Implied State Price Densities of Weather Derivatives

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State Price Densities (SPD)

European call option:

$$C(K) = \mathrm{e}^{-r\tau} \int \max(x - K, 0) f(x) dx, \qquad (1)$$

r risk-free interest rate, τ time to maturity, K strike price, $f(x)$
State Price Density

Literature

Applicable to any risk asset, Breeden and Litzenberger (1976)

- Parametric estimation: Abadir and Rockinger (2003)
- Semiparametric estimation: Ait-Sahalia and Lo (1998)
- Univariate nonparametric estimation
  - Kernel methods: Ait-Sahalia and Duarte (2003); Yatchew and Härdle (2006), Fan and Mancini (2009)
  - Mixture models: Giacomini et al. (2008), Yuan (2009)
  - Curve fitting method: Rubinstein (1994), Jackwerth & Rubinstein (1996), and Teng & Liechty (2009)

Stochastic risk factors

- Financial Asset Prices data: Härdle & Hlávka (2009)
- Interest rate data: Ren (2007), Li & Zhao (2009)
- Nontradable assets: Weather -> NEW!

Pricing of non-tradable assets

The underlying is not "tradable"

- Indifference pricing
- General Equilibrium Theory for incomplete markets
- Pricing via no arbitrage arguments: adequate equivalent martingale measure

Algorithm

Econometrics

$$T_t$$
$$\downarrow$$
$$X_t = T_t - \Lambda_t$$
$$\downarrow$$
$$X_t = \beta^\top X_{t-L} + \varepsilon_t, \varepsilon_t = \sigma_t e_t$$
$$\downarrow$$
$$e_t = \frac{\hat{X}_t}{\hat{\sigma}_t} \sim \mathrm{N}(0,1)$$

Fin. Mathematics.

$$CAR(p)$$
$$\downarrow$$
$$F_{CAT(t,\tau_1,\tau_2)} = \mathsf{E}^{Q_\lambda}\left[g(\Lambda_t, \mathbf{X}_t, \lambda_t)\right]$$
$$\downarrow$$
$$MPR : \lambda_t$$

But: market is incomplete and illiquid!

Goals

- How to estimate SPD for weather derivatives? Bayesian Quadrature
- Can we compare/connect it with the SPD implied from Equity/Bond markets? Economic Implications

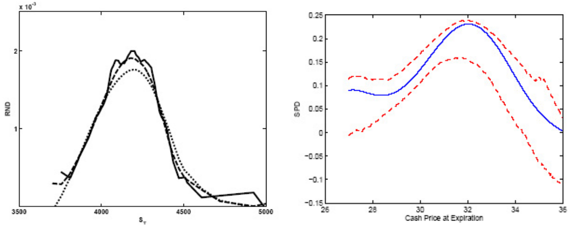

Figure 1: SPD from DAX option data (left). SPD of the interest rate with 95% pointwise Confidence intervals. (right)

Outline

1. Motivation ✓
2. Weather Derivatives
3. Bayesian Quadrature approaches
4. Empirical analysis
5. Conclusion

Weather derivatives

Hedge weather related risk exposures

- Payments based on weather related measurements
- Underlying: temperature, rainfall, wind, snow, frost

Chicago Mercantile Exchange (CME)

- Monthly/seasonal/weekly temperature Futures/Options
- 24 US, 6 Canadian, 9 European, 3 Australian, 3 Asian cities
- From 2.2 billion USD in 2004 to 15 billion USD through March 2009

Weather derivatives

Temperature CME products

- HDD$(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \max(c - T_t, 0) dt$
- CDD$(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \max(T_t - c, 0) dt$
- CAT$(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} T_t dt$, where $T_t = \frac{T_{t,max} + T_{t,min}}{2}$
- AAT$(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \widetilde{T}_t dt$, where $\widetilde{T}_t = \frac{1}{24} \int_1^{24} T_{t_i} dt_i$ and T_{t_i} denotes the temperature of hour t_i, (also referred to as C24AT index).

where c denotes the baseline ($65^\circ F$, $18^\circ C$), and T_t is average temperature on date t.

The quadrature approximation

Use a mixture model with Dirac measures,

$$f_N(x|w,\theta) = w_1\delta_{\theta_1}(x) + \cdots + w_N\delta_{\theta_N}(x),$$

with weights $w = \{w_1, \ldots, w_N\}$ and locations $\theta = \{\theta_1, \ldots, \theta_N\}$ for a non-negative integer (smoothing) parameter N.

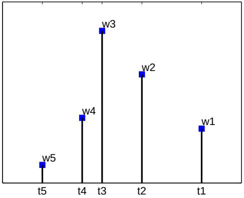

Figure 2: A quadrature plot

European options

Call option in (1) with $f(x) = f_N(x|w, \theta)$:

$$e^{-r\tau} \sum_{n=1}^{N} w_n \max(\theta_n - K, 0),$$

Put option:

$$e^{-r\tau} \sum_{n=1}^{N} w_n \max(K - \theta_n, 0).$$

Likelihood

- $C_{ij}^N(w,\theta)$ price under $f_N(w,\theta)$ with type $i \in (P,C)$, and strike K_j.
- $y = \{y_{ijk}\}, k = 1 \ldots K$ observed prices and assume

$$y_{ijk} = C_{ij}^N(w,\theta)e^{\varepsilon_{ijk}}$$

  with random variables ε_{ijk} i.i.d.$\sim$N$(0,\sigma^2)$ resulting from market friction and the approximation discrepancy.
- The likelihood:

$$L(y|w,\theta,\sigma) = \prod_{i,j,k} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{\left\{\log y_{ijk} - \log C_{ij}^N(w,\theta)\right\}^2}{2\sigma^2}\right].$$

See y_{ijk}

Prior and posterior distributions

- Prior distributions:
  - $w \sim D(\bar{w})$, where D is a Dirichlet distribution with a hyper-parameter $\bar{w}$.
  - $\theta \sim U(\mathcal{A})$, where U is a uniform distribution,

$$\mathcal{A} = \{\theta_1 > \cdots > \theta_N > 0, \theta_1 > K_{\max},\ \theta_N < K_{\min}\},$$

    and $K_{\max}$ and $K_{\min}$ are the max and the min of strike prices.
  - $\sigma^2 \sim IG(\alpha, \beta)$, where IG is an Inverse Gamma distribution with hyper-parameters α and β.
- The posterior distribution is

$$p(w, \theta, \sigma^2 | y) \propto L(y | w, \theta, \sigma^2) p(w | \bar{w}) p(\theta | K_{\max}, K_{\min}) p(\sigma^2 | \alpha, \beta).$$

MCMC algorithm

Start with random w, θ and σ^2. Do until convergence:

1. Sample $w_n \sim U(T_n)$, where T_n is an open interval for $n = 0, \ldots, N$ derived from the slice sampling (Neal, 2003).
2. Sample $\theta_n \sim U(S_n)$, where S_n is an open interval for $n = 0, \ldots, N$ derived from the slice sampling.
3. Sample $\sigma^2 \sim IG\left(\alpha + m/2,\ \beta + \sum_{i,j,k} \left\{\log y_{ijk} - \log C_{ij}^N(w,\theta)\right\}^2/2\right)$, where m is the total number of options in the data.

Data

Can we make money?

WD type	Trading date	Measurement Period		
	t	τ_1	τ_2	Futures
NewYork-HDD	20101026	20101001	20101031	171.00
	20101029	20101001	20101031	170.00
	20101102	20101001	20101031	171.00
	20101103	20101101	20101130	484.00
	20101104	20101101	20101130	480.00
	20101118	20101101	20101130	491.00
		20101201	20101231	830.00
		20110101	20110131	936.00
		20110201	20110228	797.00
		20110301	20110331	671.00
		20110401	20110430	367.00
		20110501	20110531	222.00

Table 1: New York future contracts listed at CME

HDD futures

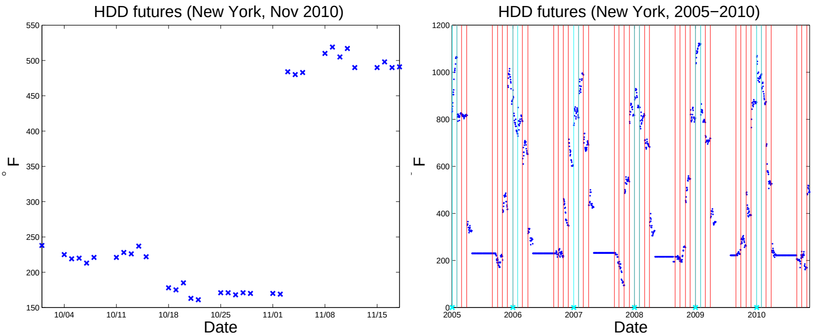

Figure 3: HDD futures for New York, Oct-Nov 2010 (left), 2005-2010 (right). Source: CME, Bloomberg

Option data

WD type	Date	No.Call	No.Put
NY-HDD	20101101	7	7
	20101102	7	7
	20101103	7	7
	20101104	7	7
	20101105	7	8
	20101108	8	8
	20101109	9	8
	20101110	9	9
	20101111	9	9
	20101112	9	9
	20101115	9	9

Table 2: New York option contracts listed at CME 2010/10/6-2010/11/18, call, put, future

Fitted quadrature model for NY-HDD Nov options

Put-red Call-blue Actual-cross Fitted-square for NY-HDD Nov options

Implied SPD for NY-HDD Nov. options

Dynamics of θ and w

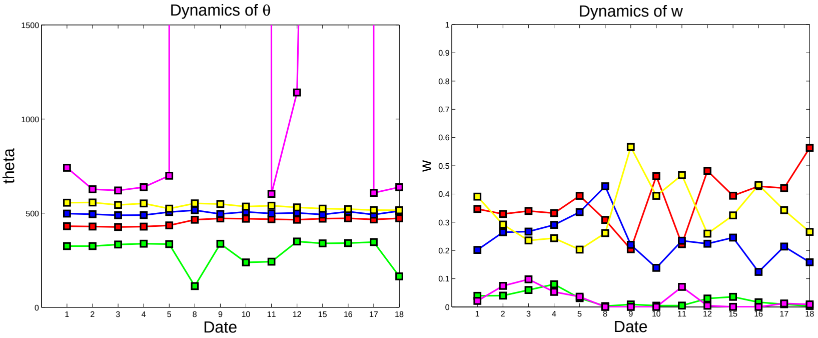

Figure 4: Dynamics of θ (left), Dynamics of w (right)

Economic implication

- SPD of WD deviate from lognormal, left or right skewed depending on maturities
- SPD's shapes depending on the volatility level of futures and investor's expectations?

Conclusion

- Our method has successfully calibrate the implied SPD to recover (illiquid) HDD options.
  - Robust & simple & fast method -> avoiding overfitting
  - Statistical inference
- Further investigation:
  - Modeling the dynamics of the implied SPD
  - Compare it with the Mixture of lognormals
  - Investigating dependence structure of the risk factors among different markets.
  - Forecasting, Pricing and Hedging

References

Y. Ait-Sahalia and A. W. Lo (1998)
Nonparametric Estimation of SPD implicit in Financial Asset Prices
J. of Finance, 53(3), 1998

Y. Ait-Sahalia and J. Duarte (2003)
Nonparametric option pricing under shape restrictions
Journal of Econometrics, Elsevier, 116(1-2), 9-47

J. Fan, L. Mancini (2009)
Option Pricing With Model-Guided Nonparametric Methods
Journal of the American Statistical Association, 104(488), 1351-1372.

References

R.A. Giacomini, A. Gottschiling, C. Haefke, and H. White (2008).
Mixtures of t -distributions for finance and forecasting
Journal of Econometrics, 144, 175-192.

W. K. Härdle and B. López Cabrera (2011)
Implied market price of weather risk
J. Applied Mathematical Finance, forthcoming

W.K. Härdle and Z. Hlávka (2009)
Dynamics of State Price Densities
Journal of Econometrics, 150(1): 2009, 1-15

References

J. Jackwerth, and M. Rubinstein (1996).
Recovering probability distributions from option prices
Journal of Finance, 51, 1611-1631.

H. Li and F. Zhao (2009)
Nonparametric Estimation of State-Price Densities Implicit in Interest Rate Cap Prices
Review Financial Studies, 22(11), 4335-4376.

Y. Ren (2005)
Nonparametric Estimation of State-Price Densities in Interest Rate Options
Thesis, University of Queen's University, Canada

References

M. Rubinstein (1994)
Implied binomial trees
Journal of Finance 69, 771-818.

H-W. Teng and J. Liechty (2009)
Bayesian Quadrature Approaches to State Price Density Estimation
Working paper, Pennsylvania State University

A. Yatchew and W.K. Härdle (2006)
Nonparametric state price density estimation using constrained least squares and the bootstrap
Journal of Econometrics, Elsevier, 133(2), 579-599

References

M. Yuan (2009)
State Price density Estimation via Nonparametric Mixtures
The Annals of Applied Statistics, 3 (3), 963-984

K.M. Abadir and M. Rockinger (2003)
Density Functionals, With An Option-Pricing Application
Econometric Theory, Cambridge University Press, 19(05), 778-811

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