TEDAS - Tail Event Driven ASset Allocation: equity and mutual funds' markets

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Strategies comparison: hedge funds' indices

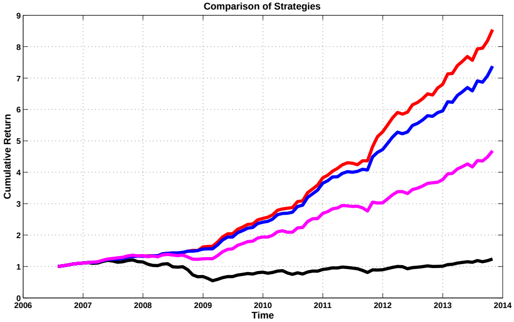

Figure 1: Strategies' cumulative returns' comparison: TEDAS Basic, S&P500 Buy-and-hold, TEDAS Naïve, OGARCH Mean-Variance

- Härdle et al. (2014)
  - TEDAS applied to hedge funds' indices performs better than benchmark models
- Limitation of using hedge indices as portfolio assets
- Application of TEDAS approach to Global mutual funds' data and German stock market

- Comparison of the TEDAS with more benchmark strategies:
  - 60/40 portfolio
  - Risk Parity (equal risk portfolio contribution)
  - Mean-Variance strategy
- TEDAS parameters optimisation

Outline

1. Motivation ✓
2. TEDAS framework
3. Data
4. Empirical Results
5. Discussion: choice of τ-spine
6. Conclusions

At time period $t = n, ..., l$

1. Consider a data vector $Y \in \mathbb{R}^n$ of core-asset returns and a matrix $X \in \mathbb{R}^{n \times p}$ of satellites' returns, $p > n$; n is equal to width of moving window
2. Obtain log-returns sample τ-quantiles (τ-spine) $\hat{q}_{\tau_{j,t}} \stackrel{\text{def}}{=} F_n^{-1}(\tau)$ from the core log-returns edf F_n, where $\tau_{j=1,...,5} = (0.05, 0.15, 0.25, 0.35, 0.50)$
3. Determine core-asset return r_t, select $\tau_{j,t}$ according to the right-side $\hat{q}_{\tau_{j,t}}$ in: $r_t \leq \hat{q}_{\tau_{1,t}}$ or $\hat{q}_{\tau_{1,t}} < r_t \leq \hat{q}_{\tau_{j,t}}$

4 ALQR for $\hat{\beta}_{\tau_{j,t},\lambda_t}$ using the observations $X \in \mathbb{R}^{l-n+1,\dots,t\times p}$, $Y \in \mathbb{R}^{l-n+1,\dots,l}$ ▸ Details

5 Depending on TEDAS gestalt apply corresponding approach for volatility modeling and weights optimisation to satellites with $\hat{\beta}_{\tau_{j,t},\lambda_t} \neq 0$

Rebalancing of portfolio:

- one of inequalities in step 3 holds
  - sell the core portfolio and buy satellites (step 4) with estimated weights (step 5)
  - stay "in cash" if there are no adversely moving satellites (step 4)
- no one of inequalities holds: invest in the core portfolio
- period (t+1), if no one of inequalities (step 3) holds, we return to the core portfolio

TEDAS gestalt

TEDAS Basic

- DCC vola ▸ Details
- CF-VaR Optimisation ▸ Details

TEDAS Naïve

- Equal weights

TEDAS Hybrid

- Volatility: sample covariance matrix
- Mean-variance optimisation of weights ▸ Details

Small and mid caps German stocks

- MDAX
  - 50 medium-sized German public limited companies and foreign companies primarily active in Germany from traditional sectors
  - Ranks after the DAX30 based on market capitalisation and stock exchange turnover
- SDAX
  - The selection index for smaller companies from traditional sectors
  - 50 stocks from the Prime Standard
- TecDAX
  - Comprises the 30 largest technology stocks below the DAX

Size premium

- Banz (1981) and Reinganum (1981): the US small cap stocks outperformed large-cap stocks (in 1936-1975)
- Fama, French (1992, 1993): a size premium of 0.27% per month in the US over the period 1963-1991
- Results are robust:
  - for stock price momentum by Jegadeesh , Titman (1993) and Carhart (1997)
  - for liquidity by Pastor, Stambaugh (2003) and Ibbotson, Hu (2011)
  - for industry factors, high leverage, low liquidity by Menchero et al. (2008)

Why small and mid cap stocks?

- Strong absolute returns
- Diversification benefits
- High risk-adjusted returns

German stocks' data

- Frankfurt Stock Exchange (Xetra), weekly data
  - 125 stocks - SDAX (48), MDAX (47) and TecDAX (50) as on 20140801
  - DAX index
- Span: 20121221 - 20141127 (100 trading weeks)
- Source: Datastream

Mutual Fund

- Open-End: buy and sell the shares, meet the demand for customers
- Unit Investment Trust: exchange-traded fund (ETF), Fixed/ unmanaged Portfolio
- Closed-End: fixed number of shares, not redeemable by the fund, buy and sell on the exchange

Why Mutual Funds?

- Importance of MF
  - $30 trillion worldwide, 15 trillion in U.S in 2013
  - 88% investment companies managed asset by holding MF
- Big data: 76 200 MFs worldwide in 2013
- Diversification

Mutual Funds' Data

- Monthly data
  - 2616 Mutual funds
  - S&P500
- Span: 19980101 - 20131201 (192 months)
- Source: Datastream

TEDAS approach:German stocks' results

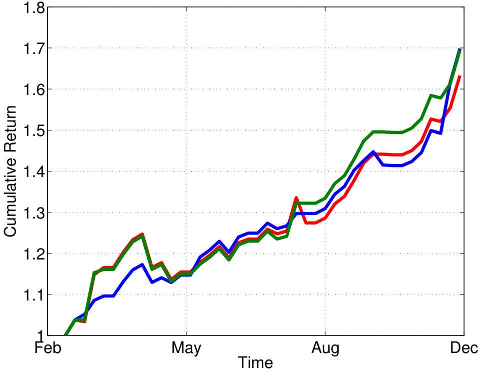

Figure 2: Strategies' cumulative returns' comparison: TEDAS Basic, TEDAS Naïve, TEDAS Hybrid

TEDAS_strategies

TEDAS approach:German stocks' results

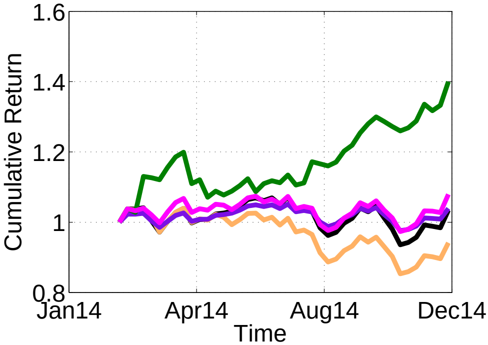

Figure 3: Strategies' cumulative returns' comparison (with transaction costs 1% of portfolio value): TEDAS Hybrid, DAX Buy-and-hold, 60/40, Risk-parity, OGARCH Mean-Variance

▸ Risk-parity details ▸ OGARCH details

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TEDAS approach:German stocks' results

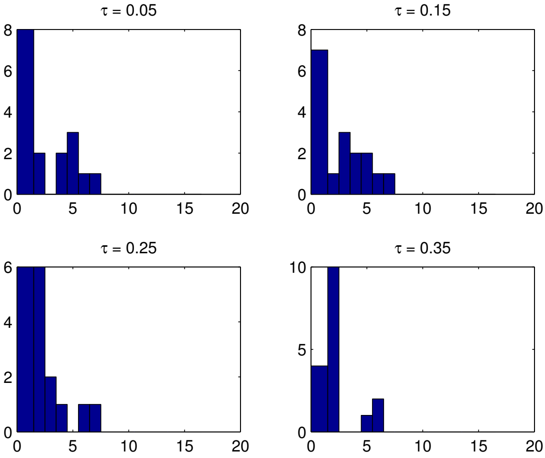

Figure 4: Frequency of the number of selected variables for 4 different τ

TEDAS approach:German stocks results

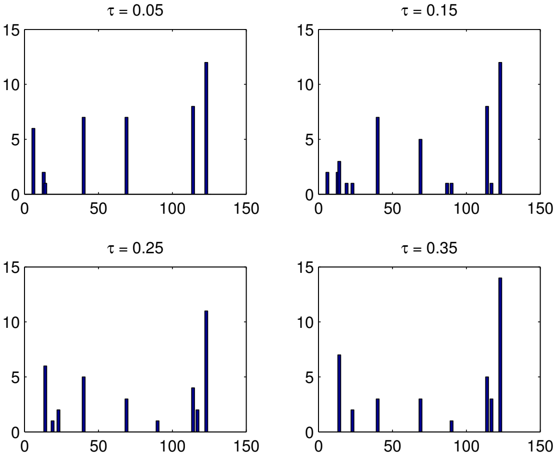

Figure 5: The frequency of stocks

Selected Stocks

Table 1: The selected German Stocks for $\tau = 0.05$

Top 5 influential Stocks	Frequency	Index	Industry
Sartorius Aktiengesellschaft	12	TecDAX	Provision of laboratory and process technologies and equipment
XING AG	8	TecDAX	Online business communication services
Surteco SE	7	SDAX	Household Goods & Home Construction
Kabel Deutschland Holding AG	7	MDAX	Cable-based telecommunication services
Biotest AG	6	MDAX	Producing biological medications

-$\widehat{\beta}$ in each window, $\tau = 0.05$

Figure 6: Different $-\widehat{\beta}$ in application; $\tau = 0.05$ ▸ Selected Stocks

TEDAS approach:Mutual Funds results

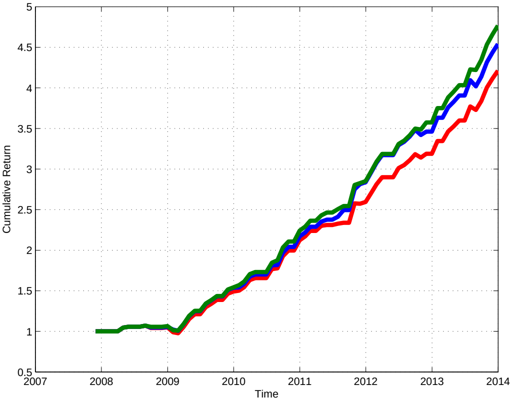

Figure 7: Strategies' cumulative returns' comparison: TEDAS Basic, TEDAS Naïve, TEDAS Hybrid TEDAS_strategies

TEDAS approach:Mutual Funds results

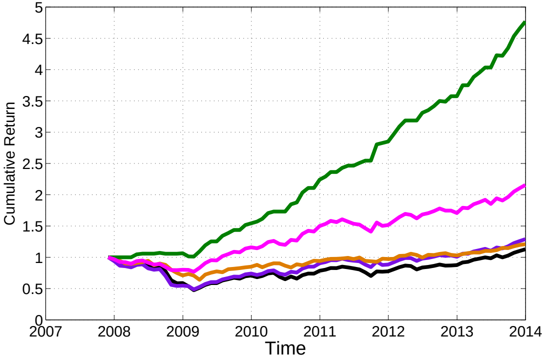

Figure 8: Strategies' cumulative returns' comparison (with transaction costs 1% of portfolio value): TEDAS Hybrid, DAX Buy-and-hold, 60/40, Risk-parity, OGARCH Mean-Variance

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TEDAS approach:Mutual Funds results

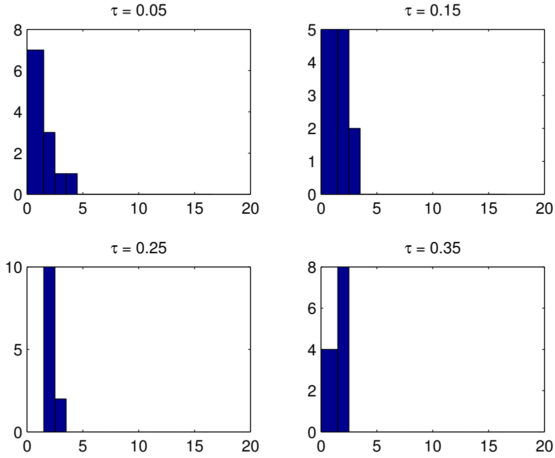


Figure 9: Frequency of the number of selected variables for 4 different τ

TEDAS approach:Mutual Funds results

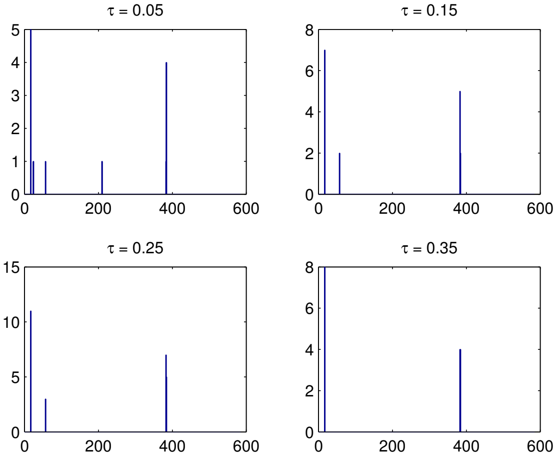


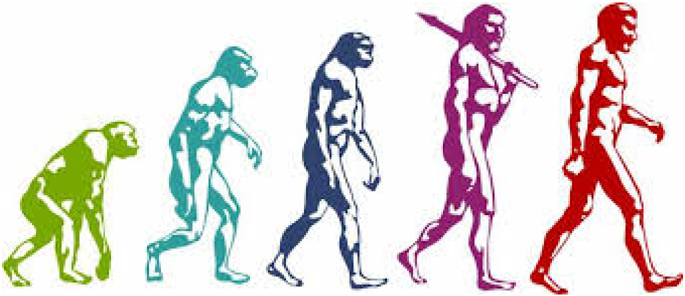
Figure 10: The frequency of mutual funds

Selected Mutual Funds

Table 2: The selected Mutual Funds for $\tau = 0.05$

Top 5 influential Stocks	Frequency	Market
Blackrock Eurofund Class I	12	U.S.
Pimco Funds Long Term United States Government Institutional Shares	8	U.S.
Prudential International Value Fund Class Z	4	U.S.
Artisan International Fund Investor Shares	3	U.S.
American Century 2OTH Century International Growth Investor Class	1	U.S.

How to choose optimal τ-spine?



Generation of different τ-spines

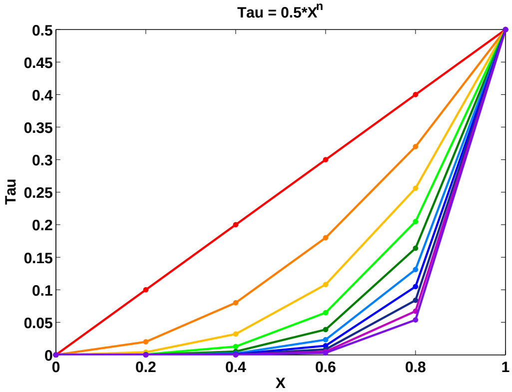


Figure 11: Generation of 10 sets of τ-spines

TEDAS Basic with different τ-spines

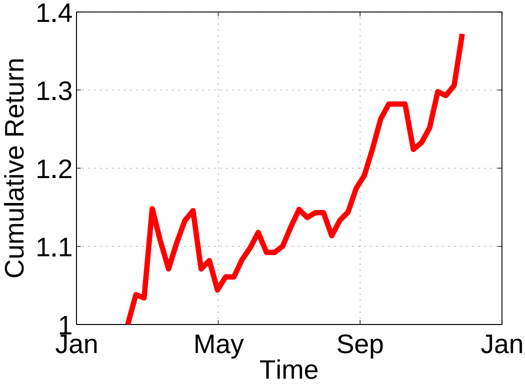


Figure 12: Cumulative return for TEDAS Basic with various τ-spines

TEDAS Basic with different τ-spines

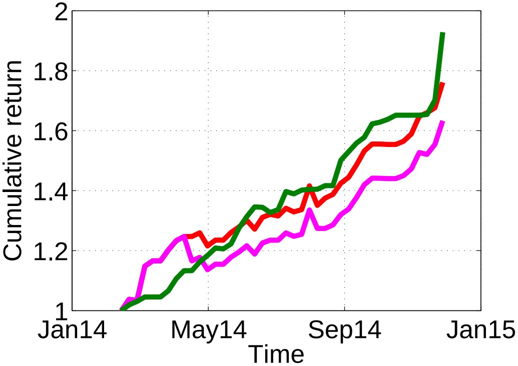


Figure 13: TEDAS Basic cumulative returns' for τ-spines:

$\tau_{j=1,\ldots,5} = (0, 0.002, 0.0233, 0.1311, 0.5)$,

$\tau_{j=1,\ldots,5} = (0.05, 0.15, 0.25, 0.35, 0.5)$,

$\tau_{j=1,\ldots,50} = (0.01, 0.02, 0.03 \ldots 0.49, 0.5)$

What is the best τ-spine?

Monte Carlo simulations

- $Y_i = \hat{q}_{\tau_i}$ $\tau_{j=1,2,3} = (0.05, 0.15, 0.35)$, $n = 100$, $Y_t \sim \text{ALD}(\mu, \sigma, \tau)$; ▸ Details
- $X_i \sim \text{N}(0, \Omega)$, $n = 100$ for every τ, $p = 150$, $\beta = (-5, -2, -1, 3, 1, 0.5, 0, ..., 0)$, $\varepsilon_i \sim \text{N}(0, \sigma^2)$; $\lambda_n = 0.25\sqrt{\|\hat{\beta}^{\text{init}}\|_0 \log(n \vee p)}(\log n)^{0.1/2}$, $\hat{\omega}_j = 1/|\hat{\beta}_j^{\text{init}}| \wedge \sqrt{n}$; $\hat{\beta}_j^{\text{init}}$;
- $\Omega_{i,j} = 0.5^{|i-j|}$, $\sigma = 0.1, 0.5, 1$ (three levels of noise);

What is the best τ-spine?

- for $\hat{\beta}^{\text{init}}$ estimator $\hat{\beta}_{\tau,\hat{\lambda}}$ from the model (2) is used, where $\hat{\lambda}$ is chosen according to the BIC criterion

$$\text{BIC}_{\lambda_n,\tau} \stackrel{\text{def}}{=} \log \left\{ n^{-1} \cdot \sum_{i=1}^{n} \rho_\tau (Y_i - X_i^\top \hat{\beta}_\tau) \right\} + \frac{\log(n)}{2n} \cdot \widehat{\text{df}}(\lambda_n)$$

- Apply one of TEDAS modification with different τ-spines
- Choose that τ-spine, which gives the highest wealth

$$W_i = \sum_{j=1}^{d} w_j x_{i,\tau} \, ,$$

- TEDAS approach performs better than traditional benchmark strategies
- TEDAS outperforms for
  - different regions (global and Germany),
  - various assets
  - alternative time periods (daily, weekly and monthly),
  - big data and small data
- Results for 3 modifications of TEDAS are robust
- Discussion:
  - How to choose optimal τ-spine?

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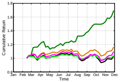

Lasso Shrinkage

Linear model: $Y = X\beta + \varepsilon$; $Y \in \mathbb{R}^n$, $X \in \mathbb{R}^{n\times p}$, $\beta \in \mathbb{R}^p$, $\{\varepsilon_i\}_{i=1}^n$ i.i.d., independent of $\{X_i; i = 1, ..., n\}$

The optimization problem for the lasso estimator:

$$\begin{aligned} \hat{\beta}^{\text{lasso}} = & \arg\min_{\beta\in\mathbb{R}^p} f(\beta) \\ & \text{subject to} \quad g(\beta) \geq 0 \end{aligned} \tag{1}$$

where

$$f(\beta) = \frac{1}{2}(y - X\beta)^\top (y - X\beta)$$
$$g(\beta) = t - \|\beta\|_1$$

where t is the size constraint on $\|\beta\|_1$ Back to "Strategies"

Lasso Duality

If (1) is convex programming problem, then the Lagrangian is

$$L(\beta, \lambda) = f(\beta) - \lambda g(\beta).$$

and the primal-dual relationship is

$$\underbrace{\underset{\beta}{\text{minimize}} \ \sup_{\lambda \geq 0} \ L(\beta, \lambda)}_{\text{primal}} \geq \underbrace{\underset{\lambda \geq 0}{\text{maximize}} \ \inf_{\beta} \ L(\beta, \lambda)}_{\text{dual}}$$

Then the dual function $L^*(\lambda) = \inf_{\beta} L(\beta, \lambda)$ is

$$L^*(\lambda) = \frac{1}{2} y^\top y - \frac{1}{2} \hat{\beta}^\top X^\top X \hat{\beta} - t \frac{(y - X\hat{\beta})^\top X \hat{\beta}}{\|\hat{\beta}\|_1}$$

with $(y - X\hat{\beta})^\top X\hat{\beta} / \|\hat{\beta}\|_1 = \lambda$ Back to "Strategies"

Quantile Regression

The loss $\rho_\tau(u) = u\{\tau - \mathbf{I}(u < 0)\}$ gives the (conditional) quantiles $F^{-1}_{y|x}(\tau) \stackrel{\text{def}}{=} q_\tau(x)$

Minimize

$$\hat{\beta}_\tau = \arg \min_{\beta \in \mathbb{R}^p} \sum_{i=1}^{n} \rho_\tau(Y_i - X_i^\top \beta).$$

Re-write:

$$\underset{(\xi,\zeta)\in\mathbb{R}_+^{2n}}{\text{minimize}} \quad \left\{ \tau 1_n^\top \xi + (1-\tau) 1_n^\top \zeta | X\beta + \xi - \zeta = Y \right\}$$

with ξ, ζ are vectors of "slack" variables Back to "Strategies"

Adaptive Lasso Procedure

The adaptive Lasso (Zou, 2006) yields a sparser solution and is less biased.

L_1 - penalty replaced by a re-weighted version; $\hat{\omega}_j = 1/|\hat{\beta}_j^{\text{init}}|^{\gamma}$, $\gamma = 1$, $\hat{\beta}^{\text{init}}$ is from (1)

The adaptive lasso estimates are given by:

$$\hat{\beta}_{\lambda}^{\text{adapt}} = \arg \min_{\beta \in \mathbb{R}^p} \sum_{i=1}^{n} (Y_i - X_i^{\top} \beta)^2 + \lambda \|\hat{\omega}^{\top} \beta\|_1$$

(Bühlmann, van de Geer, 2011): $\hat{\beta}_j^{\text{init}} = 0$, then $\hat{\beta}_j^{\text{adapt}} = 0$

Back to "Strategies"

Simple and Adaptive Lasso Penalized QR

Simple lasso-penalized QR optimization problem is:

$$\hat{\beta}_{\tau,\lambda} = \arg \min_{\beta \in \mathbb{R}^p} \sum_{i=1}^{n} \rho_\tau(Y_i - X_i^\top \beta) + \lambda \|\beta\|_1 \tag{2}$$

Adaptive lasso-penalized QR model uses the re-weighted penalty:

$$\hat{\beta}_{\tau,\lambda}^{\text{adapt}} = \arg \min_{\beta \in \mathbb{R}^p} \sum_{i=1}^{n} \rho_\tau(Y_i - X_i^\top \beta) + \lambda \|\hat{\omega}^\top \beta\|_1 \tag{3}$$

Adaptive lasso-penalized QR procedure can ensure oracle properties for the estimator ▸ Details

▸ Back to "Strategies"

Cornish-Fisher VaR Optimization

The alternative asset allocation (Favre, Galeano, 2002)

$$\begin{aligned} \underset{w \in \mathbb{R}^d}{\text{minimize}} \quad & W_t\{-q_\alpha(w_t) \cdot \sigma_p(w_t)\} \\ \text{subject to} \quad & w_t^\top \mu = \mu_p, \;\; w_t^\top 1 = 1, \;\; w_{t,i} \geq 0 \end{aligned}$$

here $W_t \stackrel{\text{def}}{=} W_0 \cdot \prod_{j=1}^{t-1} w_{t-j}^\top (1 + r_{t-j})$, $\tilde{w}$, W_0 initial wealth,
$\sigma_p^2(w) \stackrel{\text{def}}{=} w_t^\top \Sigma_t w_t$,

$$q_\alpha(w) \stackrel{\text{def}}{=} z_\alpha + (z_\alpha^2 - 1)\frac{S_p(w)}{6} + (z_\alpha^3 - 3z_\alpha)\frac{K_p(w)}{24} - (2z_\alpha^3 - 5z_\alpha)\frac{S_p(w)^2}{36},$$

here $S_p(w)$ skewness, $K_p(w)$ kurtosis, z_α is $\mathrm{N}(0,1)$ α-quantile If $S_p(w)$, $K_p(w)$ zero, then obtain Markowitz allocation

Back to "Strategies"

The Dynamic Conditional Correlations Model (cDCC, with Aielli correction)

The DCC (1,1) model separately estimates a series of univariate GARCH models and their correlation: $r_t|\mathcal{F}_{t-1} \sim \mathrm{N}(0, D_t R_t D_t)$, where

$$
\begin{aligned}
D_t^2 &= \mathrm{diag}(\omega_i) + \mathrm{diag}(\alpha_i) \odot r_{t-1} r_{t-1}^\top + \mathrm{diag}(\beta_i) \odot D_{t-1}^2, \\
\varepsilon_t &= D_t^{-1} r_t, \\
Q_t &= S \odot (\imath\imath^\top - A - B) + A \odot \{P_{t-1}\varepsilon_{t-1}\varepsilon_{t-1}^\top P_{t-1}\} + B \odot Q_{t-1}, \\
R_t &= \{\mathrm{diag}(Q_t)\}^{-1} Q_t \{\mathrm{diag}(Q_t)\}^{-1}
\end{aligned}
$$

where r_t is an $d \times 1$ vector of returns t, D_t is an $d \times d$ diagonal matrix of standard deviations σ_{it}, $i = 1, \ldots, d$, modeled by univariate GARCH, ε_t is an $d \times 1$ vector of standardized returns with $\varepsilon_{it} \stackrel{\mathrm{def}}{=} r_{it}\sigma_{it}^{-1}$, $\imath$ is a vector of ones; $P_{t-1} \stackrel{\mathrm{def}}{=} \{\mathrm{diag}(Q_t)\}^{1/2}$

The DCC Model - Continued

- the correlation targeting gives $S = (1/T)\sum_{t=1}^{T} \varepsilon_t \varepsilon_t^\top$
- then provided that $Q_0 = \varepsilon_0 \varepsilon_0^\top$ is positive definite, each subsequent Q_t will also be positive definite
- the procedure will yield consistent but inefficient estimates of the parameters: the log-likelihood function

$$L(\theta, \phi) = -\frac{1}{2} \sum_{t=1}^{T} \left(n \log(2\pi) + 2 \log |D_t| + \log |R_t| + \varepsilon_t^\top R_t^{-1} \varepsilon_t \right),$$

where θ denotes the parameters in D and ϕ denotes additional correlation parameters in R, is maximized by parts

Back to "Strategies"

The DCC Model - Continued

The log-likelihood is rewritten:

$$L(\theta,\phi) = L_V(\theta) + L_C(\theta,\phi),$$

where the volatility part is the sum of individual GARCH likelihoods jointly maximized by separately maximizing each term

$$\begin{aligned} L_V(\theta) &= -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + \log|D_t|^2 + r_t^\top D_t^{-2} r_t\right) \\ &= -\frac{1}{2}\sum_{t=1}^{T}\sum_{i=1}^{d}\left(\log(2\pi) + \log(\sigma_{it}^2) + \frac{r_{it}^2}{\sigma_{it}^2}\right), \end{aligned}$$

and the correlation part is

$$L_C(\theta,\phi) = -\frac{1}{2}\sum_{t=1}^{T}\left(\log|R_t| + \varepsilon_t^\top R_t^{-1}\varepsilon_t - \varepsilon_t^\top \varepsilon_t\right).$$

Mean-Variance Optimization

Markowitz diversification rule:

$$\begin{aligned} \underset{w \in \mathbb{R}^d}{\text{minimize}} \quad & w^\top \Sigma w \\ \text{subject to} \quad & w^\top \bar{r} = r_T, \\ & \sum_{i=1}^{d} w_i = 1, \\ & w_i \geq 0 \end{aligned}$$

where w_i, $i = 1, \ldots, d$ are weights, $\Sigma \in \mathbb{R}^{d \times d}$ is the covariance matrix for d portfolio asset returns $\bar{r}_i$, r_T is the "target" return for the portfolio. Back to "Strategies"

Risk Parity (Equal risk contribution)

Let $\sigma(w) = \sqrt{w^\top \Sigma w}$ be the risk of portfolio w. The Euler decomposition gives us:

$$\sigma(w) = \sum_{i=1}^{n} \sigma_i(w) = \sum_{i=1}^{n} w_i \frac{\sigma(w)}{\sigma(w_i)}$$

where $w_i \frac{\sigma(w)}{\sigma(w_i)}$ is the marginal risk contribution and $\sigma_i(w) = w_i \frac{\sigma(w)}{\sigma(w_i)}$ the risk contribution of i-th asset. The idea of ERC strategy is to find risk balanced porfolio, such that:

$$\sigma_i(w) = \sigma_j(w)$$

i.e. the risk contribution is the same for all assets of the portfolio

Back to "Strategies"

The Orthogonal GARCH Model

- Y_t is a time-dependent matrix of asset returns, $\Gamma_t = B_t \in \mathbb{R}^{p \times p}$ is the matrix of standardized eigenvectors of $\frac{1}{n} Y_t^\top Y_t$ ordered according to decreasing magnitude of eigenvalues

- $F_t = P_t \stackrel{\text{def}}{=} Y_t \Gamma_t$ is the matrix of principal components of Y_t

- retaining only the first k most important factors f and introducing noise terms u_i gives
  $y_j = b_{j1} f_1 + b_{j2} f_2 + \ldots + b_{jk} f_k + u_i$ or $Y_t = F_t B_t^\top + U_t$

- then $\Sigma_t = \text{Var}(Y_t) = \text{Var}(F_t B_t^\top) + \text{Var}(U_t) = B_t \Delta_t B_t^\top + \Omega_t$, where $\Delta_t = \text{Var}(F_t)$ is a diagonal matrix of principal component variances at t: can be separately modeled by univariate GARCH processes ▸ Back to "Strategies"

The Orthogonal GARCH Model - continued

- B_t does not change much from day to day and can be approximated by B_{t-1} without introducing large errors in the calculation of the covariance matrix;
- Ω_t assumed to be constant and diagonal: it may be calculated from residuals $E_t = Y_t - F_t B_t^\top$, where each ω_j^2 on the diagonal is equal to $\omega_j^2 = \frac{1}{n}\sum_{i=1}^{n}(y_{ij} - f_i^\top \tilde{b}_j)^2$ with $\tilde{B} = B^\top$;
- the rule how to choose k can be based on the "proportion of total variation" explained by the first k principal components, which is calculated as the ratio of the sum of the first k eigenvalues of the matrix $\frac{1}{n} Y_t^\top Y_t$ to the sum of all p eigenvalues of this matrix Back to "Strategies"

Oracle Properties of an Estimator

An estimator has oracle properties if (Zheng et al., 2013):

- it selects the correct model with probability converging to 1;
- the model estimates are consistent with an appropriate convergence rate (He, Shao, 2000);
- estimates are asymptotically normal with the same asymptotic variance as that knowing the true model

Back to "Simple and Adaptive Lasso Penalized QR"

Asymetric Laplace Distribution

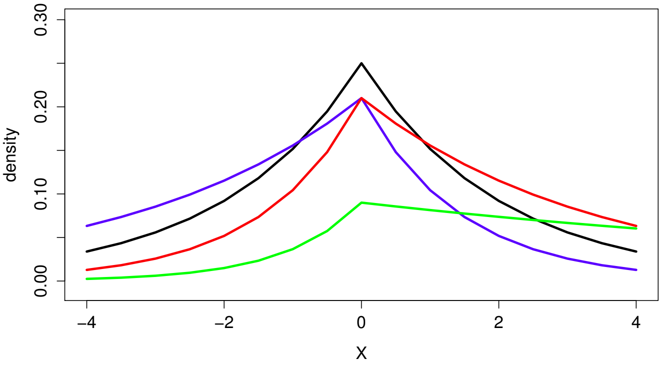

Figure 14: Standart ALD: $\tau = 0.3$,$\tau = 0.5$,$\tau = 0.7$, $\tau = 0.1$

Quantile regression using ALD

- Yu & Moyeed(2001)
  $Y_i \sim \text{ALD}(\mu, \sigma, \tau)$, if its pdf is given by

$$f(y|\mu,\sigma,\tau) = \frac{\tau(1-\tau)}{\sigma} exp \left\{ \rho_\tau \frac{(y-\mu)}{\sigma} \right\}$$

  where μ is location, σ - scale and τ-skewness parameters, and loss function $\rho_\tau(u) = u\{\tau - \mathbf{I}(u < 0)\}$

- Sanches et. al (2013)

$$y_i = x_i^\top \beta_\tau + \varepsilon_i, \; i = 1, \dots, n$$

  Re-write:

$$Y_i|U_i = u_i \sim \text{N}(x_i\beta_\tau + \theta u_i, p_\tau^2 \sigma u_i)$$
$$U_i \sim \text{Exp}(\sigma), \; i = 1, \dots, n$$

  here $\theta = \frac{1-2\tau}{\tau(1-\tau)}$ and $p_\tau^2 = \frac{2}{\tau(1-\tau)}$ Back to "Choice of τ-spine"

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