TENET: Tail-Event-driven NETwork Risk

Wolfgang Karl Härdle
Natalia Sirotko-Sibirskaya
Weining Wang

Ladislaus von Bortkiewicz Chair of Statistics
C.A.S.E. - Center for Applied Statistics and Economics
Humboldt–Universität zu Berlin
http://lvb.wiwi.hu-berlin.de
http://www.case.hu-berlin.de

What is Systemic Risk?

"Financial institutions are **systemically important** if the failure of the firm to meet its obligations to creditors and customers would have significant adverse consequences for the financial system and the broader economy".

Daniel Tarullo, Federal Reserve Board

CoVaR as a Systemic Risk Measure

Step 1. Estimate linear quantile regressions

$$X_{j,t} = \alpha_j + \gamma_j M_{t-1} + \varepsilon_{j,t},$$
$$X_{i,t} = \alpha_{i|j} + \gamma_{i|j} M_{t-1} + \beta_{i|j} X_{j,t} + \varepsilon_{i|j,t},$$

where

- $X_{i,t}$ is the log return of a financial institution i,
- M_{t-1} are lagged macroprudential variables.

Adrian and Brunnermeier (2011)

Macroprudential variables

CoVaR as a Systemic Risk Measure

Step 2. Generate predicted values under assumption $F^{-1}_{\varepsilon_{j,t}}(\tau|M_{t-1}) = 0$ and $F^{-1}_{\varepsilon_{i|j,t}}(\tau|M_{t-1}, X_{j,t}) = 0$, $\tau = (0,1)$,

$$\begin{aligned}
\widehat{\text{VaR}}^{\tau}_{j,t} &= \hat{\alpha}_j + \hat{\gamma}_j M_{t-1}, \\
\widehat{\text{CoVaR}}^{\tau}_{i|j,t} &= \hat{\alpha}_{i|j} + \hat{\gamma}_{i|j} M_{t-1} + \hat{\beta}_{i|j} \widehat{\text{VaR}}^{\tau}_{j,t}, \\
&= \widehat{\text{VaR}}^{\tau}_{i|X_j = \widehat{\text{VaR}}^{\tau}_{j,t}} + \hat{\beta}_{i|j} \widehat{\text{VaR}}^{\tau}_{j,t}.
\end{aligned}$$

Adrian and Brunnermeier (2011)

Elements of Systemic Risk

- Network Effects
- Single Institution's Contribution to Systemic Risk
- Single Institution's Exposure to Systemic Risk

Challenges

- Linear tail behavior
  - Adrian and Brunnermeier (2011)
  - Acharya et al. (2012)
  - Brownlees and Engle (2012)
- Linear tail behavior in **high dimensions**
  - Hautsch, Schaumburg, and Schienle (2014)
- **Non-linear** tail behavior in **ultra-high dimensions**
  - Method by Fan, Härdle, Wang, and Zhu (2014)

Non-Linearity

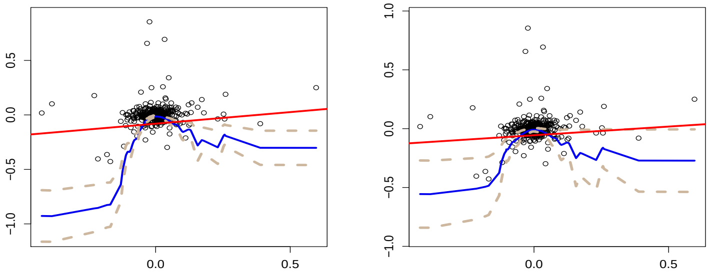

Figure 1: Bank of America (BOA) and Citi (C) weekly returns 0.05 (left) and 0.1 (right) quantile functions, y-axis = BOA returns, x-axis = C returns. Local linear quantile regression and Linear quantile regression. 95% confidence band, $T = 546$, weekly returns, 2005.01.31-2010.01.31. Chao, Härdle and Wang (2014).

Outline

1. Motivation ✓
2. Statistical Methodolody
3. Systemic Risk Modelling
4. Empirical Analysis
5. Conclusion
6. References

Model Components

- **Tail Behavior:** Generalized Quantile Regression
- **Non-Linearity:** Single-Index Model
- **Ultra-High Dimensions:** Variable Selection

Generalized Quantile Regression

Let $\{X_i, Y_i\}_{i=1}^n$ be independent r. v., $X \in \mathbb{R}^p$, $\tau \in (0,1)$.

$$q_\tau = \arg\min \mathsf{E}\{\rho_\tau(Y - \theta)\}$$

where $\rho_\tau(\cdot)$ is an asymmetric loss function

$$\rho_\tau(u) = |u|^\alpha |\mathbf{1}(u \le 0) - \tau|,$$

with $\alpha = 1$ corresponding to a quantile and $\alpha = 2$ corresponding to an expectile regression.

$$q_\tau(x) = \arg\min \mathsf{E}\{\rho_\tau(Y - \theta) | X = x\}$$

Asymmetric Loss Functions

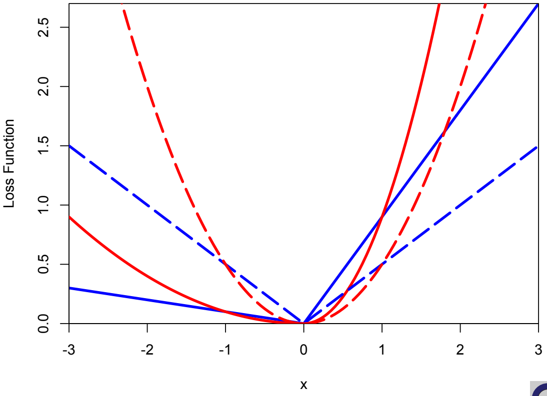

Figure 2: Asymmetric Loss Functions for Quantile and Expectile, $\tau = 0.9$: a solid line, $\tau = 0.5$: a dashed line.

Single-Index Model

Let $\{X_i, Y_i\}_{i=1}^n$ be independent r. v., $X \in \mathbb{R}^p$.

$$Y_i = g(\beta^\top X_i) + \varepsilon_i,$$

where

- $g(\cdot)$ is the link function,
- $\beta \in \mathbb{R}^p$ is the vector of index parameters,
- $p = \mathcal{O}\{\exp(n^\alpha)\}$ for some $\alpha \in (0, 1)$.

Variable Selection

$$\widehat{\beta} = \arg\min_{g,g',\beta} n^{-1} \sum_{j=1}^{n} \sum_{i=1}^{n} \rho\Big\{Y_i - g(\beta^\top X_j) - g'(\beta^\top X_j) X_{ij}^\top \beta\Big\}\omega_{ij}(\beta) + \textstyle\sum_{l=1}^{p} \gamma_\lambda(|\beta_l|^\theta),$$

where

- $X_{ij} = X_i - X_j$,
- $\omega_{ij}(\beta) \stackrel{\text{def}}{=} \dfrac{K_h(X_{ij}^\top \beta)}{\sum_{i=1}^{n} K_h(X_{ij}^\top \beta)}$,
- $\theta \geq 0$,
- $\gamma_\lambda(t)$ is some nondecreasing function concave for $t \in [0, +\infty)$ with a continuous derivative on $(0, +\infty)$.

▸ Numerical Procedure

Adaptive LASSO

$\cdots \sum_{l=1}^{p} \gamma_\lambda(|\beta_l|^\theta) = \lambda \sum_{l=1}^{p} w_l |\beta_l|,$

where

- λ is a penalty term,
- $\theta = 1$,
- $w_l = 1/|\widehat{\beta_l^0}|^\delta$ are weights, $l = 1, \ldots, p, \delta > 0$,
- $\widehat{\beta}^0$ is an initial estimator of β.

Zou (2006), Wu and Liu (2009)

Smoothly Clipped Absolute Deviation (SCAD)

$$\gamma_\lambda(|\beta|) = \lambda|\beta|\mathbf{1}(0 \leq |\beta| < \lambda) + \frac{a\lambda-(\beta^2+\lambda^2)/2}{a-1}\times$$
$$\mathbf{1}(\lambda \leq |\beta| \geq a\lambda) + \frac{(a+1)\lambda^2}{2}\mathbf{1}(|\beta| > a\lambda),$$

where

- $a > 2$, e.g., $a = 3.7$ as in Fan and Li (2001).

Fan and Li (2001), Kim, Choi, and Oh (2008)

Lambda

- Empirical choice of λ: $\lambda_n = 0.25\sqrt{||\beta_0||}\log n \vee p(\log n)^{0.5}$
- λ for ultra-high dimensions (Wang and Leng (2007))
- Schwarz Information Criteron (SIC)
  (Schwarz (1978), Koenker, Ng, and Portnoy (1994))

$$\mathrm{SIC}(\lambda) = \log[n^{-1}\sum_{i=1}^{n}\rho_\tau\{Y_i - f(X_i)\}] + \frac{\log n}{2n}\,\mathrm{df}$$

where df is a measure of the effective dimensionality of the fitted model.

Effective dimension

Bandwidth

Symmetrized nearest neighbor estimation implies

$$\widehat{m}_h(X_0) = (nh)^{-1} \sum_{i=1}^{n} Y_i K_h\{F_n(X_i) - F_n(x_0)\}$$

where

- $\widehat{m}(x)$ denotes an estimator of the regression function,
- h is some bandwidth tending to zero.

Härdle and Carroll (1989)

Elements of Systemic Risk

- Network Effects
- Network-based Systemic Risk Measures
  - Single Institution's Contribution to Systemic Risk
  - Single Institution's Exposure to Systemic Risk

Spill-Over Effects

Step 1. Estimate linear quantile regressions and generate predicted values under assumption $F^{-1}_{\varepsilon_{j,t}}(\tau | M_{t-1}) = 0$ at $\tau \in (0,1)$

$$\begin{aligned} X_{j,t} &= \alpha_j + \gamma_j^\top M_{t-1} + \varepsilon_{j,t}, \\ \widehat{\text{VaR}}^{\tau}_{j,t} &= \hat{\alpha}_j + \widehat{\gamma}_j^\top M_{t-1}, \end{aligned}$$

where

- $X_{j,t}$ is the log-return of company j,
- M_{t-1} are macroprudential variables as in Adrian and Brunnermeier (2011).

Spill-Over Effects

Step 2. Estimate single-index-model-based quantile regressions with variable selection

$$X_{j,t} = g(\beta_{j|-j}^{\top} \widetilde{X}_{-j,t}) + \varepsilon_{j,t},$$

where

- $\widetilde{X}_{-j,t} = \{M_{t-1}, X_{-j,t}\}$ and $-j$ refers to all the financial institutions in a sample except for a financial institution j,
- $\hat{\beta}$'s determine the spill-over effects from companies $-j$ to a company j, i. e. the network structure.

Value-at-Risk

$$\text{Classical approach: } \widehat{\text{VaR}}^{\tau}_{i,t} = \hat{\alpha}_i + \hat{\gamma}_i M_{t-1},$$

$$\widehat{\text{CoVAR}}_{\text{AB}}\text{: } \widehat{\text{CoVaR}}^{\tau}_{j|i,t} = \hat{\alpha}_{j|i} + \hat{\gamma}_{j|i} M_{t-1} + \hat{\beta}_{j|i} \widehat{\text{VaR}}^{\tau}_{i,t},$$

$$\widehat{\text{CoVAR}}_{\text{TENET}}\text{: } \widehat{\text{CoVaR}}^{\tau}_{j|-j,t} = \hat{g}(\widehat{\beta}^{\top}_{j|-j} \widehat{X}_{-j,t}),$$

where

- $\widehat{X}_{-j,t} = \{M_{t-1}, \widehat{\text{VaR}}^{\tau}_{-j,t}\}$.

Contribution to Systemic Risk

$$C_{i,t} = \left(1 + \frac{O_{i,t}}{T_t}\right)\widehat{\text{VaR}}_{i,t},$$
$$C^a_{i,t} = \left(1 + \frac{O^a_{i,t}}{T^a_t}\right)\widehat{\text{VaR}}_{i,t},$$

where

- $O_{i,t} = O_{i \to -i} = \sum_{i=1}^{p} \widehat{\beta}_{-i,t}$,
- $T_t = \sum_{i=1}^{p} \widehat{\beta}_{i,t}$,
- $O^a_{i,t} = O^a_{i \to -i} = \sum_{i=1}^{p} d_{-i,t}$,
- $T^a_t = \sum_{i=1}^{p} d_{-i,t}$,
- $d_{i,t} = \begin{cases} 1 & \text{if } \widehat{\beta}_{-i,t} \neq 0 \\ 0 & \text{otherwise} \end{cases}$

Exposure to Systemic Risk

$$E_{i,t} = \left(1 + \frac{I_{i,t}}{T_t}\right)\widehat{\text{VaR}}_{i,t},$$

$$E^a_{i,t} = \left(1 + \frac{I^a_{i,t}}{T^a_t}\right)\widehat{\text{VaR}}_{i,t},$$

where

- $I_{i,t} = I_{i \leftarrow -i,t} = \sum_{i=1}^{p} \widehat{\beta}_{-i,t}$,
- $T_t = \sum_{i=1}^{p} \widehat{\beta}_{i,t}$,
- $I^a_{i,t} = I^a_{i \leftarrow -i,t} = \sum_{i=1}^{p} d_{-i,t}$,
- $T^a_t = \sum_{i=1}^{p} d_{-i,t}$,
- $d_{i,t} = \begin{cases} 1 & \text{if } \widehat{\beta}_{-i,t} \neq 0 \\ 0 & \text{otherwise} \end{cases}$

Dataset

- Data: asset log returns on 200 financial companies and 7 macroprudential variables.
- Companies classified by SIC codes to: Depositories (100), Insurance (56), Broker-Dealers (25) and Others (19).
- Time period: January 1, 2006 - September 1, 2012, $T = 1669$.
- Frequency: daily.

Firms

Macroprudential variables

Summary Statistics

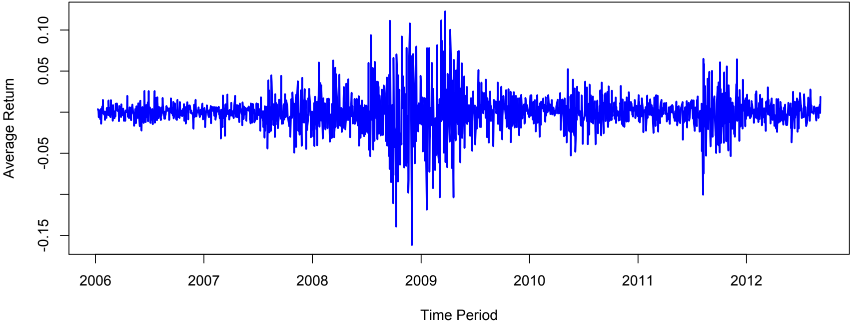

Figure 3: Average asset log returns on 200 financial companies, 1, January, 2006 - 1, September, 2012.

Average Lambda

Network Analysis

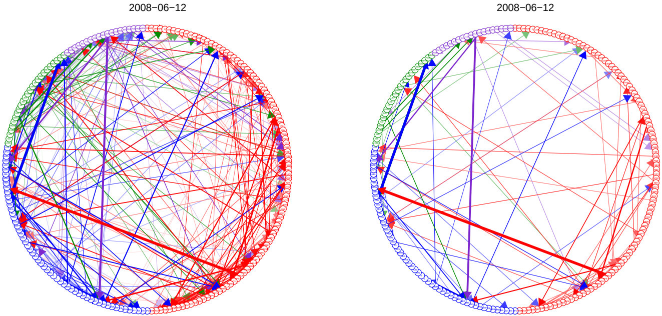

Figure 4: Example of a risk network formed by 200 financial institutions without (left) and with (right) thresholding of coefficients based on the rule: $\widehat{\beta} = 0$ if $|\widehat{\beta}| \leq 1/250 \sum_{i=1}^{l=250} \beta_{(i)}$, $\widehat{\beta} \neq 0$, otherwise; Depositories (red), Insurance (blue), Broker-Dealers (green), Others (purple); T = 1669, $\tau = 0.05$, window size $n = 125$.

Total Connectedness Matrix

	x_{d_1}	$\cdots$	x_{d_n}	x_{i_1}	$\cdots$	x_{i_n}	$\cdots$	x_{o_1}	$\cdots$	x_{o_n}	*To*
x_{d_1}	0	$\cdots$	β_{d_1,d_n}	β_{d_1,i_1}	$\cdots$	β_{d_1,i_n}	$\cdots$	β_{d_1,o_1}	$\cdots$	β_{d_1,o_n}	$\sum_j^p \beta_{1j}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
x_{d_n}	β_{d_n,d_1}	$\cdots$	0	β_{d_n,i_1}	$\cdots$	β_{d_n,i_n}	$\cdots$	β_{d_n,o_1}	$\cdots$	β_{d_n,o_n}	
x_{i_1}	β_{i_1,d_1}	$\cdots$	β_{i_1,d_n}	0	$\cdots$	β_{i_1,i_n}	$\cdots$	β_{i_1,o_1}	$\cdots$	β_{i_1,o_n}	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
x_{i_n}	β_{i_n,d_1}	$\cdots$	β_{i_n,d_n}	β_{i_n,i_1}	$\cdots$	0	$\cdots$	β_{i_n,o_1}	$\cdots$	β_{i_n,o_n}	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
x_{o_1}	β_{o_1,d_1}	$\cdots$	β_{o_1,d_n}	β_{o_1,i_1}	$\cdots$	β_{o_1,i_n}	$\cdots$	0	$\cdots$	β_{o_1,o_n}	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
x_{o_n}	β_{o_n,d_1}	$\cdots$	β_{o_n,d_n}	β_{o_n,i_1}	$\cdots$	β_{o_n,i_n}	$\cdots$	β_{o_n,o_1}	$\cdots$	0	
From	$\sum_i^p \beta_{i1}$		$\cdots$			$\cdots$		$\cdots$			$\sum_{i,j}^p \beta_{ij}$

Figure 5: Total Connectedness Matrix $p \times p$ at each time point, Depositories: d, Insurance: i, Broker-Dealers: b, Others: o with sample sizes noted as $x_{xi} \ldots x_{xn}$.

Total Connectedness

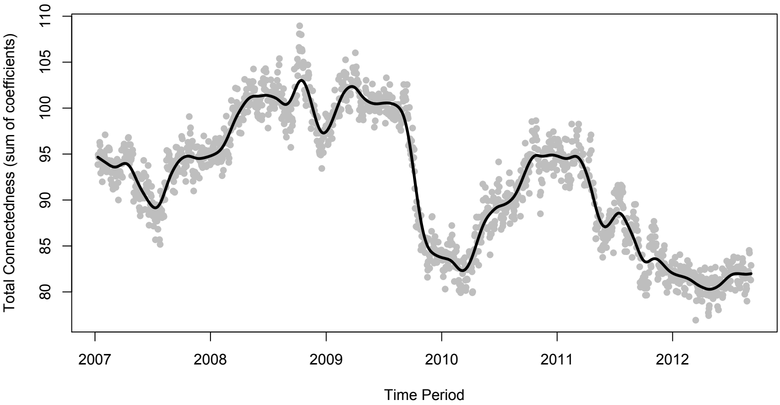

Figure 6: Total Connectedness (sum of $|\beta|$ coefficients), a smoothed curve and original data, T $= 1669$, $\tau = 0.05$, window size $n = 125$.

Total Connectedness

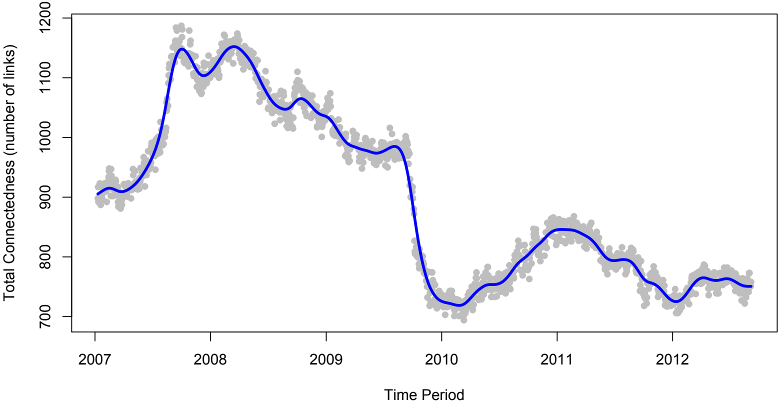

Figure 7: Total Connectedness (sum of number of links), a smoothed curve and original data, T = 1669, $\tau = 0.05$, window size $n = 125$.

Average Lambda

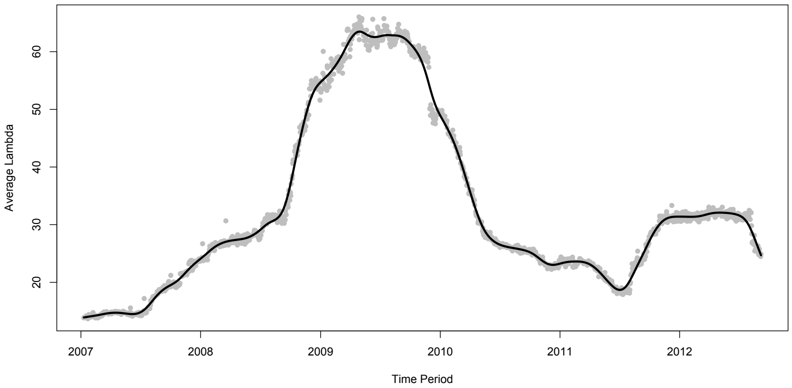

Figure 8: Averaged penalization parameter λ, a smoothed curve and original data, T = 1669, $\tau = 0.05$, window size $n = 125$.

Summary

Network Analysis

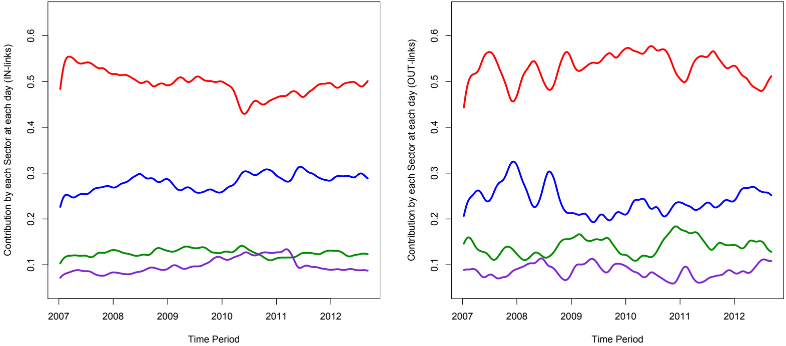

Figure 9: Proportion of each Sector in Total Connectedness (sum of $|\beta|$'s), IN (left), OUT (right), Depositories (red), Insurance (blue), Broker-Dealers (green), Others (purple), T $= 1669$, $\tau = 0.05$, window size $n = 125$.

Network Analysis

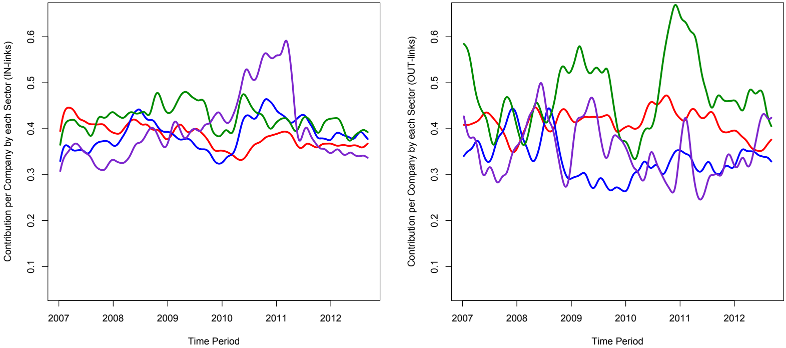

Figure 10: Exposure (left) and contribution (right) per Company in each Sector, Depositories (red), Insurance (blue), Broker-Dealers (green), Others (purple), T = 1669, $\tau = 0.05$, window size $n = 125$.

Network Analysis

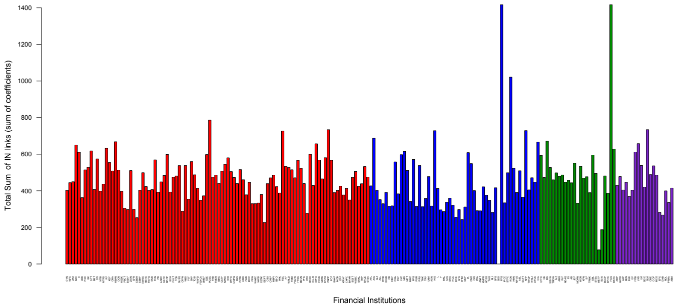

Figure 11: Total IN links (sum of $|\beta|$'s), full-sample results; Depositories (red), Insurance (blue), Broker-Dealers (green), Others (purple), $T = 1669$, $\tau = 0.05$, window size $n = 125$.

IN-links: Summary

Network Analysis

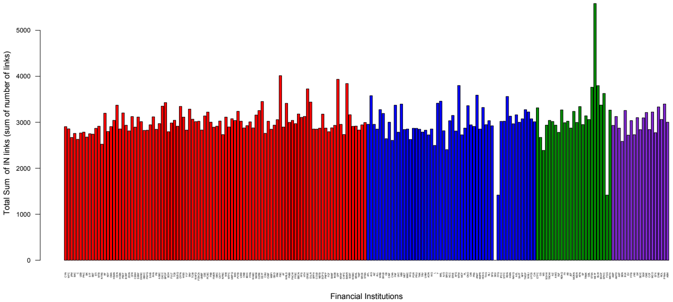

Figure 12: Total IN links (sum of number of links), full-sample results; Depositories (red), Insurance (blue), Broker-Dealers (green), Others (purple), T $= 1669$, $\tau = 0.05$, window size $n = 125$.

IN-links: Summary

Network Analysis

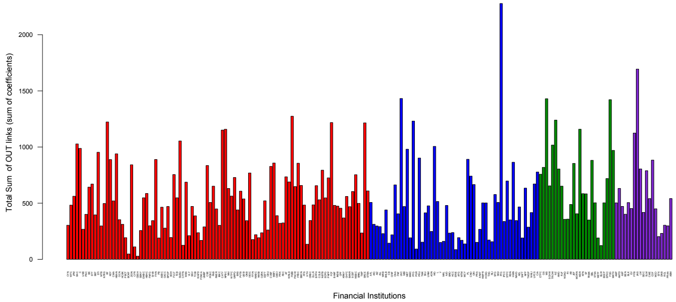

Figure 13: Total OUT links (sum of $|\beta|$'s), full-sample results; Depositories (red), Insurance (blue), Broker-Dealers (green), Others (purple), T $= 1669$, $\tau = 0.05$, window size $n = 125$.

▸ OUT-links: Summary

Network Analysis

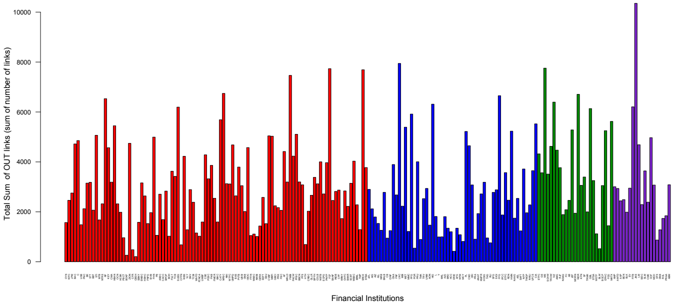

Figure 14: Total OUT links (sum of number of links), full-sample results; Depositories (red), Insurance (blue), Broker-Dealers (green), Others (purple), T = 1669, $\tau = 0.05$, window size $n = 125$.

OUT-links: Summary

Network Analysis

	Dep	Ins	Br-Deal	Oth
Total IN ($\sum \beta$'s)	54449.59	30872.66	13794.39	10788.62
Total IN (links	297081.00	174162.00	73110.00	57805.00
$\sum \beta$'s/links IN	0.18	0.18	0.19	0.19
Total OUT (β's)	58452.53	26446.72	15533.89	9472.13
Total OUT (links)	318640.00	151328.00	78890.00	53300.00
$\sum \beta$'s/links OUT	0.18	0.17	0.20	0.18

Table 1: Summary of IN- and OUT- links, full-sample results, T = 1669, $\tau = 0.05$, window size $n = 125$.

Network Analysis

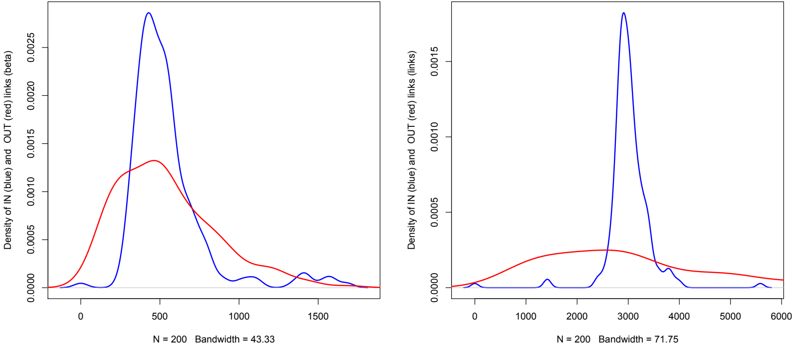

Figure 15: Densities of IN- and OUT- links, full-sample results, T $= 1669$, $\tau = 0.05$, window size $n = 125$.

Link Function Dynamics

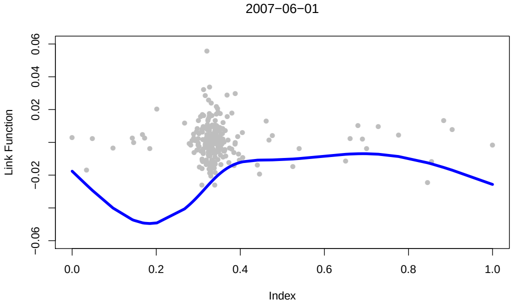

Figure 16: Link function dynamics for JPM, June, 1, 2007 - August, 1, 2008, $\tau = 0.05$, window size $n = 125$.

Conclusion

"The whole is simpler than the sum of its parts."

Williard J. Gibbs

TENET: Tail Event driven NETwork risk

Wolfgang Karl Härdle
Natalia Sirotko-Sibirskaya
Weining Wang

Ladislaus von Bortkiewicz Chair of Statistics
C.A.S.E. - Center for Applied Statistics and Economics
Humboldt–Universität zu Berlin
http://lvb.wiwi.hu-berlin.de
http://www.case.hu-berlin.de

Network Analysis: IN-links

	Ticker	Ratio IN	Total βs	Total Links
180	FGG	0.99	1416.99	1417
144	FFG	0.99	1416.99	1417
147	RDN	0.28	1021.09	3562
48	PCBC	0.24	786.25	3223
192	SLM	0.23	733.99	3097
87	PNFP	0.25	733.87	2875
152	NFP	0.23	728.83	3076
122	HIG	0.25	728.37	2855
72	FBC	0.18	726.54	4014
102	AIG	0.19	687.36	3578

Table 2: Financial institutions classified according to the total sum of β-IN coefficients, subsample of 10.

Return

Network Analysis: IN-links

	Ticker	Ratio IN	Total βs	Total Links
176	WETF	0.01	77.25	5584
72	FBC	0.18	726.54	4014
91	CPF	0.10	425.74	3935
94	BANR	0.09	349.72	3843
131	WTM	0.06	242.98	3801
177	BLW	0.04	187.27	3797
175	STSA	0.13	494.79	3763
81	CRBC	0.16	599.66	3727
179	EPHC	0.10	386.51	3628
137	ANAT	0.08	290.53	3590

Table 3: Financial institutions classified according to the total sum of number of IN links, subsample of 10.

Return

Network Analysis: IN-links

	Ticker	Ratio IN	Total βs	Total Links
180	FGG	0.99	1416.99	1417
144	FFG	0.99	1416.99	1417
147	RDN	0.28	1021.09	3562
159	MS	0.28	671.50	2390
87	PNFP	0.25	733.87	2875
122	HIG	0.25	728.37	2855
48	PCBC	0.24	786.25	3223
189	JNS	0.24	657.57	2730
192	SLM	0.23	733.99	3097
152	NFP	0.23	728.83	3076

Table 4: Financial institutions classified according to the ratio of total sum of beta coefficients to total sum of number of IN links, subsample of 10.

Return

Network Analysis: OUT-links

	Ticker	Ratio OUT	Total βs	Total Links
144	FFG	0.34	2278.29	6652
189	JNS	0.16	1694.21	10351
111	LNC	0.18	1432.52	7945
159	MS	0.18	1429.95	7756
180	FGG	0.98	1421.37	1446
75	FMBI	0.17	1274.22	7466
162	IVZ	0.19	1239.72	6396
115	PFG	0.20	1230.95	5917
14	RF	0.18	1223.46	6533
88	FCF	0.15	1217.92	7736

Table 5: Financial institutions classified according to total sum of β-OUT coefficients, subsample of 10.

Return

Network Analysis: OUT-links

	Ticker	Ratio OUT	Total βs	Total Links
189	JNS	0.16	1694.21	10351
111	LNC	0.18	1432.52	7945
159	MS	0.18	1429.95	7756
88	FCF	0.15	1217.92	7736
99	SRCE	0.15	1215.23	7690
75	FMBI	0.17	1274.22	7466
53	NPBC	0.17	1158.28	6744
170	WDR	0.17	1158.48	6708
144	FFG	0.34	2278.29	6652
14	RF	0.18	1223.46	6533

Table 6: Financial institutions classified according to total sum of number of OUT links, subsample of 10.

Return

Network Analysis: OUT-links

	Ticker	Ratio OUT	Total βs	Total Links
180	FGG	0.98	1421.37	1446
144	FFG	0.34	2278.29	6652
123	L	0.28	514.96	1815
126	AJG	0.26	479.85	1809
186	BLK	0.25	505.35	1988
177	BLW	0.23	122.82	526
196	EFX	0.23	202.69	872
23	CFR	0.22	110.98	483
158	GS	0.22	819.06	3567
193	CME	0.22	540.86	2389

Table 7: Financial institutions classified according to the ratio of total sum of beta coefficients to total sum of number of OUT links, subsample of 10.

Return

Numerical procedure

1. Given $\widehat{\beta}^{(t)}$, standardize $\widehat{\beta}^{(t)}$ so that $\|\widehat{\beta}^{(t)}\| = 1$, $\widehat{\beta}_1^{(t)} > 0$. Then compute

$$(\widehat{a}_j^{(t)}, \widehat{b}_j^{(t)}) \stackrel{\text{def}}{=} \mathop{\arg\min}_{(a_j, b_j)'\text{s}} \sum_{i=1}^{n} \rho\big(Y_i - a_j - b_j X_{ij}^\top \widehat{\beta}^{(t)}\big) \omega_{ij}(\widehat{\beta}^{(t)}),$$

where

- $\widehat{\beta}_0$ initial estimator of β^*,
- $X_{ij} = X_i - X_j$,
- $a_j = g(\beta^\top X_j)$,
- $b_j = g'(\beta^\top X_j)$,
- $\omega_{ij}(\widehat{\beta}_0^{(t)}) \stackrel{\text{def}}{=} \dfrac{K_h(X_{ij}^\top \beta_0^{(t)})}{\sum_{i=1}^{n} K_h(X_{ij}^\top \beta_0^{(t)})}$,
- $t = 1, 2, \ldots$ are iterations.

Numerical procedure

2. Given $(\widehat{a}_j^{(t)}, \widehat{b}_j^{(t)})$, solve

$$\widehat{\beta}^{(t+1)} = \arg\min_{\beta} n^{-1} \sum_{j=1}^{n} \sum_{i=1}^{n} \rho\big(Y_i - \widehat{a}_j^{(t)} - \widehat{b}_j^{(t)} X_{ij}^{\top} \beta\big)\omega_{ij}(\widehat{\beta}^{(t)}),$$

$$+ \textstyle\sum_{l=1}^{p} \widehat{d}_l^{(t)} |\beta_l|.$$

where

- $\widehat{d}_l^{(t)} = \gamma_\lambda(|\widehat{\beta}_l^{(t)}|)$,
- $\omega_{ij}(.)$ are from the step before.

Return

Effective dimension

Let $\{X_i, Y_i\}_{i=1}^n$ be independent r. v.
Given X, let $Y_i \sim (\mu(X), \sigma^2)$, where $\mu(X)$ is the true mean and σ^2 is the common variance.

$$\mathrm{df}(\hat{f}) = \sum_{i=1}^{n} \frac{\mathrm{Cov}\{\hat{f}(X_i), Y_i\}}{\sigma^2}.$$

Under certain mild conditions an unbiased estimator of df is

$$\mathrm{df}(\hat{f}) = \sum_{i=1}^{n} \frac{\partial \hat{f}(X_i)}{\partial Y_i}$$

Stein (1981)

Return

Financial firms

The financial frims	
1. Wells Fargo & Co (WFC)	15. Franklin Resources Inc. (BEN)
2. JP Morgan Chase & Co (JPM)	16. The Travelers Companies, Inc. (TRV)
3. Bank of America Corp (BAC)	17. AFLAC Inc. (AFL)
4. Citigroup Inc (C)	18. Prudential Financial, Inc. (PRU)
5. American Express Company (AXP)	19. State Street Corporation (STT)
6. U.S. Bancorp (USB)	20. The Chubb Corporation (CB)
7. The Goldman Sachs Group, Inc. (GS)	21. BB&T Corporation (BBT)
8. American International Group, Inc. (AIG)	22. Marsh & McLennan Companies, Inc. (MMC)
9. MetLife, Inc. (MET)	23. The Allstate Corporation (ALL)
10. Capital One Financial Corp. (COF)	24. Aon plc (AON)
11. BlackRock, Inc. (BLK)	25. CME Group Inc. (CME)
12. Morgan Stanley (MS)	26. The Charles Schwab Corporation (SCHW)
13. PNC Financial Services Group Inc. (PNC)	27. T. Rowe Price Group, Inc. (TROW)
14. The Bank of New York Mellon Corporation (BK)	28. Loews Corporation (L)

Financial firms

29. SunTrust Banks, Inc. (STI)	44. Lincoln National Corporation (LNC)
30. Fifth Third Bancorp (FITB)	45. Affiliated Managers Group Inc. (AMG)
31. Progressive Corp. (PGR)	46. Cincinnati Financial Corp. (CINF)
32. M&T Bank Corporation (MTB)	47. Equifax Inc. (EFX)
33. Ameriprise Financial Inc. (AMP)	48. Alleghany Corp. (Y)
34. Northern Trust Corporation (NTRS)	49. Unum Group (UNM)
35. Invesco Ltd. (IVZ)	50. Comerica Incorporated (CMA)
36. Moody's Corp. (MCO)	51. W.R. Berkley Corporation (WRB)
37. Regions Financial Corp. (RF)	52. Fidelity National Financial, Inc. (FNF)
38. The Hartford Financial Services Group, Inc. (HIG)	53. Huntington Bancshares Incorporated (HBAN)
39. TD Ameritrade Holding Corporation (AMTD)	54. Raymond James Financial Inc. (RJF)
40. Principal Financial Group Inc. (PFG)	55. Torchmark Corp. (TMK)
41. SLM Corporation (SLM)	56. Markel Corp. (MKL)
42. KeyCorp (KEY)	57. Ocwen Financial Corp. (OCN)
43. CNA Financial Corporation (CNA)	58. Arthur J Gallagher & Co. (AJG)

Financial firms

59. Hudson City Bancorp, Inc. (HCBK)	74. Commerce Bancshares, Inc. (CBSH)
60. People's United Financial Inc. (PBCT)	75. Signature Bank (SBNY)
61. SEI Investments Co. (SEIC)	76. Jefferies Group, Inc. (JEF)
62. Nasdaq OMX Group Inc. (NDAQ)	77. Rollins Inc. (ROL)
63. Brown & Brown Inc. (BRO)	78. Morningstar Inc. (MORN)
64. BOK Financial Corporation (BOKF)	79. East West Bancorp, Inc. (EWBC)
65. Zions Bancorp. (ZION)	80. Waddell & Reed Financial Inc. (WDR)
66. HCC Insurance Holdings Inc. (HCC)	81. Old Republic International Corporation (ORI)
67. Eaton Vance Corp. (EV)	82. ProAssurance Corporation (PRA)
68. Erie Indemnity Company (ERIE)	83. Assurant Inc. (AIZ)
69. American Financial Group Inc. (AFG)	84. Hancock Holding Company (HBHC)
70. Dun & Bradstreet Corp. (DNB)	85. First Niagara Financial Group Inc. (FNFG)
71. White Mountains Insurance Group, Ltd. (WTM)	86. SVB Financial Group (SIVB)
72. Cullen-Frost Bankers, Inc. (CFR)	87. First Horizon National Corporation (FHN)
73. Legg Mason Inc. (LM)	88. E-TRADE Financial Corporation (ETFC)

Financial firms

89. SunTrust Banks, Inc. (STI)	104. Valley National Bancorp (VLY)
90. Mercury General Corporation (MCY)	105. KKR Financial Holdings LLC (KFN)
91. Associated Banc-Corp (ASBC)	106. Synovus Financial Corporation (SNV)
92. Credit Acceptance Corp. (CACC)	107. Texas Capital BancShares Inc. (TCBI)
93. Protective Life Corporation (PL)	108. American National Insurance Co. (ANAT)
94. Federated Investors, Inc. (FII)	109. Washington Federal Inc. (WAFD)
95. CNO Financial Group, Inc. (CNO)	110. First Citizens Bancshares Inc. (FCNCA)
96. Popular, Inc. (BPOP)	111. Kemper Corporation (KMPR)
97. Bank of Hawaii Corporation (BOH)	112. UMB Financial Corporation (UMBF)
98. Fulton Financial Corporation (FULT)	113. Stifel Financial Corp. (SF)
99. AllianceBernstein Holding L.P. (AB)	114. CapitalSource Inc. (CSE)
100. TCF Financial Corporation (TCB)	115. Portfolio Recovery Associates Inc. (PRAA)
101. Susquehanna Bancshares, Inc. (SUSQ)	116. Janus Capital Group, Inc. (JNS)
102. Capitol Federal Financial, Inc. (CFFN)	117. MBIA Inc. (MBI)
103. Webster Financial Corp. (WBS)	118. Healthcare Services Group Inc. (HCSG)

Financial firms

119. The Hanover Insurance Group Inc. (THG)	134. BancorpSouth, Inc. (BXS)
120. F.N.B. Corporation (FNB)	135. Privatebancorp Inc. (PVTB)
121. FirstMerit Corporation (FMER)	136. United Bankshares Inc. (UBSI)
122. FirstMerit Corporation (FMER)	137. Old National Bancorp. (ONB)
123. RLI Corp. (RLI)	138. International Bancshares Corporation (IBOC)
124. StanCorp Financial Group Inc. (SFG)	139. First Financial Bankshares Inc. (FFIN)
125. Trustmark Corporation (TRMK)	140. Westamerica Bancorp. (WABC)
126. IberiaBank Corp. (IBKC)	141. Northwest Bancshares, Inc. (NWBI)
127. Cathay General Bancorp (CATY)	142. Bank of the Ozarks, Inc. (OZRK)
128. National Penn Bancshares Inc. (NPBC)	143. Huntington Bancshares Incorporated (HBAN)
129. Nelnet, Inc. (NNI)	144. Euronet Worldwide Inc. (EEFT)
130. Wintrust Financial Corporation (WTFC)	145. Community Bank System Inc. (CBU)
131. Umpqua Holdings Corporation (UMPQ)	146. CVB Financial Corp. (CVBF)
132. GAMCO Investors, Inc. (GBL)	147. MB Financial Inc. (MBFI)
133. Sterling Financial Corp. (STSA)	148. ABM Industries Incorporated (ABM)

Financial firms

149. Glacier Bancorp Inc. (GBCI)	164. Citizens Republic Bancorp, Inc (CRBC)
150. Selective Insurance Group Inc. (SIGI)	165. Horace Mann Educators Corp. (HMN)
151. Park National Corp. (PRK)	166. DFC Global Corp. (DLLR)
152. Flagstar Bancorp Inc. (FBC)	167. Navigators Group Inc. (NAVG)
153. FBL Financial Group Inc. (FFG)	168. Boston Private Financial Holdings, Inc. (BPFH)
154. Astoria Financial Corporation (AF)	169. American Equity Investment Life Holding Co. (AEL)
155. World Acceptance Corp. (WRLD)	170. BlackRock Limited Duration Income Trust (BLW)
156. First Midwest Bancorp Inc. (FMBI)	171. Columbia Banking System Inc. (COLB)
157. PacWest Bancorp (PACW))	172. Safety Insurance Group Inc. (SAFT)
158. First Financial Bancorp. (FFBC)	173. National Financial Partners Corp. (NFP)
159. BBCN Bancorp, Inc. (BBCN)	174. NBT Bancorp, Inc. (NBTB)
160. Provident Financial Services, Inc. (PFS)	175. Tower Group Inc. (TWGP)
161. FBL Financial Group Inc. (FFG)	176. Encore Capital Group, Inc. (ECPG)
162. WisdomTree Investments, Inc. (WETF)	177. Pinnacle Financial Partners Inc. (PNFP)
163. Hilltop Holdings Inc. (HTH)	178. First Commonwealth Financial Corp. (FCF)

Financial firms

179. BancFirst Corporation (BANF)	190. Berkshire Hills Bancorp Inc. (BHLB)
180. Independent Bank Corp. (INDB)	191. Brookline Bancorp, Inc. (BRKL)
181. Infinity Property and Casualty Corp. (IPCC)	192. National Western Life Insurance Company (NWLI)
182. Central Pacific Financial Corp. (CPF)	193. Tompkins Financial Corporation (TMP)
183. Kearny Financial Corp. (KRNY)	194. BGC Partners, Inc. (BGCP)
184. Chemical Financial Corporation (CHFC)	195. Epoch Investment Partners, Inc. (EPHC)
185. Banner Corporation (BANR)	196. United Fire Group, Inc (UFCS)
186. State Auto Financial Corp. (STFC)	197. 1st Source Corporation (SRCE)
187. Radian Group Inc. (RDN)	198. Citizens Inc. (CIA)
188. SCBT Financial Corporation (SCBT)	199. S&T Bancorp Inc. (STBA)
189. WesBanco Inc. (WSBC)	

Return

Macroprudential variables

1. VIX
2. Short term liquidity spread (liquidity)
3. Daily change in the 3-month Treasury maturities (3MT)
4. Change in the slope of the yield curve (yield)
5. Change in the credit spread (credit)
6. Daily Dow Jones U.S. Real Estate index returns (D_J)
7. S&P500 returns (S&P)

Source: Adrian and Brunnermeier (2011), Datastream.

Return to Introduction

Return to Empirical Analysis

References

Adrian, T. and Brunnermeier, M. K.
CoVaR
Staff Reports 348, Federal Reserve Bank of New York, 2011

Berkowitz, J., Christoffersen, P. and Pelletier, D.
Evaluating value-at-risk models with desk-level data
Working Paper 010, North Carolina State University, Department of Economics, 2009

Bickel, P. J., Ritov, Y. and Tsybakov, A. B.
Hierarchical selection of variables in sparse high-dimensional regression
Borrowing Strength: Theory Powering Applications - A Festschrift for Lawrence D. Brown. IMS Collections, v. 6, 56-69. 2010

References

Bickel, P. J., Ritov, Y. and Tsybakov, A. B.
Simultaneous analysis of lasso and dantzig selector
Ann. Statist. 37(4), 1705-1732. 2009

Bisias, D., Flood, M., Lo, A. W., and Valavanis
A Survey of Systemic Risk Analytics
Annual Review of Financial Economics, Vol. 4: 255-296, 2012.

Bradic, J., Fan, J. and Wang, W.
Penalized composite quasi-likelihood for ultrahigh dimensional variable selection
J. R. Statist. Soc. B 73(3): 325-349. 2011

Campbell, J. Y., Lo, A. W. and MacKinlay, A. C.
The Econometrics of Financial Markets
Princeton University Press, 1996

References

Carroll,R. J., and Härdle, W. K.
Symmetrized nearest neighbor regression estimates
Statistics and Probability Letters 7 (4), 315 - 318.

Chao, S. K., Härdle, W. K. and Wang, W.
Quantile regression in Risk Calibration
In Handbook for Financial Econometrics and Statistics (Cheng-Few Lee, ed.). Springer Verlag, forthcoming, SFB 649 DP 2012-006.

Engle, R. F. and Manganelli, S.
CAViaR: Conditional Autoregressive Value at Risk by Regression Quantiles
J. Bus. Econ. Stat. 22: 367-381, 2004

References

Fan, J. and Li, R.
Variable selection via nonconcave penalized likelihood and its oracle properties
J. Amer. Statist. Assoc. 96: 1348-1360. 2001

Franke, J., Härdle, W. and Hafner, C.
Statistics of Financial Markets
Springer, 2004

Härdle, W. and Stoker, T. M.
Investigating smooth multiple regression by the method of average derivatives
J. Amer. Statist. Assoc., 84: 986-995. 1989

References

Hautsch, N., Schaumburg, J. and Schienle, M.
Financial Network System Risk Contributions
Discussion Paper 2011-072, CRC 649, Humboldt-Universität zu Berlin, 2011

Huber, P. J.
Robust estimation of a location parameter
Ann. Math. Statist., 35(1): 73-101. 1964

Hull C. J.
Risk Management and Financial Institutions – 2nd ed.
Prentice Hall, 2010

References

- Kai, B., Li, R. and Zou, H.
  Local composite quantile regression smoothing: an efficient and safe alternative to local polynomial regression
  J. R. Statist. Soc. B 72: 49-69. 2010

- Kai, B., Li, R. and Zou, H.
  New Efficient Estimation and Variable Selection Methods for Semiparametric Varying-Coefficient Partially Linear Models
  Ann. Statist., 39(1): 305-332. 2011

- Kim, Y., Choi, H. and Oh, H.
  Smoothly clipped absolute deviation on high dimensions
  J. Amer. Statist. Assoc. 103, 1656-1673. 2008

References

- Koenker, R. and Bassett, G. W.
  Regression quantiles
  Econometrica. 46: 33-50. 1978
- Koenker, R. and Bassett, G. W.
  Robust tests for heteroscedasticity based on regression quantiles
  Econometrica 50: 43-61. 1982
- Koenker, R. and Hallock, K. F.
  Quantile regression
  Journal of Econometric Perspectives 15(4): 143-156. 2001

References

- Kong, E. and Xia, Y.
  Variable selection for the single-index model
  Biometrika. 94: 217-229. 1994
- Leider J.
  A Quantile Regression Study of Climate Change in Chicago
  available on www.siam.org, 2012
- Leng, C., Xia, Y. and Xu, J.
  An adaptive estimation method for semiparametric models and dimension reduction
  WSPC-Proceedings. 2010

References

Li, Y. and Zhu, J.
L1- norm quantile regression
J. Comput. Graph. Stat., 17: 163-185. 2008

Newey, W. and Powell, J.
Asymmetric least squares estimation and testing
Econometrica 55: 819-847. 1987

Koenker R.
Quantile Regression
Cambridge Univ. Press, 2000

References

Ruppert, D., Sheather, S. J. and Wand, M. P.
An effective bandwidth selector for local least squares regression.
J. Amer. Statist. Assoc. 90: 1257-1270. 1995

Schnabel, S. and Eilers, P.
Optimal expectile smoothing
Comput. Stat. Data. An. 53(12): 4168-4177. 2009

Serfling, R. J.
Approximation theorems of mathematical statistics
Wiley, New York. 1980

References

- Tibshirani, R.
  Regression shrinkage and selection via the lasso
  J. R. Statist. Soc. B 58(1): 267-288. 1996

- Wang, Q. and Yin, X.
  A nonlinear multi-dimensional variable selection methods for highdimensional data: sparse mave
  Comput. Stat. Data. An., 52: 4512-4520. 2008

- Wu, T. Z., Yu, K. and Yu, Y.
  Single-index quantile regression
  J. Multivariate Anal., 101: 1607-1621. 2010

References

- Xia, Y., Tong, H., Li, W. and Zhu, L.
  An adaptive estimation of dimension reduction space
  J. R. Statist. Soc. B 64, Part 3, pp. 363-410, 2002

- Yu, K. and Jones, M. C.
  Local linear quantile regression
  J. Amer. Statist. Assoc. 98: 228-237, 1998

- Yuan, M.
  GACV for Quantile Smoothing Splines
  Comput. Stat. Data. An., 5, 813-829. 2006

References

- Yuan, M. and Lin, Y.
  Model selection and estimation in regression with grouped variables
  J. R. Statist. Soc. B 68(1): 49-67. 2006

- Zeng, P., He, T. H. and Zhu, Y.
  A lasso-type approach for estimation and variable selection in single-index models
  J. Comput. Graph. Stat., 21: 92-109. 2012

- Zhao, P. and Yu, B.
  On model selection consistency of Lasso
  J. Machine Learning Res., 7, 2541-2567. 2006

References

Zou, H.
The adaptive Lasso and its oracle properties
J. Amer. Statist. Assoc. 101(476): 1418-1429. 2006