Risk-based versus target-based portfolio strategies in the cryptocurrency market

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Correlations between traditional assets and cryptos

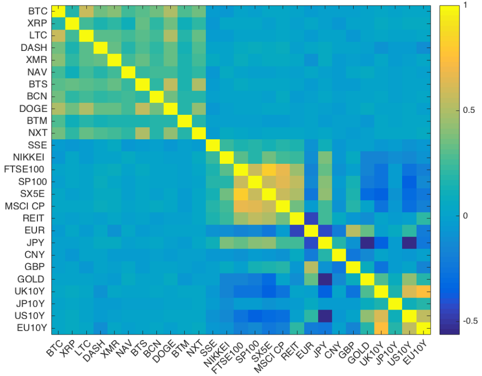

Figure 1: Correlations between cryptos and conventional financial assets, daily returns CCPDescriptiveStat

Financial relevance

Year	Price USD	Simple return (in %)
2009	0.00	
2010	0.29	
2011	4.99	+1,565.01
2012	13.59	+172.07
2013	739.10	+5,338.56
2014	317.40	−57.06
2015	428.00	+34.85
2016	952.15	+122.47
2017	14,165.57	+1,387.75
(2018)	6,973.69	−50.77

Table 1: Recently high realised returns: BTC

Challenge of crypto investment: high risk

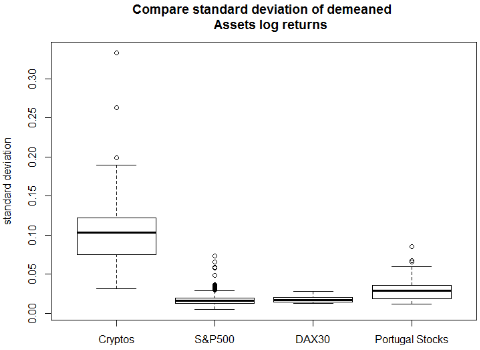

Figure 2: Crypto currencies have higher volatilities than stocks, highlighting the importance of risk management when investing to them LIBRObox1

Challenge of crypto investment: low trading volume

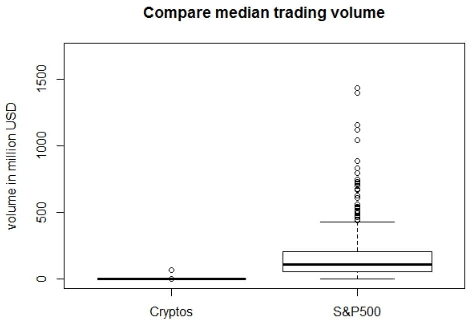

Figure 3: Crypto currencies have much lower trading volume compared to traditional assets LIBRObox2

Benefit of crypto investment

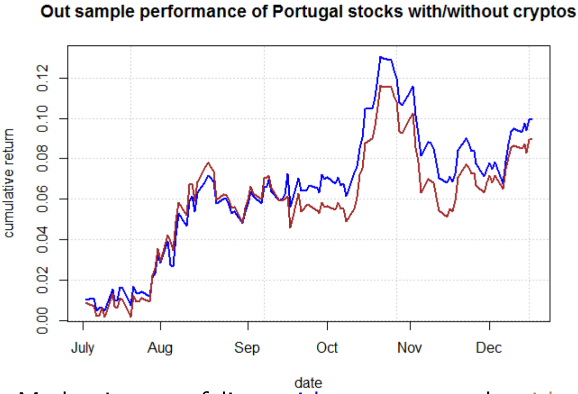

Figure 4: Markowitz portfolios with cryptos and without cryptos

LIBROreturn

Challenge of crypto investment

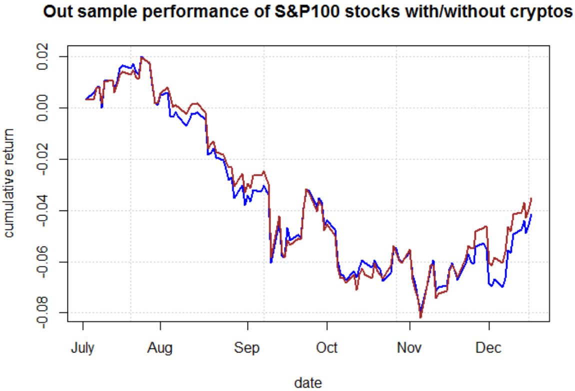

Figure 5: Markowitz portfolio with cryptos and without cryptos
LIBROreturn

Is Markowitz rule appropriate?

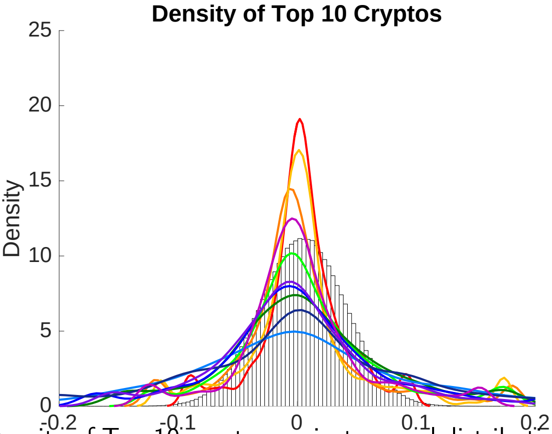

Figure 6: Density of Top-10 cryptos against normal distribution (time span is 2015-01-01 to 2017-12-31.) CCSHistReturnsDensity

Cryptos from an investment viewpoint

- Elendner et al. (2016) & Yermack (2014): Cryptos show low correlation with traditional assets
- Eisl et al. (2015), Briere (2015): Bitcoin improves the risk-return trade-off of portfolios.
- Chen et al. (2016): Analyzing dynamics of CRIX
- Trimborn, Li and Härdle (2018): Liquidity constrained risk-return portfolios in crypto markets
- Lee, Li and Wang (2018): Risk and return characteristics using portfolios with CRIX constituents

Objectives

- Out-of-sample performance analysis – is there the best individual asset allocation model?
- Diversification of models – do the combinations of models outperform individual ones?
- Do portfolio and risk concentration depend on the investor objective function?
- Do liquidity constraints affect the portfolio performance?
- Diversification effects of inclusion of CCs in various concepts of diversification

Outline

1. Motivation ✓
2. Methodology
3. Data
4. Empirical results
5. Conclusion

Investment strategies

Model	Reference	Abbreviation
Equally weighted	DeMiguel et al. (2009)	EW
Risk-return-oriented strategies		
Mean – Var – max Sharpe	Jagannathan and Ma (2003)	MV – S
Return-oriented strategies		
Risk – Return – max return	Markowitz (1952)	RR – max ret
Risk-oriented strategies		
Mean – Var – min var	Merton (1980)	MinVar
Equal Risk Contribution	Roncalli et al. (2010)	ERC
Mean – CVaR – min risk	Rockafellar and Uryasev (2000)	MinCVaR
Maximum Diversification	Rudin and Morgan(2006)	MD
Combination of models		
Naïve combination	Schanbacher (2015)	COMB NAÏVE
Combination bootstrap	Schanbacher (2014)	COMB

Table 2: List of asset allocation models

▸ MV ▸ ERC ▸ Mean-CVaR ▸ COMB ▸ LIBRO

Maximum diversification portfolio with Portfolio diversification index (PDI)

$$
\begin{aligned}
\max_{w \in \mathbb{R}^N} \quad & \mathrm{PDI}_P(w) \\
\text{s.t.} \quad & w^\top 1_N = 1, \\
& w_i \geq 0
\end{aligned}
\tag{1}
$$

$$
PDI_P(w) = 2 \sum_{i=1}^{N} i W_i - 1, \tag{2}
$$

where $W_i = \frac{\lambda_i}{\sum_{i=1}^{N} \lambda_i}$ are the relative strengths of the i-th principal portfolio.

Investment universe

- 16 traditional assets
  - Equity indices: S&P100, FTSE100, SSE, NIKKEI225, SX5E
  - 10 Years government bonds: EU, UK, JP, CN, USA
  - Real estate & commodities: GOLD, MSCI ACWI COMMOD PRODUCERS, FTSE EPRA (NAREIT DEV REITS)
  - FIAT: EUR, GBP, CNY, YEN
- 55 crypto-currencies (97%/ 61% of Entire Market Cap)
- Sources: thecrix.de, *Bloomberg*
- Time span 2015-01-01 to 2017-12-31 (781 trading days/24 moving windows)

Investment Universe

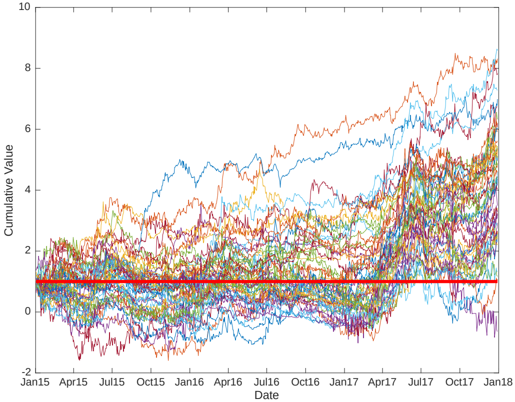

Figure 7: 55 crypto-currencies' cumulative return compared with the initial investment – thick red line (returns are 95% winsorized)

Efficient frontiers: significant shift with CC

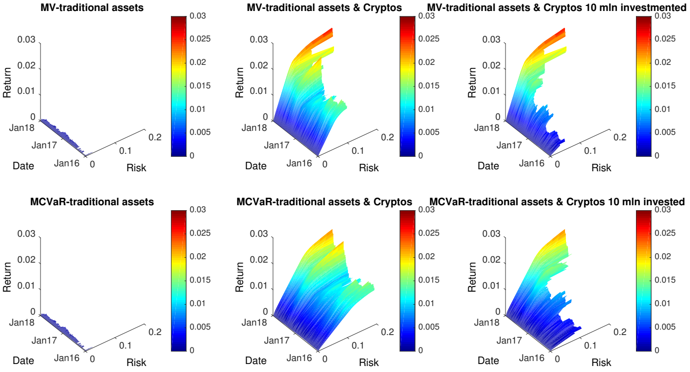

Figure 8: Efficient frontiers build on daily basis: CCPEfficient_surface

Performance of portfolio strategies

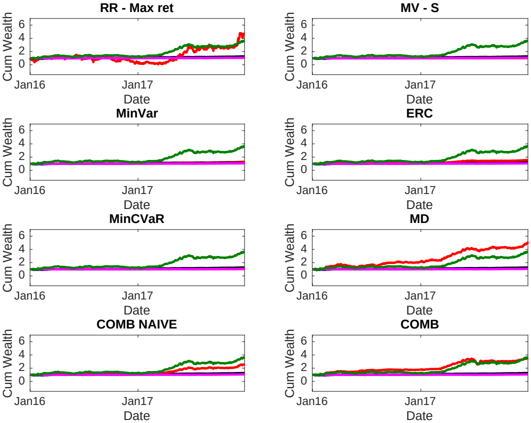

Figure 9: Cumulative wealth: S&P100, EW, MV-S–TrA, EW–TrA and corresponding Allocation strategy

CCPPerformance

Performance of portfolio strategies – LIBRO

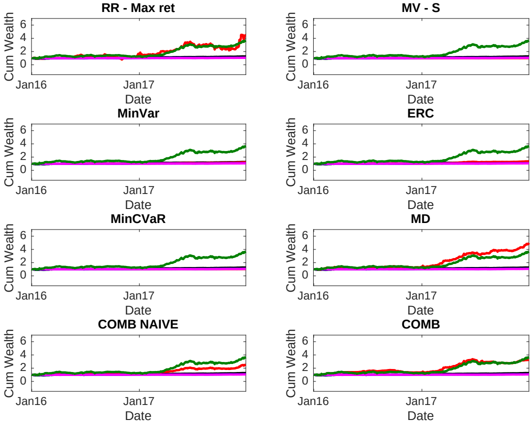

Figure 10: Cumulative wealth ($M = 10^6$ US$): S&P100, EW, MV-S–TrA, EW–TrA and corresponding Allocation strategy CCPPerformance

Capital allocation

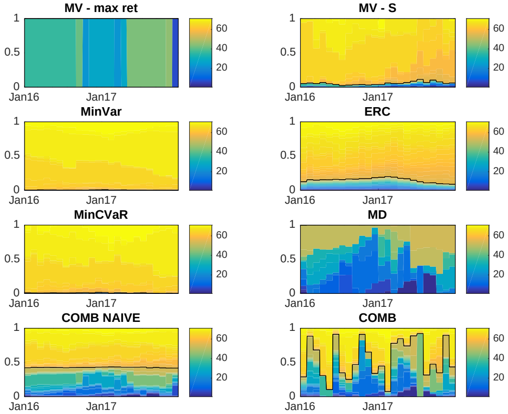

Figure 11: Dynamics in the capital composition w/o liquidity constraints

CCPWeights

Capital allocation - LIBRO

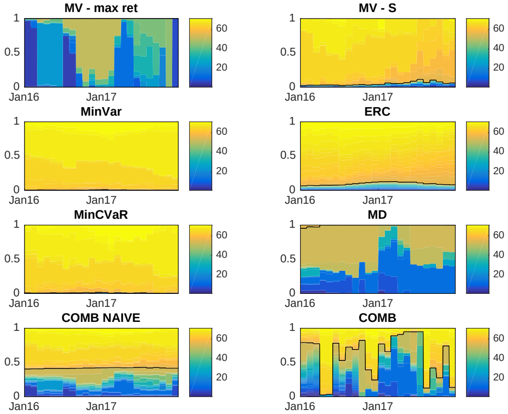

Figure 12: Dynamics in the capital composition ($M = 10^7$ US$)

CCPWeights

Portfolio risk allocation

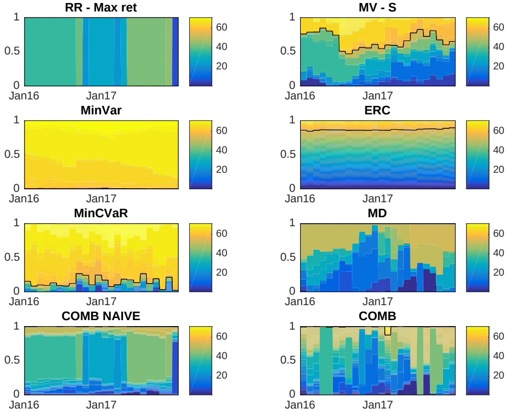

Figure 13: Dynamics of risk contributions for portfolio strategies

CCPRisk_contribution

Portfolio risk allocation - LIBRO

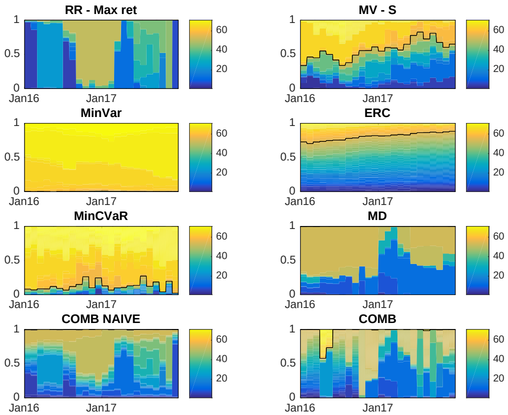

Figure 14: Dynamics of risk contributions for portfolio strategies ($M = 10^7$ US\$)

CCPRisk_contribution

Risk contribution and capital composition: results

- Very significant disparities in risk contributions and capital composition between different rules
- (Almost) no cryptos in global non-constraint minimum risk portfolio
- LIBRO approach affects risk and capital composition of portfolios

Portfolios' performance

Measures

Allocation Strategy	CW		SR		ASR		CEQ		TURNOVER	
	No const	10 mln	No const	10 mln	No const	10 mln	No const	10 mln	No const	10 mln
S&P100	1.261	1.261	0.080	0.080	0.079	0.079	0.000	0.000	0.000	0.000
EW TrA	1.069	1.069	0.048	0.048	0.047	0.047	0.004	0.004	4.824	4.824
MV-S TrA	1.052	1.052	0.068	0.068	0.068	0.068	0.000	0.000	1.359	1.359
EW	3.644	3.644	0.132	0.132	0.132	0.132	0.004	0.004	1.102	1.102
MinVar	1.001	1.001	0.065	0.071	0.065	0.072	0.001	0.002	3.924	3.710
MinCVaR	1.024	1.020	0.048	0.040	0.048	0.040	0.000	0.000	5.987	4.490
ERC	1.558	1.373	0.158	0.145	0.157	0.145	0.001	0.001	1.167	1.245
MD	5.147	4.964	0.158	0.177	0.158	0.179	0.007	0.007	4.408	5.748
RR-Max ret	4.703	4.455	0.003	0.003	0.003	0.003	0.000	0.000	0.229	0.761
MV-S	1.214	1.211	0.119	0.125	0.120	0.125	0.000	0.000	3.211	2.395
COMB NAÏVE	2.613	2.524	0.126	0.134	0.127	0.135	0.003	0.003	2.281	1.202
COMB	3.542	3.254	0.126	0.117	0.125	0.117	0.004	0.004	0.881	1.201

Table 3: Performance measures for monthly rebalancing frequency ($l = 21$)

Difference of SR and CEQ

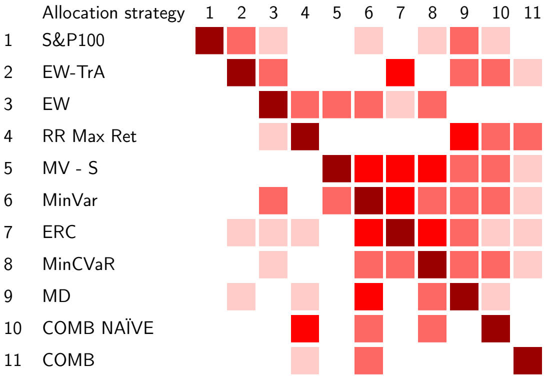

Table 4: p-value of the difference between the SR (lower triangle) and CEQ (upper triangle) of all strategies with each other with significance codes 0.01, 0.05 and 0.1 (without liquidity constraints) SR difference

Portfolios strategies: diversification effects

▸ Measures

Allocation Strategy	DR^2		Effective N		PDI	
	No const	10 mln	No const	10 mln	No const	10 mln
MV - S TrA	5.70	5.70	3.37	3.37	5.19	5.19
RR - Max ret	1.00	1.00	1.00	1.90	1.00	1.00
MinVar	13.65	13.51	3.48	3.47	25.40	25.40
MinCVaR	15.22	14.90	4.07	4.07	25.40	25.40
ERC	11.42	12.36	17.63	14.97	25.42	25.42
MD	2.99	2.41	4.08	3.01	25.93	25.90
MV -S	9.05	9.36	3.70	3.76	25.41	25.41
COMB NAÏVE	3.95	4.58	12.55	12.72	25.46	25.45
COMB	4.09	4.26	8.58	14.84	24.80	25.36

* All diversification measures are calculated based on in-sample data and averaged over the period 20150101-20171130

Table 5: Measures of diversification for monthly rebalancing

Conclusion I

- Out-of-sample performance:
  - MD and Max Return strategies show the most promising results
  - CC as portfolio components yield little variance reduction: application of CC in target return portfolio strategies
  - The inclusion of CC is strongly related to investment objectives (utility function)
- Bootstrap combination of models outperforms individual ones in many aspects

Conclusion II

- Capital and risk portfolio compositions are not robust
- LIquidity Bounded Risk-return Optimization (LIBRO) approach for CC portfolios:
  - improves risk-adjusted performance
  - strengthens diversification effects
- CC enhance diversification benefits in comparison with only conventional assets' portfolio

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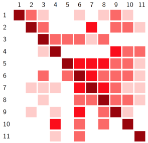

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Mean-Variance Asset Allocation

Log returns $X_t \in \mathbb{R}^p$:

$$
\begin{aligned}
\min_{w \in \mathbb{R}^p} \quad & \sigma_P^2(w) \stackrel{\text{def}}{=} w^\top \Sigma w \\
\text{s.t.} \quad & \mu_P(w) = r_T, \\
& w^\top 1_N = 1, \quad w_i \geq 0
\end{aligned}
\tag{3}
$$

where $\Sigma \stackrel{\text{def}}{=} \mathsf{E}_{t-1}\{(X-\mu)(X-\mu)^\top\}$ is the sample covariance matrix of returns, $\mu_P(w) \stackrel{\text{def}}{=} w^\top \mu$, $\mu \stackrel{\text{def}}{=} \mathsf{E}_{t-1}(X)$ is the portfolio mean and r_T – "target" return

▸ Back to "Methodology" ▸ MV LIBRO

Risk Parity (Equal risk contribution)

$\sigma_P(w) = \sqrt{w^\top \Sigma w}$ is the Euler decomposition of volatility, then:

$$\sigma_P(w) \stackrel{\text{def}}{=} \sum_{i=1}^{N} \sigma_i(w) = \sum_{i=1}^{N} w_i \frac{\partial \sigma_P(w)}{\partial w_i} \tag{4}$$

where $\frac{\partial \sigma_P(w)}{\partial w_i}$ is the marginal risk contribution and $\sigma_i(w) = w_i \frac{\partial \sigma_P(w)}{\partial w_i}$ is the risk contribution of i-th asset. In ERC portfolio:

$$\sigma_i(w) = \sigma_j(w) = \frac{1}{N} \tag{5}$$

Back to "Methodology"

Conditional VaR optimization

Given $\alpha < 0.05$ risk level, the CVaR optimized portfolio weights w are calculated as:

$$\min_{w \in \mathbb{R}^N} \mathrm{CVaR}_\alpha(w), \quad \text{s.t. } \mu_P(w) = r_T, w^\top 1_p = 1, w_i \geq 0, \tag{6}$$

$$\mathrm{CVaR}_\alpha(w) = -\frac{1}{1-\alpha} \int_{w^\top X \leq -\mathrm{VaR}_\alpha(w)} w^\top X f(w^\top X | w) dw^\top X, \tag{7}$$

where $\frac{\partial}{\partial w^\top X} F(w^\top X|w) = f(w^\top X|w)$ is pdf of the portfolio returns portfolio weights w $\mathrm{VaR}_\alpha(w)$ – α-quantile of the cdf

Back to "Methodology"

Averaging of portfolio models

- Consider m asset allocation models with weights $W_t = (w_t^1 \dots w_t^m)$, then individual shares:

$$\begin{aligned} \pi &= (\pi^1 \dots \pi^m), \\ \text{s.t. } \pi^\top 1_m &= 1 \end{aligned} \tag{8}$$

- the combined portfolio weight

$$w^{comb} = \sum_{i=1}^{m} \pi^i w^i \tag{9}$$

- Naïve combination: $\pi^i = \frac{1}{m}$ for all $i = 1 \dots m$

Back to "Methodology"

Averaging of portfolio models: bootstrap approach

- The probability that model i outperforms all other models:

$$\hat{\pi}^i = \frac{1}{B}\sum_{b=1}^{B} s_{i,b} \tag{10}$$

$$s_{i,b} = \begin{cases} 1, & \text{if } l_{i,b} > l_{j,b} \text{ for } i \neq j \\ 0, & \text{otherwise} \end{cases} \tag{11}$$

B - number of independent bootstrap samples

l - loss function optimizing CEQ: $l(w) = w^\top \hat{\mu} - \frac{\gamma}{2} w^\top \hat{\Sigma} w$

Back to "Methodology"

Liquidity Bounded Risk-return Optimization (LIBRO) I

Daily Trading Volume (TV):

$$TV = \sum_{i=1}^{n} p_i \cdot q_i \qquad (12)$$

- n - the number of trades
- p_i - the price of assets of trade i
- q_i - the number of assets of trade i

Back to "Methodology"

LIBRO II

- the market value held in asset i

$$Mw_i \leq TV_i \cdot f_i, \tag{13}$$

- f_i - controls the speed of clearing the position on asset i
- M - investment amount

$$w_i \leq \frac{TV_i \cdot f_i}{M} = \widehat{a}_i \tag{14}$$

Back to "Methodology"

LIBRO III

$$w_i \leq \frac{TV_i \cdot f_i}{M} = \widehat{a}_i \tag{15}$$

$$\widehat{a}_i = \frac{\widehat{Liq}_i}{M} \cdot c \tag{16}$$

- $\widehat{Liq}_i = TV_i \cdot f_i$ is a liquidity, $\widehat{Liq}_i \in \mathbb{R}_0^+ \backslash \{\infty\}$ for asset i
- c is the factor controlling the amount of permitted short-selling

Back to "Methodology"

LIBRO IV

MV LIBRO optimization problem is then:

$$\begin{aligned} \min_{w \in \mathbb{R}^p} \quad & \sigma_P^2(w) \overset{\text{def}}{=} w^\top \Sigma w \\ \text{s.t.} \quad & \mu_P(w) = r_T, \\ & w^\top 1_p = 1, \\ & 0 \leq w_i \leq \widehat{a}_i \end{aligned} \tag{17}$$

▸ Back to "Methodology" ▸ MV

Evaluation of Portfolios' Performance

- Certainty-EQuivalent (CEQ) return

$$\widehat{CEQ}_{i,\gamma} = \hat{\mu}_i - \frac{\gamma}{2}\hat{\sigma}_i{}^2 \tag{18}$$

where γ reflects the investor's risk aversion

- Turnover

$$Turnover_i = \frac{1}{T-L}\sum_{t=1}^{T-L}\sum_{j=1}^{N}|\hat{w}_{j,t+1} - \hat{w}_{j,t+}| \tag{19}$$

where $w_{j,t+}$ is the portfolio weight before rebalancing at $t+1$, L - length of moving window Back to "Results"

Evaluation of Portfolios' Performance

- Sharpe Ratio (SR)

$$\widehat{SR_i} = \frac{\hat{\mu}_i}{\hat{\sigma}_i{}^2} \tag{20}$$

- Adjusted Sharpe Ratio (ASR)

$$\widehat{ASR_i} = SR_i \left[1 + \left(\frac{S}{6} \right) SR_i - \left(\frac{K}{24} \right) SR_i^2 \right] \tag{21}$$

where S - skewness and K - excess kurtosis.

Back to "Results"

P-values

Ledoit ans Wolf (2008)

Let X_i and X_j - returns produced by strategies i and j and $\nu = (\mu_i, \mu_j, \mathsf{E}(X_i^2), \mathsf{E}(X_j^2))^\top$

- Difference of CEQ and SR

$$
\begin{aligned}
f_{\text{CEQ}}(\nu) &= \mu_i - \frac{\gamma}{2}(\mathsf{E}(X_i^2) - \mu_i^2) - \mu_j + \frac{\gamma}{2}(\mathsf{E}(X_j^2) - \mu_j^2) \\
f_{\text{SR}}(\nu) &= \frac{\mu_i}{\sqrt{\mathsf{E}(X_i^2) - \mu_i^2}} - \frac{\mu_j}{\sqrt{\mathsf{E}(X_j^2) - \mu_j^2}} \qquad (22)
\end{aligned}
$$

Back to "P-values"

P-values ctd.

- Delta method: if $\sqrt{T-L}(\hat{\nu}-\nu) \xrightarrow{\mathcal{L}} \mathrm{N}(0,\Psi)$, then

$$\sqrt{T-L}(\hat{f}-f) \xrightarrow{\mathcal{L}} \mathrm{N}(0, \nabla^\top f(\nu)\Psi\nabla f(\nu)), \tag{23}$$

  where ∇f is a derivative of f

- Standard Error for $\hat{f}$:

$$SE(\hat{f}) = \sqrt{\frac{\nabla^\top f(\nu)\Psi\nabla f(\nu)}{T-L}} \tag{24}$$

- Solutions for consistent estimator for $\hat{\Psi}$: HAC and Bootstrap inference

Back to "P-values"

P-values ctd.

- HAC inference

$$\Psi_{T-L} = \frac{T-L}{T-L-4} \sum_{j=-T+L+1}^{T-L-1} k \, \frac{n}{S_{T-L}} \, \hat{\Gamma}_{T-L}(n) \tag{25}$$

$$\hat{\Gamma}_{T-L}(j) = \begin{cases} \frac{1}{T-L} \sum_{t=n+1}^{T-L} \hat{y}_t \widehat{y_{t-n}}^{\top} & \text{for } j \geq 0 \\ \frac{1}{T-L} \sum_{t=-n+1}^{T-L} \widehat{y_{t+n}} \hat{y}_t^{\top} & \text{for } j \geq 0 \end{cases} \tag{26}$$

- $\hat{y}^{\top} ?t = (x_{ti} - \hat{\mu}_i, x_{tj} - \hat{\mu}_j, x_{ti}^2 - \mathrm{E}(X_i^2), x_{tj}^2 - \mathrm{E}(X_j^2))$
- $k(\cdot)$ is a kernel, S_{T-L} is a bandwidth

Back to "P-values"

P-values ctd.

- A two-sided p-value for H_0: $f = 0$

$$\hat{p} = 2\Phi \frac{|\hat{f}|}{\mathrm{SE}(\hat{f})} \tag{27}$$

Back to "P-values"

Measures of diversification

- Effective N

$$N_{Eff}(w) = \frac{1}{\sum_{i=1}^{N} w_i^2} \tag{28}$$

- Diversification Ratio (DR)

$$DR(w) = \frac{w^\top \sigma}{\sqrt{w^\top \Sigma w}} = \frac{w^\top \sigma}{\sigma_P(w)} \tag{29}$$

Back to "Results"

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