Localizing Temperature Residuals

Wolfgang Karl Härdle

Brenda López Cabrera

Weining Wang

Ladislaus von Bortkiewicz Chair of Statistics
C.A.S.E. - Center for Applied Statistics and Economics
Humboldt-Universität zu Berlin
http://lvb.wiwi.hu-berlin.de
http://www.case.hu-berlin.de

Weather

- Influences our daily lives and choices
- Impact on corporate revenues and earnings
- Meteorological institutions: business activity is weather dependent
  - British Met Office: daily beer consumption gain 10% if temperature increases by 3° C
  - If temperature in Chicago is less than 0° C consumption of orange juice declines 10% on average

Examples

- Natural gas company suffers negative impact in mild winter
- Construction companies buy weather derivatives (rain period)
- Cloth retailers sell fewer clothes in hot summer
- Salmon fishery suffer losses by increase of sea temperatures
- Ice cream producers hedge against cold summers
- Disney World (rain period)

What are Weather Derivatives?

Hedge weather related risk exposures

- Payments based on weather related measurements
- Underlying: temperature, rainfall, wind, snow, frost

Chicago Mercantile Exchange (CME)

- Monthly/seasonal/weekly temperature Future/Option contracts
- 24 US, 6 Canadian, 9 European and 3 Asian-Pacific cities (Tokyo & Osaka since 2008 and Hiroshima since 2009)
- From 2.2 billion USD in 2004 to 15 billion USD through March 2009

Figure 1: CME offers weather contracts on 42 cities throughout the world

Types of Weather Derivatives

- CME products
  - HDD$(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \max(18^\circ\mathrm{C} - T_t, 0)dt$
  - CDD$(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \max(T_t - 18^\circ\mathrm{C}, 0)dt$
  - CAT$(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} T_t dt$, where $T_t = \frac{T_{t,max} + T_{t,min}}{2}$
  - AAT$(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \widetilde{T}_t dt$, where $\widetilde{T}_t = \frac{1}{24} \int_1^{24} T_{t_i} dt_i$ and T_{t_i} denotes the temperature of hour t_i, (also referred to as C24AT index).

Outline

1. Motivation ✓
2. Weather Dynamics
3. Fitting $\hat{\sigma}_t$: 1-2 Steps
4. Local Temperature Residual

Weather Dynamics: Asian Data

Temperature Market (CME): Tokyo and Osaka

AAT Index

CME data on weather derivatives for 20081008-20090702:

	Trading Period		Measurement Period		Index	
Code	First-trade	Last-trade	τ_1	τ_2	CME[1]	AAT[2]
F9	20080203	20090202	20090101	20090131	200.2	181.0
G9	20080303	20090302	20090201	20090228	220.8	215.0
H9	20080403	20090402	20090301	20090331	301.9	298.0
J9	20080503	20090502	20090401	20090430	460.0	464.0
K9	20080603	20090602	20090501	20090531	592.0	621.0

Table 1: Osaka AAT contracts listed on CME. Source: Bloomberg. [1] prices of AAT Futures as listed on CME, [2] AAT index values computed from the historical temperature data.

Algorithm

$$
\begin{array}{cc}
\text{Econometrics} & \text{Fin. Mathematics} \\
T_t & CAR(3) \\
\downarrow & \downarrow \\
X_t = T_t - \Lambda_t & F_{CAT(t,\tau_1,\tau_2)} \\
\downarrow & \downarrow \\
X_{t+3} = a^\top X_t + \sigma_t \varepsilon_t & MPR \\
\downarrow & \\
\hat{\varepsilon}_t = \frac{\hat{X}_t}{\hat{\sigma}_t} \sim N(0,1) &
\end{array}
$$

Asian Temperature

Temperature: $T_t = X_t + \Lambda_t$

Seasonal function with trend: $\Lambda_t = a_0 + a_1 t + a_2 cos\left\{\frac{2\pi(t-a_3)}{365}\right\}$

- $\hat{a}_0$: average temperature, $\hat{a}_1$: global warming

City	Period	$\hat{a}_0$	$\hat{a}_1$	$\hat{a}_2$	$\hat{a}_3$
Tokyo	19730101-20081231	15.76	7.82e-05	10.35	-149.53
Osaka	19730101-20081231	15.54	1.28e-04	11.50	-150.54
Beijing	19730101-20081231	11.97	1.18e-04	14.91	-165.51
Taipei	19920101-20090806	23.21	1.68e-03	6.78	-154.02

Table 2: Seasonality estimates of daily average temperatures in Asia. All coefficients are nonzero at 1% significance level. Data source: Bloomberg

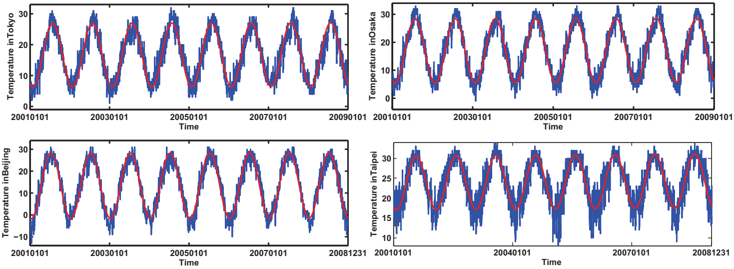


Figure 2: Seasonality effect (red line) and daily average temperatures for Tokyo Narito International Airport (upper left), Osaka Kansai International Airport (upper right), Beijing (lower left), Taipei (lower right).

AR(p): $X_{t+p} = \sum_{i=1}^{p} \beta_i X_{t+p-i} + \sigma_t \varepsilon_t$

City	Tokyo(p=3)	Osaka(p=3)	Beijing(p=3)	Taipei(p=3)
β_1	0.668	0.748	0.741	0.808
β_2	-0.069	-0.143	-0.071	-0.228
β_3	0.079	-0.079	0.071	0.063

Table 3: Coefficients of AR(p) , Model selection: AIC

(Squared) Residuals: China - Taiwan

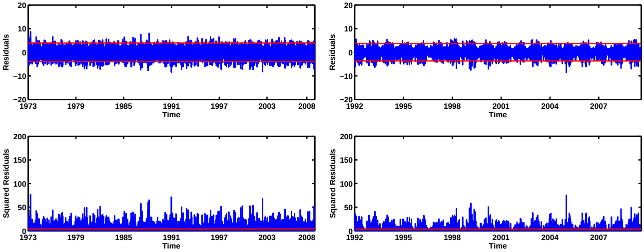


Figure 3: Residuals $\hat{\varepsilon}_t$ (up) and squared residuals $\hat{\varepsilon}_t^2$ (down) of the AR(p) (Beijing (left), Taipei (right)). No rejection of H_0 that the residuals are uncorrelated at 0% significance level, according to the modified Li-McLeod Portmanteau test

Seasonal Volatility: China - Taiwan

Close to zero ACF for residuals and highly seasonal ACF for squared residuals of AR(p)

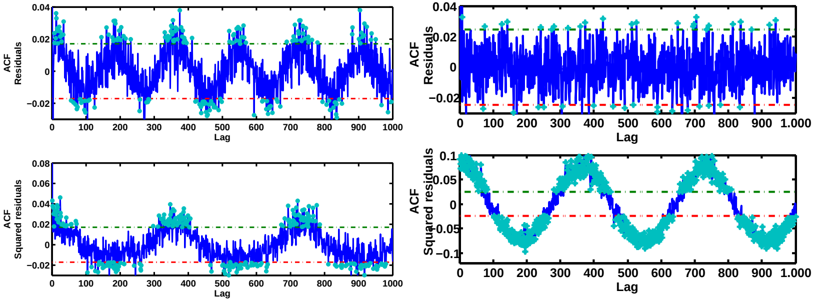


Figure 4: ACF for residuals $\hat{\varepsilon}_t$ (up) and squared residuals $\hat{\varepsilon}_t^2$ (down) of the AR(p) for Beijing (left), Taipei (right).

Calibration of Seasonal Variance: σ_t^2

Calibration of daily variances of residuals AR(3) for 36 years:

- 2 Steps: Fourier truncated series + GARCH(p,q) $\hat{\sigma}^2_{t,FTSG}$

$$\begin{aligned}\sigma_t^2 &= c_1 + \sum_{i=1}^{16}\left\{c_{2i}\cos\left(\frac{2i\pi t}{365}\right) + c_{2i+1}\sin\left(\frac{2i\pi t}{365}\right)\right\}\\ &+ \alpha_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2 \end{aligned} \tag{1}$$

- 1 Step: Local linear Regression (LLR) $\hat{\sigma}^2_{t,LLR}$, $Y_i = \hat{\varepsilon}^2_{t_i}$

$$\min_{a,b}\sum_{i=1}^{n}\left\{Y_i - a(t) - b(t)(t_i - t)\right\}^2 K\left(\frac{t_i - t}{h}\right) \tag{2}$$

Calibration of Seasonal Variance: σ_t^2

Calibration of daily variances of residuals AR(3) for 36 years:

	$\hat{c}_1$	$\hat{c}_2$	$\hat{c}_3$	$\hat{c}_4$	$\hat{c}_5$	$\hat{c}_6$	$\hat{c}_7$	α	β
Tokyo	3.91	-0.08	0.74	-0.70	-0.37	-0.13	-0.14	0.09	0.50
Osaka	3.40	0.76	0.81	-0.58	-0.29	-0.17	-0.07	0.04	0.52
Beijing	3.95	0.70	0.82	-0.26	-0.50	-0.20	-0.17	0.03	0.33
Taipei	3.54	1.49	1.62	-0.41	-0.19	0.03	-0.18	0.06	0.33

Table 4: First 7 Coefficients of σ_t^2 and $GARCH(p = 1, q = 1)$. The coefficients in black are significant at 1% level.

Seasonal Variance $\hat{\sigma}^2_{t,FTSG}$: China - Taiwan

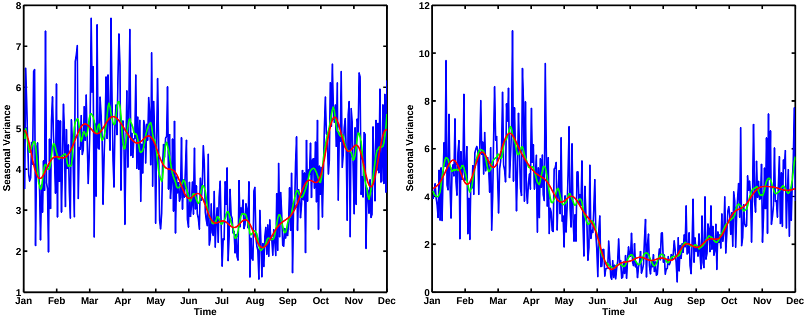


Figure 5: Daily empirical variance (blue line), $\hat{\sigma}^2_{t,FTSG}$ (red line) and $\hat{\sigma}^2_{t,LLR}$ (green line) using Epanechnikov Kernel and bandwidth $h = 4.49$ for Beijing (left), Taipei (right).

ACF of (Squared) Residuals after Correcting Seasonal Volatility: China - Taiwan

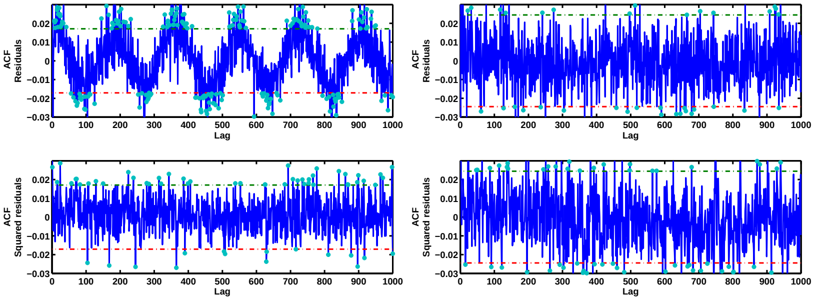


Figure 6: (Down) Up: ACF for temperature (squared) residuals $\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,LLR}}$ for Beijing (left), Taipei (right)

Residuals $\left(\frac{\hat{\varepsilon}_t}{\hat{\sigma}_t}\right)$ become normal

City		$\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,FTS}}$	$\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,FTSG}}$	$\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,LLR}}$
Tokyo	Jarque Bera	**6.49**	**5.30**	**4.68**
	Kurtosis	3.59	3.53	3.49
	Skewness	-0.14	-0.13	-0.13
Osaka	Jarque Bera	**7.25**	**6.35**	**6.25**
	Kurtosis	3.12	3.09	3.04
	Skewness	-0.34	-0.33	-0.32
Beijing	Jarque Bera	**8.03**	**7.67**	**6.98**
	Kurtosis	3.41	3.38	3.35
	Skewness	-0.30	-0.30	-0.29
Taipei	Jarque Bera	**12.47**	**11.57**	**11.00**
	Kurtosis	3.46	3.39	3.34
	Skewness	-0.39	-0.39	-0.39

Table 5: Skewness, kurtosis and values of Jarque Bera test statistics (365 days). Critical value at at 5% significance level is 5.99, at 1% – 9.21.

Residuals $(\frac{\hat{\varepsilon}_t}{\hat{\sigma}_t})$ become normal:

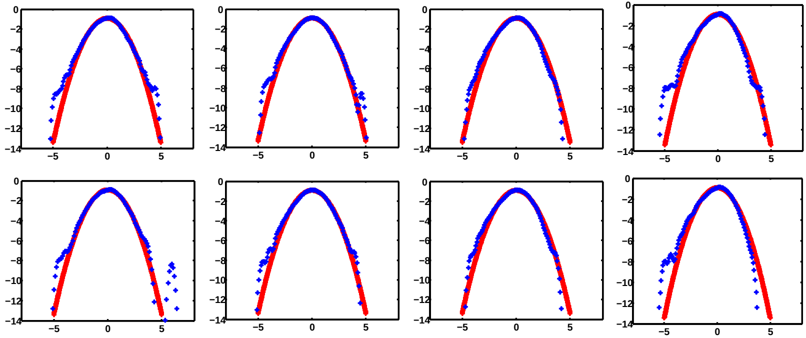


Figure 7: Log of Kernel smoothing density estimate (blue line) vs Log of Normal Kernel (red line) for $\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,LLR}}$ (upper) and $\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,FTSG}}$ (lower) of Tokyo (left), Osaka (left middle), Beijing (right middle), Taipei (right)

Temperature Dynamics

Temperature time series:

$$T_t = \Lambda_t + X_t$$

with seasonal function Λ_t. X_t can be seen as a discretization of a continuous-time process AR(p) (CAR(p)).

This stochastic model allows CAR(p) futures/options pricing.

Stochastic Pricing

Ornstein-Uhlenbeck process $\mathbf{X}_t \in \mathbb{R}^p$:

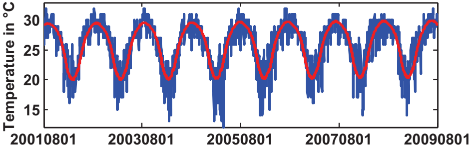
$$d\mathbf{X}_t = A\mathbf{X}_t dt + \mathbf{e}_p \sigma_t dB_t$$

$\mathbf{e}_k$: kth unit vector in $\mathbb{R}^p$ for $k = 1, \ldots, p$, $\sigma_t > 0$,
A: $(p \times p)$-matrix

$$A = \begin{pmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & \ldots & \ldots & 0 & 1 \\ -\alpha_p & -\alpha_{p-1} & \ldots & & -\alpha_1 \end{pmatrix}$$

Analysis of Weather Dynamics in Kaohsiung

1. Seasonal function with trend:

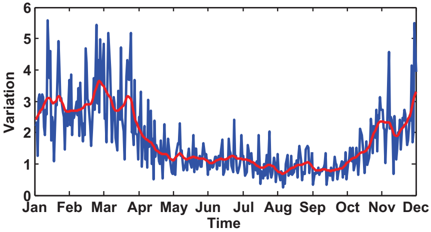


2. AR(p) process, by AIC p=3

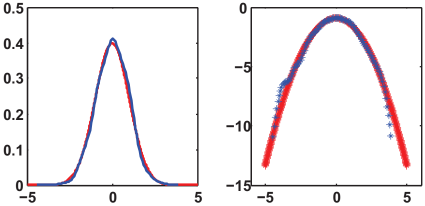
$$\hat{\beta}_1 = \ 0.77, \quad \hat{\beta}_2 = -0.12, \quad \hat{\beta}_3 = \ 0.04.$$

CAR(p) $\quad \hat{\alpha}_1 = -2.24, \quad \hat{\alpha}_2 = -1.59, \quad \hat{\alpha}_3 = -0.31.$

3. Seasonal volatility: Local Linear Regression (LLR)



4. Normality of residuals: kurtosis=3.31, skewness=-0.22, JB=4.41.



Local Temperature Residual

Normality of ε_t requires estimating the function $\theta(t) = \left\{\sigma_t^2\right\}$ with $t(day) = 1 \cdots 365, j(year) = 0 \cdots J$. Recall:

$$\begin{aligned} X_{t,j} &= T_{t,j} - \Lambda_t \\ X_{t,j} &= \sum_{l=1}^{L} \beta_l X_{t-l,j} + \sigma_t \varepsilon_{t,j} \\ \varepsilon_{t,j} &\sim N(0,1), i.i.d. \end{aligned} \tag{3}$$

Adaptation Scale

Fix $s \in 1, 2, \ldots, 365$, sequence of ordered weights:
$W^{(k)} = (w(s, 1, h_k), w(s, 2, h_k), \ldots, w(s, 365, h_k))^\top$.

Define $w(s, t, h_k) = K_{h_k}(s - t), (h_1 < h_2 < \ldots < h_K)$.

$$
\begin{aligned}
\hat{\varepsilon}_{365j+t} &= X_{365j+t} - \sum_{l=1}^{L} \hat{\beta}_l X_{365j+t-l} \\
\tilde{\theta}_k(s) &= \arg\min_{\theta \in \Theta} \sum_{t=1}^{365} \sum_{j=0}^{J} \{\log(2\pi\theta)/2 + \hat{\varepsilon}_{t,j}^2/2\theta\} w(s, t, h_k), \\
&\stackrel{\text{def}}{=} \arg\min_{\theta \in \Theta} -L(W_s^{(k)}, \theta)
\end{aligned}
$$

Local Temperature Residuals

$$
\begin{aligned}
\tilde{\theta}_s^k &= \sum_{t,j} \hat{\varepsilon}_{t,j}^2 w(s,t,h_k) \Big/ \sum_{t,j} w(s,t,h_k) \\
&= \sum_{t} \hat{\varepsilon}_{t}^2 w(s,t,h_k) \Big/ \sum_{t} w(s,t,h_k)
\end{aligned}
$$

with $\hat{\varepsilon}_t \stackrel{\text{def}}{=} (J+1)^{-1} \sum_{j=0}^{J} \hat{\varepsilon}_{t,j}^2.$

Mirror Observations

To remedy the boundary bias, use mirrored observations:

Assume $h_K < 365/2$, then the observations look like $\hat{\varepsilon}^2_{-364}, \hat{\varepsilon}^2_{-363}, \ldots, \hat{\varepsilon}^2_{0}, \hat{\varepsilon}^2_{1}, \ldots, \hat{\varepsilon}^2_{730}$, where

$$\begin{aligned}\hat{\varepsilon}^2_t &\stackrel{\text{def}}{=} \hat{\varepsilon}^2_{365+t}, -364 \leq t \leq 0 \\ \hat{\varepsilon}^2_t &\stackrel{\text{def}}{=} \hat{\varepsilon}^2_{t-365}, 366 \leq t \leq 730\end{aligned}$$

Parametric Exponential Bounds

$$
\begin{aligned}
L(\tilde{\theta}, \theta^*) &\stackrel{\text{def}}{=} L(\tilde{\theta}) - L(\theta^*) = N\mathcal{K}(\tilde{\theta}, \theta^*) \\
\mathcal{K}(\theta, \theta^*) &= -\{\log(\theta/\theta^*) + 1 - \theta^*/\theta\}/2
\end{aligned}
$$

For any $\Im > 0$,

$$
\begin{aligned}
&\mathrm{P}_{\theta^*}\{L(\tilde{\theta}, \theta^*) > \xi\} \le 2\exp(-\xi) \\
&\mathsf{E}_{\theta^*} L(\tilde{\theta}, \theta^*)^r \le \tau_r,
\end{aligned}
$$

where $\tau_r = 2r \int_{\xi \ge 0} \xi^{r-1} exp(-\xi) d\xi$.

LMS Procedure

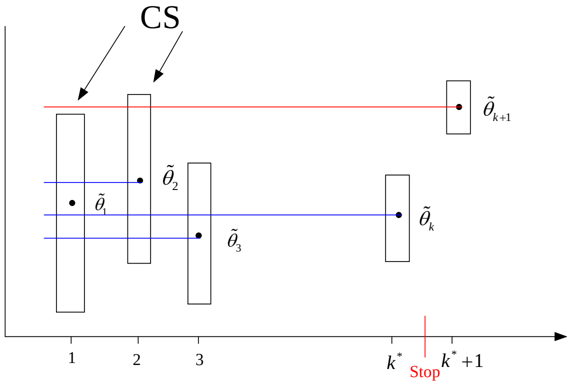
Aim: Construct an estimate $\hat{\theta} = \hat{\theta}(s)$, on the base of $\tilde{\theta}_1(s), \tilde{\theta}_2(s), \ldots, \tilde{\theta}_K(s)$.

- Start with $\hat{\theta}_1 = \tilde{\theta}_1$.
- For $k \geq 2$, $\tilde{\theta}_k$ is accepted and $\hat{\theta}_k = \tilde{\theta}_k$ if $\tilde{\theta}_{k-1}$ was accepted and

$$L(\tilde{\theta}_\ell, \tilde{\theta}_k) \leq \xi_\ell, \ell = 1, \ldots, k-1$$

  $\hat{\theta}_k$ is the the latest accepted estimate after the first k steps.

Illustration



"Propagation" Condition

$$\mathsf{E}_{\theta^*} \frac{|L(W^{(k)}, \tilde{\theta}_k, \theta^*)|^r}{\tau_r} \leq \alpha, \tag{4}$$

where $k = 1, \ldots, K$.

Sequential Choice of Critical Values

- Consider first only ξ_1 letting $\xi_2 = \ldots = \xi_{K-1} = \infty$. Leads to the estimates $\hat{\theta}_k(\xi_1)$ for $k = 2, \ldots, K$.
- The value ξ_1 is selected as the minimal one for which

$$\sup_{\theta^*} \mathrm{E}_{\theta^*,\lambda(.)} \frac{|L\{W^{(k)}, \tilde{\theta}_k, \hat{\theta}_k(\xi_1)\}|^r}{\tau_r} \le \frac{\alpha}{K-1}, k = 2, \ldots, K.$$

- Set $\xi_{k+1} = \ldots = \xi_{K-1} = \infty$ and fix ξ_k lead the set of parameters $\xi_1, \ldots, \xi_k, \infty, \ldots, \infty$ and the estimates $\hat{\theta}_m(\xi_1, \ldots, \xi_k)$ for $m = k+1, \ldots, K$. Select ξ_k s.t.

$$\sup_{\theta^*} \mathrm{E}_{\theta^*,\lambda(.)} \frac{|L\{W^{(k)}, \tilde{\theta}_m, \hat{\theta}_m(\xi_1, \xi_2, \ldots, \xi_k)\}|^r}{\tau_r} \le \frac{k\alpha}{K-1},$$

$m = k+1, \ldots, K$.

Critical Values

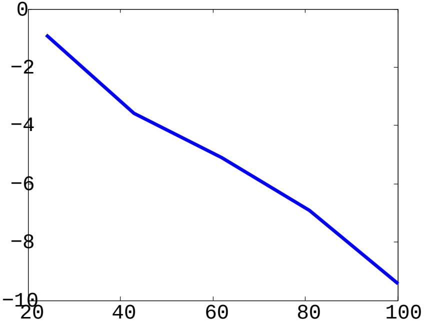


Figure 8: Log critical values $\theta^* = 1$, $\alpha = 0.05$, $r = 0.5$, $MC = 5000$

Bound for Critical Values

Suppose $0 < \mu \leq h_{k-1}/h_k \leq \mu_0 < 1$.

Let $\theta(.) = \theta^*$, for all $t \in (0, 365)$. There is a constant $a_0 > 0$ depending on r and μ_0, μ, s.t.

$$\xi_k = a_0 \log K + 2 \log(nh_k/\alpha) + 2r \log(h_K/h_k)$$

ensures (4).

"Small Modeling Bias" Condition

$$\Delta(W^{(k)}, \theta) = \sum_{i=1}^{365} \mathcal{K}\{\theta(i), \theta\} \mathbf{1}\{w(s, i, h_k) > 0\}$$

"Small Modeling Bias" Condition:

$$\Delta(W^{(k)}, \theta) \le \Delta, \forall k < k^*$$

"Small Modeling Bias" Property

Then, it holds for any estimate $\tilde{\theta}_k$ and θ satisfies "SMB":

$$\mathsf{E}_{\theta(.)} \log\{1 + |L(\tilde{\theta}_k, \theta)|^r / \tau_r\} \leq \Delta + 1,$$

"Stability" Property

Stability: the attained quality of estimation during "propagation" can not get lost at further steps.

Theorem

$$L(W^{(k^*)}, \tilde{\theta}_{k^*}, \hat{\theta}_{\hat{k}}) \mathbf{1}\{\hat{k} > k^*\} \leq \xi_{k^*}$$

"Oracle" Property

Combing the "propagation" and "stability" results yields

Theorem

Let $\Delta(W^{(k)}, \theta) \leq \Delta$ *for some* $\theta \in \Theta$ *and* $k \leq k^*$*. Then*

$$E_{\theta(.)} \log\left\{1 + \frac{|L(W^{(k^*)}, \tilde{\theta}_{k^*}, \theta)|^r}{\tau_r(W^{(k^*)})}\right\} \leq \Delta + 1$$

$$E_{\theta(.)} \log\left\{1 + \frac{|L(W^{(k^*)}, \tilde{\theta}_{k^*}, \hat{\theta})|^r}{\tau_r(W^{(k^*)})}\right\} \leq \Delta + \alpha$$

$$+ \log\{1 + \frac{\xi_k^*}{\tau_r}\}$$

References

F.E. Benth and J.S. Benth and S. Koekebakker
Putting a Price on Temperature
Scandinavian Journal of Statistics 34: 746-767, 2007

F.E. Benth and W.K. Härdle and B. López Cabrera
Pricing of Asian Temperature Risk
Working Paper SFB649, 2009-046

P.J. Brockwell
Continuous-time ARMA Processes
Handbook of Statistics 19: 248-276, 2001

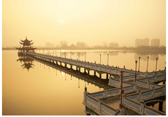
References

W.K. Härdle and B.López Cabrera
Implied Market Price of Weather Risk
Working Paper SFB649, 2009-001

U. Horst and M. Müller
On the Spanning Property of Risk Bonds Priced by Equilibrium
Mathematics of Operations Research 4: 784-807, 2007

W.K. Härdle, J. Franke, C.M. Hafner
Statistics of Financial Markets: an Introduction
Springer Verlag, Heidelberg, 2008

Localizing Temperature Residuals



Ladislaus von Bortkiewicz Chair of Statistics
C.A.S.E. - Center for Applied Statistics and Economics
Humboldt-Universität zu Berlin
http://lvb.wiwi.hu-berlin.de
http://www.case.hu-berlin.de

温度风险

Appendix A

Li-McLeod Portmanteau Test– modified Portmanteau test statistic Q_L to check the uncorrelatedness of the residuals:

$$Q_L = n \sum_{k=1}^{L} r_k^2(\hat{\varepsilon}) + \frac{L(L+1)}{2n},$$

where r_k, $k = 1, \ldots, L$ are values of residuals ACF up to the first L lags and n is the sample size. Then,

$$Q_L \sim \chi^2_{(L-p-q)}$$

Q_L is χ^2distributed on $(L - p - q)$ degrees of freedom where p,q denote AR and MA order respectively and L is a given value of considered lags.

Appendix B

Proof **CAR(3)** $\approx$ **AR(3)**
Let

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\alpha_3 & -\alpha_2 & -\alpha_1 \end{pmatrix}$$

- use $B_{t+1} - B_t = \varepsilon_t$
- substitute iteratively into X_1 dynamics:

Appendix B

$$
\begin{aligned}
X_{1(t+1)} - X_{1(t)} &= X_{2(t)}dt + \sigma_t \varepsilon_t \\
X_{2(t+1)} - X_{2(t)} &= X_{3(t)}dt + \sigma_t \varepsilon_t \\
X_{3(t+1)} - X_{3(t)} &= -\alpha_1 X_{1(t)}dt - \alpha_2 X_{2(t)}dt - \alpha_3 X_{3(t)}dt + \sigma_t \varepsilon_t \\
X_{1(t+2)} - X_{1(t+1)} &= X_{2(t+1)}dt + \sigma_{t+1} \varepsilon_{t+1} \\
X_{2(t+2)} - X_{2(t+1)} &= X_{3(t+1)}dt + \sigma_{t+1} \varepsilon_{t+1} \\
X_{3(t+2)} - X_{3(t+1)} &= -\alpha_1 X_{1(t+1)}dt - \alpha_2 X_{2(t+1)}dt \\
&\quad -\alpha_3 X_{3(t+1)}dt + \sigma_{t+1} \varepsilon_{t+1} \\
X_{1(t+3)} - X_{1(t+2)} &= X_{2(t+2)}dt + \sigma_{t+2} \varepsilon_{t+2} \\
X_{2(t+3)} - X_{2(t+2)} &= X_{3(t+2)}dt + \sigma_{t+2} \varepsilon_{t+2} \\
X_{3(t+3)} - X_{3(t+2)} &= -\alpha_1 X_{1(t+2)}dt - \alpha_2 X_{2(t+2)}dt \\
&\quad -\alpha_3 X_{3(t+2)}dt + \sigma_{t+2} \varepsilon_{t+2}
\end{aligned}
$$