The Econometrics of CRIX

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Currencies - Cigarettes, USD, Cryptos

- Anything can be a currency
- Anyone can offer a currency

Figure 1: Cigarette trading in postwar Germany ([1])

Figure 2: Friedrich A. Hayek ([2])

Econometric Analysis
Digital Economy

- Amazon
- Paypal
- Google Wallet
- Cryptocurrencies
- Ripple
Cryptocurrencies

- Decentralized, virtual, low transaction costs

- NYSE, Andreesen Horowitz, DFJ: Coinbase funding (75 M$)
- Nasdaq: company-wide utilization of blockchain technology
- Citigroup: own coin development
- PBOC: working on digital currency
- Switzerland Zug: first city accepts Bitcoin payments
Pokémon Go and Cryptocurrency

- Each creature could have an asset based crypto-tokens that could be traded in blockchain.
- Pokémon and BTC: PokéBits

Source: steemit, Bitcoin.com
Market Capitalization

CoinMarketCap
Econometric Analysis
CRypto IndeX - CRIX

- high market capitalization
- covers approximately 30 cryptos
  - different liquidity rules
  - model selection criteria
- CRIX family
  - CRIX
  - ECRIX (Exact CRIX)
  - EFCRIX (Exact Full CRIX)

Motivation

CRrypto IndeX - CRIX

- 290 cryptos
- Prices, capitalization, volume
- As of 20160815, overview of CRIX: hu.berlin/crix
  - Users: 1911
  - Page views: 3920
  - average time: 00:01:17

Econometric Analysis
Challenge

1. What’s the dynamics of CRIX?
2. How stable is the CRIX model over time?
The Econometrics of CRIX
Outline

1. Motivation ✓
2. Data
3. ARIMA Model
4. Stochastic Volatility Model
5. Multivariate GARCH Model
6. Nutshell

All QuantLets from www.quantlet.de
Three Indices

Figure 3: The daily value of indices in the CRIX family from 01/08/2014 to 06/04/2016: CRIX, ECRIX and EFCRIX.
Data Description

Figure 4: The log returns of CRIX index from 01/08/2014 to 06/04/2016.
Figure 5: Histogram and QQ plot of CRIX returns from 01/08/2014 to 06/04/2016.

Econometric Analysis
First Approach

The ARIMA($p, d, q$) with $d = 1$ is,

$$\Delta y_t = a_1 \Delta y_{t-1} + a_2 \Delta y_{t-2} + \ldots + a_p \Delta y_{t-p}$$
$$+ \varepsilon_t + b_1 \varepsilon_{t-1} + b_2 \varepsilon_{t-2} + \ldots + b_q \varepsilon_{t-q}$$

or

$$a(L) \Delta y_t = b_L \varepsilon_t$$

- $\Delta y_t = y_t - y_{t-1}$, can be replaced by $\Delta^d y_t$ if necessary.
- $L$ is the lag operator, $\varepsilon_t \sim N(0, \sigma^2)$
Box-Jenkins Procedure

1. Identification of lag orders
2. Parameter estimation
3. Diagnostic checking
Step 1: Lag Orders

$p$-value for stationarity tests: ADF test (null hypothesis: unit root) of 0.01; KPSS test (null hypothesis: stationary) of 0.1.

Figure 6: The sample ACF and PACF of CRIX returns from 01/08/2014 to 06/04/2016.
Step 1: Lag Orders - ctd

<table>
<thead>
<tr>
<th>ARIMA model selected</th>
<th>AIC</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARIMA(2,0,0)</td>
<td>-2469</td>
<td>-2451</td>
</tr>
<tr>
<td>ARIMA(2,0,2)</td>
<td>-2474</td>
<td>-2448</td>
</tr>
<tr>
<td>ARIMA(2,0,3)</td>
<td>-2473</td>
<td>-2442</td>
</tr>
<tr>
<td>ARIMA(4,0,2)</td>
<td>-2476</td>
<td>-2441</td>
</tr>
<tr>
<td>ARIMA(2,1,1)</td>
<td>-2459</td>
<td>-2441</td>
</tr>
<tr>
<td>ARIMA(2,1,3)</td>
<td>-2464</td>
<td>-2438</td>
</tr>
</tbody>
</table>

Table 1: The ARIMA model selection with AIC and BIC.
**Step 2: Parameter Estimation**

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Estimate</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>intercept $c$</td>
<td>-0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$a_1$</td>
<td>-0.70</td>
<td>0.11</td>
</tr>
<tr>
<td>$a_2$</td>
<td>-0.75</td>
<td>0.12</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.70</td>
<td>0.14</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.64</td>
<td>0.13</td>
</tr>
</tbody>
</table>

**Log likelihood** 1243.12

Table 2: Estimation result of ARIMA(2,0,2) model. econ_arima
Step 3: Diagnostic Checking

- Diagnostic plot of ARIMA(2,0,2) model
- Significant \( p \)-values of Ljung-Box test statistic
- Model residuals are independent
ARIMA Model Forecast

With ARIMA(2,0,2) model, we predict CRIX returns for next 30 days.

Figure 7: CRIX returns and predicted values. The confidence bands are red dashed lines.
Discussion

- We build an ARIMA(2,0,2) model for the CRIX return series to model intertemporal dependence.
- ACF of model residuals has no significant lags as evidenced in Step 3: Diagnostic Checking.
- Further work: Homoskedasticity or Heteroskedasticity.
Volatility Clustering

Figure 8: The squared ARIMA(2,0,2) residuals of CRIX returns.

Econometric Analysis
ARCH Model

ARCH\( (q) \) model,

\[
\begin{align*}
\varepsilon_t &= Z_t \sigma_t \\
Z_t &\sim \mathcal{N}(0, 1) \\
\sigma_t^2 &= \omega + \alpha_1 \varepsilon_{t-1}^2 + \ldots + \alpha_p \varepsilon_{t-p}^2
\end{align*}
\]

\( \varepsilon_t \) is the ARIMA model residual
\( \sigma_t^2 \) is the variance of \( \varepsilon_t \) conditional on the information available at time \( t \).
Heteroskedasticity effect

- Two approaches:
  - ARCH LM test (null hypothesis: no ARCH effects) of $\varepsilon_t$
  - Ljung-Box test for $\varepsilon_t^2$
- Both $p$-values smaller than $2.2e-16$.
- Next step: determine lag order $q$ of ARCH model
Lag order $q$

Figure 9: The ACF and PACF of squared ARIMA(2,0,2) residuals from 01/08/2014 to 06/04/2016.
Lag Order \( q \) - ctd

<table>
<thead>
<tr>
<th>Model</th>
<th>Log Likelihood</th>
<th>AIC</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARCH(1)</td>
<td>1281.7</td>
<td>-2567.4</td>
<td>-2558.6</td>
</tr>
<tr>
<td>ARCH(2)</td>
<td>1283.4</td>
<td>-2560.8</td>
<td>-2547.6</td>
</tr>
<tr>
<td>ARCH(3)</td>
<td>1291.6</td>
<td>-2575.2</td>
<td>-2557.5</td>
</tr>
<tr>
<td>ARCH(4)</td>
<td>1288.8</td>
<td>-2567.5</td>
<td>-2545.4</td>
</tr>
</tbody>
</table>

Table 3: The ARCH model selection with AIC and BIC.
## ARCH Estimation

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Estimates</th>
<th>Standard deviation</th>
<th>Ljung-Box test statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega$</td>
<td>0.001</td>
<td>0.000</td>
<td>16.798*</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.195</td>
<td>0.042</td>
<td>4.589*</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.054</td>
<td>0.037</td>
<td>1.469</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>0.238</td>
<td>0.029</td>
<td>8.088*</td>
</tr>
</tbody>
</table>

Table 4: Estimation result of ARIMA(2,0,2)-ARCH(3) model, with significant level is 0.1%.
GARCH Model

The standard GARCH\((p, q)\) model is,

\[
\varepsilon_t = Z_t \sigma_t \\
Z_t \sim N(0, 1)
\]

\[
\sigma_t^2 = \omega + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2
\]

with the condition that

\[
\omega > 0; \quad \alpha_i \geq 0, \beta_i \geq 0; \quad \sum_{i=1}^{p} \beta_i + \sum_{j=1}^{q} \alpha_j < 1
\]

This ensures that the GARCH model is strictly stationary with finite variance.

Continuous-time GARCH model

Econometric Analysis
Lag Orders $p, q$

- Normally up to GARCH(2, 2) model is used in practice.
- In particular, the orders of $p = q = 1$ is sufficient in most cases.

<table>
<thead>
<tr>
<th>GARCH models</th>
<th>Log likelihood</th>
<th>AIC</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>GARCH(1,1)</td>
<td>1305.355</td>
<td>-4.239</td>
<td>-4.210</td>
</tr>
<tr>
<td>GARCH(1,2)</td>
<td>1309.363</td>
<td>-4.249</td>
<td>-4.213</td>
</tr>
<tr>
<td>GARCH(2,1)</td>
<td>1305.142</td>
<td>-4.235</td>
<td>-4.199</td>
</tr>
<tr>
<td>GARCH(2,2)</td>
<td>1309.363</td>
<td>-4.245</td>
<td>-4.202</td>
</tr>
</tbody>
</table>

Table 5: Comparison of GARCH model, orders up to $p = q = 2$.  

Econometric Analysis
GARCH Estimation

**GARCH(1,2) model,**

\[
\begin{align*}
\varepsilon_t &= Z_t \sigma_t, \quad Z_t \sim N(0, 1) \\
\sigma_t^2 &= \omega + \beta_1 \sigma_{t-1}^2 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2
\end{align*}
\]

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Estimates</th>
<th>Standard deviation</th>
<th>Ljung-Box test statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\omega)</td>
<td>9.91e-05</td>
<td>4.75e-05</td>
<td>2.08*</td>
</tr>
<tr>
<td>(\alpha_1)</td>
<td>1.65e-01</td>
<td>3.72e-02</td>
<td>4.45***</td>
</tr>
<tr>
<td>(\beta_1)</td>
<td>8.07e-02</td>
<td>8.24e-02</td>
<td>0.98</td>
</tr>
<tr>
<td>(\beta_2)</td>
<td>6.51e-01</td>
<td>8.20e-02</td>
<td>7.94***</td>
</tr>
</tbody>
</table>

Table 6: Estimation result of ARIMA(2,0,2)-GARCH(1,2) model. * represents significant level of 5% and *** of 0.1%.
GARCH Estimation II

- GARCH(1,1) model is sufficient in most cases,

\[ \varepsilon_t = Z_t \sigma_t, \quad Z_t \sim N(0, 1) \]
\[ \sigma_t^2 = \omega + \beta_1 \sigma_{t-1}^2 + \alpha_1 \varepsilon_{t-1}^2 \]

- All parameters are significant:

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Estimates</th>
<th>Standard deviation</th>
<th>Ljung-Box test statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega )</td>
<td>5.32e-05</td>
<td>2.25e-05</td>
<td>2.37*</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>1.20e-01</td>
<td>2.79e-02</td>
<td>4.32***</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>8.32e-02</td>
<td>3.99e-02</td>
<td>20.85***</td>
</tr>
</tbody>
</table>

Table 7: Estimation result of ARIMA(2,0,2)-GARCH(1,1) model. \* represents significant level of 5% and \*\*\* of 0.1%.

Econometric Analysis
GARCH Estimation II - ctd

With no significant correlations for any lag, GARCH(1,1) is sufficient enough to explain the heteroskedasticity effect.

Figure 10: The ACF and PACF of squared ARIMA(2,0,2) residuals from 01/08/2014 to 06/04/2016.
GARCH Model Residual

- Kolmogorov-Smirnov test of ARIMA-GARCH model residuals.
- The small $p$-value rejects the null hypothesis that the residuals are drawn from the normal distribution.
- Sample data exhibits leptokurtosis.

<table>
<thead>
<tr>
<th>Model</th>
<th>Kolmogorov distance</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARIMA-GARCH</td>
<td>0.50</td>
<td>2.86e − 10</td>
</tr>
</tbody>
</table>

Table 8: Test of model residuals of ARIMA(2,0,2)-GARCH(1,1) process.
GARCH Model Residual - ctd

Figure 11: The QQ plots of model residuals of ARIMA-GARCH process.

Econometric Analysis
$t$-GARCH Estimation

- Impose $Z_t \sim t(d)$ to replace the normal assumption of $Z_t$
- $\xi$ controls the height and fat-tail of density function, therefore different shape of distribution function.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Estimates</th>
<th>Standard deviation</th>
<th>T test</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega$</td>
<td>8.39e − 05</td>
<td>5.45e − 05</td>
<td>1.54</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>2.82e − 01</td>
<td>1.46e − 01</td>
<td>1.93∗</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>7.90e − 01</td>
<td>6.12e − 02</td>
<td>12.91***</td>
</tr>
<tr>
<td>$\xi$</td>
<td>2.58e + 00</td>
<td>3.62e − 01</td>
<td>7.11***</td>
</tr>
</tbody>
</table>

Table 9: Estimation result of ARIMA(2,0,2)-$t$-GARCH(1,1) model. ∗ represents significant level of 10% and ∗∗∗ of 0.1%.

Econometric Analysis
**t-GARCH Model Estimation - ctd**

![ACF and PACF plots](image)

**Figure 12:** The ACF and PACF plots for model residuals of ARIMA(2,0,2)-t-GARCH(1,1) process.

**Econometric Analysis**
$t$-GARCH Model Residual

Figure 13: The QQ plots of model residuals of ARIMA-$t$-GARCH process.

Econometric Analysis
EGARCH Model

- The introduced GARCH model successfully solve the problem of volatility clustering, but cannot capture the leverage effect.
- The exponential GARCH (EGARCH) model with standard innovations,

\[ \varepsilon_t = Z_t \sigma_t \]

\[ Z_t \sim N(0, 1) \]

\[ \log(\sigma_t^2) = \omega + \sum_{i=1}^{p} \beta_i \log(\sigma_{t-i}^2) + \sum_{j=1}^{q} g_j(Z_{t-j}) \]

with the condition that

\[ g_j(Z_t) = \alpha_j Z_t + \phi_j(|Z_{t-j}| - E|Z_{t-j}|), \quad j = 1, 2, \ldots, q \]
**$t$-EGARCH Estimation**

- Fit a $\text{EGARCH}(1,1)$ model with student $t$ distributed innovation term.
- The estimation results of the $\text{ARIMA}(2,0,2)-t-\text{EGARCH}(1,1)$ model is,

<table>
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<th>Standard deviation</th>
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<tr>
<td>$\omega$</td>
<td>9.91e−05</td>
<td>4.75e−05</td>
<td>2.08*</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>1.65e−01</td>
<td>3.72e−02</td>
<td>4.45*</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>8.07e−02</td>
<td>8.24e−02</td>
<td>0.98</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>6.51e−01</td>
<td>8.20e−02</td>
<td>7.94*</td>
</tr>
</tbody>
</table>

Table 10: Estimation result of $\text{ARIMA}(2,0,2)-t-\text{EGARCH}(1,1)$ model. * represents significant level of 5% and ** of 0.1%. 

Econometric Analysis
$t$-EGARCH Model Residual

Figure 14: The QQ plots of model residuals of ARIMA-$t$-EGARCH process.

Econometric Analysis
GARCH Model Selection

<table>
<thead>
<tr>
<th>GARCH models</th>
<th>Log likelihood</th>
<th>AIC</th>
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</thead>
<tbody>
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<tr>
<td>t-GARCH(1,1)</td>
<td>1309.363</td>
<td>-4.249</td>
<td>-4.213</td>
</tr>
<tr>
<td>t-EGARCH(1,1)</td>
<td>1305.142</td>
<td>-4.235</td>
<td>-4.199</td>
</tr>
</tbody>
</table>

Table 11: Comparison of the variants of GARCH model.
MGARCH Model

Consider the error term $\varepsilon_t$ with $E(\varepsilon_t) = 0$, and conditional covariance matrix $H_t$ is $(d \times d)$ positive definite,

$$\varepsilon_t = H_t^{\frac{1}{2}} \eta_t$$

$H_t^{\frac{1}{2}}$ can be obtained by Cholesky factorization of $H_t$.

$\eta_t$ is an iid innovation vector such that,

$$E(\eta_t) = 0$$
$$\text{Var}(\eta_t) = E(\eta_t \eta_t^\top) = \mathcal{I}_d$$

with $\mathcal{I}_d$ is the identity matrix with order of $d$. 
DCC-GARCH Model

- Different specification of $H_t$ yields various parametric formulations: VEC, BEKK, CCC, DCC etc.
- Dynamic Conditional Correlation (DCC) model: conditional correlation $\rho_{ij}$ between the $i$-th and $j$-th component is the $ij$-th element of the matrix $P_t$

$$
H_t = D_t P_t D_t
$$

$$
P_t = (I \odot Q_t)^{-\frac{1}{2}} Q_t (I \odot Q_t)^{-\frac{1}{2}}
$$

with

$$
Q_t = (1 - a - b)S + a \varepsilon_{t-1} \varepsilon_{t-1}^T + b Q_{t-1}
$$

- The diagonal matrix $D_t$ is the conditional variance matrix.
- $S$ is unconditional matrix of $\varepsilon_t$
DCC-GARCH Model Estimation

Figure 15: The standard error of DCC-GARCH model, with CRIX (upper), ECRIX (middle) and EFCRIX (lower).
DCC-GARCH Model Estimation - ctd

- All the estimated parameters are statistically significant except for the constant terms,

\[
\sigma_{CRIX,t}^2 = 0.123 \varepsilon_{CRIX,t-1}^2 + 0.832 \sigma_{CRIX,t-1}^2
\]

\[
\sigma_{ECRIX,t}^2 = 0.123 \varepsilon_{ECRIX,t-1}^2 + 0.832 \sigma_{ECRIX,t-1}^2
\]

\[
\sigma_{EFCRIX,t}^2 = 0.124 \varepsilon_{EFCRIX,t-1}^2 + 0.831 \sigma_{EFCRIX,t-1}^2
\]

\[
Q_t = (1 - 0.268 - 0.571)S + 0.268 \varepsilon_{t-1}^T \varepsilon_{t-1} + 0.571 Q_{t-1}
\]

- The unconditional covariance matrix \( S \),

\[
S = \begin{pmatrix}
0.994 & 0.994 & 0.994 \\
0.994 & 0.994 & 0.993 \\
0.994 & 0.993 & 0.994
\end{pmatrix}
\]
Figure 16: The estimated volatility (black) and realized volatility (grey) using DCC-GARCH model, for example CRIX.
Figure 17: The dynamic autocorrelation between three CRIX indices: CRIX, ECRIX and EFCRIX estimated by DCC-GARCH model.
Figure 18: The dynamic autocorrelation between three CRIX indices: CRIX, ECRIX and EFCRIX estimated by DCC-GARCH model.
DCC-GARCH Model Diagnostics

Figure 19: The comparison of ACF between premodel squared residuals and DCC squared residuals, for example CRIX.
DCC-GARCH Model Diagnostics - ctd

Figure 20: The comparison of PACF between premodel squared residuals and DCC squared residuals, for example CRIX.
GARCH Option Pricing Model

- Option pricing models
  - Black-Scholes model
  - GARCH models: superior in describing asset return dynamics.

- For instance Heston and Nandi (2000), HN model in brief.
  - a closed form expression for European option prices
  - GARCH models with Gaussian innovations
HN model

- In the HN model, the asset return dynamic under the risk neutral measure $\mathbb{Q}$ is,

$$\log\left(\frac{S_t}{S_{t-1}}\right) = r - \frac{\sigma_t^2}{2} + \sigma_t Z_t$$

$$\sigma_t^2 = \omega_{hn} + \beta_{hn}\sigma_{t-1}^2 + \alpha_{hn}(Z_{t-1} - \gamma_{hn}\sigma_{t-1})^2$$

- $r$ is risk-free interest rate
- $Z_t$ is a standard Gaussian innovation
- Risk neutral GARCH parameter: $\theta_{hn} = \{\omega_{hn}, \beta_{hn}, \alpha_{hn}, \gamma_{hn}\}$
- $S_t$ is the return to estimate.
HN model - ctd

☐ The call option $C_t$ at time $t$, with strike price $K$ and time to maturity $\tau$ is worth,

$$
C_t = \exp(-r\tau)f_{hn}(1) \left[ \frac{1}{2} + \frac{1}{\pi} \int_{0}^{\infty} \mathcal{R} \left\{ \frac{K^{-i\phi}f_{hn}(i\phi + 1)}{i\phi f_{hn}(1)} \right\} d\phi \right] 
$$

$$
- \exp(-r\tau)K \left[ \frac{1}{2} + \frac{1}{\pi} \int_{0}^{\infty} \mathcal{R} \left\{ \frac{K^{-i\phi}f_{hn}(i\phi)}{i\phi} \right\} d\phi \right] 
$$

\(\mathcal{R}\{\}\) denotes the real part of a complex number

\(f_{hn}(\phi)\) is the conditional moment generating function at time $t$

$$
f_{hn}(\phi) = E_Q \left[ \exp \left\{ \phi \log(S_t) \right\} \middle| \mathcal{F}_t \right] = S_t^\phi \exp(A_t + B_t\sigma_{t+1}^2) 
$$
HN model - ctd

- The coefficients $A_t$ and $B_t$ are computed backward starting from the terminal condition $A_T = B_T = 0$.
- The HN model recursive equations are,

$$A_t = A_{t+1} + \phi r + B_{t+1} \omega_{hn} - \frac{1}{2} \log(1 - 2\alpha_{hn} B_{t+1})$$

$$B_t = \phi \left( \gamma_{hn} - \frac{1}{2} \right) - \frac{\gamma_{hn}^2}{2} + \beta_{hn} B_{t+1} + \frac{1/2(\phi - \gamma_{hn})^2}{1 - 2\alpha_{hn} B_{t+1}}$$
Nutshell

- ARIMA model is implemented for removing the intertemporal dependence.
- Volatility models such as ARCH, GARCH and EGARCH are applied to eliminate the effect of heteroskedasticity.
- The $t$-GARCH(1,1) is introduced to deal with the fat-tail properties of GARCH residuals.
- DCC-GARCH(1,1) exhibits time varying covariances between three CRIX indices.
- Outlook: GARCH option pricing model, eg. HN GARCH model.
The Econometrics of CRIX

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References

Cigarette trading in postwar Germany, Bundesarchiv, Bild 183-R79014 / CC-BY-SA.

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**COGARCH Model**

- Irregularly spaced data: continuous-time GARCH model.
- The GARCH(1, 1) model diffusion limit satisfies,

\[
\begin{align*}
  dG_t &= \sigma_t dW_t^{(1)} \\
  d\sigma_t^2 &= \theta(\gamma - \sigma_t^2) + \rho\sigma_t^2 dW_t^{(2)}
\end{align*}
\]

- \( G_t \) is the log return \( r_t \) to estimate.
- \( \left\{ W_t^{(1)} \right\}_{t \geq 0} \) and \( \left\{ W_t^{(2)} \right\}_{t \geq 0} \) are two independent Brownian motions.
- \( \theta, \gamma \) and \( \rho \) are parameters.