Pricing Temperature around the Globe

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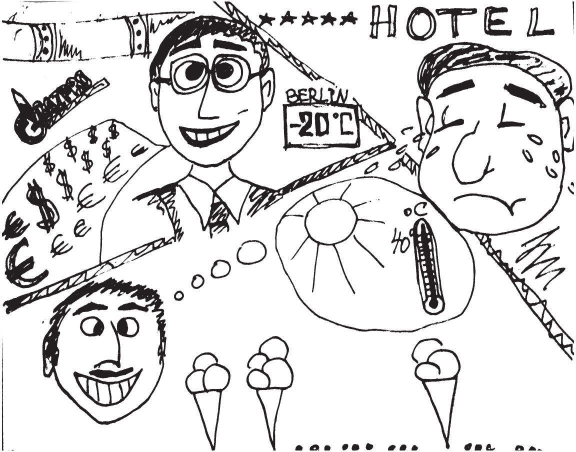
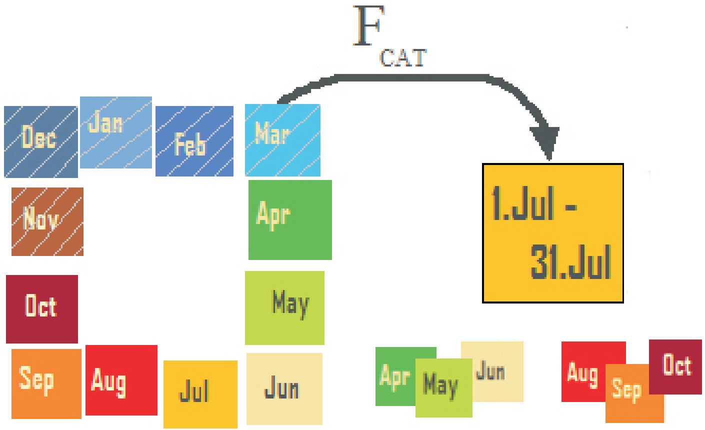


"Everybody complains about weather but nobody does anything about it." Mark Twain


F
CAT
Dec
Jan
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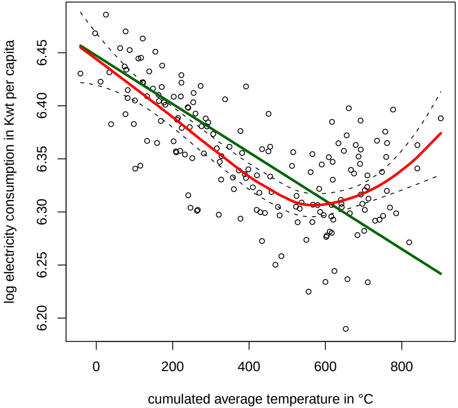


Figure 1: Estimated dependency of log electricity consumption in Germany on cumulative average temperature: 19960101-20100731

Future Contracts on Temperature

- transfer weather related risks,
- number of agents limited,
- weather non-tradable, insurance nature,
- geographically separated markets, spatial relationship.

Figure 2: Contracts on 9 cities in Europe are traded on CME

Pricing Model

- Econometrics of temperature
  - seasonal function

$$\Lambda(t) = a + bt + \sum_{j=1}^{k} \left\{ c_{2j-1} \sin\left(\frac{2j\pi}{365}\right) + c_{2j} \cos\left(\frac{2j\pi}{365}\right) \right\}, \quad (1)$$

  - p-dimensional Ornstein-Uhlenbeck process $\mathbf{X}(t)$:

$$d\mathbf{X}(t) = A\mathbf{X}(t)dt + \mathbf{e}_p \sigma(t) dB(t),$$

  - $B(t)$ is Brownian motion, $\mathbf{e}_p$ pth column of I_p, $\sigma(t)$ seasonal variation.

Pricing Model

Price of a future on Cumulative Average Temperature index (CAT):

$$
\begin{aligned}
F_{CAT(t,\tau_1,\tau_2)} = & \int_{\tau_1}^{\tau_2} \Lambda(u)du + \mathbf{a}(t)\mathbf{X}(t) \\
& + \int_{t}^{\tau_1} \theta(u)\sigma(u)\mathbf{a}(u)\mathbf{e}_p du \\
& + \int_{\tau_1}^{\tau_2} \theta(u)\sigma(u)\mathbf{e}_1^\top A^{-1} \left[\exp\left\{A(\tau_2 - u)\right\} - I_p\right] \mathbf{e}_p du.
\end{aligned} \tag{2}
$$

with $\mathbf{a}(t) = \mathbf{e}_1^\top A^{-1} \left[\exp\left\{A(\tau_2 - t)\right\} - \exp\left\{A(\tau_1 - t)\right\}\right]$

- MPR $\theta(t)$ unknown for specific locations.

Pricing Temperature around the Globe

- $\theta(t)$ varies across locations.
- Connect $\theta(t)$ to a known risk factor!
- Spatial model to interpolate for other locations.

How can FPCA help pricing an arbitrary location?

Outline

1. Motivation $\checkmark$
2. Functional Principal Components (FPCA)
3. Geographically Weighted Regression (GWR)
4. Empirical Findings
5. Outlook

- RP for $t = \tau_1$, ith location and jth contract month (RP_{ij}) assuming (1),
- a functional regression set up with scalar response: $w(t) \stackrel{def}{=} \mathbf{e}_1^\top A^{-1} \left[\exp\{A(\tau_2 - t)\} - I_p\right] \mathbf{e}_p$ and $\theta_w(t) = \theta(t) w(t)$:

$$RP_{ij}(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \theta_w^i(u) \sigma_{ij}(u) du + \varepsilon_{ij}.$$

- Regress RP on PC scores of $\sigma(t)$ for dimension reduction.

Functional Principal Components

1. Decompose

$$\sigma_{ij}(t) = \{\sigma_{ij}(t) - \bar{\sigma}_i(t)\} + \bar{\sigma}_i(t),$$

$\sigma_{ij}(t)$ variation curve for ith location and jth month,

$\bar{\sigma}_i(t)$ average curve for ith location.

$$\begin{aligned} RP_{ij} &= \int_{\tau_1}^{\tau_2} \theta_w^i(u)\bar{\sigma}_i(u)du \\ &\quad + \int_{\tau_1}^{\tau_2} \theta_w^i(u) \underbrace{\{\sigma_{ij}(u) - \bar{\sigma}_i(u)\}}_{\text{FPCA for } \sigma_{ij}} du. \end{aligned}$$

2. Perform FPCA: derive scores for temperature variation

PC Scores

- PC scores for functions $\sigma_{ij}(t)$ $i = 1, \ldots, 9$ (9 cities), $j = 1, \ldots, 7$ (7 traded months):

$$c_{ijk} = \int \xi_{ik}(t) \left\{ \sigma_{ij}(t) - \bar{\sigma}_i(t) \right\} dt,$$

  c_{ijk} scores for K largest eigenvalues, $\xi_{ik}(t)$ orthonormal eigenfunctions of $Cov\{\sigma(\cdot)\}$ operator.
- Collect scores capturing the variance in the data in matrix C.

3. Regress the response RP_{ij} at $t = \tau_1$ on the PC scores (Ramsay & Silverman, 2008):

$$RP_{ij} = \beta_{i0} \quad + \quad \int \sum_{k=1}^{K} \beta_{ik} \xi_{ik}(t) \{\sigma_{ij}(t) - \bar{\sigma}_i(t)\} dt + \nu_{ij}$$

for ith location and jth month,

- Functional form of MPR: $\theta_w^i(t) = \sum_k \beta_{ik} \xi_{ik}(t)$
- Need spatial model for regression on PC scores.

Spatial Specification: GWR

Why GWR (Anselin et al., 2004)?

- distance based weights,
- nonstationarity over space,
- local nature of spatial dependence.

GWR: the Model

$$
\begin{aligned}
W_i^{\frac{1}{2}} RP &= W_i^{\frac{1}{2}} C\beta_i + \varepsilon_i, \\
RP &= (RP_{1,1}, RP_{2,1}, \dots, RP_{n,1}, RP_{1,2}, \dots \ \dots, RP_{n,7})^{\top}, \\
C &= \begin{pmatrix} c_{1,1,1} & \dots & c_{1,1,K} \\ c_{2,1,1} & \dots & c_{1,1,K} \\ \dots & \dots & \dots \\ c_{n,7,1} & \dots & c_{n,7,K} \end{pmatrix} \\
n &- \text{total number of locations} \\
K &- \text{number of PC scores}
\end{aligned}
$$

GWR: the Model

$$
\begin{array}{rcl}
W_i & = & diag(w_i), \;\; i = 1, \dots, n \\
w_i & = & diag\left\{\exp\left(\dfrac{-d_{i1}}{h}\right), \dots, \exp\left(\dfrac{-d_{in}}{h}\right)\right\}, \\
d_{il}, \;\; l = 1, \dots, n & - & \text{distances to } i\text{th city}, \\
h & = & \underset{h \in H}{\arg\min} \displaystyle\sum_{m=1}^{7n} \left\{RP_m - \widehat{RP}_{\neq m}(h)\right\}^2,
\end{array}
$$

Temperature Data

City	First Date	Last Date	First F_{CAT} Trade
Amsterdam	19730101	20090930	20030401
Berlin	19480101	20090930	20030401
Barcelona	19730101	20090930	20050401
Essen	19700101	20090930	20050401
London	19730101	20090930	20030401
Madrid	19730101	20090930	20050401
Paris	19730101	20090930	20030401
Rome	19730101	20090930	20050401
Stockholm	19730101	20090930	20030401

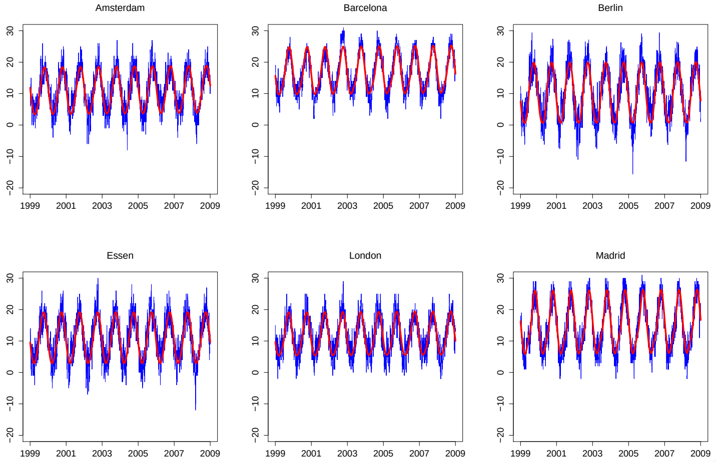
Table 1: Average Temperatures without 29th February. Source Bloomberg and DWD.

Volatility Functions $\sigma_i(t)$

Fit to data

- Seasonality $\Lambda(t)$,
- AR(p) process with seasonally heteroscedastic errors,
- estimate RP_{ij}, $i = 1, \ldots, 9$ and $j = 1, \ldots, 7$,
- estimate $\sigma_i(t)$, $t \in [1, 365]$ using residual standard deviation for each day of year and smooth by Fourier series.

Seasonality



Seasonality

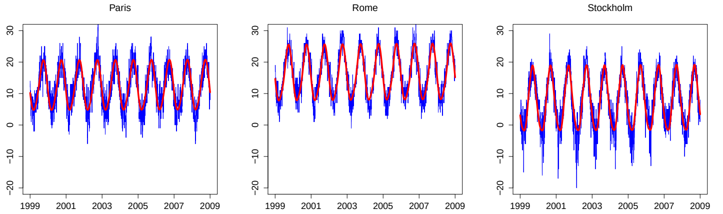


Figure 3: Daily average temperatures $T(t)$ (blue) and seasonality $\Lambda(t)$ (red).

	Essen		London		Madrid	
	estimate	t.stat	estimate	t.stat	estimate	t.stat
a	10.62	169.54	10.75	213.63	13.94	282.09
b	$1 \cdot 10^{-5}$	1.77	$8 \cdot 10^{-5}$	12.69	$1 \cdot 10^{-4}$	14.96
c_1	-2.35	-52.99	-2.47	-69.49	-3.29	-94.29
c_2	-7.75	-174.73	-6.37	-178.88	-8.91	-254.98
c_3	0.49	11.06	0.78	21.85	1.67	47.78
c_4	–	–	0.26	7.44	0.25	7.24
c_5	–	–	–	–	-0.18	-5.29
c_6	–	–	–	–	-0.34	-9.77

Table 2: Estimated Parameters of seasonality (1) for Essen, London, Madrid

AR(p)

	Amsterdam		Barcelona		Berlin	
	estimate	t.stat	estimate	t.stat	estimate	t.stat
α_1	0.89	103.47	0.70	81.30	0.92	138.10
α_2	-0.19	-16.87	0.03	3.18	-0.20	-22.83
α_3	0.09	10.57	0.02	1.60	0.08	11.85
α_4	–	–	0.03	3.79	–	–

Table 3: Estimated Parameters of AR(p) for Amsterdam, Barcelona, Berlin

RP

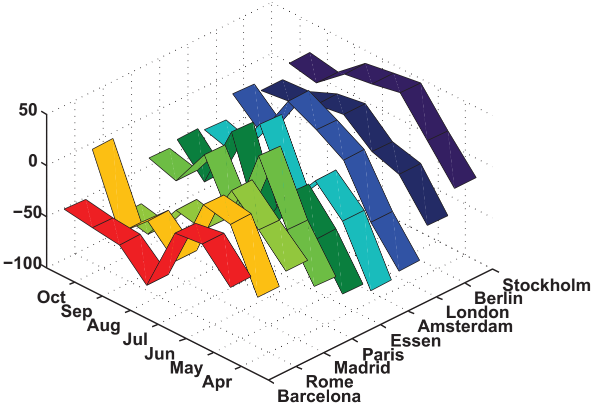
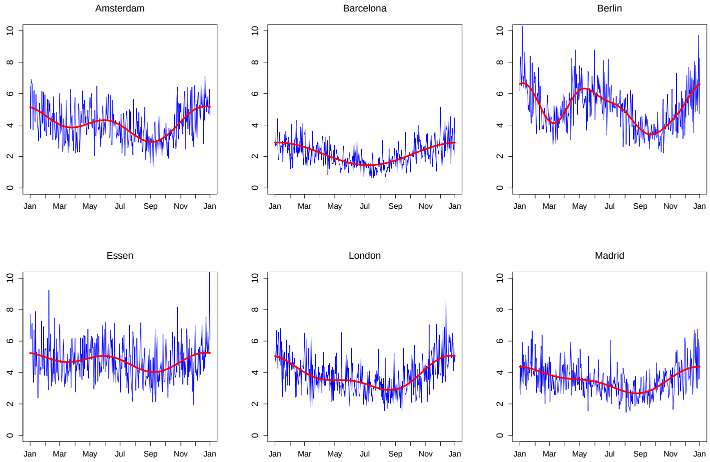


Figure 4: Average RP for traded locations computed according to (2)

Seasonal Variation



Seasonal Variation

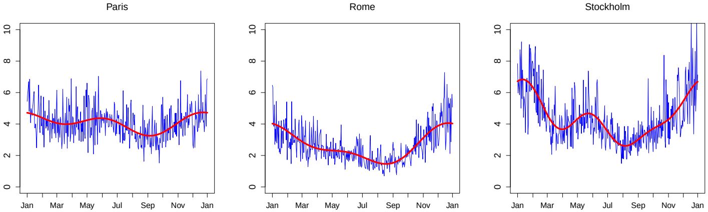
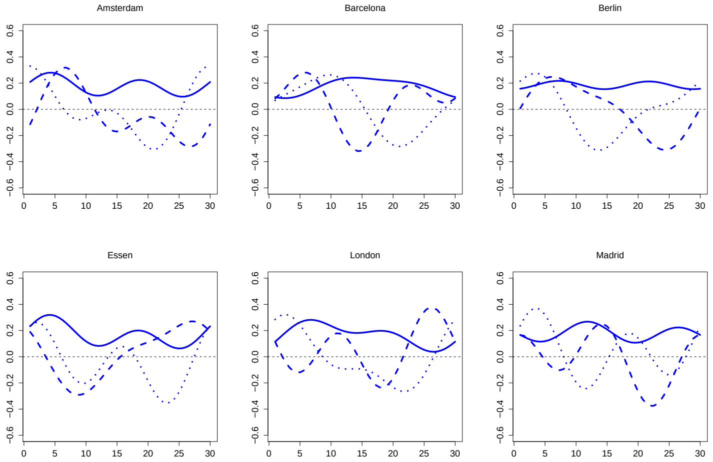


Figure 5: Estimated $\sigma(t)$ (blue) and smoothed by Fourier series (red).

Eigenfunctions



Eigenfunctions

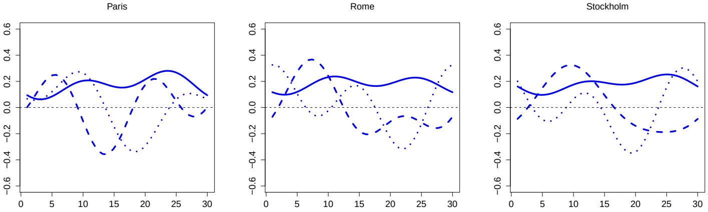
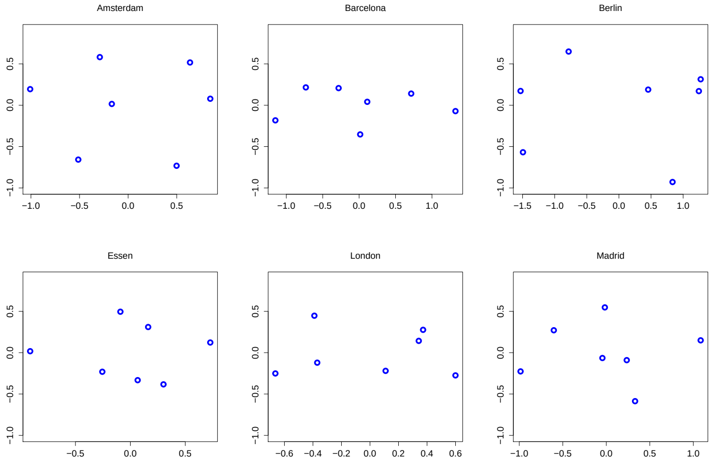


Figure 6: FPCA weight functions: eigenfunction ξ_1 (solid), ξ_2 (dashed), ξ_3 (dotted).

FPCA Scores


Amsterdam
Barcelona
Berlin
Essen
London
Madrid
−1.0
−0.5
0.0
0.5
1.0
−1.5
−0.6
−0.4
−0.2
0.2
0.4
0.6

FPCA Scores

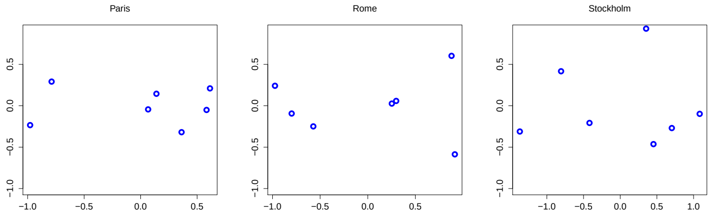
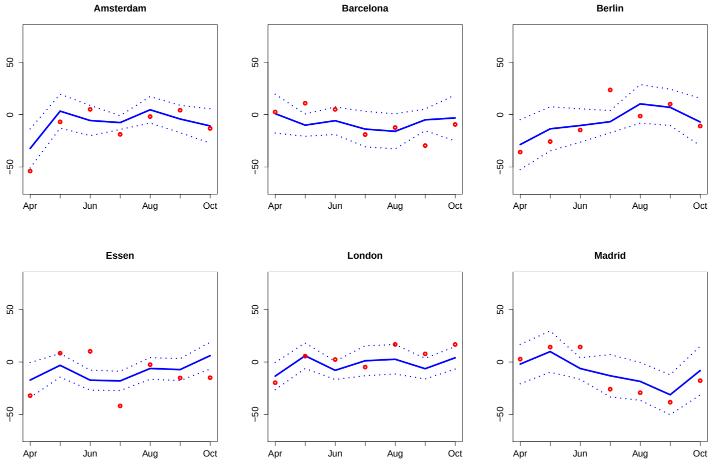


Figure 7: FPCA scores c_{ij1} and c_{ij2}

GWR Estimation Results



GWR Estimation Results

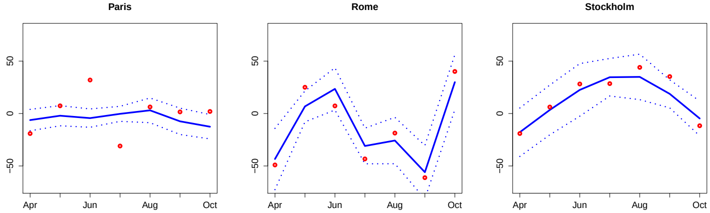


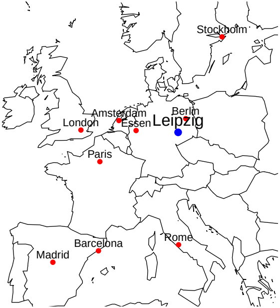
Figure 8: RP (red) and fitted values with 95% CI (blue) returned by GWR.

City	Coordinates	β_0	β_1	β_2	β_3	R^2_{loc}
Amsterdam	(52.37,4.89)	-7.53	-2.61	18.45	28.56	0.35
Barcelona	(41.38,2.17)	-7.55	6.24	-2.96	14.90	0.34
Berlin	(52.52,13.41)	-7.02	-7.05	17.53	-1.89	0.55
Essen	(51.47,7.01)	-8.98	-2.50	16.77	26.09	0.34
London	(51.51,-0.09)	-1.97	-3.87	16.21	27.86	0.30
Madrid	(40.42,-3.70)	-9.87	13.11	-9.33	51.69	0.57
Paris	(48.86,2.35)	-4.28	-0.86	15.77	22.18	0.26
Rome	(41.89,12.48)	-13.68	21.47	-38.14	-91.18	0.88
Stockholm	(59.33,18.07)	13.11	-18.43	18.49	-49.71	0.85

Table 4: Estimated Parameters of GWR

Example: Hedging weather risk in electricity demand

- An electricity provider in Leipzig transfers risk via CAT futures.
- What RP one would pay for F_{CAT}?



Out-of-Sample Forecast: Leipzig

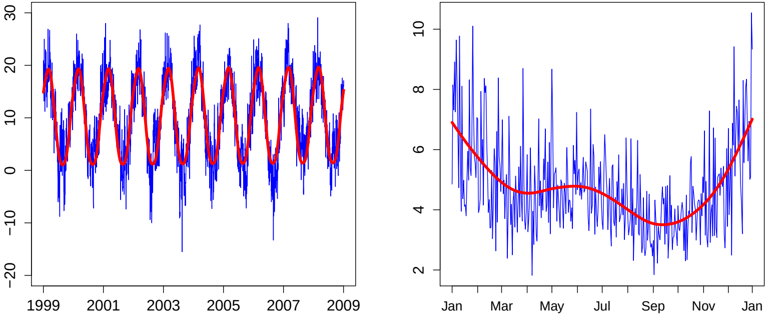


Figure 9: Λ_t, σ_t for Leipzig.

Out-of-Sample Forecast: Leipzig

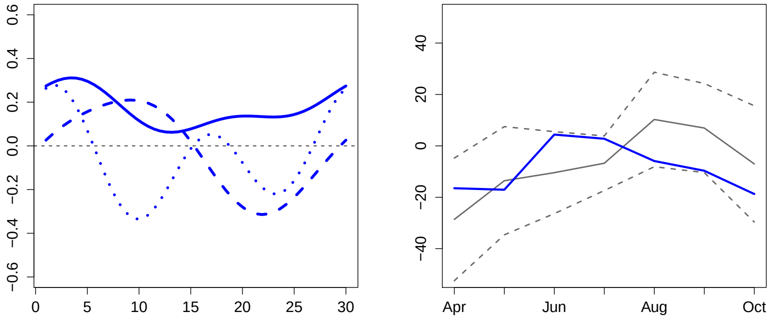
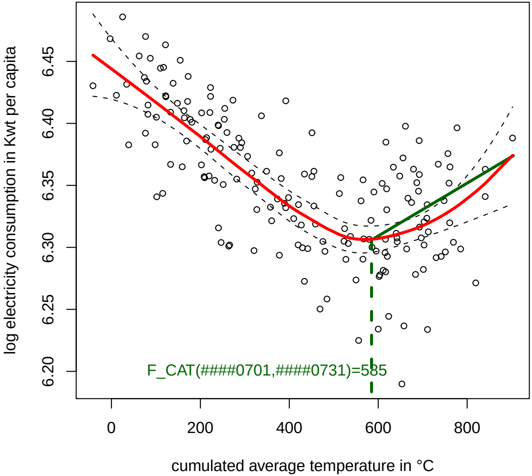


Figure 10: ξ and the resulting forecast for *RP* for Leipzig.


log electricity consumption in Kwt per capita
6.20
6.25
6.30
6.35
6.40
6.45
F_CAT(####0701,####0731)=585
0
200
400
600
800
cumulated average temperature in °C

Example: Hedging weather risk in electricity demand

- c – marginal costs of meeting additional log demand of 1% per person
- b – estimated marginal effects of 1°C CAT on log demand starting from threshold F_{CAT}
- α – number of WD hold, t – tick value of WD (for traded futures on Europe – 20EUR)

exposure	benefits
$\approx cb(CAT - F_{CAT})$	$\alpha t(CAT - F_{CAT})$

- hedging, s.t.

$$cb \leq \alpha t$$

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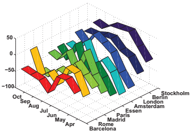

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