# Risk Patterns and Correlated Brain Activities

Alena Myšičková
Piotr Majer
Song Song
Peter N. C. Mohr
Wolfgang K. Härdle
Hauke R. Heekeren

C.A.S.E. Centre for Applied Statistics and Economics
Humboldt-Universität zu Berlin
Freie Universität Berlin
Max Planck Institute for Molecular Genetics
http://lvb.wiwi.hu-berlin.de
http://www.languages-of-emotion.de
http://www.molgen.mpg.de

# Risk Perception

- Which part is activated during *risk related decisions* ?
- Can statistical analysis help to detect this area?
- Response curve (to stimuli)? classify “risky people”?

# Risk Perception

- Survey conducted by Max Planck Institute

- 22 young, native German, right-handed and healthy volunteers

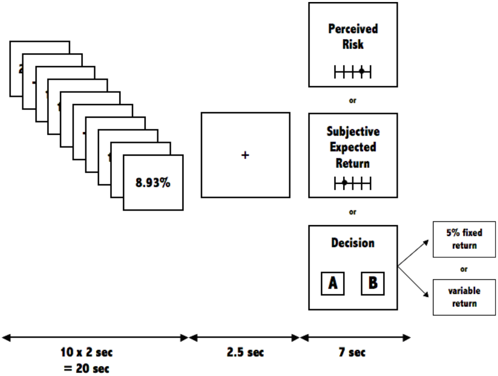
  3 subjects with extensive head movements ($> 5mm$)

  2 subjects with different stimulus frequency

  $n = 22 - (3 + 2) = 17$

- Experiment
  - Risk Perception and Investment Decision (▸ RPID) task ($\times 81$)
  - fMRI images every 2.5 sec.
  - Analysis of the first part ($\times 45$)

# Risk Perception




Returns Pause Decision

# Risk Perception

- functional Magnetic Resonance Imaging

- Measuring Blood Oxygenation Level Dependent (BOLD) effect every 2-3 sec
  High-dimensional, high frequency & large data set

# Risk Perception

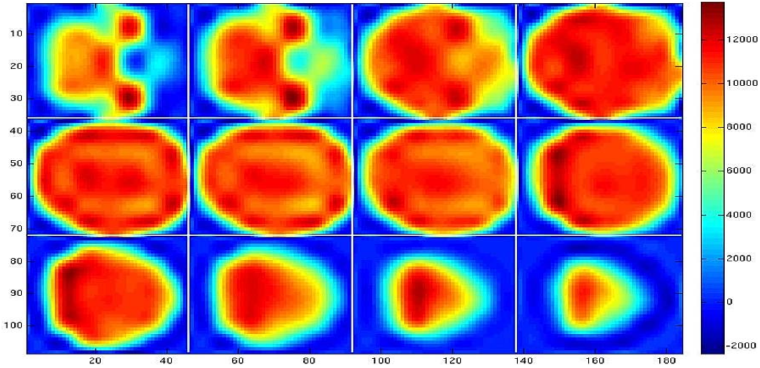
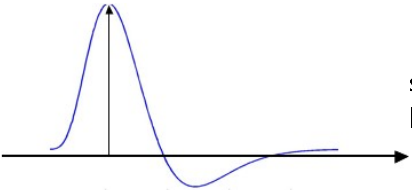


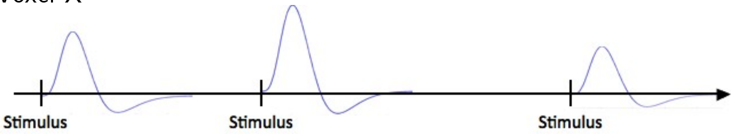

Figure 1: fMRI image observed every 2.5 sec, 12 horizontal slices of the brain's scan, $91 \times 92 \times 71 (x, y, z)$ data points of size 22 MB; scan resolution: $2 \times 2 \times 2mm^3$ fMRI

# fMRI



Is there a significant reaction to specific stimuli in the hemodynamic response?

Voxel X



# fMRI methods

- Voxel-wise GLM ‣ Voxel-wise GLM
  - linear model for each voxel separately
  - strong a priori hypothesis necessary
- Dynamic Semiparametric Factor Model (DSFM)
  - Use a "time & space" dynamic approach
  - Separate low dim time dynamics from space functions
  - Low dim time series exploratory analysis

# Outline

1. Motivation ✓
2. DSFM
3. Results vs. Subject's Behaviour
4. Conclusion
5. Future Perspectives
6. References
7. Appendix

# Notation

$$\underbrace{(X_{1,1},Y_{1,1}),\ldots,(X_{J,1},Y_{J,1})}_{t=1},\ldots,\underbrace{(X_{1,T},Y_{1,T}),\ldots,(X_{J,T},Y_{J,T})}_{t=T},$$

$X_{j,t} \in \mathbb{R}^d,\ Y_{j,t} \in \mathbb{R}$
$T$ - the number of observed time periods
$J$ - the number of the observations in a period $t$
$\mathrm{E}(Y_t|X_t) = F_t(X_t)$

**Quantify $F_t(X_t)$. How does it move?**

# Dynamic Semiparametric Factor Model

$$\mathsf{E}(Y_t|X_t) = \sum_{l=0}^{L} Z_{t,l} m_l(X_t) = Z_t^\top m(X_t) = Z_t^\top A^* \Psi$$

$$Z_t = (\mathbf{1}, Z_{t,1}, \ldots, Z_{t,L})^\top \text{low dim (stationary) time series}$$

$$m = (m_0, m_1, \ldots, m_L)^\top, \text{tuple of functions}$$

$$\Psi = \{\psi_1(X_t), \ldots, \psi_K(X_t)\}^\top, \psi_k(x) \text{ space basis}$$

$$A^* : (L+1) \times K \text{ coefficient matrix}$$

# DSFM Estimation

$$Y_{t,j} = \sum_{l=0}^{L} Z_{t,l} m_l(X_{t,j}) + \varepsilon_{t,j} = Z_t^\top A^* \psi(X_{t,j}) + \varepsilon_{t,j}$$

- $\psi(x) = \{\psi_1(x), \ldots, \psi_K(x)\}^\top$ tensor $B$-spline basis

$$(\widehat{Z}_t, \widehat{A^*}) = \arg \min_{Z_t, A^*} \sum_{t=1}^{T} \sum_{j=1}^{J} \left\{ Y_{t,j} - Z_t^\top A^* \psi(X_{t,j}) \right\}^2 \tag{1}$$

- Minimization by Newton-Raphson algorithm

# B-Splines

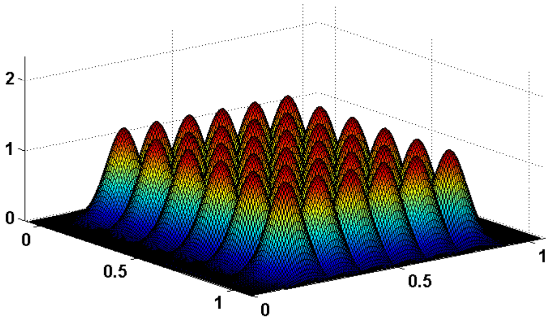



Figure 2: $B$-splines basis functions; order of $B$-splines: quadratic; number of knots: $6 \times 6 = 36$ B-Splines

# DSFM Estimation

- Selection of $L$ by explained variance

$$EV(L) = 1 - \frac{\sum_{t=1}^{T}\sum_{j=1}^{J}\left\{Y_{t,j} - \sum_{l=0}^{L} Z_{t,l} m_l(X_{t,j})\right\}^2}{\sum_{t=1}^{T}\sum_{j=1}^{J}\left\{Y_{t,j} - \bar{Y}\right\}^2}$$

number of $B$-splines (equally spaced) knots: $K = 12 \times 14 \times 14$

| $L = 2$ | $L = 4$ | $L = 5$ | $L = 10$ | $L = 20$ |
|---|---|---|---|---|
| 92.07 | 92.25 | 92.29 | 93.66 | 95.19 |

Table 1: $EV$ in percent of the model with different numbers of factors $L$, averaged over all 17 analyzed subjects.

# Panel DSFM

$$Y^i_{t,j} = \sum_{l=0}^{L} (Z^i_{t,l} + \alpha^i_{t,l}) m_l(X_{t,j}) + \varepsilon_{t,j},{}^{i}, \quad 1 \leq j \leq J,\ 1 \leq t \leq T,$$

- $n = 17$ weakly/strongly risk-averse subjects

- $Y_{t,j}$ - BOLD signal; $X_j$ voxel's index
  $\alpha^i_{t,l}$ - fixed individual effect; Residual Analysis

- Identification condition: $\mathrm{E}\left\{ \sum_{i=1}^{n} \sum_{l=0}^{L} \alpha^i_{t,l} m_l(X_{t,j}) | X_{t,j} \right\} = 0$

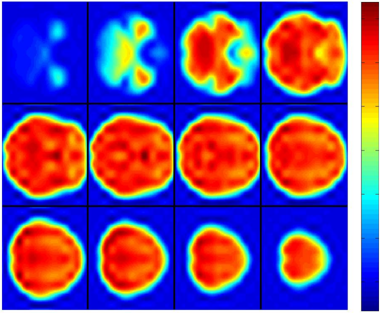
# Panel DSFM Estimation

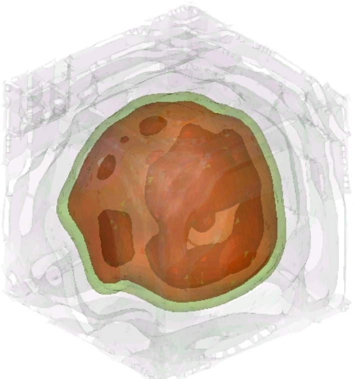
Feasible estimation algorithm:

1. Average $Y^i_{t,j}$ over subjects $i$ to obtain $\bar{Y}_{t,j}$
2. Estimate factors $m_l$ for the "average brain" [via (1)]
3. Given $\widehat{m}_l$, for $i$, estimate $Z^i_{t,l}$

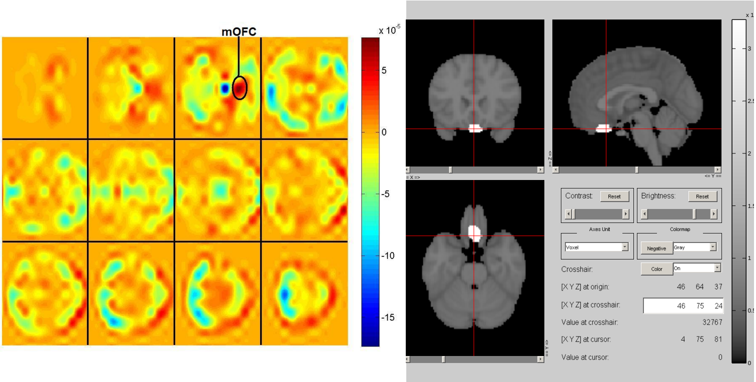
$$Y^i_{t,j} = \sum_{l=0}^{L} Z^i_{t,l} \widehat{m}_l(X_{t,j}) + \varepsilon^i_{t,j}$$

- 26h - computing time; CPU - $2 \times 2.8$GHz; data set of size 24.31 GB

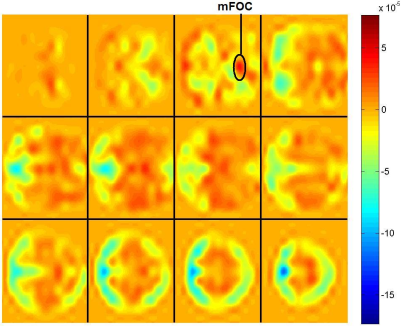


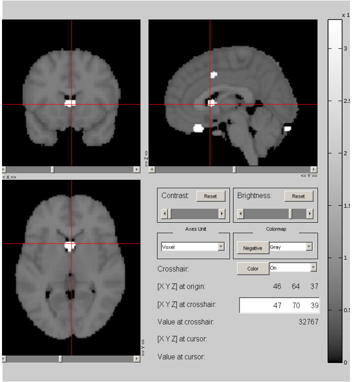


Estimated constant factor $\widehat{m}_0(X) = \sum_{k=1}^{K} \widehat{a}_{0,k}\psi_k(X)$ with $L = 20$

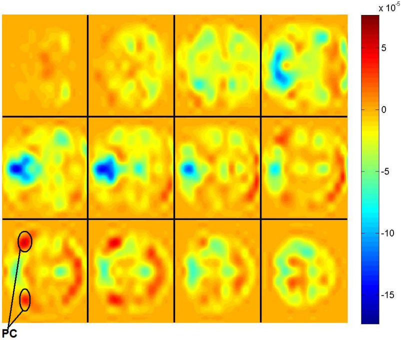


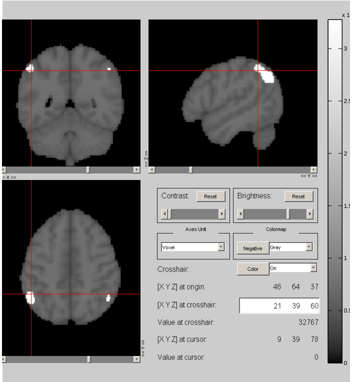

Estimated factor $\widehat{m}_5(X) = \sum_{k=1}^{K} \widehat{a}_{5,k}\psi_k(X)$ with $L = 20$
(MOFC = Medial orbitofrontal cortex)



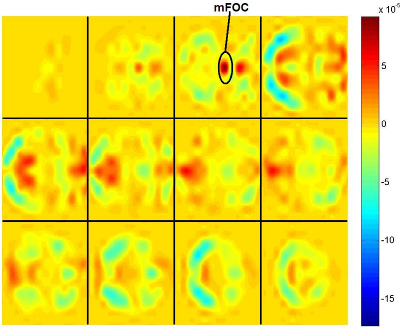



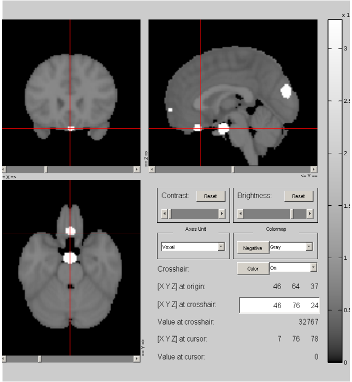

Estimated factor $\widehat{m}_9(X) = \sum_{k=1}^{K} \widehat{a}_{9,k}\psi_k(X)$ with $L = 20$



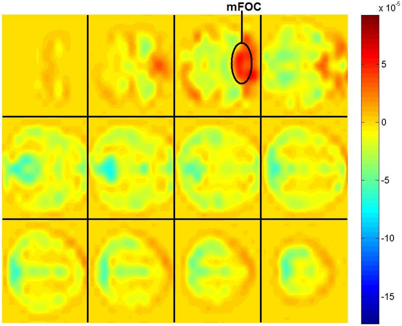



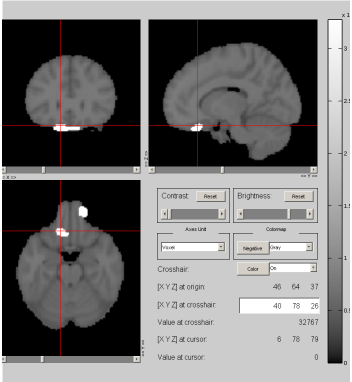

Estimated factor $\widehat{m}_{12}(X) = \sum_{k=1}^{K} \widehat{a}_{12,k}\psi_k(X)$ with $L = 20$
(PC = Paretial Cortex)



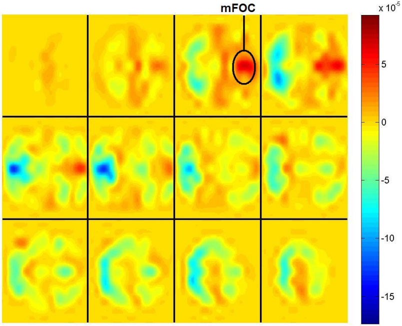



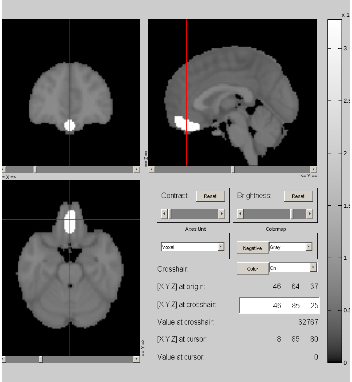

Estimated factor $\widehat{m}_{16}(X) = \sum_{k=1}^{K} \widehat{a}_{16,k}\psi_k(X)$ with $L = 20$




Estimated factor $\widehat{m}_{17}(X) = \sum_{k=1}^{K} \widehat{a}_{17,k}\psi_k(X)$ with $L = 20$




Estimated factor $\widehat{m}_{18}(X) = \sum_{k=1}^{K} \widehat{a}_{18,k}\psi_k(X)$ with $L = 20$

# Estimated Factor Loading $\widehat{Z}_5$

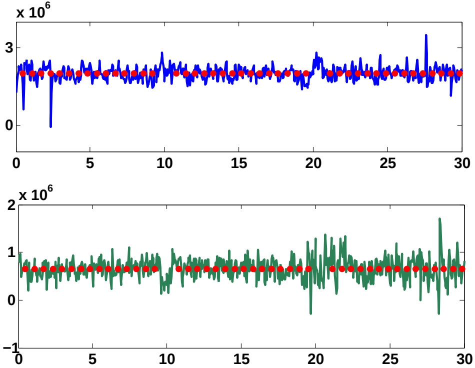



Figure 3: Estimated factor loading $\widehat{Z}_5$ for subjects within 30 minutes: 12 (upper panel) and 19 (lower panel) with $L = 20$; red dots denote stimulus

# Estimated Factor Loading $\widehat{Z}_9$

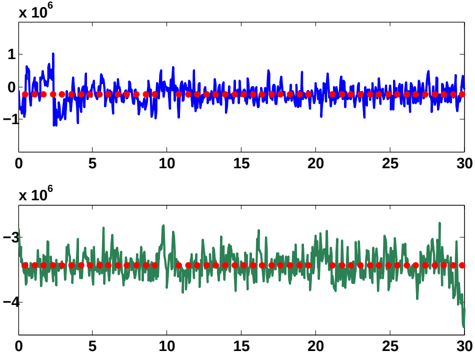



Figure 4: Estimated factor loading $\widehat{Z}_9$ for subjects within 30 minutes: 12 (upper panel) and 19 (lower panel) with $L = 20$; red dots denote stimulus

# Estimated Factor Loading $\widehat{Z}_{12}$

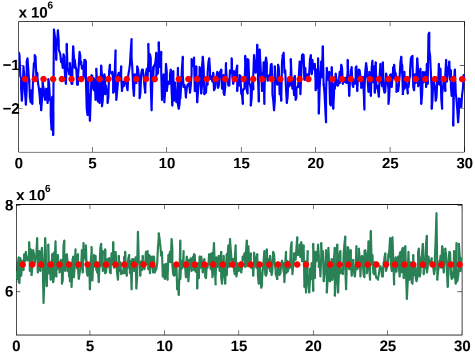



Figure 5: Estimated factor loading $\widehat{Z}_{12}$ for subjects within 30 minutes: 12 (upper panel) and 19 (lower panel) with $L = 20$; red dots denote stimulus

# Estimated Factor Loading $\widehat{Z}_{16}$

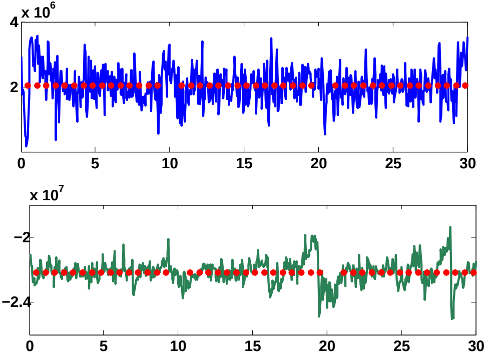



Figure 6: Estimated factor loading $\widehat{Z}_{16}$ for subjects within 30 minutes: 12 (upper panel) and 19 (lower panel) with $L = 20$; red dots denote stimulus

# Estimated Factor Loading $\widehat{Z}_{17}$

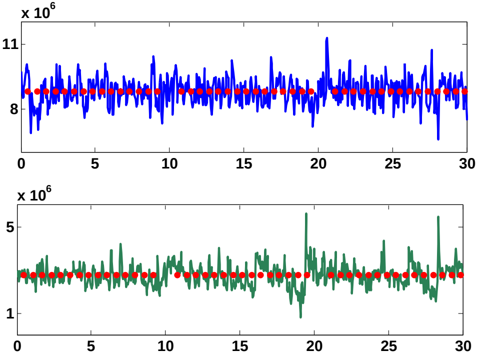



Figure 7: Estimated factor loading $\widehat{Z}_{17}$ for subjects within 30 minutes: 12 (upper panel) and 19 (lower panel) with $L = 20$; red dots denote stimulus

# Estimated Factor Loading $\widehat{Z}_{18}$

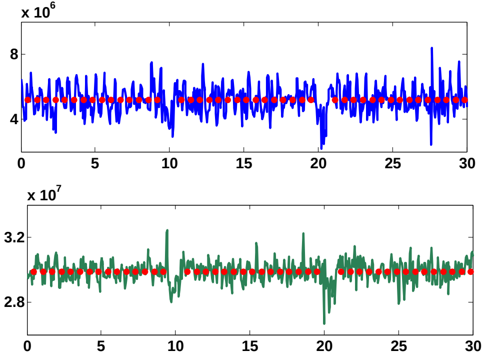



Figure 8: Estimated factor loading $\widehat{Z}_{18}$ for subjects within 30 minutes: 12 (upper panel) and 19 (lower panel) with $L = 20$; red dots denote stimulus

# Reaction to the stimulus

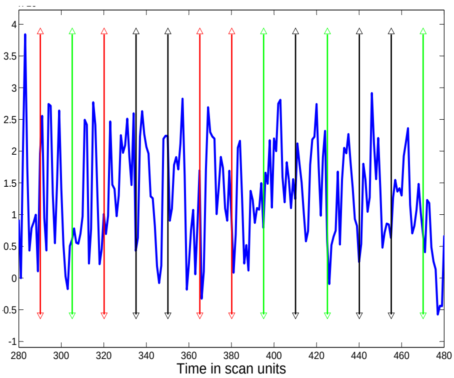



Figure 9: Detailed view of factor loading $\widehat{Z}_1$ for subject 12 with vertical lines in time points of stimuli of 3 different task: decision (red), subjective expected return (green) and perceived risk (black)

# Reaction to the stimulus

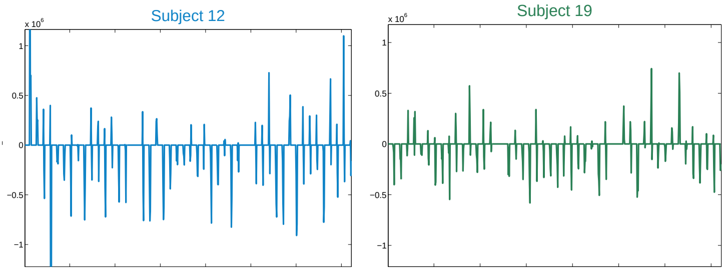



Figure 10: Reaction to stimulus $\overline{\Delta}\widehat{Z}^i_{s,l} = \frac{1}{3}\sum_{\tau=1}^{3}\Delta\widehat{Z}^i_{s+\tau,l}$, where $\Delta\widehat{Z}^i_{t,l} \stackrel{\text{def}}{=} \widehat{Z}^i_{s+t,l} - \widehat{Z}_{s,l}$, $t = 1, 2, 3$, $s$ is the time of stimulus for factors loadings $\widehat{Z}^i_{t,12}$, for subjects 12 (left) and 19 (right) during the experiment (45 stimuli).

# Risk attitude

- Subject's risk perception $\widetilde{R}_{i,s}$ - Risk Metrics
  - standard deviation
  - empirical frequency of loss (negative return)
  - difference between highest an lowest return (range)
  - coefficient of range (range/mean)
  - empirical frequency of ending below 5%
  - coefficient of variation (standard deviation/mean)

- Different subject - different risk perception

  fitted by correlation between risk metrics of return streams and $R_{i,j,s}$ - answers for "perceived risk" task $Q1$, $N = 27$

# Risk attitude

- Subjective expected return $\widetilde{m}_{i,s}$ - ▸ Return Ratings
  - recency (higher weights on later returns)
  - primacy (higher weights on earlier returns)
  - below 0% (higher weights on returns below 0%)
  - below 5% (higher weights on returns below 5%)
  - mean

- Selecting return ratings for each subject individually

  best model selected by prediction power of one-leave-out cross validation procedure, $N = 27$

# Risk attitude

- Each subject $i$ has $(R_i, m_i)$
- Risk-return choice model

$$V_i(x_s) = m_i(x_s) - \beta_i R_i(x_s), \quad 1 \leq i \leq n, 1 \leq s \leq 27$$

$x_s$ - return stream, $m_i$-subjective expected return, $R_i$ - perceived risk , $V_i$ - subjective value (unobserved), 5% - risk free return

- $\beta$ Risk attitude parameter

# Risk attitude

- Estimation of individual risk attitude by logistic regression

$$\begin{aligned} \mathrm{P}\left\{\text{risky choice}|(m,R)\right\} &= \frac{1}{1+\exp(m-\beta R-5)} \\ \mathrm{P}\left\{\text{sure choice}|(m,R)\right\} &= 1-\frac{1}{1+\exp(m-\beta R-5)} \end{aligned}$$

risky choice - unknown return, sure choice - fixed, 5% return

- $\widehat{\beta}$ derived by maximum likelihood method

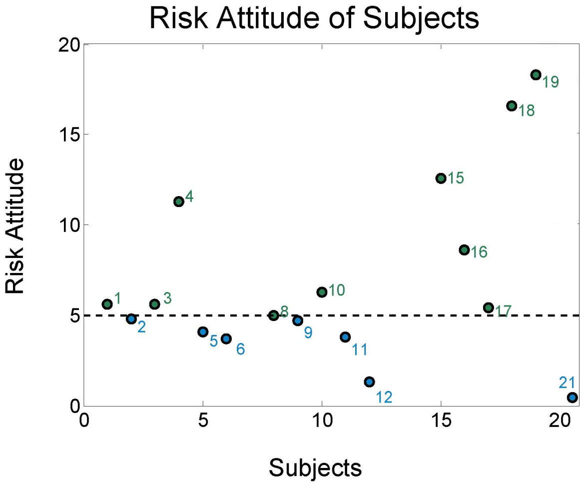



Figure 11: Risk attitude $\widehat{\beta}_i$ for 17 subjects; modeled by the softmax function from individuals' decisions, estimated by ML method ▸ Mohr et. al.

# SVM Classification Analysis

- Support Vector Machines (SVM)
  17 subjects, 20 factor loading time series per subject
- Leave-one-out method to train and estimate classification rate
  SVM with Gaussian kernel; $(R, C)$ chosen to maximize classification rate
- Weakly/strongly risk-averse subjects differ in reaction to stimulus $\Delta\widehat{Z}^{i}_{t,l}$ Reaction to Stimulus

# SVM Classification Analysis

1. factors attributed to risk patterns: $l = 5, 9, 12, 16, 17, 18$
2. only "Decision under Risk" ($Q3$) stimulus
3. average reaction to $s$ stimulus $\overline{\Delta}\widehat{Z}^{i}_{s,l} = \frac{1}{3}\sum_{\tau=1}^{3}\Delta\widehat{Z}^{i}_{s+\tau,l}$

**SVM** input data: volatility of $\overline{\Delta}\widehat{Z}^{i}_{s,l}$ over all $Q3$

| Std | | Estimated | |
| --- | --- | --- | --- |
| | | Strongly | Weakly |
| Data | Strongly | 1.00 | 0.00 |
| | Weakly | 0.14 | 0.86 |

Table 2: Classification rates of the SVM method, **without** knowing the subject's estimated risk attitude SVM Scores

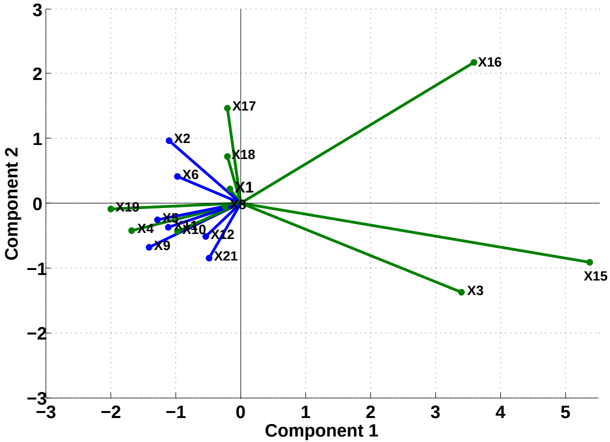



Figure 12: Normalized Principal Component Analysis on volatility of $\overline{\Delta \widehat{Z}}_{s,l}^{i}$ after stimulus for weakly/strongly risk-averse subjects; variance explained by the first and second components: 72%, 85%, respectively

# Conclusion

- Factors $\widehat{m}$ identify activated areas, neurological reasonable
- Estimated factor loadings show differences for individuals with different risk attitudes (e.g. 12 vs. 19)
- SVM classification analysis of measurements in $\widehat{Z}_{t,l}$, $l = 5, 9, 12, 16, 17, 18$ after stimulus, can distinguish weakly/strongly risk-averse individuals with high classification rate, **without** knowing the subject's answers

# Future Perspectives

- Comparison with the PCA/ICA (PARAFAC) approach
- Analysis of the second part of the experiment (under assumption of independency) to "generate" larger number of subjects
- Improvement of the classification criterion
- Penalized DSFM with seasonal effects

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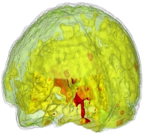

C.A.S.E. Centre for Applied Statistics and Economics
Humboldt-Universität zu Berlin
Freie Universität Berlin
Max Planck Institute for Molecular Genetics
http://lvb.wiwi.hu-berlin.de
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# References


Joule, J. P.
On the Mechanical Equivalent of Heat
*British Association Rep., trans. Chemical Section*, 35, 1845.

Mayer, R.
Remarks on the Forces of Nature
*The Benjamin/Cummings Publishing Company, London*, 1841.

Mohr, P., Biele G., Krugel, L., Li S., Heekeren, H. ▸ Risk attitudes
Neural foundations of risk-return trade-off in investment decisions
*NeuroImage*, 49: 2556-2563, 2010.

# References


Park, B., Mammen, E., Härdle, W. and Borak, S.
Time Series Modelling with Semiparametric Factor Dynamics
*J. Amer. Stat. Assoc.*, 104(485): 284-298, 2009.

Ramsay, J. O. and Silverman, B. W.
Functional Data Analysis
*New York: Springer*, 1997.

Woolrich, M., Ripley, B., Brady, M., Smith, S.
Temporal Autocorrelation in Univariate Linear Modelling of FMRI Data
*NeuroImage*, 21: 2245-2278, 2010

# Voxel-wise GLM ▸ fMRI methods

- FEAT - FMRI Expert Analysis Tool by Department of Clinical Neurology, University of Oxford

- GLM framework

$$Y = XB + \eta, \tag{2}$$

$Y$ - single voxel BOLD time series, $X$ - design matrix (regressors, i.e. visual, auditory)

- Significant, active areas ($B$) selected by *z-scores* $\equiv \frac{B_i - 0}{\sqrt{\mathrm{Var}(B_i)}}$ and grouping (20 neighbors) scheme

# Voxel-wise GLM

fMRI methods

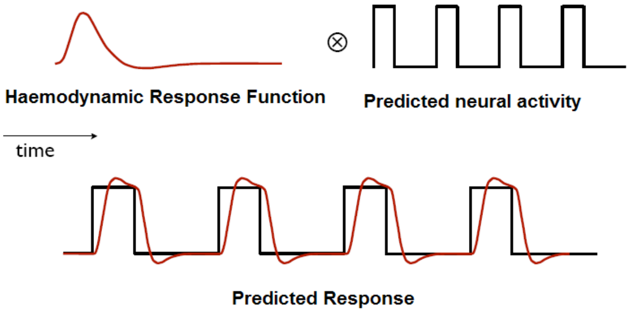


Figure 13: Predicted reaction to the stimulus as a convolution of a stimulus signal (predicted neural response) and a haemodynamic response function as an example of the elements of design matrix $X$ (2). Figure modified from FEAT - FMRI.

# B-Splines

▸ B-Splines

Univariate **B-spline** basis $\Psi = \{\psi_1(X), \ldots, \psi_K(X)\}^\top$ is a series of $\psi_k(X)$ functions defined by $x_0 \leq x_2 \leq \ldots \leq x_{K-1}$, $K$ knots and order $p$, i.e. for $p = 2$ (quadratic)

$$\psi_j(x) = \begin{cases} \frac{1}{2}(x - x_j)^2 & \text{if } x_j \leq x < x_{j+1} \\ \frac{1}{2} - (x - x_{j+1})^2 + (x - x_{j+1}) & \text{if } x_{j+1} \leq x < x_{j+2} \\ \frac{1}{2}\left\{1 - (x - x_{j+2})^2\right\} & \text{if } x_j \leq x < x_{j+1} \\ x & \text{otherwise} \end{cases}$$

# B-Splines

▸ B-Splines

- Knots $K$ and order $p$ has to be specified in advance ($EV$ criterion); $K$ corresponds to bandwidth

- In higher dimensions, for $dim(X) = d > 1$

$$\Psi = \{\psi_1(X_1), \ldots, \psi_{K_1}(X_1)\} \times \ldots \times \{\psi_1(X_d), \ldots, \psi_{K_d}(X_d)\}$$

- Flexible and computationally efficient approach to capture various spatial structures

# Residual Analysis

PDSFM

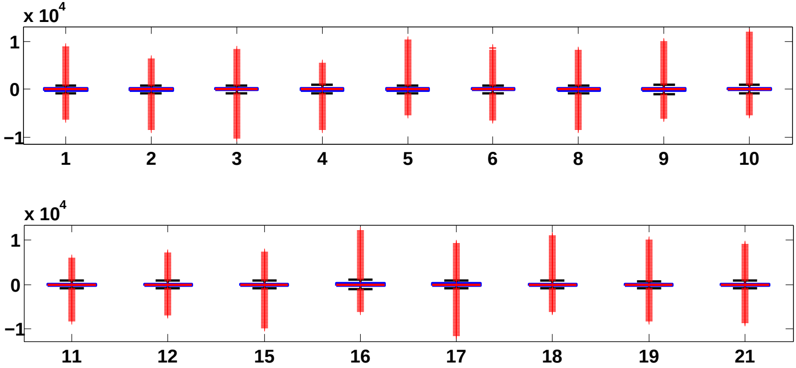


Figure 14: Boxplots of random subsets (size $3 \times 10^7$) from $\varepsilon^i_{t,j}$ ($4.3 \times 10^9$ points) for all 17 analyzed subjects. Kurtosis exceeds 10

# Residual Analysis

PDSFM

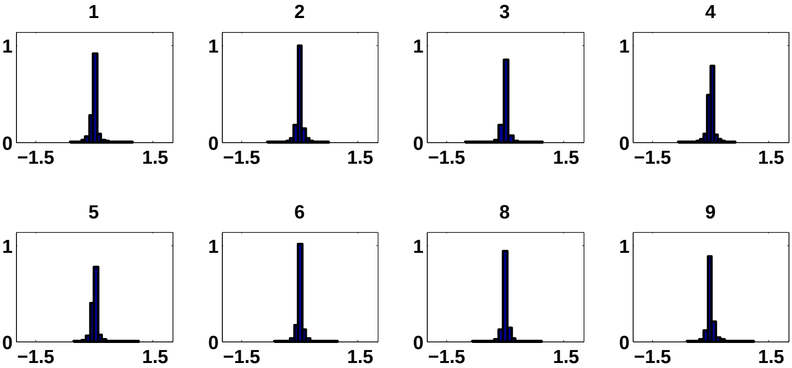


Figure 15: Histograms of random subsets (size $3 \times 10^7$) from $\varepsilon^i_{t,j}$ ($4.3 \times 10^9$ points) for subjects $i = 1, 2, 3, 4, 5, 6, 8, 9$, respectively. Normality hypothesis (**KS test**) for standardized $\varepsilon^i_{t,j}$ rejected for all subjects, $\alpha = 5\%$

# Residual Analysis

PDSFM

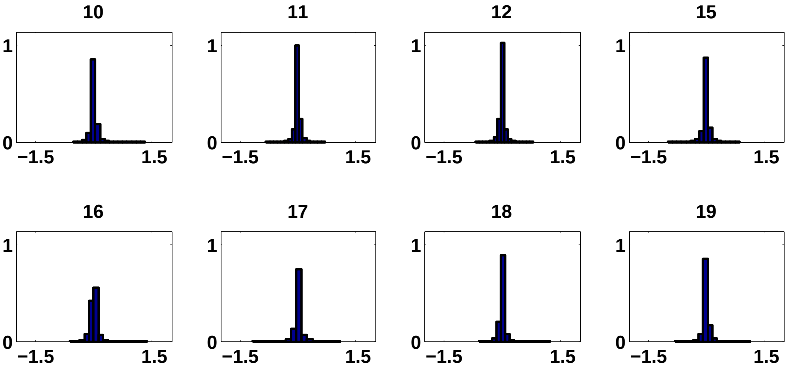


Figure 16: Histograms of random subsets (size $3 \times 10^7$) from $\varepsilon^i_{t,j}$ ($4.3 \times 10^9$ points) for subjects $i = 10, 11, 12, 15, 16, 17, 18, 19$ respectively

# Residual Analysis

▸ PDSFM

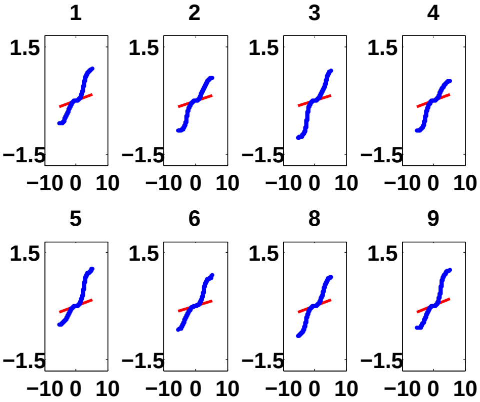


Figure 17: QQplots of random subsets (size $3 \times 10^7$) from $\varepsilon^i_{t,j}$ ($4.3 \times 10^9$ points) for subjects $i = 1, 2, 3, 4, 5, 6, 8, 9$, respectively

# Residual Analysis

▸ PDSFM

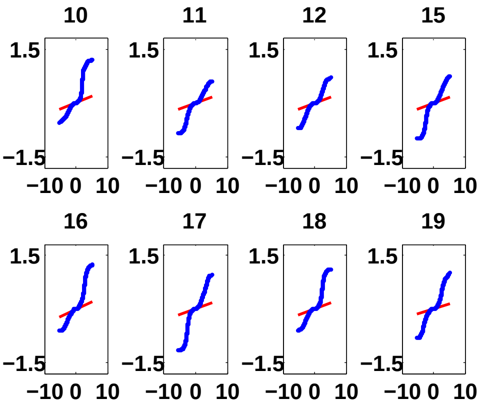


Figure 18: QQplots of random subsets (size $3 \times 10^7$) from $\varepsilon^i_{t,j}$ ($4.3 \times 10^9$ points) for subjects $i = 10, 11, 12, 15, 16, 17, 18, 19$ respectively

# Reaction to stimulus

▸ SVM Analysis

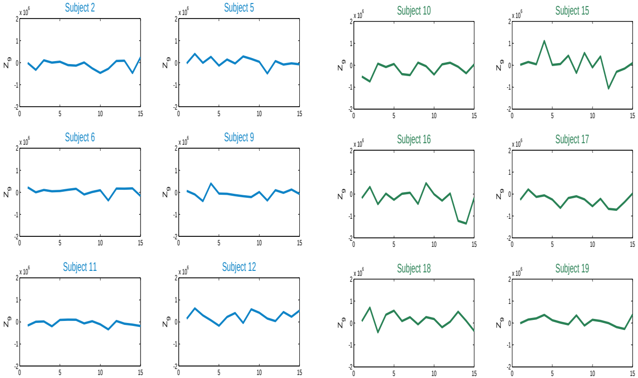


Figure 19: Averaged reaction $\overline{\Delta}\widehat{Z}^{i}_{s,9}$ to stimulus for all 15 $Q3$ questions for weakly/strongly risk-averse individuals

# Reaction to stimulus

SVM Analysis

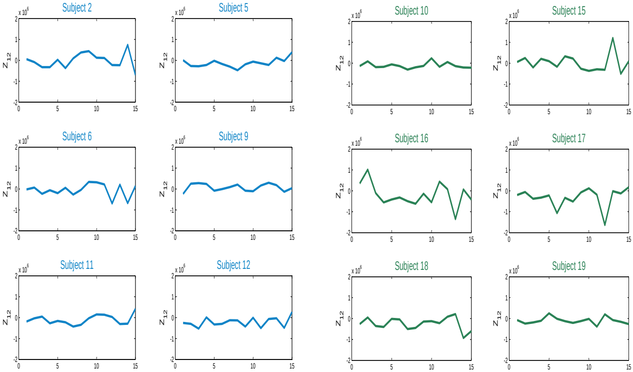


Figure 20: Averaged reaction $\overline{\Delta}\widehat{Z}^{i}_{s,12}$ to stimulus for all 15 $Q3$ questions for weakly/strongly risk-averse individuals

# Return Ratings

Risk Attitude

$r_i$, $i = 1, \ldots, 10$ denotes sequence of random returns in each trial
Subjective Expected Return (**SER**) models:

- Mean

$$SER = \frac{\sum_{i=10-m}^{10} r_i}{m}$$

  $m$-number of returns remembered, $2 \leq m \leq 10$

- Recency

$$SER = \frac{\sum_{i=10-m}^{10} r_i p}{\sum_{i=10-m}^{10} p}, \quad p = (i - 9 + m)^g$$

  $g$ - weighting parameter of returns, $0 < g < 1$

# Return Ratings

Risk Attitude

- Primacy

$$SER = \frac{\sum_{i=10-m}^{10} r_i p}{\sum_{i=10-m}^{10} p}, \quad p = (11-i)^g$$

$m$-number of returns remembered, $2 \leq m \leq 10$

$g$ - weighting parameter of returns, $0 < g < 1$

- Overweight $< 0\%$

$$SER = \frac{\sum_{i=10-m}^{10} r_i p}{\sum_{i=10-m}^{10} p}, p = \begin{cases} 1, & \text{if } r_i \geq 0 \\ 1+w, & \text{otherwise} \end{cases}$$

$w$ - additional weight of returns , $0 < w < 1$; $1 \leq m \leq 9$

# Return Ratings

Risk Attitude

- Overweight $< 5\%$

$$SER = \frac{\sum_{i=10-m}^{10} r_i p}{\sum_{i=10-m}^{10} p}, p = \begin{cases} 1, & \text{if } r_i \geq 5 \\ 1 + w, & \text{otherwise} \end{cases}$$

$w$ - additional weight of returns , $0 < w < 1$; $1 \leq m \leq 9$

- Parameters fitted by Cross Validation over all 27 trials

# Return Ratings

▸ Risk Attitude

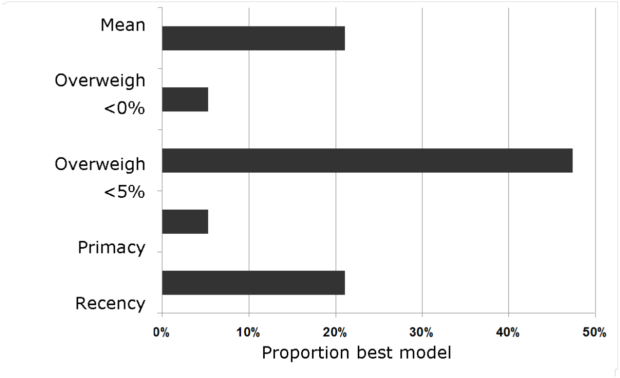


Figure 21: Distribution of return ratings over analyzed subjects

# Risk Metrics

▸ Risk Attitude

**Risk perception** - risk metrics used by individuals

- Standard deviation of a return sequence
- Empirical frequency of loss (negative returns / all returns)
- Range - difference between highest an lowest return in a sequence
- Coefficient of range (range / mean)
- Empirical frequency of ending below 5% (returns $< 5\%$ / all returns)
- Coefficient of variation (standard deviation / mean)

# Risk Metrics

▸ Risk Attitude

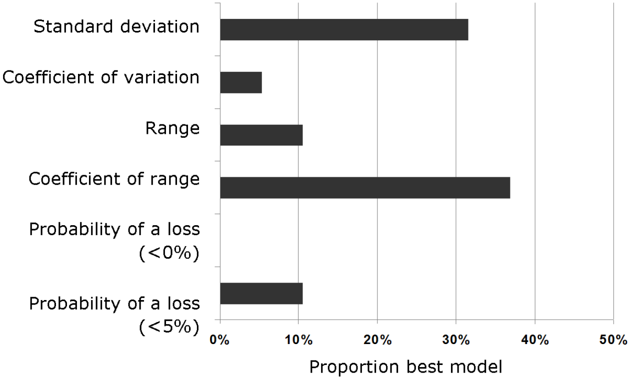


Figure 22: Distribution of risk metrices over analyzed subjects

# SVM Scores

▸ SVM Classification

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Strongly | | | | | |
| $i$ | 1 | 3 | 4 | 8 | 10 | 15 | 16 | 17 | 18 | 19 |
| $\beta$ | 5.6 | 5.6 | 11.3 | 5.0 | 6.3 | 12.6 | 8.6 | 5.4 | 16.6 | 18.3 |
| Score | 0.02 | 0.43 | 0.43 | 0.32 | 0.58 | 0.40 | 0.44 | 0.23 | 0.68 | 0.59 |
| | | | | | Weakly | | | | | |
| $i$ | 2 | 5 | 6 | 9 | 11 | 12 | 21 | | | |
| $\beta$ | 4.8 | 4.1 | 3.7 | 4.7 | 3.8 | 1.3 | 1.8 | | | |
| Score | 0.32 | −1.03 | −0.32 | −0.44 | −0.79 | −0.04 | −0.08 | | | |

Table 3: Estimated risk attitude and SVM scores (obtained **without** knowing the subject's answers)

# SVM Scores

▸ SVM Classification

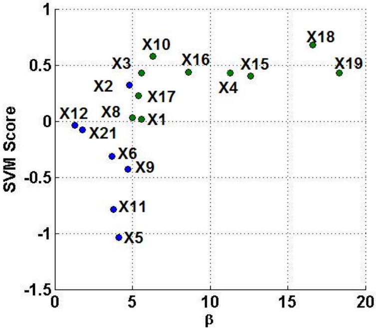


Figure 23: Scatter plot of $\widehat{\beta}_i$ vs SVM scores

# Risk Metrics

Risk Attitude

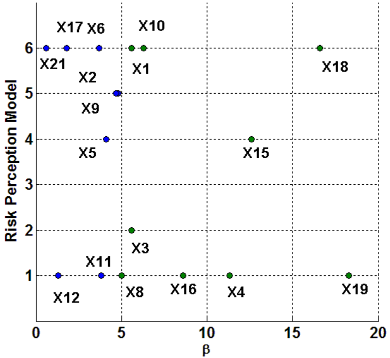


Figure 24: Scatter plot of $\widehat{\beta}_i$ vs risk perception models (vertical line). 1 - Standard deviation, 2 - Coefficient of variation, 3 - Empirical frequency of loss; 4 - Empirical frequency of ending below 5%, 5 - Coefficient of range, 6 - Coefficient of variation.

# Subject Answers

▸ Risk Attitude

| $i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Ans | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 |
| mean | 12 | 6 | 9 | 9 | 6 | 12 | 6 | 9 | 12 |
| std | 6 | 1 | 5 | 9 | 5 | 1 | 12 | 1 | 5 |
| $i$ | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| Ans | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 |
| mean | 6 | 12 | 9 | 9 | 12 | 6 | 9 | 12 | 6 |
| std | 1 | 5 | 5 | 9 | 1 | 9 | 1 | 5 | 9 |
| $i$ | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 |
| Ans | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 |
| mean | 6 | 12 | 9 | 6 | 12 | 9 | 12 | 6 | 9 |
| std | 1 | 9 | 5 | 5 | 1 | 9 | 1 | 9 | 1 |

Table 4: Answers of subject 2 (1 -**risky** choice, 0 -**fixed** return), the mean and the standard deviation for the displayed return streams $x_s$, $1 \leq s \leq 27$ used in question type 3.

# RPID task

Risk Perception

$$\begin{aligned} x_1 &= (6.53, 6.88, 5.46, 4.94, 7.91, 5.28, 6.01, 5.97, 6.48, 4.54) \\ x_2 &= (24.14, -2.14, 10.28, 12.28, 8.29, -4.23, 1.50, 11.91, 7.93, 20.04) \end{aligned} \tag{3}$$

- Return streams (in (3): mean=6, 12; STD=1, 9)
- Question of type 1, 2 or 3
- Answer given by the right hand on a pad