Credit Risk Calibration based on CDS Spreads

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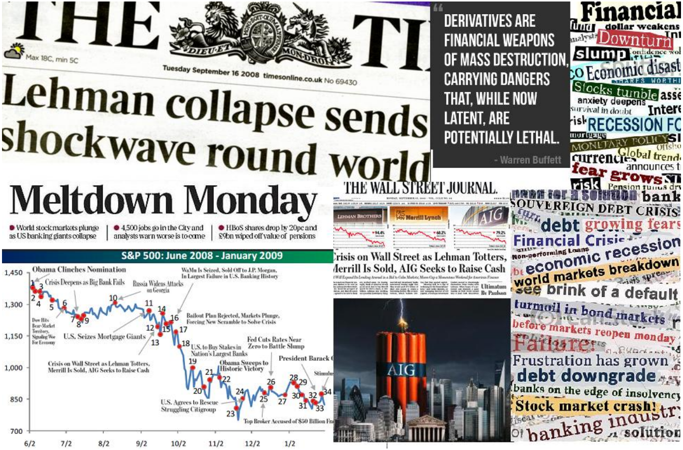
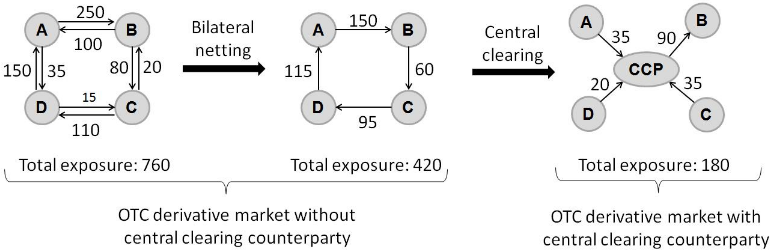
The impact of the subprime crisis



The consequences out of the financial crisis

The Concept of Central Counterparty (CCP)

Central Counterparty interposes itself between counterparties and becomes the buyer to every seller and the seller to every buyer.



Risk Mangement of CCP

Main focus: credit risk

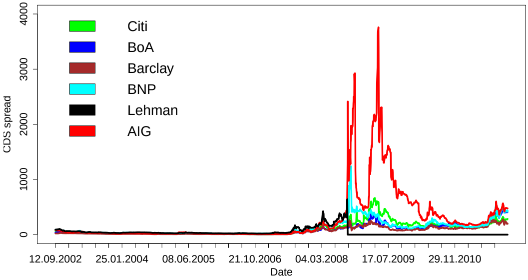
Membership Requirements	e.g. minimum requirement of rating, minimum capital requirement,...
Variation margin	Margin based on daily changes in market value of the cleared product
Initial margin	Margin based on potential future exposure (via stress test, e.g. largest 5 days decline)
Default Funds	Funds based on loss given default of single largest clearing member or simultaneous defaults of second and third largest

Credit Risk Calibration by CCP

Is CCP in the position to monitoring the spillover of credit risk by its members?

Credit Risk Calibration: How to measure credit risk spillover effects?

High upward and downward co-movements in CDS spreads during the period 2007-2009.



Risk measures

- Value at Risk (VaR)

$$\mathrm{VaR}^{\alpha}_{t+d} = \inf\{x \in \mathbb{R} \ : \ \mathrm{P}(X_{t+d} \le x \mid \mathcal{F}_t) \ge \alpha\}$$

where $X_t = -\log\left(\frac{S_t}{S_{t-1}}\right)$ denotes the CDS spread log returns.

Objectives

- Marginal credit risk analysis tool based on CDS spreads
- Measure of interconnectedness: quantification of mutual effects of credit risk
- Relationship between CDS spreads in tail events: linear or non-linear?
- Uncover the relationship between CDS spreads and CDS determinants

Outline

1. Motivation $\checkmark$
2. Linear quantile regression
3. PLM Methodology
4. Empirical study
5. Conclusions

Linear Quantile Regression

$$X_{i,t} = \alpha_i + \gamma_i^\top M_{t-1} + \varepsilon_{i,t},$$
$$X_{j,t} = \alpha_{j|i} + \beta_{j|i} X_{i,t} + \gamma_{j|i}^\top M_{t-1} + \varepsilon_{j,t}.$$

M_t: state variables. $F^{-1}_{\varepsilon_{i,t}}(\tau|M_{t-1}) = 0$ and $F^{-1}_{\varepsilon_{j,t}}(\tau|M_{t-1}, X_{i,t}) = 0$.

$$\widehat{VaR}_{i,t} = \hat{\alpha}_i + \hat{\gamma}_i^\top M_{t-1},$$
$$\widehat{CoVaR}_{j|i,t} = \hat{\alpha}_{j|i} + \hat{\beta}_{j|i} \widehat{VaR}_{i,t} + \hat{\gamma}_{j|i}^\top M_{t-1}.$$

Systemic contribution of i on j:

$$\triangle \widehat{CoVaR}_{j|i,t} = \widehat{CoVaR}_{j|i,t} - \widehat{CoVaR}_{j|X_i=\text{Median},t}$$

See Adrian & Brunnermeier (2011): CoVaR (AB (2011))

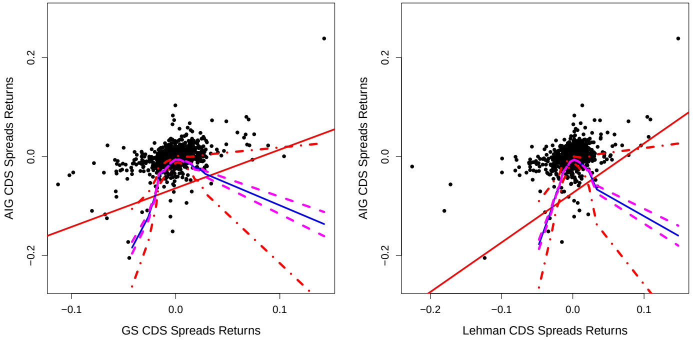


Figure 1: Quantile regression at 0.01 level on CDS spread return. Linear quantile regression line. Partial linear quantile regression estimation. The dashed lines express the asymptotic and bootstrap confidence bands at 95% level.

Partial Linear Quantile Regression:

$$X_{i,t} = \alpha_i + \gamma_i^\top M_{t-1} + \varepsilon_{i,t};$$
$$X_{j,t} = \tilde{\alpha}_{j|i} + \tilde{\beta}_{j|i}^\top M_{t-1} + l_{j|i}(X_{i,t}) + \varepsilon_{j,t}.$$

l: a general function. M_t: state variables. $F_{\varepsilon_{i,t}}^{-1}(\tau|M_{t-1}) = 0$ and $F_{\varepsilon_{j,t}}^{-1}(\tau|M_{t-1}, X_{i,t}) = 0$.

$$\widehat{VaR}_{i,t} = \hat{\alpha}_i + \hat{\gamma}_i^\top M_{t-1},$$
$$\widehat{CoVaR}_{j|i,t} = \hat{\alpha}_{j|i} + \hat{\gamma}_{j|i}^\top M_{t-1} + \hat{l}_{j|i}(\widehat{VaR}_{i,t}).$$

See Chao, Härdle & Wang (2013): Quantile Regression in Risk Calibration

State variables

M_t: 7 state variables suggested by AB and further extension:

1. VIX
2. Short term liquidity spread
3. Change in the 3M T-bill rate
4. Change in the slope of the yield curve
5. Change in the credit spread between 10 years BAA-rated bonds and the T-bond rate
6. S&P500 returns
7. Dow Jones U.S. Real Estate index returns
8. Constituent's specific stock log returns (15x)
9. Constituent's specific stock volatility log returns (15x)

Least Absolute Shrinkage and Selection Operator (LASSO)

- Selection of variables with significant effect on CDS spread returns
- The quantile regression under LASSO penalty

$$L^{LASSO}(\beta) = \sum_{i=1}^{n} \rho_\tau \left(y_i - \beta^\top x_i \right) + \lambda_n \sum_{j=1}^{p} \mid \beta_j \mid$$

where $0 \leq \tau \leq 1$ and λ_n denotes the penalty parameter.

- λ_n is chosen via generalized approximate cross-validation (GACV) suggested by Yuan (2006) and Li et al. (2007)

CDS spread returns

- Daily CDS spreads of 14 biggest derivative dealers and 1 monoline
- Overall data period: Sept 2002 - Dec 2011 ($N = 2208$)
- Segregation into two sub-periods
  - pre-shock: Sept 12 2002 - Sept 12 2008
  - shock event: Lehman Brothers filed for Chapter 11 bankruptcy protection on Sept 15 2008
  - post-shock: Sept 16 2008 - Dec 31 2011

Table 1: Descriptive statistics of CDS spread log returns

	Std. Dev	Skewness	Kurtosis	Min	Max	Autocorr.
CITI	0.023	0.871	27.203	-0.174	0.286	0.032
BOA	0.023	0.579	14.454	-0.182	0.247	0.008
BARC	0.021	1.045	24.028	-0.155	0.270	0.115
BNP	0.021	0.160	17.017	-0.192	0.214	0.117
CS	0.019	0.172	17.983	-0.168	0.182	0.065
DB	0.020	0.682	22.554	-0.156	0.252	0.143
GS	0.020	-0.040	28.865	-0.248	0.219	0.222
HSBC	0.019	-0.294	13.582	-0.147	0.151	0.067
JPM	0.019	0.453	15.169	-0.138	0.213	0.117
MS	0.023	4.678	118.434	-0.255	0.475	-0.006
RBS	0.024	1.884	87.755	-0.368	0.376	-0.072
SG	0.020	-0.209	21.404	-0.223	0.187	0.129
UBS	0.020	0.439	20.372	-0.153	0.218	0.090
LEH	0.019	-2.040	30.336	-0.226	0.148	0.138
AIG	0.024	1.106	61.673	-0.253	0.402	0.237

Estimated Coefficient: $\widehat{\beta}_{VIX}$ - pre-shock

Figure 2: $\widehat{\beta}$ of variable VIX of all 15 FI: 1-Citi, 2-BoA, 3-GS, 4-JPM, 5-MS, 6-LEH, 7-AIG, 8-SG, 9-BNP, 10-CS, 11-DB, 12-BARC, 13-HSBC, 14-RBS, 15-UBS

Estimated Coefficient: $\widehat{\beta}_{VIX}$ - post-shock

Figure 3: $\widehat{\beta}$ of variable VIX of all 15 FI: 1-Citi, 2-BoA, 3-GS, 4-JPM, 5-MS, 6-AIG, 7-SG, 8-BNP, 9-CS, 10-DB, 11-BARC, 12-HSBC, 13-RBS, 14-UBS

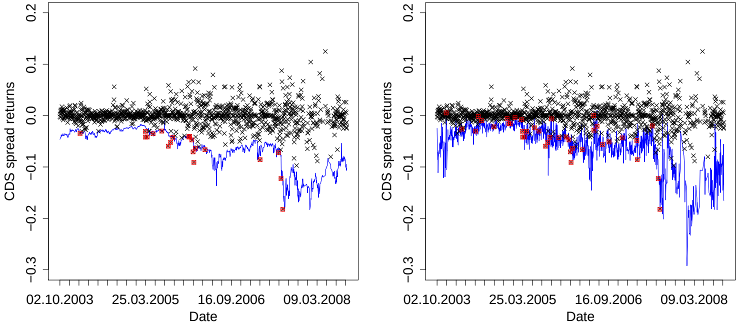


Figure 4: Backtesting results: Bank of America VaR exceedance under LASSO quantile regression (left) and under AB model (right) in pre-shock period.

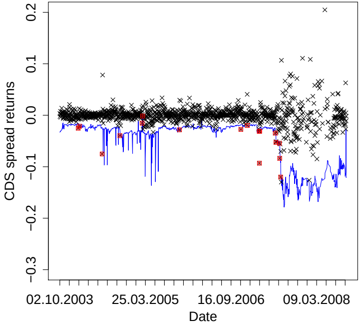


Figure 5: Backtesting results: Royal Bank of Scotland VaR exceedance under LASSO quantile regression (left) and under AB model (right) in pre-shock period.

Backtesting of calculated VaR under AB (2011)

	Exceedance	LR_{POF}	LR_{uncond}	LR_{CC}	Test Outcome
CITI	38	38.69	0	38.69	Rejected
BOA	39	41.17	0	41.17	Rejected
BARC	28	17.22	0	17.22	Rejected
BNP	33	27.17	0	27.17	Rejected
CS	46	59.90	0	59.90	Rejected
DB	47	62.76	0	62.76	Rejected
GS	45	57.08	0	57.08	Rejected
HSBC	41	46.27	0	46.27	Rejected
JPM	57	93.73	0	93.73	Rejected
MS	60	103.77	0	103.77	Rejected
RBS	40	43.70	0	43.70	Rejected
SG	31	22.99	0	22.99	Rejected
UBS	36	33.91	0	33.91	Rejected
LEH	43	51.58	0	51.58	Rejected
AIG	57	93.73	0	93.73	Rejected

Table 2: Backtesting for N=1145 observations; Test statistic: LR_{POF} for Kupiec test, LR_{uncond} for Christoffersen test, LR_{CC} for conditional coverage.

Backtesting of calculated VaR under QLPLM

	Exceedance	LR_{POF}	LR_{uncond}	LR_{CC}	Test Outcome
CITI	18	3.22	0	3.22	Not Rejected
BOA	20	5.27	0	5.27	Not Rejected
BARC	15	1.01	0	1.01	Not Rejected
BNP	19	4.19	0	4.19	Not Rejected
CS	15	1.01	0	1.01	Not Rejected
DB	22	7.73	0	7.73	Not Rejected
GS	26	13.73	0	13.73	Rejected
HSBC	18	3.22	0	3.22	Not Rejected
JPM	19	4.19	0	4.19	Not Rejected
MS	20	5.27	0	5.27	Not Rejected
RBS	18	3.22	0	3.22	Not Rejected
SG	21	6.45	0	6.45	Not Rejected
UBS	16	1.62	0	1.62	Not Rejected
LEH	33	27.17	0	27.17	Rejected
AIG	25	12.11	0	12.11	Rejected

Table 3: Backtesting for N=1145 observations; Test statistic: LR_{POF} for Kupiec test, LR_{uncond} for Christoffersen test, LR_{CC} for conditional coverage.

$\triangle$ CoVaR in pre-shock period

	Citi	BoA	BAR	DB	GS	JPM	MS	RBS	LEH	AIG	sum
Citi	-	-0.04	-0.03	-0.02	-0.03	-0.03	-0.03	-0.03	-0.04	-0.04	-0.41
BoA	-0.07	-	-0.04	-0.03	-0.05	-0.05	-0.04	-0.04	-0.04	-0.04	-0.58
BAR	-0.01	-0.04	-	-0.05	-0.03	-0.04	-0.03	-0.07	-0.03	-0.03	-0.61
DB	0.00	-0.01	-0.05	-	-0.03	-0.03	-0.03	-0.04	-0.01	-0.02	-0.37
GS	-0.05	-0.04	-0.02	-0.02	-	-0.04	-0.04	-0.03	-0.03	-0.04	-0.46
JPM	-0.05	-0.05	-0.03	-0.03	-0.04	-	-0.03	-0.03	-0.03	-0.04	-0.52
MS	-0.04	-0.03	-0.03	-0.03	-0.05	-0.03	-	-0.03	-0.03	-0.05	-0.43
RBS	-0.03	-0.02	-0.12	-0.07	-0.02	-0.04	-0.02	-	-0.03	-0.02	-0.78
LEH	-0.04	-0.04	-0.03	-0.03	-0.04	-0.04	-0.03	-0.03	-	-0.04	-0.46
AIG	-0.02	-0.02	-0.01	-0.02	-0.03	-0.03	-0.02	-0.02	-0.02	-	-0.28

Table 4: Average $\triangle$ CoVaR overview for pre-shock period.

△ CoVaR in post-shock period

	Citi	BoA	BAR	DB	GS	JPM	MS	RBS	SG	AIG	sum
Citi	-	-0.16	-0.07	-0.05	-0.15	-0.15	-0.11	-0.07	-0.08	-0.11	-0.96
BoA	-0.19	-	-0.14	-0.13	-0.20	-0.19	-0.18	-0.13	-0.16	-0.11	-1.45
BAR	-0.11	-0.15	-	-0.10	-0.12	-0.12	-0.08	-0.14	-0.13	-0.10	-1.06
DB	-0.15	-0.16	-0.13	-	-0.19	-0.18	-0.17	-0.20	-0.20	-0.16	-1.54
GS	-0.21	-0.20	-0.13	-0.15	-	-0.22	-0.18	-0.14	-0.17	-0.14	-1.53
JPM	-0.17	-0.18	-0.09	-0.12	-0.17	-	-0.17	-0.14	-0.15	-0.13	-1.32
MS	-0.11	-0.13	-0.07	-0.08	-0.17	-0.14	-	-0.10	-0.11	-0.13	-1.03
RBS	-0.10	-0.17	-0.12	-0.16	-0.17	-0.12	-0.12	-	-0.14	-0.16	-1.25
SG	-0.15	-0.25	-0.13	-0.14	-0.21	-0.24	-0.18	-0.22	-	-0.17	-1.69
AIG	-0.01	-0.03	-0.05	-0.06	-0.04	-0.03	-0.04	-0.05	-0.04	-	-0.35

Table 5: Average △ CoVaR overview for post-shock period

Average $\triangle$ CoVaR in the pre-shock period

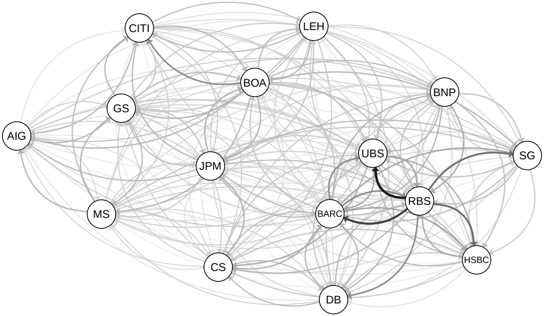


Figure 6: Network of spread spillover effects described by average $\triangle$ CoVaR

Average $\triangle$ CoVaR in the post-shock period

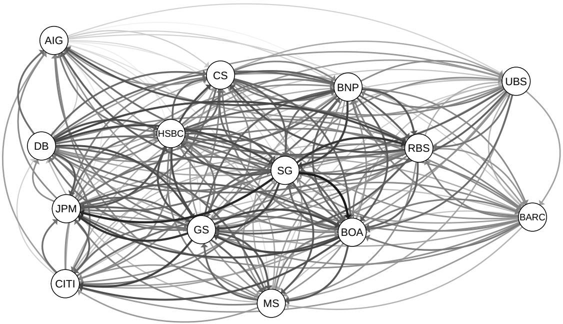


Figure 7: Network of spread spillover effects described by average $\triangle$ CoVaR

Change in $\triangle$ CoVaR during the pre-shock period

Figure 8: Network of spread spillover effects described by $\triangle$ CoVaR

Study of CDS spreads determinants

- CDS spread returns mainly described by implied volatility index VIX and real estate sector returns
- Strong positive relationship between CDS spread returns and equity volatility index
- Heterogeneous impact in regions: high sensitivity of US FIs to VIX after shock, delayed in sensitivity for European FIs.
- Effects of firm specific volatility is not as strong as market volatility indicated by VIX index

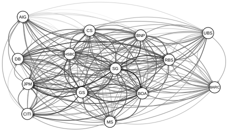
Study of $\triangle$ CoVaR

- Continental effects shown by $\triangle$ CoVaR: higher value observed between FIs from the same region
- $\triangle$ CoVaR more suitable for computing stressed VaR (VaR under data of financial crisis) rather than for CDS spread forecasting, especially in late post-shock period

Next steps:

- $\triangle$ CoVaR as risk weighting basis for transactions cleared through CCP
- $\triangle$ CoVaR of CDS index on corporate companies for estimation of portfolio potential future exposure (PFE)

Credit Risk Calibration based on CDS Spreads



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Partial Linear Model (PLM)

- The partial linearity observation implies:

$$X_{i,t} = \alpha_i + \gamma_i^\top M_{t-1} + \varepsilon_{i,t};$$
$$X_{j,t} = \tilde{\beta}_{j|i}^\top M_{t-1} + l_{j|i}(X_{i,t}) + \varepsilon_{j,t}. \quad (1)$$

l: a general function. M_t: state variables. $F^{-1}_{\varepsilon_{i,t}}(\tau|M_{t-1}) = 0$ and $F^{-1}_{\varepsilon_{j,t}}(\tau|M_{t-1}, X_{i,t}) = 0$.

- Advantages
    - Capturing nonlinear asset dependence
    - Avoid curse of dimensionality

Estimation of Partial Linear Model

- PLM model: Liang, Härdle and Carroll (1999) and Härdle, Ritov and Song (2012)

$$Y_t = \beta^\top M_{t-1} + l(X_t) + \varepsilon_t.$$

- Consider $[0,1]$ (standard rank space). Dividing $[0,1]$ into a_n equally divided subintervals I_{nt}, $a_n \uparrow \infty$. On each subinterval, $l(\cdot)$ is roughly constant.

Estimation of PLM QR

1. Linear element β:

$$\hat{\beta} =$$

$$\operatorname*{argmin}_{\beta} \min_{l_1,\dots,l_{a_n}} \sum_{t=1}^{n} \rho_\tau \left\{ Y_t - \beta^\top M_{t-1} - \sum_{m=1}^{a_n} l_m \mathbf{1}(X_t \in I_{nt}) \right\}$$

2. Nonlinear element $l(\cdot)$: With data $\{(X_t, Y_t - \hat{\beta}^\top M_{t-1})\}_{t=1}^{n}$, applying LLQR.

$\triangle$ CoVaR **in pre-shock period**

	Citi	BoA	BARC	DB	GS	JPM	MS	RBS	LEH	AIG
Citi	-	-0.37	-0.23	-0.27	-0.35	-0.32	-0.27	-0.34	-0.42	-0.45
BoA	-0.52	-	-0.33	-0.26	-0.29	-0.27	-0.21	-0.50	-0.33	-0.43
BARC	-0.42	-0.29	-	-0.35	-0.42	-0.35	-0.30	-0.46	-0.58	-0.52
DB	-0.23	-0.22	-0.52	-	-0.16	-0.21	-0.24	-0.52	-0.29	-0.50
GS	-0.27	-0.28	-0.29	-0.22	-	-0.22	-0.27	-0.61	-0.34	-0.28
JPM	-0.29	-0.25	-0.20	-0.23	-0.24	-	-0.46	-0.50	-0.45	-0.26
MS	-0.27	-0.25	-0.50	-0.36	-0.37	-0.23	-	-0.56	-0.27	-0.47
RBS	-0.32	-0.35	-1.67	-0.80	-0.16	-0.55	-0.22	-	-0.46	-0.46
LEH	-0.35	-0.29	-0.26	-0.32	-0.30	-0.25	-0.29	-0.27	-	-0.32
AIG	-0.34	-0.32	-0.36	-0.21	-0.28	-0.21	-0.27	-0.52	-0.36	-

Table 6: Minimum $\triangle$ CoVaR overview for pre-shock period which demonstrates the maximum negative effects on CDS spreads returns.

$\triangle$ CoVaR in post-shock period

	Citi	BoA	BARC	DB	GS	JPM	MS	RBS	SG	AIG
Citi	-	-0.79	-0.97	-0.79	-1.03	-1.55	-1.36	-1.06	-0.51	-1.24
BoA	-0.84	-	-0.55	-0.58	-0.83	-0.58	-1.19	-0.45	-0.65	-0.56
BARC	-1.72	-0.78	-	-0.58	-0.90	-0.46	-0.42	-0.95	-0.47	-0.74
DB	-1.41	-0.82	-0.97	-	-1.60	-1.52	-1.32	-0.74	-2.19	-1.35
GS	-0.90	-1.18	-0.63	-1.09	-	-0.73	-1.99	-1.51	-0.94	-1.66
JPM	-0.58	-0.54	-0.34	-0.42	-0.55	-	-1.07	-0.44	-0.61	-0.77
MS	-1.26	-0.94	-0.83	-1.05	-0.95	-0.89	-	-1.40	-1.14	-2.31
RBS	-0.69	-0.67	-0.39	-0.52	-0.81	-0.55	-0.47	-	-0.61	-0.64
SG	-0.89	-1.02	-0.38	-0.44	-0.90	-0.79	-0.71	-0.63	-	-0.54
AIG	-0.61	-0.41	-0.65	-0.71	-0.37	-0.49	-0.58	-0.78	-0.31	-

Table 7: Minimum $\triangle$ CoVaR overview for post-shock period which demonstrates the maximum negative effects on CDS spreads returns.

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