Systemic Weather Risk and Crop Insurance: The Case of China

Ostap Okhrin[1]
Martin Odening[2]
Wei Xu[3]

Ladislaus von Bortkiewicz Chair of Statistics
C.A.S.E. – Center for Applied Statistics
and Economics[1]
Department of Agricultural Economics[2]
Humboldt–Universität zu Berlin
SCOR SE[3] Zürich
http://lvb.wiwi.hu-berlin.de
http://www.agrar.hu-berlin.de

Motivation

- High weather sensitivity of agricultural production
- Increase of extreme weather events
- Problems with traditional (re)insurance
- Emergence of weather markets

Potential demand for weather derivatives in agriculture

Agricultural Insurance Systems

Country	Ins. coverage	Premium subsidies	Catastrophe aid	Partici-pation	Reinsu-rance
Germany	hail, suppl. ins.	none	only for uninsure-able risks	approx.35% hail <1% MPCI	pri. ins.
France	multiple peril crop ins.	60%	government aid for natu-ral disasters (drought, earthquake, flooding)	20%	pri. ins.
Greece	comprehensive ins.	50%	n.a.	n.a.	n.a.
Italy	hail, frost, drought	60% for hail 80% for MPCI	only for uninsure-able risks	n.a.	pri. ins.
Luxem-bourg	comprehensive ins.	up to 50%	n.a.	10%	n.a.
Austria	comprehensive ins.	50% for hail- and frost ins.	only for uninsure-able risks	78% hail 56% MPCI	priv. ins. exclusively
Spain	comprehensive ins.	55%	only for extreme disasters	approx. 42%	pri.and pub. ins.
Canada	multiple peril crop ins.	50%	for extreme and uninsurable disasters	50%	pri. and pub. ins.
USA	multiple peril crop ins.	35 up to 100%	only for unin-surable disasters	80%	pri. and pub. ins.

Table 1: Agricultural Insurances Systems

Pearson Correlation Coefficients vs. Distance: normal yield years

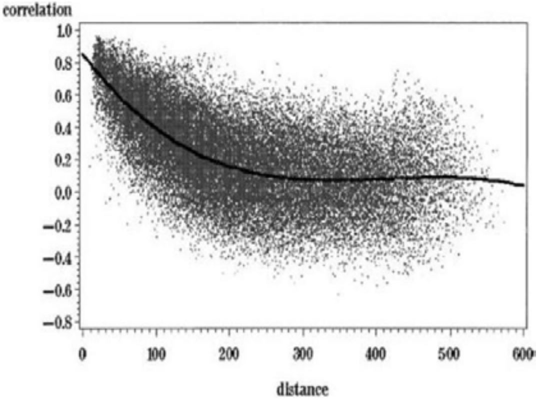


Figure 1: Goodwin, B.K.(2001)

Pearson Correlation Coefficients vs. Distance: extreme yield years

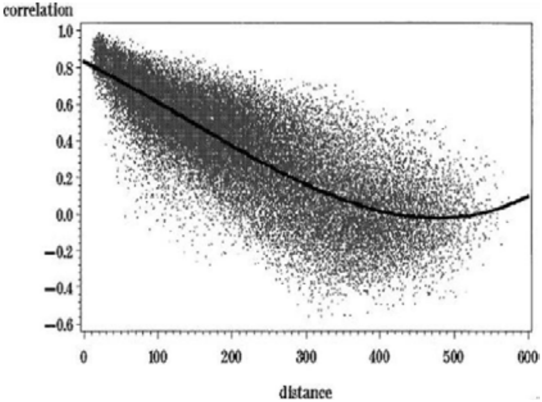


Figure 2: Goodwin, B.K.(2001)

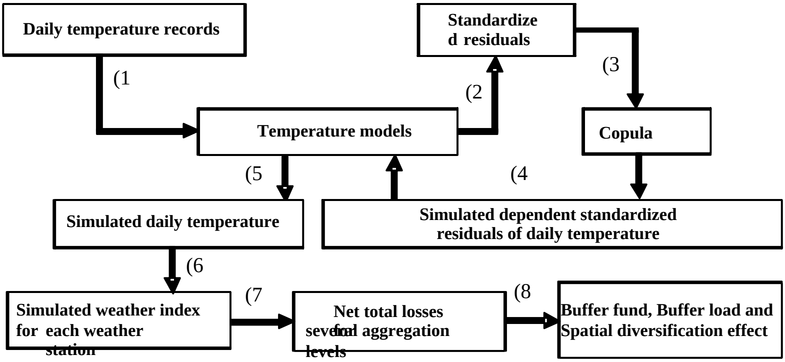
Objectives & Research Questions

- Quantification of the dependence structure of weather events at different locations
- Does the dependence of weather events fade out with increasing distance?
- Is spatial diversification of systemic weather risk possible?
- How to measure systemic weather risk correctly?

Outline

1. Motivation ✓
2. Model and Methods
3. Application
4. Conclusion

Flow Chart of the Computational Procedure



Buffer Fund

$$
\begin{aligned}
I_i &= I_i(T_i),\ L_i = f(I_i, K_i) \cdot V, \quad \Pi_i = E(L_i), \\
NTL &= \sum_{i=1}^{n} w_i \cdot (L_i - \Pi_i), \\
BF &= VaR_\alpha(NTL),\ BL_n = BF/n \\
DE &= nBL_n / \sum_{j=1}^{n} BL_j
\end{aligned}
$$

- BF – buffer fund,
- NTL – net total loss,
- L – loss,
- Π – fair premium,
- w – weight,
- I – weather index,
- K – trigger level,
- V – tick size,
- α – confidence level,
- i – region.

Indices: Growing Degree Days (GDD)

$$GDD_{i,t} = \sum_{j=\tau_{B,t}}^{\tau_{E,t}} \max\left(0, T_{i,t,j} - \widehat{T}\right),$$

where $\tau_{B,t}$ is the first of March, $\tau_{E,t}$ is October 31, where $\widehat{T}$ is the triggering temperature and is $5^\circ C$;

- Loss function for the risk of insufficient temperature

$$L_{GD_{i,t}} = \max\left(0, K_i^{GDD} - GDD_t\right) \cdot V,$$

K_i^{GDD} is the strike level being equal to 50% and the 15% quantile of the index distribution.

Indices: Frost Index (FI)

$$FI_{i,t} = \sum_{j=\tau_N}^{\tau_M} \mathbf{I}\left(T_{i,t,j} < \widehat{T}\right),$$

$$L_{FI_{i,t}} = \max\left(0, FI_{i,t} - K_i^{FI}\right) \cdot V,$$

where τ_N and τ_M denote November 1 and March 31, $\widehat{T} = 0^\circ C$ and K_i^{FI} is the strike level be equal to 50% and 85%.

Daily average temperature

$$T_{i,t} = \Delta_{i,t} + \Psi_{i,t},$$

$$\Delta_{i,t} = a_{1,i} + a_{2,i} \cdot t + a_{3,i} \cdot \cos\left(2\pi \frac{t - a_{4,i}}{365}\right),$$

$$\Psi_{i,t} = \sum_{j=1}^{J_i} b_{j,i} \cdot \Psi_{t-j,i} + \sigma_{i,t} \cdot \varepsilon_{i,t}$$

time-varying variance:

$$\sigma^2_{i,t} = d_{1,i} + d_{2,i} \cdot t + \sum_{k=1}^{K_i} \left[d_{3,k,i} \cdot \cos\left(2\pi k \frac{t}{365}\right) + d_{4,k,i} \cdot \sin\left(2\pi k \frac{t}{365}\right)\right]$$

Correlation

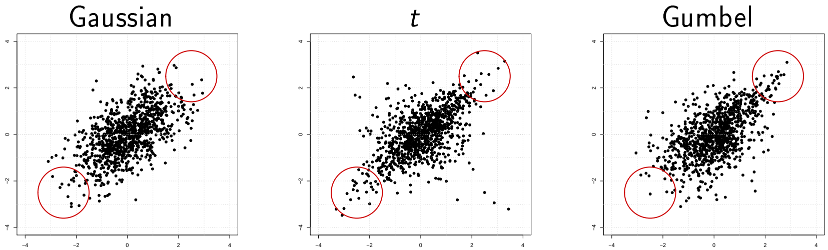


Figure 3: Scatterplots for two distributions with $\rho = 0.4$

- same linear correlation coefficient ($\rho = 0.4$)
- same marginal distributions
- rather big difference

Copula

For a distribution function F with marginals $F_{X_1}, \dots, F_{X_d}$, there exists a copula $C : [0,1]^d \to [0,1]$, such that

$$F(x_1, \dots, x_d) = \mathbf{C}\{F_{X_1}(x_1), \dots, F_{X_d}(x_d)\}.$$

Recall Archimedean Copula

Multivariate Archimedean copula $C : [0,1]^d \rightarrow [0,1]$ defined as

$$C(u_1, \ldots, u_d) = \phi\{\phi^{-1}(u_1) + \cdots + \phi^{-1}(u_d)\}, \qquad (1)$$

where $\phi : [0,\infty) \rightarrow [0,1]$ is continuous and strictly decreasing with $\phi(0) = 1$, $\phi(\infty) = 0$ and ϕ^{-1} its pseudo-inverse.

Example 1

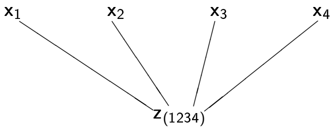
$$\begin{aligned} \phi_{Gumbel}(u,\theta) &= \exp\{-u^{1/\theta}\}, \text{ where } 1 \leq \theta < \infty \\ \phi_{Clayton}(u,\theta) &= (\theta u + 1)^{-1/\theta}, \text{ where } \theta \in [-1,\infty)\backslash\{0\} \end{aligned}$$

Disadvantages: too restrictive: single parameter, exchangeable

Hierarchical Archimedean Copulas

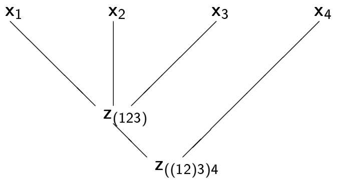
Simple AC with s=(1234)

$$C(u_1, u_2, u_3, u_4) = C_1(u_1, u_2, u_3, u_4)$$



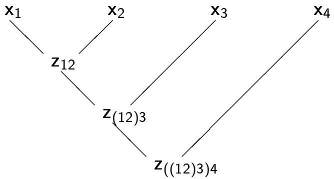
AC with s=((123)4)

$$C(u_1, u_2, u_3, u_4) = C_1\{C_2(u_1, u_2, u_3), u_4\}$$



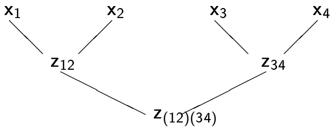
Fully nested AC with s=(((12)3)4)

$$C(u_1, u_2, u_3, u_4) = C_1[C_2\{C_3(u_1, u_2), u_3\}, u_4]$$

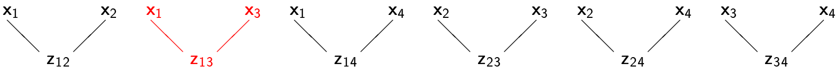


Partially Nested AC with s=((12)(34))

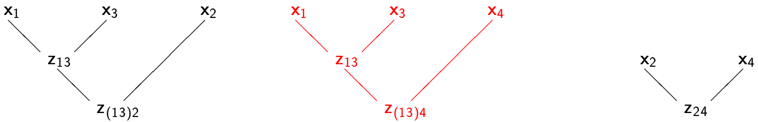
$$C(u_1, u_2, u_3, u_4) = C_1\{C_2(u_1, u_2), C_3(u_3, u_4)\}$$



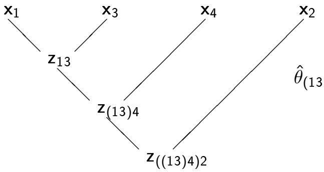
Recovering the structure (easy practice)



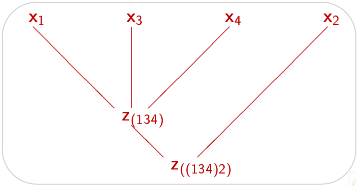
$$\max\{\hat{\theta}_{12}, \hat{\theta}_{13}, \hat{\theta}_{14}, \hat{\theta}_{23}, \hat{\theta}_{24}, \hat{\theta}_{34}\} = \hat{\theta}_{13} \quad \Rightarrow$$



$$\max\{\hat{\theta}_{(13)2}, \hat{\theta}_{(13)4}, \hat{\theta}_{24}\} = \hat{\theta}_{(13)4} \quad \Rightarrow$$



$$\hat{\theta}_{(13)} \approx \hat{\theta}_{((13)4)} \Rightarrow$$



Estimation Issues - Margins

$$
\begin{aligned}
F_j(x;\hat{\alpha}_j) &= F_j\left\{x; arg\max_\alpha \sum_{i=1}^{n} \log\ f_j(X_{ji},\alpha)\right\},\\
\hat{F}_j(x) &= \frac{1}{n+1}\sum_{i=1}^{n} \mathrm{I}(X_{ji}\le x),\\
\tilde{F}_j(x) &= \frac{1}{n+1}\sum_{i=1}^{n} K\left(\frac{x-X_{ji}}{h}\right)
\end{aligned}
$$

for $j=1,\dots,k$, where $\kappa:\mathbb{R}\to\mathbb{R}$, $\int\kappa=1$, $K(x)=\int_{-\infty}^{x}\kappa(t)dt$ and $h>0$ is the bandwidth.

$$\check{F}_j(x)\in\{\hat{F}_j(x),\tilde{F}_j(x),F_j(x;\hat{\alpha}_j)\}$$

Estimation Issues - Multistage Estimation

$$\left(\frac{\partial \mathcal{L}_1}{\partial \boldsymbol{\theta}_1^\top}, \ldots, \frac{\partial \mathcal{L}_p}{\partial \boldsymbol{\theta}_p^\top}\right)^\top = \mathbf{0},$$

$$\text{where} \quad \mathcal{L}_j = \sum_{i=1}^{n} l_j(\mathbf{X}_i)$$

$$l_j(\mathbf{X}_i) = \log\left(c(\{\phi_\ell, \boldsymbol{\theta}_\ell\}_{\ell=1,\ldots,j};\, s_j)\left[\{\check{F}_m(x_{mi})\}_{m \in s_j}\right]\right)$$

$$\text{for } j = 1, \ldots, p.$$

Theorem

Under regularity conditions, estimator $\hat{\boldsymbol{\theta}}$ is consistent and

$$n^{\frac{1}{2}}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \overset{a}{\sim} N(\mathbf{0}, \mathbf{B}^{-1}\Sigma\mathbf{B}^{-1})$$

Copula: Goodness-of-Fit Tests

Hypothesis

$$H_0 : C_\theta \in \mathcal{C}_0; \theta \in \Theta \quad vs \quad H_1 : C_\theta \notin \mathcal{C}_0; \theta \in \Theta,$$

Cramér von Mises

$$S = n \int_{[0,1]^d} \left[\widehat{C}(u_1, \ldots, u_d) - C(u_1, \ldots, u_d; \widehat{\theta}) \right]^2 d\widehat{C}(u-1, \ldots, u_d)$$

Kolmogorov-Smirnov

$$T = \sqrt{n} \sup_{u_1, \ldots, u_d \in [0,1]} |\widehat{C}(u_1, \ldots, u_d) - C(u_1, \ldots, u_d; \widehat{\theta})|$$

in practice p-values are calculated using the bootstrap methods described in Genest and Remillard (2008)

Simulation

Frees and Valdez, (1998, NAAJ), Whelan, (2004, QF), Marshal and Olkin, (1988, JASA)

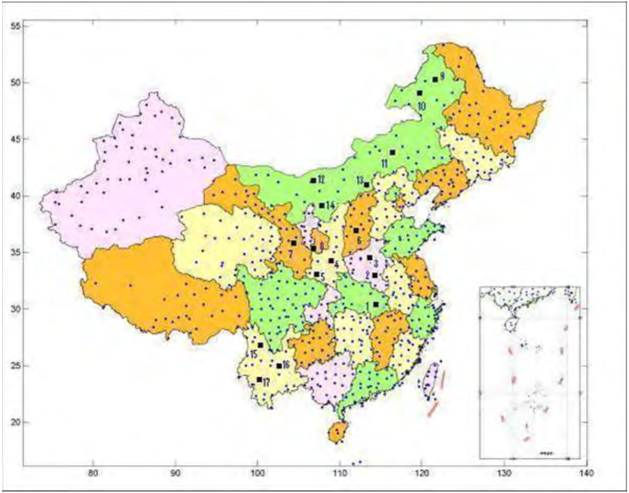
Conditional inversion method:

Let $C = C(u_1, \ldots, u_k)$, $C_i = C(u_1, \ldots, u_i, 1, \ldots, 1)$ and $C_k = C(u_1, \ldots, u_k)$. Conditional distribution of U_i is given by

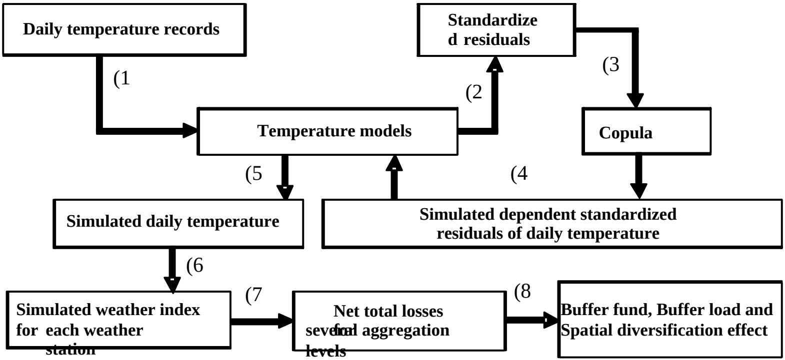
$$\begin{aligned} C_i(u_i|u_1, \ldots, u_{i-1}) &= P\{U_i \leq u_i | U_1 = u_1 \ldots U_{i-1} = u_{i-1}\} \\ &= \frac{\partial^{i-1} C_i(u_1, \ldots, u_i)}{\partial u_1 \ldots \partial u_{i-1}} \Big/ \frac{\partial^{i-1} C_{i-1}(u_1, \ldots, u_{i-1})}{\partial u_1 \ldots \partial u_{i-1}} \end{aligned}$$

- Generate i.r.v. $v_1, \ldots, v_k \sim U(0, 1)$
- Set $u_1 = v_1$
- $u_i = C_k^{-1}(v_i | u_1, \ldots, u_{i-1}) \; \forall i = \overline{2, k}$

Location of selected weather stations



Flow Chart of the Computational Procedure



Descriptive Statistics

st.	GDD	FI
1	4114.98 (198.13)	6.26 (6.07)
2	3740.56 (148.25)	19.92 (10.29)
3	3700.36 (146.95)	30.76 (12.23)
4	3517.92 (186.12)	32.22 (12.32)
5	3498.83 (144.03)	5.86 (5.18)
6	2897.29 (140.68)	75.60 (11.64)
7	2623.34 (172.30)	87.44 (12.07)
...	...	...
14	2353.13 (141.53)	117.68 (9.24)
15	2557.45 (103.70)	0.20 (0.64)
16	3113.99 (156.99)	0.26 (0.60)
17	3670.46 (105.20)	0.00 (0.00)

Table 2: Descriptives

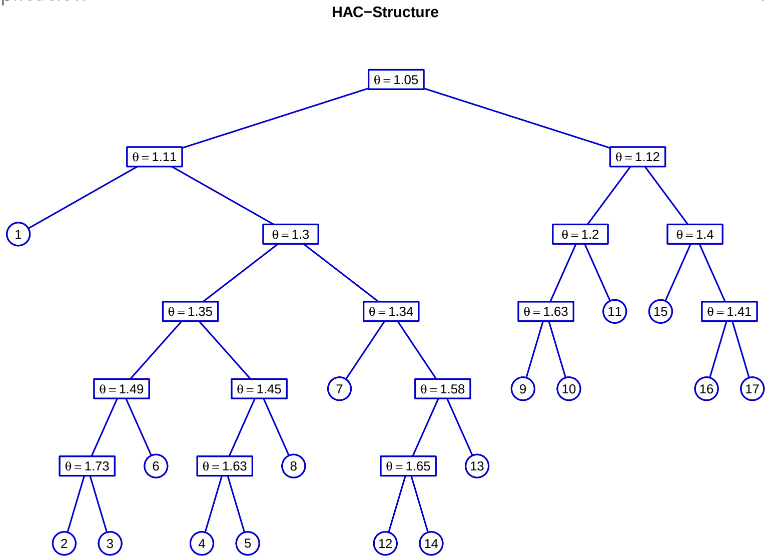
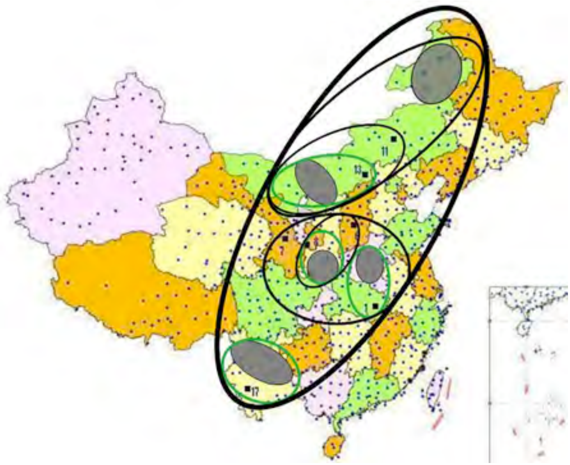
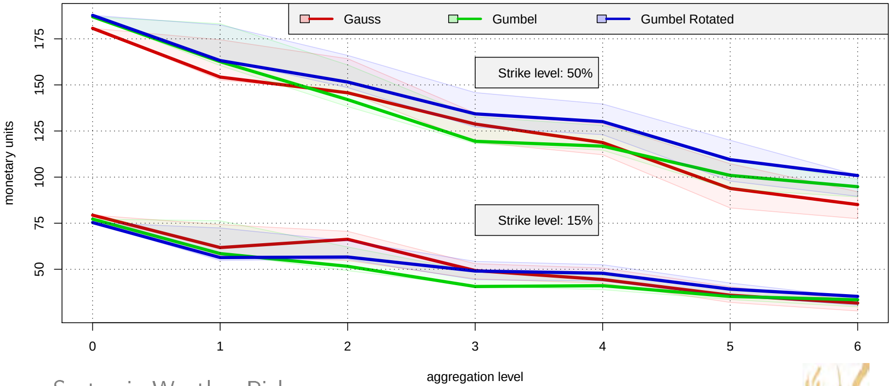

HAC−Structure
θ = 1.05
θ = 1.11
θ = 1.12
1
θ = 1.3
θ = 1.2
θ = 1.4
θ = 1.35
θ = 1.34
θ = 1.63
11
15
θ = 1.41
θ = 1.49
θ = 1.45
7
θ = 1.58
9
10
16
17
θ = 1.73
6
θ = 1.63
8
θ = 1.65
13
2
3
4
5
12
14

Illustration of Dependence Cluster



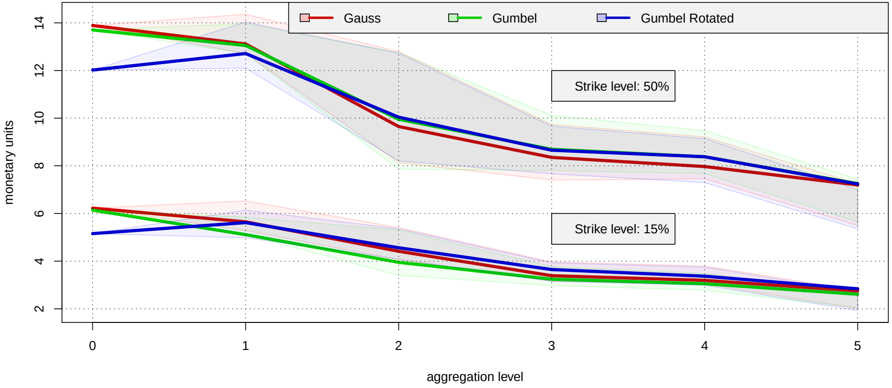
BL for Different Aggregation: GDD

Aggregation: (2) (2, 3) (1-3) (1-6, 8) (1-8) (1-8, 15-17) (1-17)



BL for Different Aggregation: FI

Aggregation: (2) (2, 3) (1-3) (1-6, 8) (1-8) (1-8, 11-14)



Fair Prices, Buffer Loads and Diversification Effects I

Type of Copula	Gaussian	Gumbel	Rotated-Gumbel
GDD Strike Level 50%			
Fair Price	58.047	58.623	58.930
Buffer Load	85.091	94.784	100.839
Diversification Effect	0.481	0.539	0.567
GDD Strike Level 15%			
Fair Price	10.598	10.275	10.332
Buffer Load	31.688	33.476	35.301
Diversification Effect	0.430	0.466	0.488

Fair Prices, Buffer Loads and Diversification Effects II

Type of Copula	Gaussian	Gumbel	Rotated-Gumbel
FI Strike Level 50%			
Fair Price	3.082	3.166	3.004
Buffer Load	7.197	7.253	7.238
Diversification Effect	0.742	0.748	0.777
FI Strike Level 15%			
Fair Price	0.611	0.593	0.603
Buffer Load	2.750	2.611	2.838
Diversification Effect	0.658	0.645	0.690

Conclusions

- Weather risk in China has a systemic component on a state level as well as on a national level
- The possibility of regional diversification depends on the type of weather index (temperature $<$ drought $<$ flooding)
- Weather risks should be globally diversified or transferred to the capital market (e.g. weather bonds)
- Linear correlation may under- or overestimate systemic weather risk
- Copulas allow a flexible modeling of the dependence structure of joint weather risks
- But: risk of misspecification

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Humboldt–Universität zu Berlin
SCOR SE[3] Zürich
http://lvb.wiwi.hu-berlin.de
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