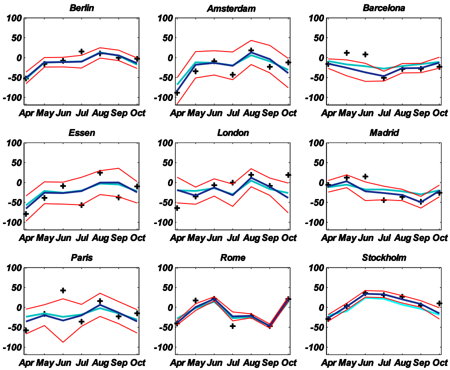


Figure 10: RP (+) and fitted values with 95% CI (red) returned by BGWR (blue) and GWR (turquoise)

Recovered $\theta(t)$: Berlin and Amsterdam

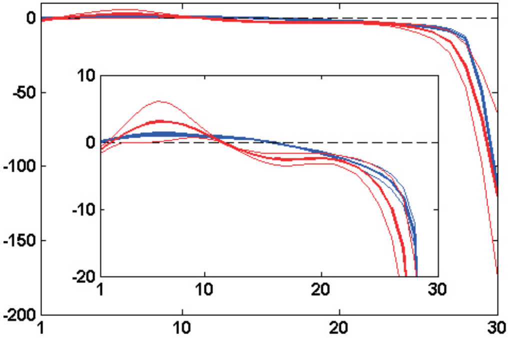


Figure 11: Recovered $\theta(t)$ for April in Berlin (blue) and Amsterdam (red)

Recovered $\theta(t)$: London and Paris

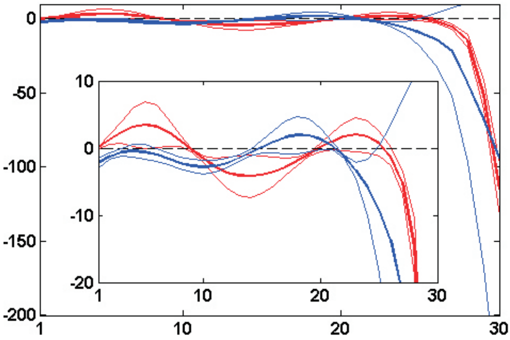


Figure 12: Recovered $\theta(t)$ for April in London (blue) and Paris (red)

BGWR Leave-One-Out Forecast

Quality of out-of-sample forecast?

leave-one-out scheme:

- take one city out of the sample,
- fit the model,
- forecast RP for the discarded location.

BGWR Leave-One-Out Forecast

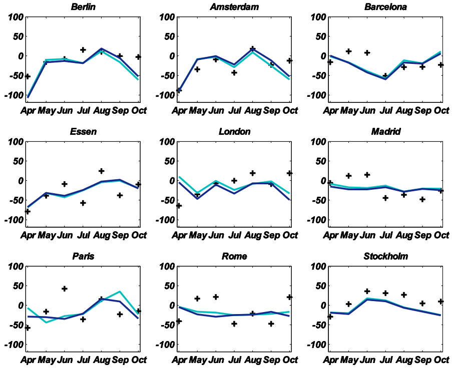


Figure 13: RP (+) and fitted values returned by BGWR (blue) and GWR (turquoise)

Out-of-Sample Forecast: Munich

- A beer producer in Munich tranfers risk via CAT futures.
- What RP he'd pay for Munich?

Out-of-Sample Forecast: Munich

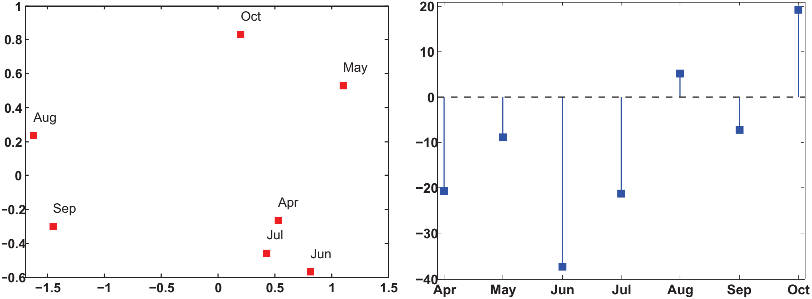


Figure 14: FPCA scores for Munich and resulting forecast for RP.

Outlook

- Expand the model to further locations (USA, Asien, Australien).
- RP interpolation
- Alternative spatial specifications.
- Other pricing models, e.g. equilibrium approach.

Literature

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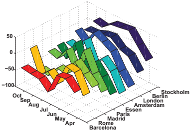

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