CoVaR with very high dimensional risk factors

Wolfgang Karl Härdle
Weining Wang
Lixing Zhu, Lining Yu

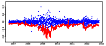
Ladislaus von Bortkiewicz Chair of Statistics
C.A.S.E. - Center for Applied Statistics and Economics
Humboldt–Universität zu Berlin
http://lvb.wiwi.hu-berlin.de
http://www.case.hu-berlin.de

Financial risk

Figure 1: Financial risk

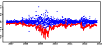
Objective

- Risk patterns depend on covariates X
- Dimensionality issues, $X \in \mathbb{R}^p, p \to \infty$
- Variable selection for Quantile Regression (QR)
- CoVaR, single index model (SIM)



Challenges

- Model of tails of conditional distribution
- Dimension reduction
- SIM estimation combined with variable selection
- Alternatives to MAVE (minimum average variance estimation)



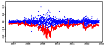
Single Index Model

- Observations $\{X_i, Y_i\}_{i=1}^n$ with

$$Y_i = g(\beta^{*\top} X_i) + \varepsilon_i, \quad (1)$$

where $g(\cdot)$ is the link function, and $\beta^* \in \mathbb{R}^p$. $\{\varepsilon_i\}_{i=1}^n$ are independent.

- p is possible large: $p \to \infty$.
- $\mathsf{E}_{Y|X=x}(\varepsilon) = 0$ for mean regression.
- $F^{-1}_{\varepsilon|X=x}(\tau) = 0$ for quantile regression.



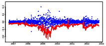
CoVaR

- *CoVaR* technique (AB)
- Two linear quantile regressions

$$\begin{aligned} X_{i,t} &= \alpha_i + \gamma_i^\top M_{t-1} + \varepsilon_{i,t}, \\ X_{j,t} &= \alpha_{j|i} + \beta_{j|i} X_{i,t} + \gamma_{j|i}^\top M_{t-1} + \varepsilon_{j,t}. \end{aligned}$$

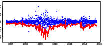
- $F^{-1}_{\varepsilon_{i,t}}(\tau | M_{t-1}) = 0$ and $F^{-1}_{\varepsilon_{j,t}}(\tau | M_{t-1}, X_{i,t}) = 0$, then

$$\begin{aligned} \widehat{VaR}^{\tau}_{i,t} &= \widehat{\alpha}_i + \widehat{\gamma}_i^\top M_{t-1}, \\ \widehat{CoVaR}^{\tau}_{j|i,t} &= \widehat{\alpha}_{j|i} + \widehat{\beta}_{j|i} \widehat{VaR}^{\tau}_{i,t} + \widehat{\gamma}_{j|i}^\top M_{t-1}. \end{aligned}$$



What is known?

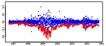
- MAVE method, Xia et al.(2002)
- Application in banking, environmental statistics
- First order "free lunch" $\sqrt{n}$ rate
- A one dimensional problem for estimating $g(\cdot)$



High dimensional SIM

- How to estimate nonzero β_l^* ?
- Which rates can we allow for p ?
- What are the consequences for estimating $g(\cdot)$?
- Combine dimension reduction with variable selection in a tail regression context.

Go to details



CoVaR application

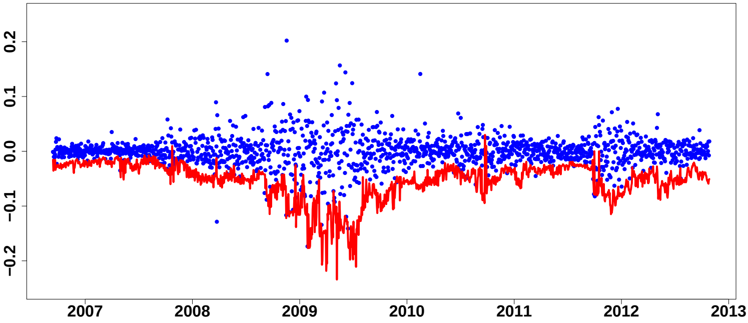
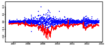


Figure 2: Log returns of CYN (blue) and the estimated $CoVaR_{SIM}$ (red), $\tau = 0.05,\ T = 1543$, window size $n = 126$, refer to (25).



Outline

1. Motivation ✓
2. Single index model
3. Simulations
4. Applications
5. Further work

A quasi-likelihood approach

Recall (1): $$\min_{\beta} \mathsf{E}_{(Y|X=x)} \rho_{\mathrm{w}}\{Y - g(\beta^{\top} x)\}, \tag{2}$$

where $\rho_{\mathrm{w}}(\cdot) \stackrel{\text{def}}{=} \sum_{k=1}^{K} \mathrm{w}_k \rho_k(\cdot)$, $(\mathrm{w_k} > 0, \rho_{\mathrm{k}}(\cdot)$ convex) and

$\mathrm{w} \stackrel{\text{def}}{=} (\mathrm{w}_1, \cdots, \mathrm{w_K})$ is chosen to be data driven, $\sum_{k=1}^{K} \mathrm{w}_k = 1$.

In degenerate case, $K = 1$:

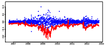
Quantile regression

$$\rho_{\mathrm{w}}(u) = \tau u \mathbf{1}\{u \in (0, \infty)\} - (1-\tau) u \mathbf{1}\{u \in (-\infty, 0)\} \tag{3}$$

Expectile regression

$$\rho_{\mathrm{w}}(u) = \tau u^2 \mathbf{1}\{u \in (0, \infty)\} + (1-\tau) u^2 \mathbf{1}\{u \in (-\infty, 0)\} \tag{4}$$

▸ Expectile-Quantile Correspondence ▸ Go to selection of weigthts



Expectile and Quantile Curves

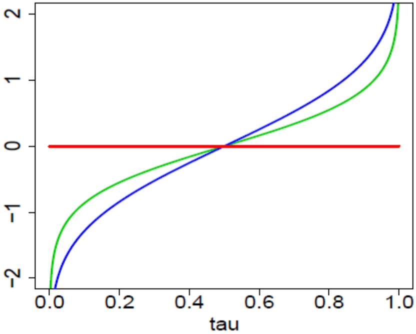


Figure 3: Expectile (green) and Quantile (blue) for $N(0,1)$.

Likelihood approximations

$$g(\beta^\top X_i) \approx g(\beta^\top x) + g'(\beta^\top x)\beta^\top(X_i - x) \tag{5}$$

Approximations

$$\begin{aligned} L_x(\beta) &\stackrel{\text{def}}{=} \mathsf{E}\,\rho_{\mathrm{w}}\{Y - g(\beta^\top x) - g'(\beta^\top x)\beta^\top(X - x)\} \\ &\quad K_h\{\beta^\top(X - x)\} \qquad (6) \\ L_{n,x}(\beta) &\stackrel{\text{def}}{=} n^{-1}\sum_{i=1}^{n} \rho_{\mathrm{w}}\{Y_i - g(\beta^\top x) - g'(\beta^\top x)\beta^\top(X_i - x)\} \\ &\quad K_h\{\beta^\top(X_i - x)\} \qquad (7) \end{aligned}$$

where $K_h(.) = K(./h)/h$ with $K(.)$ a kernel, h bandwidth

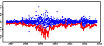
A simple trick

Minimize average contrast (w.r.t. β):

$$
\begin{aligned}
L_n(\beta) &\stackrel{\text{def}}{=} n^{-1} \sum_{j=1}^{n} L_{n,X_j}(\beta) \\
&= n^{-2} \sum_{j=1}^{n} \sum_{i=1}^{n} \rho_{\mathrm{w}} \left\{ Y_i - g(\beta^\top X_j) - g'(\beta^\top X_j)\beta^\top (X_i - X_j) \right\} \\
&\qquad K_h\{\beta^\top (X_i - X_j)\} \qquad (8)
\end{aligned}
$$

Therefore (in first approach):

$$
\widehat{\beta} \approx \arg\min_{\beta} L_n(\beta).
$$

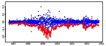


One more trick

Let $a_j = g(\beta^\top X_j)$, $b_j = g'(\beta^\top X_j)$, estimate β by:

$$\min_{(a_j, b_j)'\text{s}, \beta} n^{-1} \sum_{j=1}^{n} \sum_{i=1}^{n} \rho_{\mathrm{w}} \big(Y_i - a_j - b_j X_{ij}^\top \beta \big) \omega_{ij}(\beta), \tag{9}$$

where $X_{ij} \stackrel{\text{def}}{=} X_i - X_j$, $\omega_{ij}(\beta) \stackrel{\text{def}}{=} K_h(X_{ij}^\top \beta) / \sum_{i=1}^{n} K_h(X_{ij}^\top \beta)$.



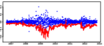
The final trick

Penalize the dimension p and estimate β by:

$$\min_{(a_j,b_j)'\mathrm{s},\beta} n^{-1} \sum_{j=1}^{n} \sum_{i=1}^{n} \rho_{\mathrm{w}}\left(Y_i - a_j - b_j X_{ij}^{\top}\beta\right)\omega_{ij}(\beta) + \sum_{l=1}^{p} \gamma_{\lambda}(|\widehat{\beta}_l^{(0)}|)|\beta_l|, \quad (10)$$

where $\gamma_{\lambda}(t)$ is some non-negative function, and $\widehat{\beta}^{(0)}$ initial estimator of β^* (linear QR with variable selection).

Go to details



How does this work?

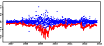
- $\widehat{\beta}^{(0)}$ initial estimator of β^* (linear QR with variable selection).
- For $t = 0, 1, 2, \cdots$, given $\widehat{\beta}^{(t)}$, standardize $\widehat{\beta}^{(t)}$, $\|\widehat{\beta}^{(t)}\| = 1$, $\widehat{\beta}_1^{(t)} = 1$, $\widehat{d}_l^{(t)} \stackrel{\text{def}}{=} \gamma_\lambda(|\widehat{\beta}_l^{(t)}|)$. Then compute

$$(\widehat{a}_j^{(t)}, \widehat{b}_j^{(t)}) \stackrel{\text{def}}{=} \underset{(a_j, b_j)'\text{s}}{\arg\min} \sum_{i=1}^{n} \rho_{\mathrm{w}}\big(Y_i - a_j - b_j X_{ij}^\top \widehat{\beta}^{(t)}\big)\omega_{ij}(\widehat{\beta}^{(t)})$$

- Given $(\widehat{a}_j^{(t)}, \widehat{b}_j^{(t)})$, solve

$$\widehat{\beta}^{(t+1)} = \underset{\beta}{\arg\min}\ n^{-1} \sum_{j=1}^{n} \sum_{i=1}^{n} \rho_{\mathrm{w}}\big(Y_i - \widehat{a}_j^{(t)} - \widehat{b}_j^{(t)} X_{ij}^\top \beta\big)\omega_{ij}(\widehat{\beta}^{(t)})$$

$$+ \textstyle\sum_{l=1}^{p} \widehat{d}_l^{(t)} |\beta_l|.$$

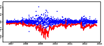


Some definitions

Let $\beta^* = (\beta^{*\top}_{(1)}, \mathbf{0}^\top)^\top$ with $\beta^*_{(1)} \stackrel{\text{def}}{=} (\beta_1, \ldots, \beta_q)^\top \neq 0$. $X_{i(1)} \stackrel{\text{def}}{=}$ sub vector of X_i corresponding to $\beta^*_{(1)}$, $X_{i(0)}$ corresponding to zero β^*. $\widehat{\beta}^0 \stackrel{\text{def}}{=} (\widehat{\beta}^{0\top}_{(1)}, \mathbf{0}^\top)^\top$.

$$\widehat{\beta}^0_{(1)} \stackrel{\text{def}}{=} \arg\min_{\beta_{(1)}} n^{-1} \sum_{j=1}^{n} \sum_{i=1}^{n} \rho_{\mathrm{w}} \left\{ Y_i - a_{j(1)} - b_{j(1)} X^\top_{ij(1)} \beta_{(1)} \right\} \omega_{ij}(\beta_{(1)})$$

where $a_{j(1)} = g(\beta^\top_{(1)} X_{j(1)})$, $b_{j(1)} = g'(\beta^\top_{(1)} X_{j(1)})$, $X_{ij(1)} = X_{i(1)} - X_{j(1)}$.



An amuse gueule of theory

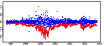
Denote $\widehat{\beta}$ as the final estimate of β^*.

Theorem

Under A 1-5, the estimators $\widehat{\beta}^0$ *and* $\widehat{\beta}$ *exist and* $\mathsf{P}(\widehat{\beta}^0 = \widehat{\beta}) \to 1$. *Moreover,*

$$\mathsf{P}(\widehat{\beta}^0 = \widehat{\beta}) \geq 1 - (p - q)\exp(-C' n^{\alpha}), \tag{11}$$

Go to details



Antipasti Theory

Theorem

Under A 1-5, $\widehat{\beta}_{(1)} \stackrel{\text{def}}{=} \left(\widehat{\beta}_l\right)_{l \in \mathcal{M}_*}$, $b \in \mathbb{R}^q$, $\|b\| = 1$:

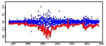
$$\|\widehat{\beta}_{(1)} - \beta^*_{(1)}\| = \mathcal{O}_p\{(\lambda D_n + n^{-1/2})\sqrt{q}\} \tag{12}$$

$$\boldsymbol{b}^\top C^{-1}_{0(1)} \sqrt{n}(\widehat{\beta}_{(1)} - \beta^*_{(1)}) \xrightarrow{\mathcal{L}} \mathrm{N}(0,\ \sigma^2_{\mathrm{w}}) \tag{13}$$

where $\sigma^2_{\mathrm{w}} = \mathrm{E}[\psi_{\mathrm{w}}(\varepsilon_i)]^2 / [\partial^2 \, \mathrm{E}\, \rho_{\mathrm{w}}(\varepsilon_i)]^2$

$$\partial^2 \, \mathrm{E}\, \rho_{\mathrm{w}}(\cdot) = \left.\frac{\partial^2 \, \mathrm{E}\, \rho_{\mathrm{w}}(\varepsilon_i - v)^2}{\partial v^2}\right|_{v=0} \tag{14}$$

Go to details



Main Course

Theorem

Under A 1-5, $\mathcal{B}_n \stackrel{\text{def}}{=} \{\widehat{\beta} = \beta^*\} : \mathsf{P}(\mathcal{B}_n) \to 1$. *Let* $\mu_j \stackrel{\text{def}}{=} \int u^j K(u) du$, $\nu_j \stackrel{\text{def}}{=} \int u^j K^2(u) du$, $j = 0, 1, \ldots$. *If* $nh^3 \to \infty$ *and* $h \to 0$, *then*

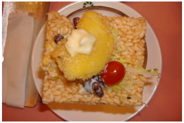
$$\sqrt{nh}\sqrt{f_{Z_{(1)}}(z)/(\nu_0 \sigma_{\mathrm{w}}^2)}\left\{\widehat{g}(x^\top \widehat{\beta}) - g(x^\top \beta^*) - \frac{1}{2}h^2 g''(x^\top \beta^*)\mu_2 \partial \,\mathsf{E}\,\psi_{\mathrm{w}}(\varepsilon)\right\} \xrightarrow{\mathcal{L}} \mathrm{N}(0,\ 1),$$

and

$$\sqrt{nh^3}\sqrt{\{f_{Z_{(1)}}(z)\mu_2^2\}/(\nu_2 \sigma_{\mathrm{w}}^2)}\left\{\widehat{g}'(x^\top \widehat{\beta}) - g'(x^\top \beta^*)\right\} \xrightarrow{\mathcal{L}} \mathrm{N}(0,\ 1).$$

Go to details

Simulation



- 3 different $\beta^*_{(1)}$: $\beta^{*\top}_{(1)} = (5, 5, 5, 5, 5)$, $\beta^{*\top}_{(1)} = (5, 4, 3, 2, 1)$ and $\beta^{*\top}_{(1)} = (5, 2, 1, 0.8, 0.2)$.
- 2 different distribution of ε: $\varepsilon_i \sim \mathrm{N}(0, 0.1)$ and $\varepsilon_i \sim t(5)$.
- 3 different τ: $\tau = 0.95$, $\tau = 0.5$ and $\tau = 0.05$.
- 2 different p: $p = 10$ and $p = 200$.
- 3 different link functions: Model 1, Model 2 and Model 3.

Link functions

- Model 1

$$Y_i = 5\cos(D \cdot Z_i) + \exp(-D \cdot Z_i^2) + \varepsilon_i, \tag{15}$$

$Z_i = X_i^\top \beta^*$, $D = 0.01$ is a scaling constant and ε_i is the error term.

- Model 2

$$Y_i = \sin\{\pi(A \cdot Z_i - B)\} + \varepsilon_i, \tag{16}$$

with the parameters $A = 0.3$, $B = 3$.

- Model 3

$$Y_i = 10\sin(D \cdot Z_i) + \sqrt{|\sin(0.5 \cdot Z_i) + \varepsilon_i|}, \tag{17}$$

with $D = 0.1$.

The estimated vs. true link functions

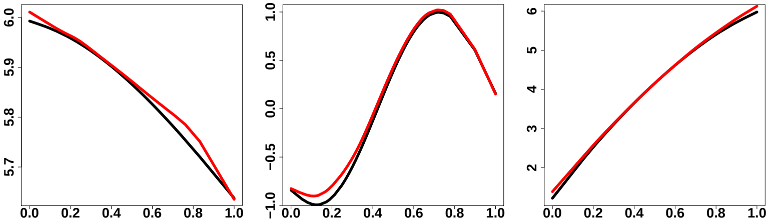
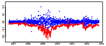


Figure 4: The true link functions (black) and the estimated link functions (red) with $\beta^{*\top}_{(1)} = (5, 5, 5, 5, 5)$, and $\varepsilon \sim \mathrm{N}(0, 0.1)$, $n = 100, p = 10, q = 5, \tau = 0.95$, model 1 (left) with $h = 1.02$, model 2 (middle) with $h = 0.15$ and model 3 (right) with $h = 0.76$.



Criteria

1. Standardized $L2$ norm:

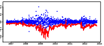
$$Dev \stackrel{\text{def}}{=} \frac{\|\beta^* - \widehat{\beta}\|_2}{\|\beta^*\|_2}$$

2. Sign consistency:

$$Acc \stackrel{\text{def}}{=} \sum_{l=1}^{p} |sign(\beta_l^*) - sign(\widehat{\beta}_l)|$$

3. Least angle:

$$Angle \stackrel{\text{def}}{=} \frac{< \beta^*, \widehat{\beta} >}{\|\beta^*\|_2 \cdot \|\widehat{\beta}\|_2}$$



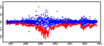
Criteria

4. Relative error:

$$Error \stackrel{\text{def}}{=} \frac{1}{n}\sum_{i=1}^{n}\left|\frac{g(Z_i)-\widehat{g}(\widehat{Z}_i)}{g(Z_i)}\right|$$

5. Average squared error:

$$ASE(h) \stackrel{\text{def}}{=} \frac{1}{n}\sum_{i=1}^{n}\left\{g(Z_i)-\widehat{g}(\widehat{Z}_i)\right\}^2$$



Criteria - Quantile regression (small p case)

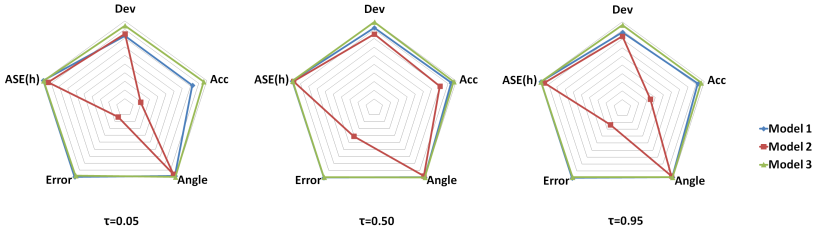
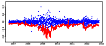


Figure 5: Criteria evaluated under different models and quantiles. $\beta^{*\top}_{(1)} = (5,5,5,5,5)$, the error ε follows a $\mathrm{N}(0, 0.1)$ distribution. In 100 simulations we set $n = 100$, $p = 10$, $q = 5$.

Criteria table



Criteria - Quantile regression (small p case)

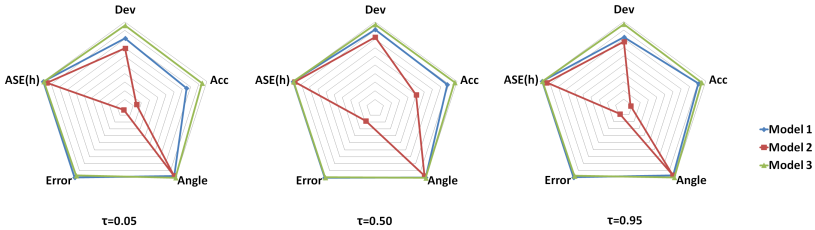


Figure 6: Criteria evaluated under different models and quantiles. $\beta_{(1)}^{*\top} = (5,5,5,5,5)$, the error ε follows a $t(5)$ distribution. In 100 simulations we set $n = 100, p = 10, q = 5$.

Criteria table

Criteria - Quantile regression (different $\beta^*_{(1)}$)

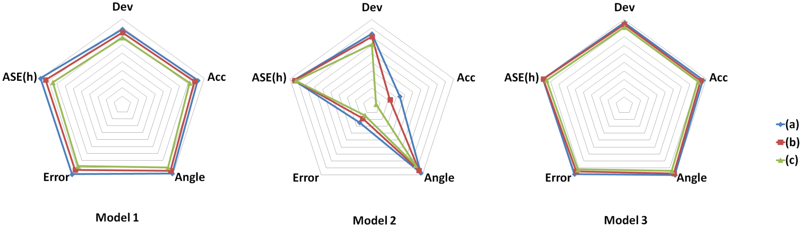
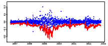


Figure 7: Criteria evaluated under three different $\beta^*_{(1)}$: (a) $\beta^{*\top}_{(1)} = (5,5,5,5,5)$, (b) $\beta^{*\top}_{(1)} = (5,4,3,2,1)$, (c) $\beta^{*\top}_{(1)} = (5,2,1,0.8,0.2)$ the error ε follows a $\mathrm{N}(0,0.1)$ distribution. In 100 simulations we set $n = 100, p = 10, q = 5, \tau = 0.95$.

Criteria table



Criteria - Quantile regression (large p case)

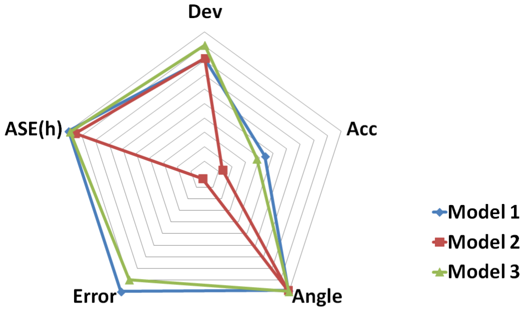
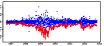


Figure 8: Criteria evaluated with different models under $p > n$ case. $\beta^{*\top}_{(1)} = (5, 5, 5, 5, 5)$, the error ε follows a $\mathrm{N}(0, 0.1)$ distribution. In 100 simulations we set $n = 100, p = 200, q = 5, \tau = 0.05$.

▸ Criteria table



Criteria - Expectile regression (small p case)

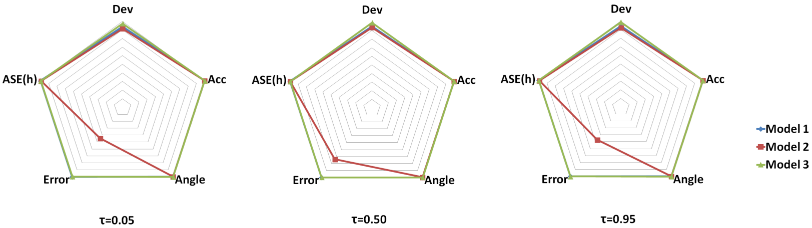
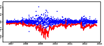


Figure 9: Criteria evaluated under different models and quantiles. $\beta^{*\top}_{(1)} = (5, 5, 5, 5, 5)$, the error ε follows a $\mathrm{N}(0, 0.1)$ distribution. In 100 simulations we set $n = 100, p = 10, q = 5$.

Criteria table



Criteria - Expectile regression (large p case)

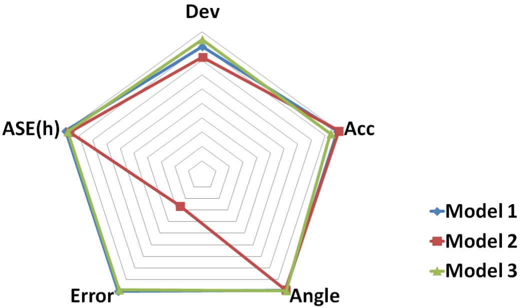
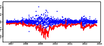


Figure 10: Criteria evaluated with different models under $p > n$ case. $\beta^{*\top}_{(1)} = (5, 5, 5, 5, 5)$, the error ε follows a $\mathrm{N}\,(0, 0.1)$ distribution. In 100 simulations we set $n = 100, p = 200, q = 5, \tau = 0.05$.

Expectile-Quantile Correspondence Criteria table



Criteria - Composite quantile regression

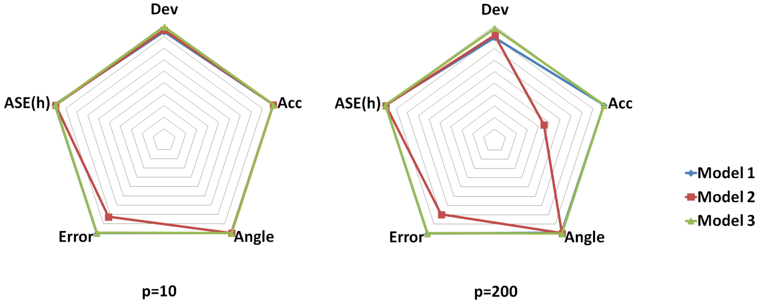
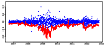


Figure 11: Criteria evaluated under different models and number of p, where $p = 10$ (left) and $p = 200$ (right), $\beta^{*\top}_{(1)} = (5, 5, 5, 5, 5)$, the error ε follows a $\mathrm{N}(0, 0.1)$ distribution. In 100 simulations we set $\tau = (0.25, 0.35, 0.5, 0.65, 0.75)$, $n = 100$, $q = 5$.

Criteria table



The estimated vs. true link functions

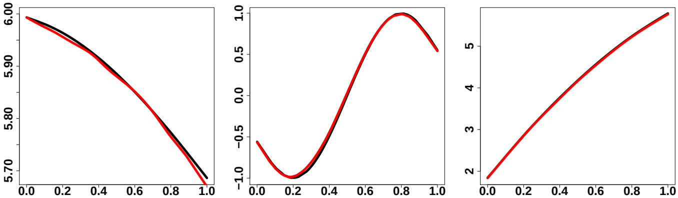


Figure 12: Plot of the true function $g(\cdot)$ (black) and the estimation (red) with $\tau = (0.25, 0.35, 0.5, 0.65, 0.75), n = 100, p = 10, q = 5$ and $\varepsilon \sim \mathrm{N}(0, 0.1)$ in different $g(\cdot)$ functions, model 1 (left) with $h = 0.45$, model 2 (middle) with $h = 0.16$ and model 3 (right) with $h = 0.15$.

The estimated vs. true link functions

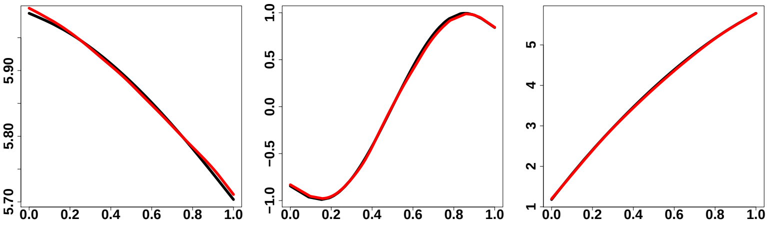
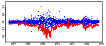


Figure 13: Plot of the true function $g(\cdot)$ (black) and the estimation (red) with $\tau = (0.25, 0.35, 0.5, 0.65, 0.75), n = 100, p = 200, q = 5$ and $\varepsilon \sim \mathrm{N}(0, 0.1)$ in different $g(\cdot)$ functions, model 1 (left) with $h = 0.28$, model 2 (middle) with $h = 0.05$ and model 3 (right) with $h = 0.13$.

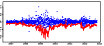


Value at Risk

- Value-at-Risk (VaR) is the most known measure for quantifying and controlling the risk of a portfolio.
- The VaR of a financial institution i at $\tau \in (0,1)$:

$$\mathrm{P}(X_{i,t} \leq VaR_{i,t}^{\tau}) \stackrel{def}{=} \tau,$$

where $X_{i,t}$ represents the asset return of financial institution i at time t.

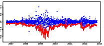


CoVaR

- Adrian and Brunnermeier (AB) (2011) proposed CoVaR.
- The CoVaR of a risk factor j given X_i at level $\tau \in (0,1)$:

$$\mathrm{P}\{X_{j,t} \le CoVaR^{\tau}_{j|i,t} | X_{i,t} = VaR^{\tau}(X_{i,t}), M_{t-1}\} \overset{def}{=} \tau,$$

here M_{t-1} is a vector of macroprudential variables.



Quantile regression

- CoVaR technique (AB)
- Two linear quantile regressions

$$
\begin{aligned}
X_{i,t} &= \alpha_i + \gamma_i^\top M_{t-1} + \varepsilon_{i,t}, && (18)\\
X_{j,t} &= \alpha_{j|i} + \beta_{j|i} X_{i,t} + \gamma_{j|i}^\top M_{t-1} + \varepsilon_{j,t}. && (19)
\end{aligned}
$$

- $F^{-1}_{\varepsilon_{i,t}}(\tau|M_{t-1}) = 0$ and $F^{-1}_{\varepsilon_{j,t}}(\tau|M_{t-1}, X_{i,t}) = 0$, then

$$
\begin{aligned}
\widehat{VaR}^{\tau}_{i,t} &= \widehat{\alpha}_i + \widehat{\gamma}_i^\top M_{t-1}, && (20)\\
\widehat{CoVaR}^{\tau}_{j|i,t} &= \widehat{\alpha}_{j|i} + \widehat{\beta}_{j|i} \widehat{VaR}^{\tau}_{i,t} + \widehat{\gamma}_{j|i}^\top M_{t-1}. && (21)
\end{aligned}
$$

Quantile regression

- Generalize (19):

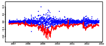
$$X_{j,t} = S^\top \beta_{j|S} + \varepsilon_{j,t}, \tag{22}$$

where $S \overset{def}{=} [M_{t-1}, R]$, R is a vector of log returns. $\beta_{j|S}$ is a $p \times 1$ vector, p large.

- $F^{-1}_{\varepsilon_{j,t}}(\tau|S) = 0$, then:

$$\widehat{CoVaR}^{\tau}_{j|\widehat{S}} = \widehat{S}^\top \widehat{\beta}_{j|S}, \tag{23}$$

where $\widehat{S} \overset{def}{=} [M_{t-1}, \widehat{V}]$, where $\widehat{V}$ is the estimated VaR in (20).



Quantile regression and SIM

- Generalize (19):

$$X_{j,t} = g(S^\top \beta_{j|S}) + \varepsilon_{j,t}, \tag{24}$$

where $S \overset{def}{=} [M_{t-1}, R]$, R is a vector of log returns. $g(\cdot)$ is a link function. $\beta_{j|S}$ is a $p \times 1$ vector, p large.

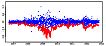
- $F^{-1}_{\varepsilon_{j,t}}(\tau|S) = 0$, then:

$$\widehat{CoVaR}^{\tau}_{j|\widehat{S}} = \widehat{g}(\widehat{S}^\top \widehat{\beta}_{j|S}), \tag{25}$$

where $\widehat{S} \overset{def}{=} [M_{t-1}, \widehat{V}]$, where $\widehat{V}$ is the estimated VaR in (20).

Dataset

- City national corp (CYN) (as an example).
- Choose 199 financial firms and 7 macroprudential variables.
- Time period is from January 6, 2006 to September 6, 2012, $T = 1669$.



Descriptive statistics of CYN

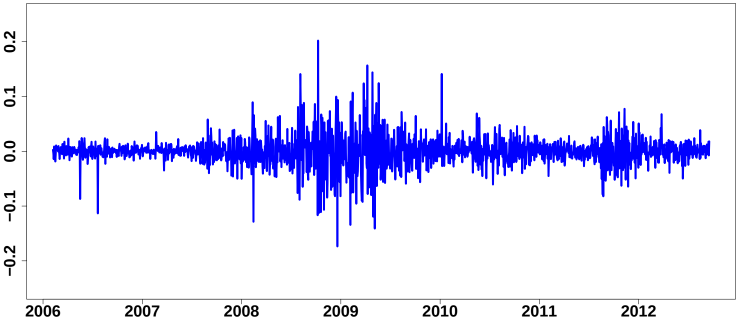


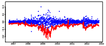
Figure 14: Log returns of CYN

Descriptive statistics of CYN

	Mean	SD	Skewness	Kurtosis
Before crisis	-4.0×10^{-4}	0.0209	0.2408	12.1977
In crisis	-9.2×10^{-5}	0.0312	0.1326	8.9544

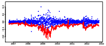
Table 1: Descriptive statistics

- Jarque Bera Test is performed: log returns of CYN are not normally distributed.
- Unit root test is conducted: log returns of CYN are stationary.



Estimation of VaR

- Window size: $n = 126$.
- 7 Macroprudential variables are applied.
- Method: quantile regression.
- $\tau = 0.05$.
- $T = 1543$ estimated VaR by moving window estimation.



Estimation of VaR

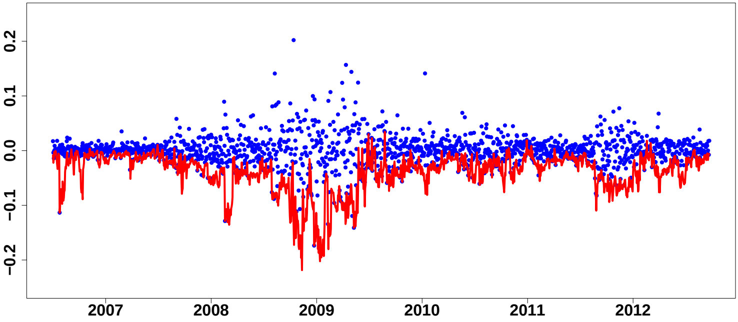
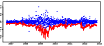


Figure 15: Log returns of CYN (blue) and *VaR* of log returns of CYN (red), $\tau = 0.05$, $T = 1543$, window size $n = 126$, refer to (20).



Estimation of VaR

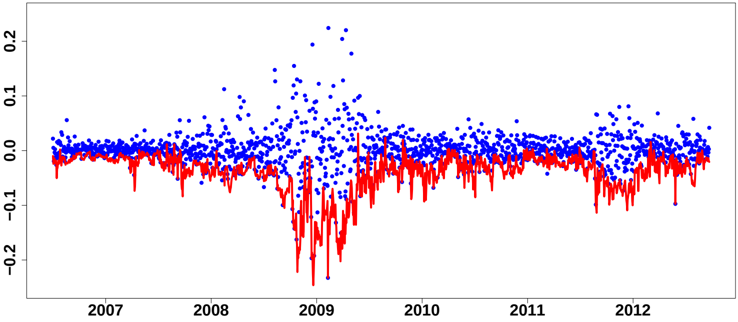
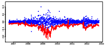


Figure 16: Log returns of JPM (blue) and *VaR* of log returns of JPM (red), $\tau = 0.05,\ T = 1543$, window size $n = 126$, refer to (20).

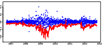


Estimation of $\mathrm{CoVaR_L}$

- Window size: $n = 126$.
- Original variables: $p = 206$. ▸ 206 covariates
- Method: $L1$-norm quantile regression.
- $\tau = 0.05$.
- Lambda: generalized approximate cross-validation criterion (GACV) (Yuan 2006):

$$GACV(\lambda) = \frac{\sum_{i=1}^{n} \rho_\tau\{y_i - f(x_i)\}}{n - df},$$

  where df is a measure of the effective dimensionality of the fitted model.
- Selected variables: Different $\widehat{q}$ in each window.
- $T = 1543$ estimated $CoVaR_L$ by moving window estimation.



Estimation of $\mathrm{CoVaR_L}$

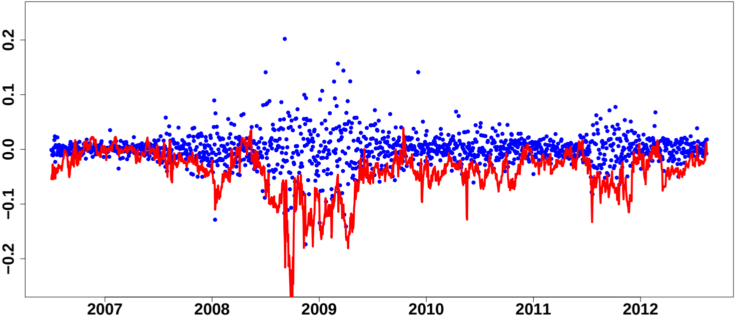
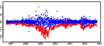


Figure 17: Log returns of CYN (blue) and the estimated $CoVaR_L$ (red), $\tau = 0.05$, $T = 1543$, window size $n = 126$, refer to (23).



$\widehat{\beta}$ in each window

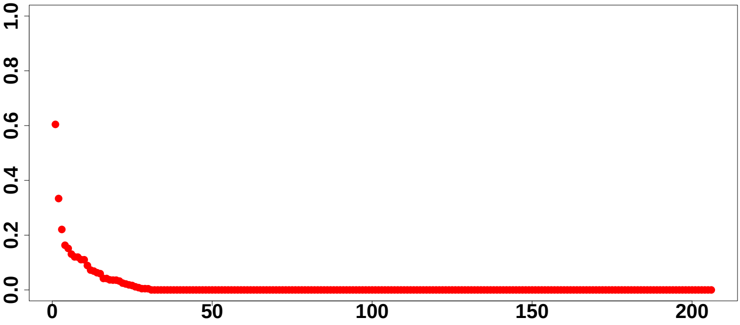
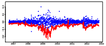


Figure 18: Different $\widehat{\beta}$ in application.



The histogram of $\widehat{q}$

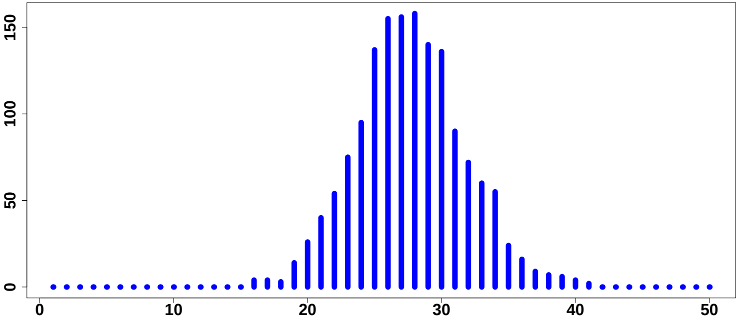


Figure 19: Frequency of the number of selected variables. Where $\widehat{q} = 28$ with frequency 158, $\widehat{q} = 27$ with frequency 156 and $\widehat{q} = 26$ with frequency 155.

The estimated value of λ

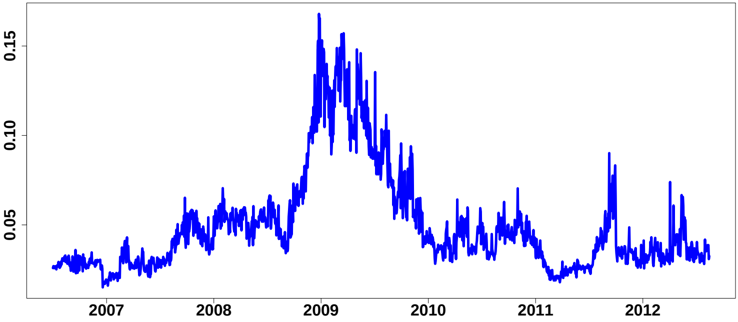


Figure 20: The $\widehat{\lambda}$ in application.

The influential variables

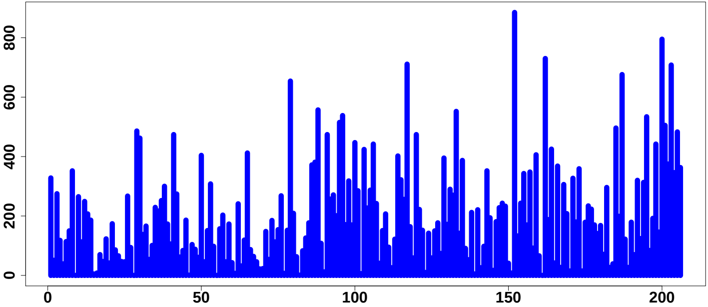
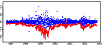


Figure 21: The frequency of the firms and macroprudential variables. The X-axis: $1-206$ variables, and the Y-axis: the frequency of the variables selected in the moving window estimation. The variable 152, i.e. "Flagstar Bancorp Inc. (FBC)" is the most frequently selected variable with frequency 885.

Go to details

Estimation of CoVaR$_{\mathsf{SIM}}$

- Window size: $n = 126$.
- Original variables: $p = 206$.
- Method: $L1$-norm quantile regression & Single index model.
- $\tau = 0.05$.
- Bandwidth: $h_\tau = h_{mean}\left[\tau(1-\tau)\varphi\{\Phi^{-1}(\tau)\}^{-2}\right]^{0.2}$, where h_{mean}: use direct plug-in methodology of a local linear regression described by Ruppert, Sheather and Wand (1995).



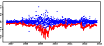
Estimation of $\text{CoVaR}_{\text{SIM}}$

- Lambda: generalized approximate cross-validation criterion (GACV) (Yuan 2006):

$$GACV(\lambda) = \frac{\sum_{i=1}^{n} \rho_\tau \{y_i - f(x_i)\}}{n - df},$$

where df is a measure of the effective dimensionality of the fitted model.

- Selected variables: Different $\widehat{q}$ in each window.
- $T = 1543$ estimated $CoVaR_{SIM}$ by moving window estimation.



Estimation of $\text{CoVaR}_{\text{SIM}}$

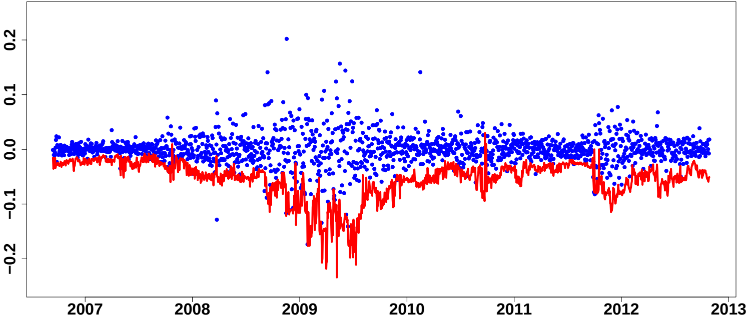


Figure 22: Log returns of CYN (blue) and the estimated $CoVaR_{SIM}$ (red), $\tau = 0.05$, $T = 1543$, window size $n = 126$, refer to (25).

$\widehat{\beta}$ in each window

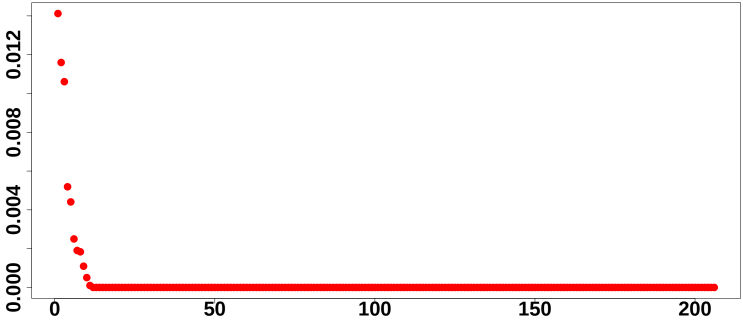


Figure 23: Different $\widehat{\beta}$ in application.

The histogram of $\widehat{q}$

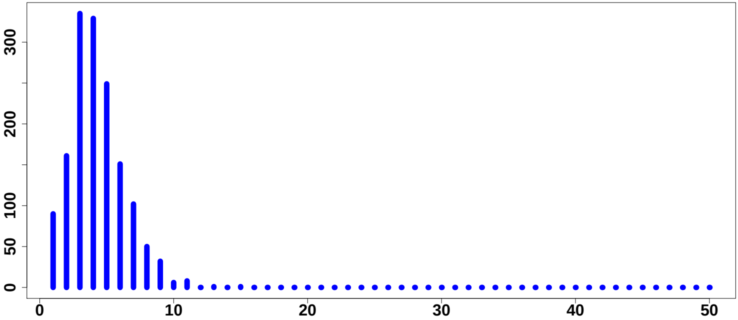


Figure 24: Frequency of the number of selected variables. Where $\widehat{q} = 3$ with frequency 335, $\widehat{q} = 4$ with frequency 329 and $\widehat{q} = 5$ with frequency 249.

The estimated value of λ

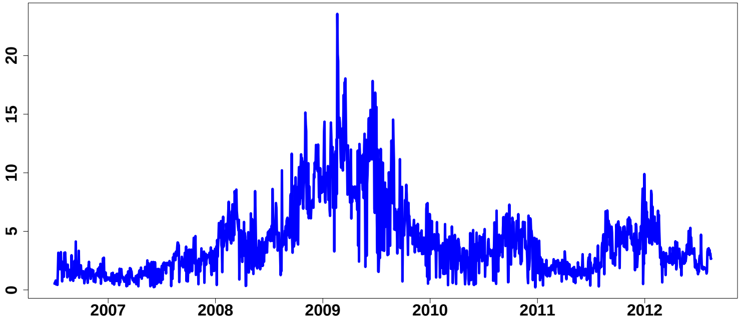
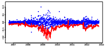


Figure 25: The $\widehat{\lambda}$ in application.



The link function

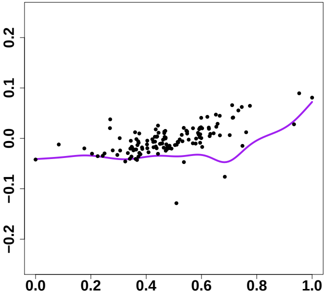


Figure 26: The link functions

The link function

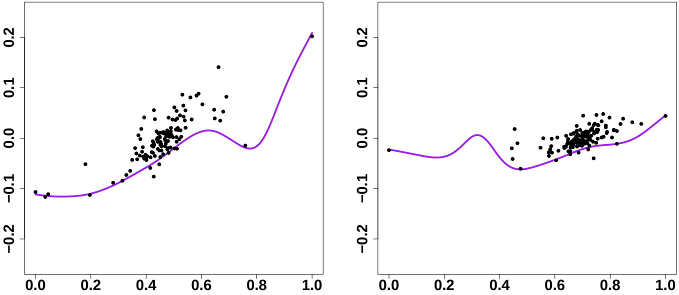


Figure 27: The estimated link function, window size $n = 126$, $\tau = 0.05$, $p = 206$. For the left graph: starting date is 20081029, $h = 0.065$, $\widehat{q} = 3$: HBAN, CNO, STSA. For the right graph: starting date is 20101230, $h = 0.058$, $\widehat{q} = 2$: FBC, RDN.

The influential variables

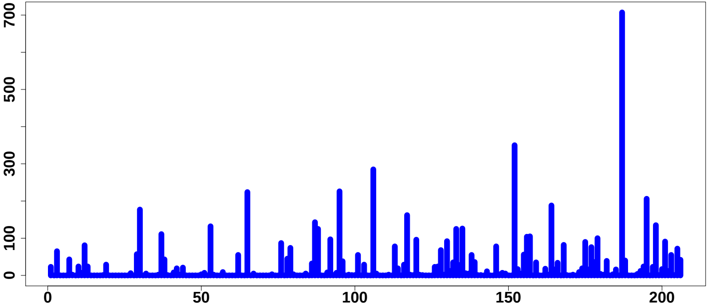
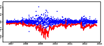


Figure 28: The frequency of the firms and macroprudential variables. The X-axis: $1-206$ variables, and the Y-axis: the frequency of the variables selected in the moving window estimation. The variable 187, i.e. "Radian Group Inc. (RDN)" is the most frequently selected variable with frequency 707.

Go to details



Backtesting

- The violation sequence:

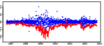
$$I_t = \begin{cases} 1, & X_{i,t} < \widehat{VaR}^{\tau}_{i,t}; \\ 0, & otherwise. \end{cases}$$

  If VaR algorithm is correct, I_t should be a martingale difference sequence.
- The CaViaR test model:

$$I_t = \alpha + \beta_1 I_{t-1} + \beta_2 VaR_t + u_t.$$

  where VaR_t can be $CoVaR_t$ in case of $CoVaR$ estimation.
- The test procedure: estimate $\widehat{\beta}_1$ and $\widehat{\beta}_2$ by logistic regression. Then Wald's test is applied. Null hypothesis: $\widehat{\beta}_1 = \widehat{\beta}_2 = 0$, i.e. I_t is a martingale difference sequence.

▸ logistic regression



Backtesting VaR

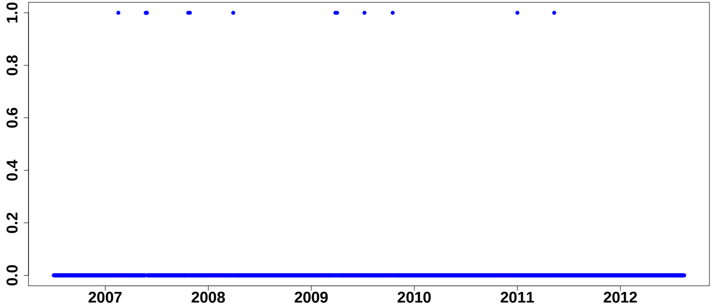


Figure 29: The top dots are the violations (i.e. $\{t : I_t = 1\}$) of $\widehat{VaR}$ of CYN, totally 14 violations, $\widehat{\tau} = 0.009$, $T = 1543$.

Backtesting CoVaR_L

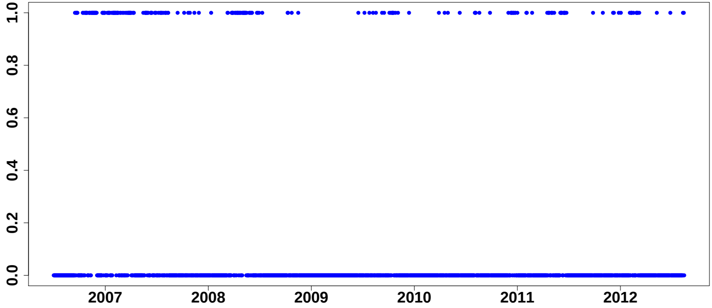
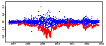


Figure 30: The top dots are the violations (i.e. $\{t : I_t = 1\}$) of $\widehat{CoVaR}_L$ of CYN, totally 231 violations, $\widehat{\tau} = 0.15,\ T = 1543$.



Backtesting CoVaR$_{SIM}$

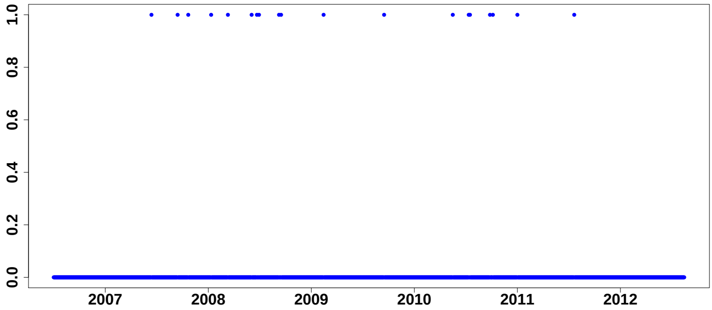
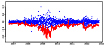


Figure 31: The top dots are the violations (i.e. $\{t : I_t = 1\}$) of $\widehat{CoVaR}_{SIM}$ of CYN, totally 19 violations, $\widehat{\tau} = 0.012$, $T = 1543$.



Backtesting results (The overall period)

- Test result

	p-value of Wald's test statistics
$\widehat{VaR}$	2.7×10^{-6}
$\widehat{CoVaR}_L$	0.00
$\widehat{CoVaR}_{SIM}$	0.54

Table 2: The CaViaR test for $\widehat{VaR}$, $\widehat{CoVaR}_L$ and $\widehat{CoVaR}_{SIM}$ for CYN, $T = 1543$, $20060706 - 20120906$.

- Only for $\widehat{CoVaR}_{SIM}$, null hypothesis can not be rejected. $\widehat{VaR}$ and $\widehat{CoVaR}_L$ algorithms perform not so well in overall period.

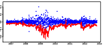
Backtesting results (The crisis period)

- Test result

	p-value of Wald's test statistics
$\widehat{VaR}$	0.99
$\widehat{CoVaR}_L$	3.2×10^{-5}
$\widehat{CoVaR}_{SIM}$	0.93

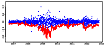
Table 3: The CaViaR test for $\widehat{VaR}$, $\widehat{CoVaR}_L$ and $\widehat{CoVaR}_{SIM}$ for CYN, $T = 350$, 20080915 – 20100208.

- Null hypothesis of $\widehat{VaR}$ and $\widehat{CoVaR}_{SIM}$ can not be rejected, therefore both $\widehat{VaR}$ and $\widehat{CoVaR}_{SIM}$ algorithms perform well during the crisis period, but $\widehat{CoVaR}_L$ performs not well.



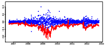
Backtesting results

- $\widehat{VaR}$ performs well only in crisis period.
- $\widehat{CoVaR}_L$ performs not well in both crisis and overall period.
- $\widehat{CoVaR}_{SIM}$ performs well in both crisis and overall period.



Conclusion of CoVaR estimation

- $CoVaR_{SIM}$ risk measure is more precise.
- $CoVaR_{SIM}$ can help us to find the most relevant influential firms.



Estimation of the median by using QR

- Window size: $n = 126$.
- 7 Macroprudential variables are applied.
- Method: quantile regression.
- $\tau = 0.5$.
- $T = 1543$ estimated median by moving window estimation.

Estimation of the median by using QR

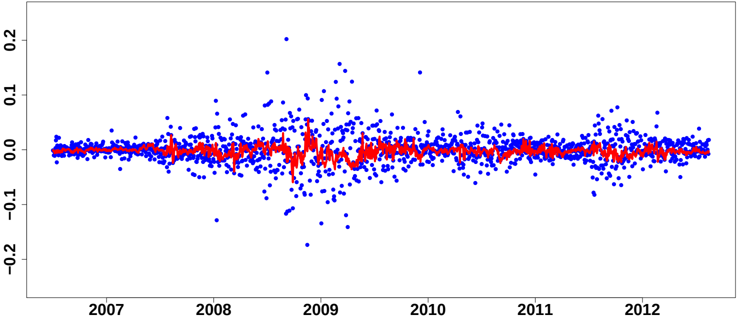
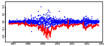
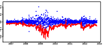


Figure 32: Log returns of CYN (blue) and the estimated median (red), $tau = 0.5$, $T = 1543$, window size $n = 126$, refer to (20).



Estimation of the median by using CQR

- Window size: $n = 126$.
- Original variables: $p = 206$.
- Method: composite quantile regression.
- $\tau = (0.25, 0.35, 0.5, 0.65, 0.75)$.
- Selected variables: Different $\widehat{q}$ in each window.
- $T = 1543$ estimated median by moving window estimation.



Estimation of the median by using CQR

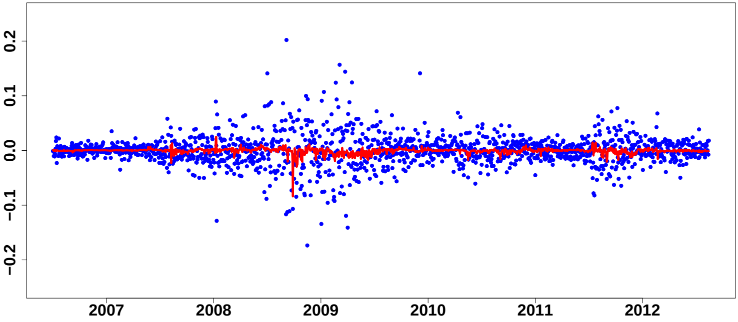


Figure 33: Log returns of CYN (blue) and the estimated median (red), $T = 1543$, window size $n = 126$, refer to (25).

The probability density function of the residuals

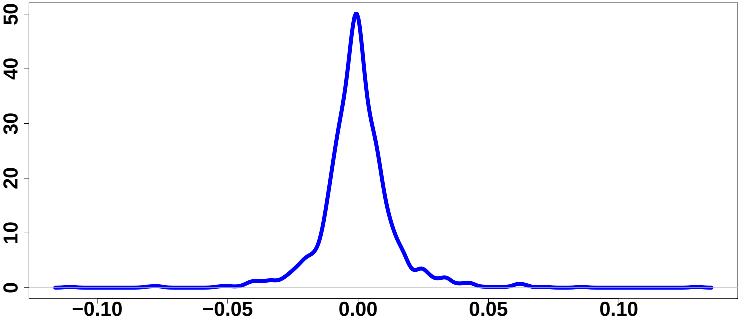
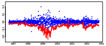


Figure 34: The probability density function of the residuals.



The cumulative distribution function of the residuals

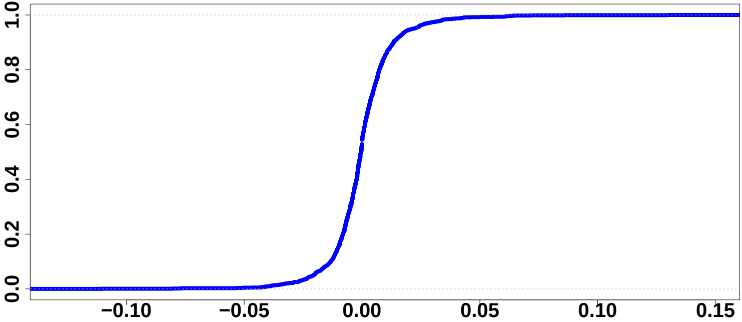
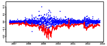


Figure 35: The cumulative distribution function of the residuals.



$\widehat{\beta}$ in each window

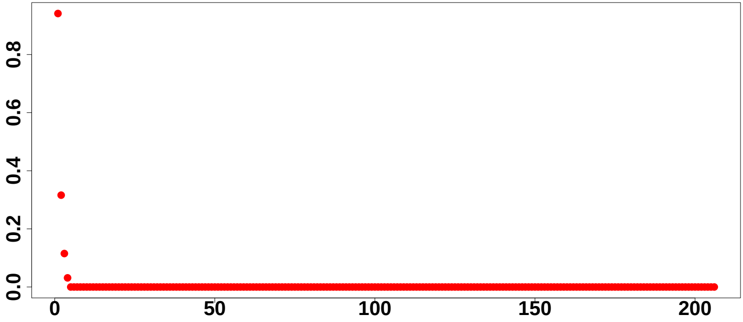


Figure 36: Different $\widehat{\beta}$ by CQR.

The histogram of $\widehat{q}$

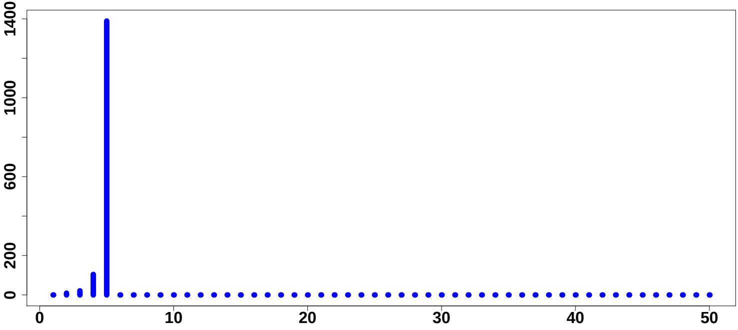


Figure 37: Frequency of the number of selected variables. Where $\widehat{q} = 5$ with frequency 1400, $\widehat{q} = 4$ with frequency 110 and $\widehat{q} = 3$ with frequency 24.

The link function

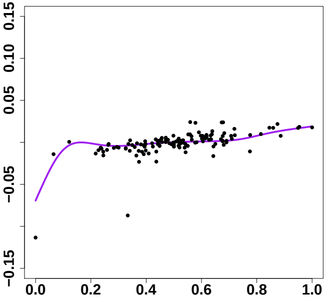


Figure 38: The link functions

The link function

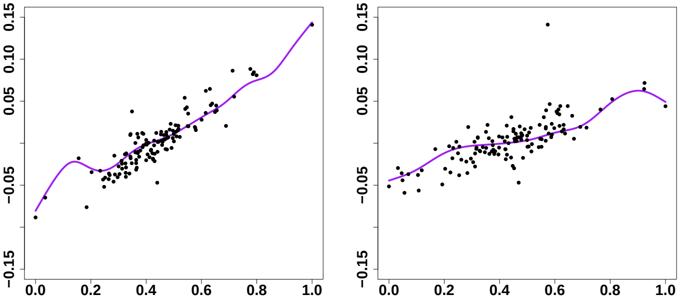


Figure 39: The estimated link function, window size $n = 126$, $\tau = (0.25, 0.35, 0.5, 0.65, 0.75)$, $p = 206$. For the left graph: starting date is 20081029, $h = 0.18$, $\widehat{q} = 3$: MTB, UNM, LNC. For the right graph: starting date is 20101230, $h = 0.26$, $\widehat{q} = 4$: MCY, AB, FBC, VIX.

The influential variables

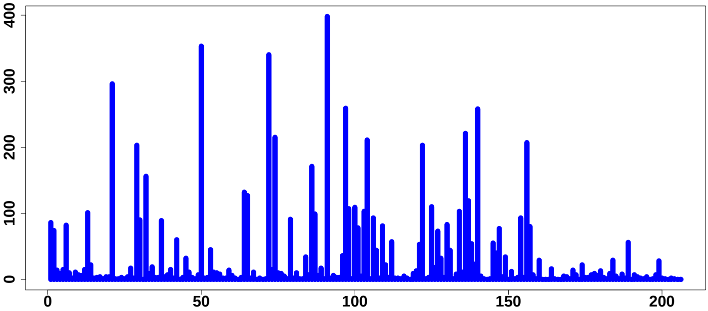
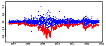


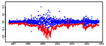
Figure 40: The frequency of the firms and macroprudential variables. The variable 91 "Associated Banc-Corp (ASBC)", the variable 50 "Comerica Incorporated (CMA)" and the variable 72 "Cullen-Frost Bankers, Inc. (CFR)" are the most frequently selected variables with frequency 398, 353 and 340, respectively.

Go to details



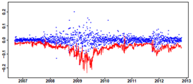
Conclusion of CQR estimation

- CQR estimates conditional median quantile.
- The residuals are symmetric, so the CQR estimation is appropriate.
- Variable selection at median level are different from variable selection at tail level.



CoVaR with very high dimensional risk factors

Wolfgang Karl Härdle
Weining Wang
Lixing Zhu, Lining Yu



Ladislaus von Bortkiewicz Chair of Statistics
C.A.S.E. - Center for Applied Statistics and Economics
Humboldt–Universität zu Berlin
http://lvb.wiwi.hu-berlin.de
http://www.case.hu-berlin.de

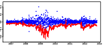
The penalty term

- Lasso, Tibshirani (1996): $\gamma_\lambda(x) = \lambda$
- SCAD, Fan and Li (2001):

$$\gamma_\lambda(x) = \lambda\{\mathbf{I}(x \leq \lambda) + \frac{(a\lambda - x)_+}{(a-1)\lambda}\mathbf{I}(x > \lambda)\},$$

- The adaptive Lasso, Zou (2006): $\gamma_\lambda(x) = \lambda|x|^{-a}$ for some $a > 0$.

Return

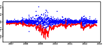


Assumptions

A1 K a cts symmetric pdf, $g(\cdot) \in C^2$.

A2 $\rho_{\mathrm{w}}(x)$ convex. Suppose $\psi_{\mathrm{w}}(x)$, subgradient of $\rho_{\mathrm{w}}(x)$: i) Lipschitz continuous; ii) $\mathsf{E}\,\psi_{\mathrm{w}}(\varepsilon_i) = 0$ and $\inf_{|v| \le c} \partial\, \mathsf{E}\,\psi_{\mathrm{w}}(\varepsilon_i - v) = C_1$.

A3 ε_i is independent of X_i. Let $Z_i = X_i^\top \beta^*$ and $Z_{ij} = Z_i - Z_j$. $C_{0(1)} \stackrel{\mathrm{def}}{=} \mathsf{E}\{g'(Z_i)^2(\mathsf{E}(X_{i(1)}|Z_i) - X_{i(1)})(\mathsf{E}(X_{i(1)}|Z_i - X_{i(1)})\}^\top\}$, and the matrix $C_{0(1)}$ satisfies $0 < L_1 \le \lambda_{\min}(C_{0(1)}) \le \lambda_{\max}(C_{0(1)}) \le L_2$ for positive constants L_1 and L_2. There exists a constant $c_0 > 0$ such that $\sum_{i=1}^n \{\|X_{i(1)}\|/\sqrt{n}\}^{2+c_0} \stackrel{p}{\to} 0$, with $0 < c_0 < 1$. Also $\|\sum_i \sum_j X_{(0)ij}\omega_{ij}X_{(1)ij}^\top \partial\,\mathsf{E}\,\psi_{\mathrm{w}}(v_{ij})\|_{2,\infty} = \mathcal{O}_p(n^{1-\alpha_1})$.

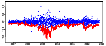


Assumptions

A4 The penalty parameter λ is chosen such that $\lambda D_n = \mathcal{O}\{n^{-1/2}\}$, with $D_n \stackrel{\text{def}}{=} \max\{d_l : l \in \mathcal{M}_*\} = \mathcal{O}(n^{\alpha_1 - \alpha_2/2})$, $d_l \stackrel{\text{def}}{=} \gamma_\lambda(|\beta_l^*|)$, $\mathcal{M}_* = \{l : \beta_l^* \neq 0\}$ be the true model. Furthermore assume $qh \to 0$ as $n \to \infty$, $q = \mathcal{O}(n^{\alpha_2})$, $p = \mathcal{O}(\exp\{n^{\delta}\})$, $nh^3 \to \infty$ and $h \to 0$. Also, $0 < \delta < \alpha < \alpha_2/2 < 1/2$, $\alpha_2/2 < \alpha_1 < 1$. For example, $\delta = 1/5$, $\alpha = 1/4$, $\alpha_2 = 3/5$, $\alpha_1 = 3/5$.

A5 The error term ε_i satisfies $\mathsf{E}\,\varepsilon_i = 0$ and $\mathrm{Var}(\varepsilon_i) < \infty$. Assume that $\mathsf{E}\left|\psi^m(\varepsilon_i)/m!\right| \leq s_0 c^m$ where s_0 and c are constants.

▸ Return



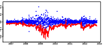
Subgradient

If $f : U \to \mathbb{R}$ is a real-valued convex function defined on a convex open set in the Euclidean space $\mathbb{R}^n$, a vector v in that space is called a subgradient at a point x_0 in U if for any x in U one has

$$f(x) - f(x_0) \geq v \cdot (x - x_0)$$

where the dot denotes the dot product.

▸ Return

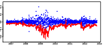


Matrix norm

Assume A is a $m \times n$ matrix

$$\|A\|_{\alpha,\beta} = \max_{x \neq 0} \frac{\|Ax\|_\beta}{\|x\|_\alpha}$$

▸ Return

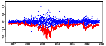


Sparsistency

The result of (11) is stronger than the oracle property defined in Fan and Li (2001) once the properties of $\widehat{\beta}^0$ are established. It was formulated by Kim et al. (2008) for the SCAD estimator with polynomial dimensionality p. It implies not only the model selection consistency and but also sign consistency (Zhao and Yu, 2006; Bickel et al., 2008, 2009):

$$\mathrm{P}\{sgn(\widehat{\beta}) = sgn(\beta^*)\} = \mathrm{P}\{sgn(\widehat{\beta}^0) = sgn(\beta^*)\} \to 1$$

Return



Selection of weights

$$\mathrm{w}_{opt} = \operatorname*{arg\,min}_{\mathrm{w}\geq 0, |\mathrm{w}|=1} \mathrm{w}^\top M \mathrm{w}, \tag{26}$$

where M is a $(K \times K)$ matrix with element in lth row and kth column as $m_{lk} = n^{-1}\sum_{i=1}^{n} \psi_l(\varepsilon_i^{(0)})\psi_k(\varepsilon_i^{(0)})$, $\psi_l(\cdot)$ and $\psi_k(\cdot)$ are the first derivative of $\rho_l(\cdot)$ and $\rho_k(\cdot)$ respectively. $\varepsilon_i^{(0)}$ is the residual induced by the initial estimate $\beta^{(0)}$.

▸ Return

The confidence interval

The $100(1-\alpha)\%$ confidence interval:

$$\Big[\widehat{g}(z) - \frac{1}{\sqrt{nh}} \cdot \frac{\sigma_{\text{w}}\sqrt{\nu_0}}{\sqrt{\widehat{f}_{Z_{(1)}}(z)}} \cdot \mathfrak{z}_{\alpha} + \frac{1}{2}h^2 \widehat{g}''(z)\mu_2 \partial \widehat{\mathsf{E}}\psi_{\text{w}}(\varepsilon);$$

$$\widehat{g}(z) + \frac{1}{\sqrt{nh}} \cdot \frac{\sigma_{\text{w}}\sqrt{\nu_0}}{\sqrt{\widehat{f}_{Z_{(1)}}(z)}} \cdot \mathfrak{z}_{\alpha} + \frac{1}{2}h^2 \widehat{g}''(z)\mu_2 \partial \widehat{\mathsf{E}}\psi_{\text{w}}(\varepsilon)\Big]$$

where $\mathfrak{z}_{\alpha}$ is the α-Quantile of the standard normal distribution, and

$\widehat{f}_{Z_{(1)}}(z) = n^{-1}\sum_{i=1}^{n} K_h(z - Z_{i(1)})$, where $Z_{i(1)} = X_{i(1)}^{\top}\widehat{\beta}_{(1)}$.

▸ Return

Expectile-Quantile Correspondence

Let $v(x)$ represents expectile regression, $l(x)$ represents quantile regression.
Fixed x, define $w(\tau)$ such that $v_{w(\tau)}(x) = l(x)$ then $w(\tau)$ is related to $l(x)$ via

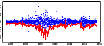
$$w(\tau) = \frac{\tau l(x) - \int_{-\infty}^{l(x)} y dF(y|x)}{2\,\mathrm{E}(Y|x) - 2\int_{-\infty}^{l(x)} y dF(y|x) - (1-2\tau) l(x)}$$

For example, $Y \sim U(-1,1)$, then $w(\tau) = \tau^2/(2\tau^2 - 2\tau + 1)$
Expectile corresponds to quantile with transformation w.

Return to 2-1 Return to 3-11

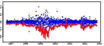
The financial firms and macroprudential variables

The financial frims:	
1. Wells Fargo & Co (WFC)	15. Franklin Resources Inc. (BEN)
2. JP Morgan Chase & Co (JPM)	16. The Travelers Companies, Inc. (TRV)
3. Bank of America Corp (BAC)	17. AFLAC Inc. (AFL)
4. Citigroup Inc (C)	18. Prudential Financial, Inc. (PRU)
5. American Express Company (AXP)	19. State Street Corporation (STT)
6. U.S. Bancorp (USB)	20. The Chubb Corporation (CB)
7. The Goldman Sachs Group, Inc. (GS)	21. BB&T Corporation (BBT)
8. American International Group, Inc. (AIG)	22. Marsh & McLennan Companies, Inc. (MMC)
9. MetLife, Inc. (MET)	23. The Allstate Corporation (ALL)
10. Capital One Financial Corp. (COF)	24. Aon plc (AON)
11. BlackRock, Inc. (BLK)	25. CME Group Inc. (CME)
12. Morgan Stanley (MS)	26. The Charles Schwab Corporation (SCHW)
13. PNC Financial Services Group Inc. (PNC)	27. T. Rowe Price Group, Inc. (TROW)
14. The Bank of New York Mellon Corporation (BK)	28. Loews Corporation (L)



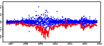
The financial firms and macroprudential variables

29. SunTrust Banks, Inc. (STI)	44. Lincoln National Corporation (LNC)
30. Fifth Third Bancorp (FITB)	45. Affiliated Managers Group Inc. (AMG)
31. Progressive Corp. (PGR)	46. Cincinnati Financial Corp. (CINF)
32. M&T Bank Corporation (MTB)	47. Equifax Inc. (EFX)
33. Ameriprise Financial Inc. (AMP)	48. Alleghany Corp. (Y)
34. Northern Trust Corporation (NTRS)	49. Unum Group (UNM)
35. Invesco Ltd. (IVZ)	50. Comerica Incorporated (CMA)
36. Moody's Corp. (MCO)	51. W.R. Berkley Corporation (WRB)
37. Regions Financial Corp. (RF)	52. Fidelity National Financial, Inc. (FNF)
38. The Hartford Financial Services Group, Inc. (HIG)	53. Huntington Bancshares Incorporated (HBAN)
39. TD Ameritrade Holding Corporation (AMTD)	54. Raymond James Financial Inc. (RJF)
40. Principal Financial Group Inc. (PFG)	55. Torchmark Corp. (TMK)
41. SLM Corporation (SLM)	56. Markel Corp. (MKL)
42. KeyCorp (KEY)	57. Ocwen Financial Corp. (OCN)
43. CNA Financial Corporation (CNA)	58. Arthur J Gallagher & Co. (AJG)



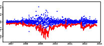
The financial firms and macroprudential variables

59. Hudson City Bancorp, Inc. (HCBK)	74. Commerce Bancshares, Inc. (CBSH)
60. People's United Financial Inc. (PBCT)	75. Signature Bank (SBNY)
61. SEI Investments Co. (SEIC)	76. Jefferies Group, Inc. (JEF)
62. Nasdaq OMX Group Inc. (NDAQ)	77. Rollins Inc. (ROL)
63. Brown & Brown Inc. (BRO)	78. Morningstar Inc. (MORN)
64. BOK Financial Corporation (BOKF)	79. East West Bancorp, Inc. (EWBC)
65. Zions Bancorp. (ZION)	80. Waddell & Reed Financial Inc. (WDR)
66. HCC Insurance Holdings Inc. (HCC)	81. Old Republic International Corporation (ORI)
67. Eaton Vance Corp. (EV)	82. ProAssurance Corporation (PRA)
68. Erie Indemnity Company (ERIE)	83. Assurant Inc. (AIZ)
69. American Financial Group Inc. (AFG)	84. Hancock Holding Company (HBHC)
70. Dun & Bradstreet Corp. (DNB)	85. First Niagara Financial Group Inc. (FNFG)
71. White Mountains Insurance Group, Ltd. (WTM)	86. SVB Financial Group (SIVB)
72. Cullen-Frost Bankers, Inc. (CFR)	87. First Horizon National Corporation (FHN)
73. Legg Mason Inc. (LM)	88. E-TRADE Financial Corporation (ETFC)



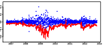
The financial firms and macroprudential variables

89. SunTrust Banks, Inc. (STI)	104. Valley National Bancorp (VLY)
90. Mercury General Corporation (MCY)	105. KKR Financial Holdings LLC (KFN)
91. Associated Banc-Corp (ASBC)	106. Synovus Financial Corporation (SNV)
92. Credit Acceptance Corp. (CACC)	107. Texas Capital BancShares Inc. (TCBI)
93. Protective Life Corporation (PL)	108. American National Insurance Co. (ANAT)
94. Federated Investors, Inc. (FII)	109. Washington Federal Inc. (WAFD)
95. CNO Financial Group, Inc. (CNO)	110. First Citizens Bancshares Inc. (FCNCA)
96. Popular, Inc. (BPOP)	111. Kemper Corporation (KMPR)
97. Bank of Hawaii Corporation (BOH)	112. UMB Financial Corporation (UMBF)
98. Fulton Financial Corporation (FULT)	113. Stifel Financial Corp. (SF)
99. AllianceBernstein Holding L.P. (AB)	114. CapitalSource Inc. (CSE)
100. TCF Financial Corporation (TCB)	115. Portfolio Recovery Associates Inc. (PRAA)
101. Susquehanna Bancshares, Inc. (SUSQ)	116. Janus Capital Group, Inc. (JNS)
102. Capitol Federal Financial, Inc. (CFFN)	117. MBIA Inc. (MBI)
103. Webster Financial Corp. (WBS)	118. Healthcare Services Group Inc. (HCSG)



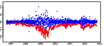
The financial firms and macroprudential variables

119. The Hanover Insurance Group Inc. (THG)	134. BancorpSouth, Inc. (BXS)
120. F.N.B. Corporation (FNB)	135. Privatebancorp Inc. (PVTB)
121. FirstMerit Corporation (FMER)	136. United Bankshares Inc. (UBSI)
122. FirstMerit Corporation (FMER)	137. Old National Bancorp. (ONB)
123. RLI Corp. (RLI)	138. International Bancshares Corporation (IBOC)
124. StanCorp Financial Group Inc. (SFG)	139. First Financial Bankshares Inc. (FFIN)
125. Trustmark Corporation (TRMK)	140. Westamerica Bancorp. (WABC)
126. IberiaBank Corp. (IBKC)	141. Northwest Bancshares, Inc. (NWBI)
127. Cathay General Bancorp (CATY)	142. Bank of the Ozarks, Inc. (OZRK)
128. National Penn Bancshares Inc. (NPBC)	143. Huntington Bancshares Incorporated (HBAN)
129. Nelnet, Inc. (NNI)	144. Euronet Worldwide Inc. (EEFT)
130. Wintrust Financial Corporation (WTFC)	145. Community Bank System Inc. (CBU)
131. Umpqua Holdings Corporation (UMPQ)	146. CVB Financial Corp. (CVBF)
132. GAMCO Investors, Inc. (GBL)	147. MB Financial Inc. (MBFI)
133. Sterling Financial Corp. (STSA)	148. ABM Industries Incorporated (ABM)



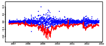
The financial firms and macroprudential variables

149. Glacier Bancorp Inc. (GBCI)	164. Citizens Republic Bancorp, Inc (CRBC)
150. Selective Insurance Group Inc. (SIGI)	165. Horace Mann Educators Corp. (HMN)
151. Park National Corp. (PRK)	166. DFC Global Corp. (DLLR)
152. Flagstar Bancorp Inc. (FBC)	167. Navigators Group Inc. (NAVG)
153. FBL Financial Group Inc. (FFG)	168. Boston Private Financial Holdings, Inc. (BPFH)
154. Astoria Financial Corporation (AF)	169. American Equity Investment Life Holding Co. (AEL)
155. World Acceptance Corp. (WRLD)	170. BlackRock Limited Duration Income Trust (BLW)
156. First Midwest Bancorp Inc. (FMBI)	171. Columbia Banking System Inc. (COLB)
157. PacWest Bancorp (PACW))	172. Safety Insurance Group Inc. (SAFT)
158. First Financial Bancorp. (FFBC)	173. National Financial Partners Corp. (NFP)
159. BBCN Bancorp, Inc. (BBCN)	174. NBT Bancorp, Inc. (NBTB)
160. Provident Financial Services, Inc. (PFS)	175. Tower Group Inc. (TWGP)
161. FBL Financial Group Inc. (FFG)	176. Encore Capital Group, Inc. (ECPG)
162. WisdomTree Investments, Inc. (WETF)	177. Pinnacle Financial Partners Inc. (PNFP)
163. Hilltop Holdings Inc. (HTH)	178. First Commonwealth Financial Corp. (FCF)



The financial firms and macroprudential variables

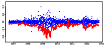
179. BancFirst Corporation (BANF)	190. Berkshire Hills Bancorp Inc. (BHLB)
180. Independent Bank Corp. (INDB)	191. Brookline Bancorp, Inc. (BRKL)
181. Infinity Property and Casualty Corp. (IPCC)	192. National Western Life Insurance Company (NWLI)
182. Central Pacific Financial Corp. (CPF)	193. Tompkins Financial Corporation (TMP)
183. Kearny Financial Corp. (KRNY)	194. BGC Partners, Inc. (BGCP)
184. Chemical Financial Corporation (CHFC)	195. Epoch Investment Partners, Inc. (EPHC)
185. Banner Corporation (BANR)	196. United Fire Group, Inc (UFCS)
186. State Auto Financial Corp. (STFC)	197. 1st Source Corporation (SRCE)
187. Radian Group Inc. (RDN)	198. Citizens Inc. (CIA)
188. SCBT Financial Corporation (SCBT)	199. S&T Bancorp Inc. (STBA)
189. WesBanco Inc. (WSBC)	



The financial firms and macroprudential variables

The macroprudential variables:
200. VIX
201. Short term liquidity spread (liquidity)
202. Daily change in the 3-month Treasury maturities (3MT)
203. Change in the slope of the yield curve (yield)
204. Change in the credit spread (credit)
205. Daily Dow Jones U.S. Real Estate index returns (D_J)
206. S&P500 returns (S&P)

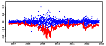
Return



The selected financial firms and macroprudential variables ($CoVaR_L$)

Top 6 influential covariates	Frequency
No. 152 Flagstar Bancorp Inc. (FBC)	885
No. 200 VIX	795
No. 162 WisdomTree Investments, Inc. (WETF)	730
No. 117 MBIA Inc. (MBI)	711
No. 203 Change in the slope of the yield curve (yield)	703
No. 187 Radian Group Inc. (RDN)	676

Return



The selected financial firms and macroprudential variables ($\text{CoVaR}_{\text{SIM}}$)

Top 6 influential covariates	Frequency
No. 187 Radian Group Inc. (RDN)	707
No. 152 Flagstar Bancorp Inc. (FBC)	350
No. 106 Synovus Financial Corporation (SNV)	285
No. 95 CNO Financial Group, Inc. (CNO)	226
No. 65 Zions Bancorp. (ZION)	224
No. 195 Epoch Investment Partners, Inc. (EPHC)	206

Return

The selected financial firms and macroprudential variables (CQR)

Top 6 influential covariates		Frequency
No. 91	Associated Banc-Corp (ASBC)	398
No. 50	Comerica Incorporated (CMA)	353
No. 72	Cullen-Frost Bankers, Inc. (CFR)	340
No. 21	BB&T Corporation (BBT)	296
No. 97	Bank of Hawaii Corporation (BOH)	259
No. 140	Westamerica Bancorp. (WABC)	258

Return

The logistic regression

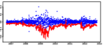
$$
\begin{aligned}
\mathrm{P}(I_t = 1 | I_{t-1}, VaR_t) &= \mathrm{P}(\alpha + \beta_1 I_{t-1} + \beta_2 VaR_t + u_t > 0 | I_{t-1}, VaR_t) \\
&= \Lambda(\alpha + \beta_1 I_{t-1} + \beta_2 VaR_t) \\
&= \frac{e^{\alpha + \beta_1 I_{t-1} + \beta_2 VaR_t}}{1 + e^{\alpha + \beta_1 I_{t-1} + \beta_2 VaR_t}}
\end{aligned}
$$

i.e.

$$
\begin{aligned}
logit(p) &= \log(\frac{p}{1-p}) \\
&= \alpha + \beta_1 I_{t-1} + \beta_2 VaR_t
\end{aligned}
$$

where $p = \mathrm{P}(I_t = 1 | I_{t-1}, VaR_t)$.

Return

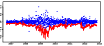


Criteria - Quantile regression (small p case)

$g(\cdot)$	τ	*Dev*	*Acc*	*Angle*	*Error*	*ASE*(h)
	0.95	1.22(0.36)	0.8(3.53)	9.874(0.079)	0.029(0.004)	0.044(0.014)
Model 1	0.50	0.74(0.25)	0.6(1.45)	9.969(0.023)	0.007(0.002)	0.003(0.002)
	0.05	1.75(0.59)	1.8(3.55)	9.829(0.123)	0.038(0.006)	0.064(0.021)
	0.95	1.68(1.88)	6.6(9.32)	9.691(0.666)	7.564(7.159)	4.769(8.771)
Model 2	0.50	1.49(1.46)	1.0(2.82)	9.780(0.401)	5.916(4.874)	1.363(2.305)
	0.05	1.50(1.73)	8.1(9.71)	9.556(0.985)	8.627(8.526)	6.145(9.168)
	0.95	0.37(0.27)	0.4(2.19)	9.989(0.016)	0.141(0.069)	0.574(0.624)
Model 3	0.50	0.11(0.08)	0.3(0.79)	9.997(0.002)	0.051(0.029)	0.076(0.049)
	0.05	0.56(0.32)	0.4(2.28)	9.978(0.025)	0.229(0.063)	0.724(0.711)

Table 4: Criteria evaluated under different models and quantiles. $\beta^{*\top}_{(1)} = (5,5,5,5,5)$, the error ε follows a $\mathrm{N}(0, 0.1)$ distribution. In 100 simulations we set $n = 100, p = 10, q = 5$. Standard deviations are given in brackets. *Dev*, *Acc*, *Angle*, *Error* and their standard deviations are reported in 10^{-1}. *ASE*(h) and its standard deviations are reported in 10^{-2}.

Return

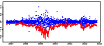


Criteria - Quantile regression (small p case)

$g(\cdot)$	τ	*Dev*	*Acc*	*Angle*	*Error*	*ASE(h)*
	0.95	1.79(0.76)	0.9(4.23)	9.720(0.191)	0.050(0.008)	0.111(0.037)
Model 1	0.50	0.91(0.39)	1.2(2.56)	9.951(0.039)	0.009(0.002)	0.004(0.003)
	0.05	1.92(0.79)	2.5(4.48)	9.746(0.128)	0.053(0.009)	0.122(0.049)
	0.95	2.31(1.88)	9.2(9.48)	9.610(0.668)	9.158(9.561)	5.643(6.561)
Model 2	0.50	1.77(1.59)	5.0(3.58)	9.712(0.487)	8.152(7.278)	1.785(2.814)
	0.05	3.07(1.06)	8.6(9.28)	9.695(0.551)	9.750(7.464)	4.643(4.462)
	0.95	0.32(0.24)	0.5(2.11)	9.987(0.016)	0.235(0.117)	0.759(0.798)
Model 3	0.50	0.29(0.11)	0.3(0.90)	9.994(0.008)	0.077(0.052)	0.081(0.085)
	0.05	0.42(0.26)	0.6(2.26)	9.982(0.019)	0.326(0.201)	0.861(0.863)

Table 5: Criteria evaluated under different models and quantiles. $\beta^{*\top}_{(1)} = (5, 5, 5, 5, 5)$, the error ε follows a $t(5)$ distribution. In 100 simulations we set $n = 100, p = 10, q = 5$. Standard deviations are given in brackets. *Dev*, *Acc*, *Angle*, *Error* and their standard deviations are reported in 10^{-1}. $ASE(h)$ and its standard deviations are reported in 10^{-2}.

Return

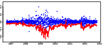


Criteria - Quantile regression (different $\beta^*_{(1)}$)

$g(\cdot)$	$\beta^*_{(1)}$	*Dev*	*Acc*	*Angle*	*Error*	*ASE(h)*
	(a)	1.22(0.36)	0.8(3.53)	9.874(0.079)	0.029(0.004)	0.044(0.014)
Model 1	(b)	1.51(0.36)	1.0(3.62)	9.861(0.092)	0.035(0.005)	0.052(0.019)
	(c)	1.72(0.38)	1.3(3.94)	9.892(0.099)	0.036(0.005)	0.059(0.023)
	(a)	1.68(1.88)	6.6(9.32)	9.691(0.666)	7.564(7.159)	4.769(8.771)
Model 2	(b)	1.85(1.95)	7.4(9.45)	9.541(0.752)	8.135(8.352)	5.731(8.928)
	(c)	2.34(2.21)	9.5(9.88)	9.432(0.856)	8.374(8.973)	7.212(9.134)
	(a)	0.37(0.27)	0.4(2.19)	9.989(0.016)	0.141(0.069)	0.574(0.624)
Model 3	(b)	0.41(0.26)	0.5(2.46)	9.981(0.019)	0.259(0.122)	0.786(0.812)
	(c)	0.53(0.28)	0.6(2.87)	9.973(0.021)	0.352(0.229)	0.814(0.921)

Table 6: Criteria evaluated under three different $\beta^*_{(1)}$: (a) $\beta^{*\top}_{(1)} = (5,5,5,5,5)$, (b) $\beta^{*\top}_{(1)} = (5,4,3,2,1)$, (c) $\beta^{*\top}_{(1)} = (5,2,1,0.8,0.2)$ the error ε follows a $\mathrm{N}(0,0.1)$ distribution. In 100 simulations we set $n = 100, p = 10, q = 5, \tau = 0.95$. Standard deviations are given in brackets. *Dev*, *Acc*, *Angle*, *Error* and their standard deviations are reported in 10^{-1}. *ASE*(h) and its standard deviations are reported in 10^{-2}.

Return

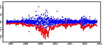


Criteria - Quantile regression (large p case)

$g(\cdot)$	*Dev*	*Acc*	*Angle*	*Error*	*ASE*(h)
Model 1	1.86(0.84)	5.6(6.92)	9.891(0.225)	0.046(0.009)	0.103(0.040)
Model 2	1.85(1.65)	9.7(8.51)	9.873(0.651)	9.731(9.516)	4.971(3.121)
Model 3	0.92(0.39)	6.2(5.72)	9.952(0.041)	1.051(0.108)	1.432(1.042)

Table 7: Criteria evaluated with different models under $p > n$ case. $\beta^{*\top}_{(1)} = (5, 5, 5, 5, 5)$, the error ε follows a $\mathrm{N}(0, 0.1)$ distribution. In 100 simulations we set $n = 100, p = 200, q = 5, \tau = 0.05$. Standard deviations are given in brackets. *Dev*, *Acc*, *Angle*, *Error* and their standard deviations are reported in 10^{-1}, *ASE*(h) and its standard deviations are reported in 10^{-2}.

Return



Criteria - Expectile regression (small p case)

$g(\cdot)$	τ	*Dev*	*Acc*	*Angle*	*Error*	*ASE*(h)
	0.95	0.60(0.25)	0.0(0.00)	9.978(0.018)	0.029(0.002)	0.006(0.002)
Model 1	0.50	0.51(0.21)	0.0(0.00)	9.985(0.013)	0.005(0.001)	0.002(0.001)
	0.05	0.61(0.25)	0.0(0.00)	9.977(0.018)	0.016(0.003)	0.010(0.003)
	0.95	0.78(0.47)	0.0(0.00)	9.958(0.056)	5.306(5.689)	0.571(0.678)
Model 2	0.50	0.64(0.43)	0.0(0.00)	9.969(0.054)	2.643(3.444)	0.249(0.435)
	0.05	0.85(0.44)	0.0(0.00)	9.954(0.055)	5.564(6.216)	0.619(0.670)
	0.95	0.14(0.07)	0.0(0.00)	9.998(0.002)	0.027(0.014)	0.024(0.024)
Model 3	0.50	0.11(0.04)	0.0(0.00)	9.999(0.001)	0.019(0.007)	0.014(0.008)
	0.05	0.26(0.08)	0.0(0.00)	9.996(0.003)	0.062(0.020)	0.116(0.064)

Table 8: Criteria evaluated under different models and quantiles. $\beta^{*\top}_{(1)} = (5,5,5,5,5)$, the error ε follows a $\mathrm{N}(0, 0.1)$ distribution. In 100 simulations we set $n = 100, p = 10, q = 5$. Standard deviations are given in brackets. *Dev*, *Acc*, *Angle*, *Error* and their standard deviations are reported in 10^{-1}. *ASE*(h) and its standard deviations are reported in 10^{-2}.

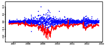
Return

Criteria - Expectile regression (large p case)

$g(\cdot)$	*Dev*	*Acc*	*Angle*	*Error*	*ASE*(h)
Model 1	1.03(0.51)	0.1(0.30)	9.927(0.087)	0.017(0.003)	0.013(0.007)
Model 2	1.77(1.29)	0.0(0.01)	9.863(0.321)	7.351(7.401)	2.698(3.011)
Model 3	0.56(0.26)	0.6(0.98)	9.951(0.137)	0.126(0.106)	1.383(0.382)

Table 9: Criteria evaluated with different models under $p > n$ case. $\beta^{*\top}_{(1)} = (5, 5, 5, 5, 5)$, the error ε follows a $\mathrm{N}(0, 0.1)$ distribution. In 100 simulations we set $n = 100, p = 200, q = 5, \tau = 0.05$. Standard deviations are given in brackets. *Dev*, *Acc*, *Angle*, *Error* and their standard deviations are reported in 10^{-1}, $ASE(h)$ and its standard deviations are reported in 10^{-2}.

▸ Return

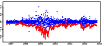


Criteria - Composite quantile regression

$g(\cdot)$	p	*Dev*	*Acc*	*Angle*	*Error*	*ASE*(h)
Model 1	*small*	0.54(0.17)	0.0(0.00)	9.983(0.009)	0.006(0.001)	0.002(0.001)
	large	1.02(0.34)	0.0(0.00)	9.934(0.038)	0.009(0.002)	0.005(0.003)
Model 2	*small*	0.40(0.17)	0.0(0.00)	9.990(0.007)	1.760(1.143)	0.073(0.049)
	large	0.81(0.46)	5.5(0.99)	9.973(0.032)	2.058(1.702)	0.179(0.126)
Model 3	*small*	0.12(0.03)	0.0(0.00)	9.999(0.001)	0.020(0.004)	0.020(0.006)
	large	0.22(0.04)	0.0(0.00)	9.997(0.002)	0.044(0.008)	0.049(0.018)

Table 10: Criteria evaluated under different models and number of p. *small* represents $p = 10$, *large* stands for $p = 200$, $\beta^{*\top}_{(1)} = (5, 5, 5, 5, 5)$, the error ε follows a $\mathrm{N}(0, 0.1)$ distribution. In 100 simulations we set $\tau = (0.25, 0.35, 0.5, 0.65, 0.75)$, $n = 100$, $q = 5$. Standard deviations are given in brackets. *Dev*, *Acc*, *Angle*, *Error* and their standard deviations are reported in 10^{-1}. *ASE*(h) and its standard deviations are reported in 10^{-2}.

Return

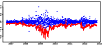


References

Adrian, T. and Brunnermeier, M. K.
CoVaR
Staff Reports 348, Federal Reserve Bank of New York, 2011

Berkowitz, J., Christoffersen, P. and Pelletier, D.
Evaluating value-at-risk models with desk-level data
Working Paper 010, North Carolina State University, Department of Economics, 2009

Bickel, P. J., Ritov, Y. and Tsybakov, A. B.
Hierarchical selection of variables in sparse high-dimensional regression
Borrowing Strength: Theory Powering Applications - A Festschrift for Lawrence D. Brown. IMS Collections, v. 6, 56-69, 2010.

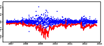


References

Bickel, P. J., Ritov, Y. and Tsybakov, A. B.
Simultaneous analysis of lasso and dantzig selector
Ann. Statist. 37(4), 1705-1732. 2009

Bradic, J., Fan, J. and Wang, W.
Penalized composite quasi-likelihood for ultrahigh dimensional variable selection
J. R. Statist. Soc. B 73(3): 325-349. 2011

Campbell, J. Y., Lo, A. W. and MacKinlay, A. C.
The Econometrics of Financial Markets
Princeton University Press, 1996

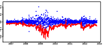


References

Chao, S. K., Härdle, W. K. and Wang, W.
Quantile regression in Risk Calibration
In Handbook for Financial Econometrics and Statistics (Cheng-Few Lee, ed.). Springer Verlag, forthcoming, SFB 649 DP 2012-006.

Engle, R. F. and Manganelli, S.
CAViaR: Conditional Autoregressive Value at Risk by Regression Quantiles
J. Bus. Econ. Stat. 22: 367-381, 2004

Fan, J. and Li, R.
Variable selection via nonconcave penalized likelihood and its oracle properties
J. Amer. Statist. Assoc. 96: 1348-1360. 2001

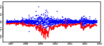


References

Franke, J., Härdle, W. and Hafner, C.
Statistics of Financial Markets
Springer, 2004

Härdle, W. and Stoker, T. M.
Investigating smooth multiple regression by the method of average derivatives
J. Amer. Statist. Assoc., 84: 986-995. 1989

Hao, L. and Naiman, Q. D.
Quantile Regression
Sage Publications, 2007

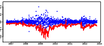


References

Hautsch, N., Schaumburg, J. and Schienle, M.
Financial Network System Risk Contributions
Discussion Paper 2011-072, CRC 649, Humboldt-Universität zu Berlin, 2011

Huber, P. J.
Robust estimation of a location parameter
Ann. Math. Statist., 35(1): 73-101. 1964

Hull C. J.
Risk Management and Financial Institutions – 2nd ed.
Prentice Hall, 2010

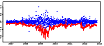


References

Kai, B., Li, R. and Zou, H.
Local composite quantile regression smoothing: an efficient and safe alternative to local polynomial regression
J. R. Statist. Soc. B 72: 49-69. 2010

Kai, B., Li, R. and Zou, H.
New Efficient Estimation and Variable Selection Methods for Semiparametric Varying-Coefficient Partially Linear Models
Ann. Statist., 39(1): 305-332. 2011

Kim, Y., Choi, H. and Oh, H.
Smoothly clipped absolute deviation on high dimensions
J. Amer. Statist. Assoc. 103, 1656-1673. 2008

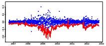


References

Koenker, R. and Bassett, G. W.
Regression quantiles
Econometrica. 46: 33-50. 1978

Koenker, R. and Bassett, G. W.
Robust tests for heteroscedasticity based on regression quantiles
Econometrica 50: 43-61. 1982

Koenker, R. and Hallock, K. F.
Quantile regression
Journal of Econometric Perspectives 15(4): 143-156. 2001

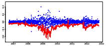


References

Kong, E. and Xia, Y.
Variable selection for the single-index model
Biometrika. 94: 217-229. 1994

Leider J.
A Quantile Regression Study of Climate Change in Chicago
available on www.siam.org, 2012

Leng, C., Xia, Y. and Xu, J.
An adaptive estimation method for semiparametric models and dimension reduction
WSPC-Proceedings. 2010

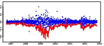


References

Li, Y. and Zhu, J.
L1- norm quantile regression
J. Comput. Graph. Stat., 17: 163-185. 2008

Newey, W. and Powell, J.
Asymmetric least squares estimation and testing
Econometrica 55: 819-847. 1987

Koenker R.
Quantile Regression
Cambridge Univ. Press, 2000

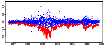


References

Ruppert, D., Sheather, S. J. and Wand, M. P.
An effective bandwidth selector for local least squares regression.
J. Amer. Statist. Assoc. 90: 1257-1270. 1995

Schnabel, S. and Eilers, P.
Optimal expectile smoothing
Comput. Stat. Data. An. 53(12): 4168-4177. 2009

Serfling, R. J.
Approximation theorems of mathematical statistics
Wiley, New York. 1980



References

Tibshirani, R.
Regression shrinkage and selection via the lasso
J. R. Statist. Soc. B 58(1): 267-288. 1996

Wang, Q. and Yin, X.
A nonlinear multi-dimensional variable selection methods for highdimensional data: sparse mave
Comput. Stat. Data. An., 52: 4512-4520. 2008

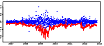
Wu, T. Z., Yu, K. and Yu, Y.
Single-index quantile regression
J. Multivariate Anal., 101: 1607-1621. 2010

References

Xia, Y., Tong, H., Li, W. and Zhu, L.
An adaptive estimation of dimension reduction space
J. R. Statist. Soc. B 64, Part 3, pp. 363-410, 2002

Yu, K. and Jones, M. C.
Local linear quantile regression
J. Amer. Statist. Assoc. 98: 228-237, 1998

Yuan, M.
GACV for Quantile Smoothing Splines
Comput. Stat. Data. An., 5, 813-829. 2006



References

Yuan, M. and Lin, Y.
Model selection and estimation in regression with grouped variables
J. R. Statist. Soc. B 68(1): 49-67. 2006

Zeng, P., He, T. H. and Zhu, Y.
A lasso-type approach for estimation and variable selection in single-index models
J. Comput. Graph. Stat., 21: 92-109. 2012

Zhao, P. and Yu, B.
On model selection consistency of Lasso
J. Machine Learning Res., 7, 2541-2567. 2006

References

Zou, H.
The adaptive Lasso and its oracle properties
J. Amer. Statist. Assoc. 101(476): 1418-1429. 2006