Mortality Model for Multi-Populations: A Semiparametric Comparison Approach

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Demographic Risk

- Low mortality, low fertility, global aging trend
- Mortality rate is the key to insurance and pension industry
Demographic key element: mortality

- Mortality rate: number of death/number of exposure, taken as the log transformation
- Mortality rate: age-specific, male and female, (region-specific)
- Mortality change is more "stable" compared to fertility

Note: In following graphs, rates in different years are plotted in rainbow palette so that the earliest years are red and so on.
Demographic Risk in Japan

Figure 1: Japan female mortality trend: 1947-2012
Demographic Risk in Japan

Figure 2: Japan fertility trend: 1947-2012
Demographic risk in China

- Small sample size: 17 years
- Aging trend is inevitable
- Regional similarities between Japan and China
Motivation

Demographic Risk in China

Figure 3: China female mortality trend: 1994-2010, Japan’s historical female mortality is displayed as grey zone.

Mortality Model for Multi-Populations
Demographic Risk in China

Figure 4: China male mortality trend: 1994-2010, Japan’s historical male mortality is displayed as grey zone.

Mortality Model for Multi-Populations
Motivation

Literature

Mortality Similarity

- Hanewald (2011): The Lee-Carter mortality index $k_t$ correlates significantly with macroeconomic fluctuations in some periods

Semiparametric Comparison Model

- Härdle and Marron (1990): Semiparametric comparison of regression curves
- Grith et al. (2013): Shape invariant model

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Multi-Population Mortality Modeling

China
- Is there mortality similarity between China and Japan?
- How can the mortality modeling and forecasting be improved via Japan?

Multi-Countries
- How do we generate a multi-population mortality model based on the common shape?
Outline

1. Motivation  ✔
2. Classic mortality models
3. Semiparametric comparison model
4. Empirical analysis
5. Reference
Lee-Carter (LC) Method

- A benchmark in demographics: Lee and Carter (1992)
- Idea: use SVD to extract a single time-varying index of mortality/fertility rate level
- Take mortality for analysis:

\[
\log\{y_t(x)\} = a_x + b_x k_t + \varepsilon_{x,t}
\]

- \(y_t(x)\) observed mortality rate at age \(x\) in year \(t\)
- \(a_x\) age pattern averaged across years
- \(b_x\) first PC reflecting how fast the mortality changes at each age
- \(k_t\) time-varying index of mortality level
- \(\varepsilon_{x,t}\) residual at age \(x\) in year \(t\)
Hyndman-Ullah (HU) Method

- Variant of LC method: presmooth, orthogonalize, forecast
- Estimate the smooth functions $s_t(x)$ through the data sets \( \{x, y_t(x)\} \) for each $t$:

\[
y_t(x) = s_t(x) + \sigma_t(x) \varepsilon_t
\]

- $s_t(x)$ smooth function
- $\sigma_t(x)$ smooth volatility function of $y_t(x)$
- $\varepsilon_t$ i.i.d. random error
Hyndman-Ullah (HU) Method

Use functional principal component analysis (FPCA)

\[ s_t(x) = \mu(x) + \sum_{k=1}^{K} \beta_{t,k} \phi_k(x) + e_t(x) \]

- \( \mu(x) \) mean of \( s_t(x) \) across years
- \( \phi_k(x) \) orthogonal basis functional PCs
- \( \beta_{t,k} \) uncorrelated PC scores
- \( e_t(x) \) is residual function with mean zero
Mortality Analysis

Figure 5: China’s female mortality decomposition by HU Method: yellow areas represent the 95% confidence intervals for the coefficients forecast.
Mortality trends comparison

- Time-varying indicator $k_t$ derived from Lee-Carter model presents similar pattern.

Figure 6: China mortality trend (short curves) vs. Japan mortality trend (long curves): female, male.
Semiparametric comparison model

two-country case

Take China and Japan for example

□ Use $k_t$ derived from LC model

$$\log\{y_t(x)\} = a_x + b_x k_t + \varepsilon_{x,t},$$

□ Infer China’s mortality trend via Japan’s trend

$$k_c(t) = \theta_1 k_j \left( \frac{t - \theta_2}{\theta_3} \right) + \theta_4,$$

- $k_c(t)$ is the time-varying indicator for China
- $k_j(t)$ is the time-varying indicator for Japan
- $\theta = (\theta_1, \theta_2, \theta_3, \theta_4)^\top$ are shape deviation parameters
Model estimation

- Estimation procedure

\[
\min_{\theta} \int_{t_c} \left\{ \hat{k}_c(u) - \theta_1 \hat{k}_j \left( \frac{u - \theta_2}{\theta_3} \right) - \theta_4 \right\}^2 w(u) du,
\]

where \( \hat{k}_c(t) \) and \( \hat{k}_j(t) \) are the nonparametric estimates of the original time-varying indicators, \( t_c \) is the China data’s time interval, and the comparison region satisfies the condition

\[
w(u) = \prod_{t_j} 1_{[a, b]} \{(u - \theta_2)/\theta_3\},
\]

where \( t_j \) is the time interval of Japan’s mortality data, \( a \geq \inf(t_j) \) and \( b \leq \sup(t_j) \).
Algorithm

- Iterate based on the scheme (3)
- Set up the prior estimates \( \theta^0 = (\theta^0_1, \theta^0_2, \theta^0_3, \theta^0_4)^T \) and the nonparametric estimates of \( \hat{k}_c(t) \) and \( \hat{k}_j(t) \)
- Update \( (\theta_1, \theta_2, \theta_3, \theta_4)^T \)
- Reach convergence
Semiparametric comparison model
multi-country case

- $k_i(t)$ is a derived time-varying mortality indicator for country $i$, with $i \in \{1, \ldots, n\}$, $n = 36$ stands for 36 countries.

- The curves can be represented in the form

$$k_i(t) = \theta_1 k_0 \left( \frac{t - \theta_2}{\theta_3} \right) + \theta_4,$$

(4)

- $k_i(t)$ is the time-varying indicator for country $i$
- $k_0(t)$ is a reference curve, understood as common trend
- $\theta = (\theta_1, \theta_2, \theta_3, \theta_4)^\top$ are shape deviation parameters
Estimation of Common Trend

- Synchronization

\[ k_i(\theta_{i3} t + \theta_{i2}) = \theta_{i1} k_0(t) + \theta_{i4} , \] (5)

- Identification conditions (normalize)

\[ T^{-1} \sum_{i=1}^{N} \theta_{i1} = T^{-1} \sum_{i=1}^{N} \theta_{i3} = 1, \] (6)

\[ T^{-1} \sum_{i=1}^{N} \theta_{i2} = T^{-1} \sum_{i=1}^{N} \theta_{i4} = 0 \] (7)

- Common trend curve

\[ k_0(t) = T^{-1} \sum_{i=1}^{N} k_t(\theta_{i3} t + \theta_{i2}) \] (8)
Initial Value and Algorithm

- Choose a group of countries with bigger sample size and set their average curve $k_0(t)$ as initial reference curve.
- Repeat the iteration of two-country case and generate initial $\theta^0$ for the other countries.
- Get the common trend based on formula (8).
- Iterate based on the above procedures.
- Update $(\theta_1, \theta_2, \theta_3, \theta_4)^\top$.
- Reach convergence.
Demographic Data

- **China**
  - Mortality: age-specific (0,90+), male and female
  - Years: 1994-2010
  - Data Source: China Statistical Year Book

- The other 35 countries
  - Mortality: age-specific (0,110+), male and female
  - Extracted ages: (0,90)
  - Years: it differs from 14 years (Chile) to 261 years (Sweden)
  - Data Source: Human Mortality Database
Mortality trends comparison

Intuitive comparison: time delay between China and Japan female mortality trend.

Figure 7: Japan trend, Japan smoothed trend, China trend and China smoothed trends of no-delay, 20-, 23- and 25- year delay respect.
Understanding $\theta$

$\theta = (\theta_1, \theta_2, \theta_3, \theta_4)^T = (1, \theta_2, 1, \theta_4)^T$

- $\theta_1$ is the general trend adjustment, possibly selected as 1.
- $\theta_2$ is the time-delay parameter
- $\theta_3$ is the time acceleration parameter, possibly selected as 1.
- $\theta_4$ is the vertical shift parameter

Figure 8: Time delay $\theta_2 = 23$

Figure 9: Vertical shift $\theta_4 = -85$

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Initial choice of $\theta_2$ and $\theta_4$

- Potential linear relation between $\theta_2$ and $\theta_4$.

Figure 10: Loss surface of $\theta_2$ and $\theta_4$.

Figure 11: Contour of $\theta_2$ and $\theta_4$. 

Mortality Model for Multi-Populations
**Time delay or vertical shift**

- Stick with time delay influence $\theta_2$, and the optimal value is obtained around 23.

**Figure 12**: Loss function of $\theta_2$ with $(\theta_1, \theta_3, \theta_4)^T = (1, 1, 0)^T$. 

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Empirical results

Goodness of Fit

Optimal $\theta = (1.160, 23.032, 1.000, -0.057)^T$

Figure 13: Goodness of Fit: Japan trend, Japan smoothed trend, China trend, China smoothed trend and fitted trend (black dots).
Forecast

Forecasting $k_t$ for China

$$k_c(t + i) = \theta_1 k_j \left\{ \frac{(t + i) - \theta_2}{\theta_3} \right\} + \theta_4,$$

(9)

- $\theta = (1.160, 23.032, 1.000, -0.057)^T$
- $t = 1994, 1995, ..., 2010; i = 1, 2, ..., 20$

Figure 14: Forecast of China’s mortality trend from 2011 to 2030.
Multi-Populations Case

Figure 15: Original mortality trend among 36 countries

Figure 16: Original mortality trend among 36 countries
Multi-Populations Case

Figure 17: Reference curve vs. original smoothed $k_i(t)$

Figure 18: Reference curve vs. shifted $k_i(t)$

Mortality Model for Multi-Populations
Multi-Populations Case

Figure 19: Italy shifted $\hat{k}_t$ according to reference curve $k_0$.

Figure 20: Norway shifted $\hat{k}_t$ according to reference curve $k_0$. 
Outlook

- Global common mortality trend
- Confidence interval for forecast with multi-populations mortality model
- Comparison with classical mortality methods
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