Tail event driven networks of SIFIs

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Motivation

- Systemic risk threatens financial stability
- Interconnectedness of financial institutions is key to understanding systemic risk
- Important research questions
  - Quantify systemic risk
  - Identify important contributors to systemic risk

Tail event driven networks of SIFIs
Outline

1. Motivation ✓
2. Adjacency matrix and systemic risk score
   2.1 Characteristics of SIFIs
   2.2 Similarity and adjacency matrix
   2.3 Systemic risk score and risk decomposition
3. Tail event driven network quantile regression
   3.1 TENQR model
   3.2 Estimation results
4. Conclusion
Characteristics of SIFIs

Systemically important financial institution (SIFI)
Financial institution whose failure might trigger a financial crisis

- List of 28 SIFIs published 2011 by FSB
- too-big-too-fail principle
- Characteristic factors
  - size
  - global activity
  - interconnectedness
  - lack of substitutes for its provided financial infrastructure
## Table 1: Overview of SIFIs

*Note: Buckets assigned by BCBS, required level of additional common equity loss absorbency*

Tail event driven networks of SIFIs
Similarity Matrix

- SIFIs are connected if they share certain degree of similarity
- Risk profile similarity

\[ \rho_{ij,t} = \frac{X_{i,t}^\top X_{j,t}}{\|X_{i,t}\| \|X_{j,t}\|} \] for \( j \neq i \), \( i = 1, \ldots, N \), \( t = 1, \ldots, T \),

with \( X_{i,t} = [\text{VaR}_{i,t}, \text{ES}_{i,t}, \text{IV}_{i,t}]^\top \)

- Value-at-risk at 95 % level
- Expected shortfall at 95 % level
- Implied volatility

- Analogous to Pearson correlation coefficient

Tail event driven networks of SIFIs
Figure 1: Risk profile similarity 2007 - 2014, blue: negative correlation, yellow: positive correlation

Tail event driven networks of SIFIs


**Adjacency matrix**

Each network is characterized by the **adjacency matrix** $A = \{a_{ij}\}$, for $i, j = 1, \ldots, N$.

$$a_{ij} = \begin{cases} 
1 & \text{if } i \text{ is directly connected to } j \\
0 & \text{if } i \text{ is not directly connected to } j 
\end{cases}$$

If $a_{ij} = a_{ji}$ for all $i$ and $j$ then the network is an **undirected network**.

\[
\begin{bmatrix}
1 & 1 & . & . & . \\
1 & 0 & 1 & . & . \\
1 & 0 & 1 & 1 & . \\
0 & 0 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 \\
\end{bmatrix}
\]
Adjacency matrix

Need for three groups to disentangle asymmetric correlations

Figure 2: Fractions of positive correlations in the similarity matrix
Adjacency matrix

- Ordered Fisher’s Z transformed correlations
  \[ \rho^* = (\rho^*_1, \rho^*_2, \ldots, \rho^*_n) \]

- Edges are constructed based on large spacings between \( \rho^*_j \) and \( \rho^*_{j-1} \), \( h \) equals sample size
  \[ \Delta_j = \Phi \left( \sqrt{h - 3\rho^*_j} \right) - \Phi \left( \sqrt{h - 3\rho^*_{j-1}} \right) \]

- Split spacing sequence \( \Delta_j \) into three homogeneous groups

Appendix: Transformation
Appendix: Classification approach

Tail event driven networks of SIFIs
Figure 3: Positive (white), negative (gray) and weak correlation (black) for 2007 - 2014.
Systemic risk score

- Quantifies degree of systemic risk in financial system
- Systemic risk score $S$ is function of compromise level of all nodes

$$S(C, A) = C^T AC$$

- $A$, adjacency matrix
- $C = (C_1, ..., C_N)^T$, compromise vector
- Level of compromise defined as nodal market capitalization (Basel III, "too-big-to-fail"-consideration)
**Risk decomposition**

- Decomposes aggregate risk $S$ into individual risk score $S_i$
- Euler’s equation to decompose first-order functions

$$S = \sum_{i=1}^{N} S_i = \frac{\partial S}{\partial C_1} C_1 + \frac{\partial S}{\partial C_2} C_2 + ... + \frac{\partial S}{\partial C_N} C_N$$

- Enables to identify source of systemic vulnerabilities
### Systemic Risk Score and Risk Decomposition

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| Systemic Risk Score      | 4746 | 4938 | 5430 | 4419 | 4514 | 4588 | 5032 | 5193 | 4942 |
| Average score (US)       | 162  | 182  | 187  | 164  | 164  | 164  | 169  | 191  | 172  |
| Average score (Europe)   | 186  | 175  | 196  | 157  | 162  | 167  | **198** | 183  | 171  |
| Average score (Asia)     | 142  | 173  | **200** | 150  | 155  | 157  | 151  | 184  | 196  |

Table 2: Systemic risk score and decomposition, red: maximum value per column

Tail event driven networks of SIFIs
Tail event driven network quantile regression

Three issues on network dynamics

- Current nodal response is related to connectedness at previous time point
- SIFIs respond stronger to negative than positive network effect
- Returns are subject to geographical proximity
Model for SIFI returns

\[ Y_{it} = \alpha_0 + \alpha_{i1} Y_{i,t-1} + \alpha_{i2}^\top W_t + \alpha_{i3} S_{it} + \nu_{it}, \text{ for } i = 1, \ldots, N, \ t = 1, \ldots, T, \]

with

- \( Y_{it}, Y_{i,t-1} \): return and autoregressive term of SIFI \( i \)
- \( W_t \): market influence (VIX, TED spread)
- \( S_{it} \): node-specific variables (log firm size, total debt to asset ratio)
- Estimation by OLS: for individual nodes or stacked groups of SIFIs
Random coefficient model

'Residual returns' may contain network information

\[ \hat{v}_{it} = \beta_{r0}(U_t) + \beta_{r1}(U_t) \sum_{j \in B_i} m_i(Y_{j,t-1}), \quad \text{for } i \in R_r \]

- $\beta_{r1}$ represents network effect
- $R_r$, with $r = 1, 2, 3$ all SIFIs from US, Europe, Asia
- $m_i(Y_{j,t-1})$ connectedness, $B_i$ neighbors of node $i$
- $\{U_t\} \sim U(0,1)$ iid sequence
Conditional quantile function

\[ Q_{\hat{\nu}_{it}}(\tau | I_{t-1}) = \beta_{r0}(\tau) + \beta_{r1}(\tau) \sum_{j \in B_j} m_i(Y_{j,t-1}) \]

- \( m_i(Y_{j,t-1}) \), connectedness of nodes within network
- **Network factor**, average impact from \( i \)-th node neighbors

\[ \sum_{j \in B_j} m_i(Y_{j,t-1}) = \frac{1}{|B_i|} \sum_{j=1}^{N} a_{ij,t-1} Y_{j,t-1} \]

- **Significance of network factor** can be statistically tested
TENQR: Estimation

Minimize objective function (Koenker and Xiao, 2006)

\[
\hat{V}_r(\tau) = \min_{\theta_r(\tau) \in \mathbb{R}^2} \sum_{t=1}^{T} \sum_{i \in \mathcal{R}_r} \rho_\tau \left\{ \hat{v}_{it} - x_{i,t-1}^\top \theta_r(\tau) \right\} \quad \text{for} \quad \tau \in (0, 1)
\]

- \( \rho_\tau(u) = u \cdot \{\tau - I(u < 0)\} \) is asymmetric loss function
- \( x_{i,t-1}^\top \) contains all relevant explanatory variables
- \( \theta_r(\tau) = \{\beta_{r0}(\tau), \beta_{r1}(\tau)\} \)
### Estimation result pooled return model

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<th>VIX</th>
<th>TEDrate</th>
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<th>debt ratio</th>
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<td>-0.0221***</td>
<td>0.0485**</td>
<td>0.0030</td>
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<td>0.0023</td>
<td>-0.0973***</td>
<td>-0.0861</td>
<td>-0.2937</td>
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</table>

Table 3: Estimation of returns on lagged returns, market- and node-specific covariates for each geographic region (daily data on SIFI returns 01.01.2007 - 31.12.2015)
Distribution of residuals

Figure 4: QQ plots of the absolute residuals from the individual regressions for JP Morgan, Unicredit and Bank of China and the Gaussian distribution (full sample estimation)

Tail event driven networks of SIFIs
Quantilogram

Figure 5: Quantilograms of residuals from the individual regressions for JP Morgan, Unicredit and Bank of China with the 10%, 50% and 90% quantiles of network factor (full sample estimation)
Figure 6: Slopes from quantile regressions of residuals grouped by geographic regions on network factor (full sample). Colored area shows 95% confidence band, horizontal lines depict OLS parameters.
Estimation results

Time variation of network effect

Figure 7: Moving window estimation (90 days) of $\beta_1(\tau)$ in quantile regression aggregate for geographic regions

Tail event driven networks of SIFIs
Conclusion

- Systemic risk depends on interdependence of SIFIs in stress situations
- TENQR method allows to isolate network factor and to study joint dynamics
- Network topology allows precise insight into management of systemic risk
- Supervisors may identify high risk contributors and predict their impact in an interconnected financial system
Tail event driven networks of SIFIs

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Tail event driven networks of SIFIs
Appendix

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Tail event driven networks of SIFIs
Appendix

Fisher’s Z transformation

- **Transformation:**

  \[ \rho^*_j = \frac{1}{2} \log \left( \frac{1 + \rho_j}{1 - \rho_j} \right) \]

- Transformed correlations are approximately normal with constant
  \[ \text{Var}(\rho^*_j) = \frac{1}{(h-3)} \] (\(h = \text{sample size}\))
Estimation of break fractions $\hat{\theta}_1, \hat{\theta}_2$

Minimize the total sum of squared residuals

$$(\hat{\theta}_1, \hat{\theta}_2) = \arg\min_{\theta_1,2 \in [\theta, \bar{\theta}]} \sum_{j=1}^{[\theta_1n]} (\Delta(j) - \bar{\Delta}_S)^2 + \sum_{j=[\theta_1n]+1}^{[\theta_2n]} (\Delta(j) - \bar{\Delta}_M)^2 + \sum_{j=[\theta_2n]+1}^{n} (\Delta(j) - \bar{\Delta}_L)^2$$

$$\bar{\Delta}_S = \frac{1}{[\theta_1n]} \sum_{j=1}^{[\theta_1n]} \Delta(j),$$

with

$$\bar{\Delta}_M = \frac{1}{[\theta_2n] - [\theta_1n]} \sum_{j=[\theta_1n]+1}^{[\theta_2n]} \Delta(j),$$

$$\bar{\Delta}_L = \frac{1}{n - [\theta_2n]} \sum_{j=[\theta_2n]+1}^{n} \Delta(j).$$

- $\Delta(j)$ ordered spacings, $[\theta n]$ integer part of $\theta n$ and $\bar{\theta} = 0.1 = 1 - \bar{\theta}$
- $\theta_1$: fraction of highly negative correlations
- $\theta_2$: fraction of highly positive correlations
Appendix 8-6

Cross-quantilogram, Han et al. (2016)

Capture of serial dependence between the two series at different conditional quantile levels

\[ \varrho(\tau_1, \tau_2)(k) = \frac{\sum_{t=k+1}^{T} \varphi_{\tau_1}(y_{1t} - \tilde{y}_{1,\tau_1}) \varphi_{\tau_2}(y_{2,t-k} - \tilde{y}_{2,\tau_2})}{\sqrt{\sum_{t=k+1}^{T} \varphi_{\tau_1}^2(y_{1t} - \tilde{y}_{1,\tau_1}) \sum_{t=k+1}^{T} \varphi_{\tau_2}^2(y_{2,t-k} - \tilde{y}_{2,\tau_2})}} \]

where \( \varphi_{\tau}(u) = I(u < 0) - \tau \), with \( \tau \in (0, 1) \)