Time-varying Limit Order Book Networks

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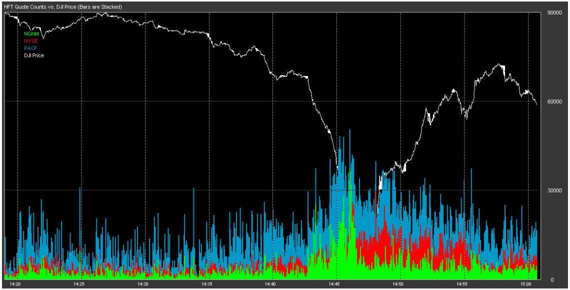
High Frequency Trading



- Rapid-fire trading makes decisions in *m*s.
- 10 seconds of HFT in super slow motion.

HF Data Overview

- Take the best ask price on one single trading day as example,

	Microsoft	Pfizer	Citigroup
NumObs ($*10^3$)	584.55	427.51	472.90

Table 1: *NumObs* denotes the number of tick-by-tick observations.

- Size of data sample, including 9 stocks over 2 months: 23.4 gigabyte (GB)

	Microsoft	JP & Morgan	Citigroup
Size, GB	5.0	3.3	3.5

Data Source

- Reconstructed limit order book (LOB) with NASDAQ traded stocks.
- Source: lobsterdata.com
  - NASDAQ'S historical TotalView - ITCH files
  - Detailed event information

LOBSTER
Limit Order Book System – The Efficient Reconstructor

Nasdaq

LOB Data Structure

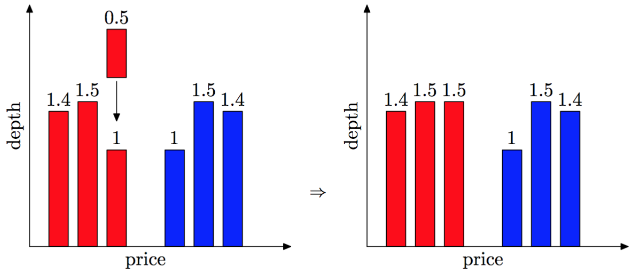
- Indicators for the type of event causing an update of the limit order book. e.g. 34713 sec = 9:38:55am; Direction−1: sell

Time (sec)	Event Type	Order ID	Size	Price	Direction
⋮	⋮	⋮	⋮	⋮	⋮
34713.685155243	1	206833312	100	118600	-1
34714.133632201	3	206833312	100	118600	-1
⋮	⋮	⋮	⋮	⋮	⋮

- Evolution of the limit order book up to the requested number of levels.

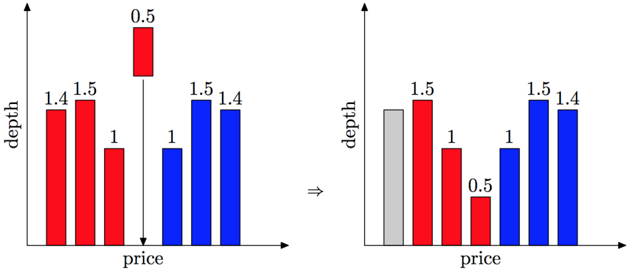
Ask Price 1	Ask Size 1	Bid Price 1	Bid Size 1	Ask Price 2	Ask Size 2	Bid Price 2	Bid Size 2	...
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1186600	9484	118500	8800	118700	22700	118400	14930	...
1186600	9384	118500	8800	118700	22700	118400	14930	...
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

LOB - Normal Limit Order



- arrival of a buy limit order with price 1000 and size 0.5 to be placed.

LOB - Aggressive Limit Order



- arrival of a buy limit order with price 1001 and size 0.5 to be posted inside of the current spread. ▸ more details: passive limit order

Research Interest

- How does the order flows interact with price dynamics, and further affect the market behavior?
- Are the impacts on price responding to incoming ask and bid market/limit orders widely symmetric? If not, how does the heterogeneous market impact caused by bid and ask order for various stocks affect the whole market?
- How to measure the impact of market/limit order quantitatively?

Challenges

1. Construct LOB network in the presence of microstructure noise (MN) and non-synchronous trading.
   - propose volume synchronization algorithm.
2. Extend current literature where the connectedness measures are often estimated by MA representation of VAR systems and restricted to Gaussian innovations
   - combine bootstrap-based generalized impulse response analysis with network construction.
3. Efficient programming to speed up numerical computation ($<$ 1 day).

LOB Network

Network structure with sparsity to find relevant intercon-nections

Individual stock network

- **stock $A \overset{impact}{\Longleftrightarrow}$ stock B**
- **with and without order flows**

LOB network

- **Asymmetric market sell/buy pressure**
- **Own-price/cross-price market impact**

- Focus on the dynamics of LOB networks and their evolution.

Programming Micro-Benchmarks

- Benchmark times relative to Cpp (smaller is better)

	Cpp gcc 4.8.5	Matlab R 2018a	Python 3.6.3	R 3.3.1
iteration-pi-sum	1.00	1.01	14.75	8.92
recursion-fibonacci	1.00	18.69	100.77	608.81
parse-integers	1.00	229.56	19.98	50.90
matrix-statistics	1.00	8.10	17.93	20.35
matrix-multiply	1.00	1.16	1.18	8.74
userfunc-mandelbrot	1.00	10.07	132.38	333.03

Outline

All QuantLets from www.quantlet.de

Microstructure Noise

- High Frequency (HF) data is contaminated by market frictions.
  - stale prices due to infrequent trading.
  - price discreteness, rounding
  - bid-ask bounds and misprints.
- Tick size is one cent for any stock over 1 USD.
- More liquid stocks have small tick sizes.

Basic Model

- MN contaminated Y_t (log price) with latent X_t,

$$Y_t = X_t + \varepsilon_t, \quad t \geq 0$$

  with $\mathrm{E}(\varepsilon_t | X) = 0$.

- Efficient log price X_t is semi-martingale, Delbaen and Schachermayer (1994),

$$X_t = X_0 + \int_0^t a_s ds + \int_0^t \sigma_s dW_s$$

- $(a_s)_{s \geq 0}$ càdlàg drift process, $(\sigma_s)_{s \geq 0}$ càdlàg volatility process.

Estimation Strategy

- Pre-averaging mitigates MN,
  - transaction or quote data are averaged over short time periods ranging from 30s to 5m.
  - the resulting average approximates the efficient price process much better than raw data.
- Portfolio selection using HF data
- Match price and size.

Pre-averaged Estimator

- Choose the "kernel" of $g(x) = x \wedge (1 - x)$, Podolskij et al. (2009), Christensen et al. (2010). More details of model setting
- Then pre-average returns with g,

$$\overline{Y}_i^n = \sum_{j=1}^{k_n-1} g\left(\frac{j}{k_n}\right) \Delta_{i+j}^n Y$$

$$= -\sum_{j=0}^{k_n-1} \left\{ g\left(\frac{j+1}{k_n}\right) - g\left(\frac{j}{k_n}\right) \right\} Y_{i+j}^n$$

for $i = 0, \ldots, n - k_n + 1$.

Size Intensity

- In contrast to a moderate time interval for price to reduce the MN, the time interval for trading volumes should be small enough to capture the trading crowd.
- Define size intensity as $\tilde{S}_{t_j}$, t_j denotes the time stamp of jth LOB.

$$\tilde{S}_{t_j} = S_{t_j}(t_{j+1} - t_j)$$

- The *size intensity* can be summed up over a given time interval and can be matched with returns.

Volume Synchronization Algorithm

1. Set equally-spaced time intervals $T_0 + k\Delta T$, $k = 0, 1, 2, \ldots, K$
2. Define the price and size at time $T_0 + k\Delta T$ as

$$\begin{aligned}
\tilde{P}_{T_0+k\Delta T} &= P_{t_m}, \;\; t_m = \max\{t_j;\;\; t_j \le T_0 + k\Delta T\} \\
\tilde{S}_{T_0+k\Delta T} &= \sum_{T_0+(k-1)\Delta T \le t_j \le T_0+k\Delta T} S_{t_j}(t_{j+1} - t_j)
\end{aligned}$$

3. The changes of the log values are

$$\begin{aligned}
\Delta p_{T_0+k\Delta T} &= \log \tilde{P}_{T_0+k\Delta T} - \log \tilde{P}_{T_0+(k-1)\Delta T} \\
\Delta s_{T_0+k\Delta T} &= \log \tilde{S}_{T_0+k\Delta T} - \log \tilde{S}_{T_0+(k-1)\Delta T}
\end{aligned}$$

Volume Synchronization Algorithm - ctd

4. Pre-averaging both $\Delta p_{T_0+k\Delta T}$ and $\Delta s_{T_0+k\Delta T}$ by

$$\Delta \tilde{p}_{T_0+k\Delta T} = \sum_{j=0}^{J} g_j \Delta p_{T_0+j\Delta T}$$

$$\Delta \tilde{s}_{T_0+k\Delta T} = \sum_{j=0}^{J} g_j \Delta s_{T_0+j\Delta T}$$

where $g_j \geq 0$ and $\sum_{j=0}^{J} g_j = 1$.

hfhd_dataclean

Variables

- Prepare data in this way helps to:
  - alleviate microstructure noise
  - match the price to the size
  - solve non-synchronicity.
- Record the mid price on the first level, the ask and bid sizes on the first 3 levels,

$$y_t^{(n)\top} = [\Delta\tilde{p}_t^{(n)}, \Delta\tilde{s}_t^{a1(n)}, \Delta\tilde{s}_t^{a2(n)}, \Delta\tilde{s}_t^{a3(n)}, \Delta\tilde{s}_t^{b1(n)}, \Delta\tilde{s}_t^{b2(n)}, \Delta\tilde{s}_t^{b3(n)}]$$

This yields a vector,

$$Y_t^\top = [y_t^{(1)\top}, y_t^{(2)\top}, \dots, y_t^{(N)\top}]$$

that is stacked by $y_t^{(n)T}$ for different N stocks.

Penalized VAR Approach

- The VAR(p) model of Y_t,

$$
\begin{aligned}
Y_t &= \nu + A_1 Y_{t-1} + A_2 Y_{t-2} + \cdots + A_p Y_{t-p} + u_t \qquad (1) \\
&= \nu + (A_1, A_2, \ldots, A_p) \left(Y_{t-1}^\top, Y_{t-2}^\top, \ldots, Y_{t-p}^\top \right)^\top + u_t
\end{aligned}
$$

- Assume (1) satisfies,
  1. The roots of $|I_K - \sum_{j=1}^{p} A_j z^j| = 0$ lie outside unit circle.
  2. u_t are i.i.d innovations;
     each element has bounded $(4+\delta)$th moment, $\delta > 0$.
  3. $\|\Sigma_u\|_2 < \infty$ and $\| (A_1, A_2, \ldots, A_p) \|_2 < \infty$.

Penalized VAR Approach

- For the multivariate case,

$$vec(Y) = (Z^\top \otimes I_K)vec(A) + vec(U) \tag{2}$$

- and

$$\begin{aligned} Y &= (Y_1, Y_2, \ldots, Y_T) \qquad & A = (A_1, A_2, \ldots, A_p) \\ Z_t &= (y_t, y_{t+1}, \ldots, y_{t-p+1})^\top \qquad & Z = (Z_0, Z_1, \ldots, Z_{T-1}) \end{aligned}$$

where A_i are $(K \times K)$, $K = 7N$.

- The compact form is equivalent to,

$$\mathbf{y} = (Z^\top \otimes I_K)\beta + \mathbf{u} = \mathbf{x}\beta + \mathbf{u}$$

Estimation Strategy

- Elastic net:

$$\arg\min_{\beta} \left(\|\mathbf{y} - \mathbf{x}\beta\|_2^2 + \alpha_{1,T}\|\beta\|_1 + \alpha_{2,T}\|\beta\|_2^2 \right)$$

equivalent to:

$$\arg\min_{A_1, A_2, \ldots, A_p} \sum_{t=1}^{T} \| Y_t - \sum_{j=1}^{p} A_j Y_{t-j} \|_2^2 + \alpha_{1,T} \sum_{j=1}^{p} \| vec(A_j) \|_1 + \alpha_{2,T} \sum_{j=1}^{p} \| vec(A_j) \|_2^2$$

  - $\|M\|_p$ depends on whether M is a vector or a matrix.
  - To avoid confusion, $vec(M)$ used to transform the object within $\|\|_p$ into a vector.
  - Reference: Basu et al. (2015); Liang and Schienle (2017).

Generalized Impulse Response Function

- Assume shocks hitting at j-th equation.
- Define the generalized impulse response GI, as shock δ_{jt} on j-th equation of y_t at horizon l,

$$GI(l, \delta_{jt}, \mathcal{F}_{t-1}) = \mathsf{E}(y_{t+l} | u_{jt} = \delta_{jt}, \mathcal{F}_{t-1}) - \mathsf{E}(y_{t+l} | \mathcal{F}_{t-1})$$

- $\mathsf{E}(y_{t+1} | u_{jt} = \delta_{jt}, \mathcal{F}_{t-1})$ represents the expectation conditional on the history $\mathcal{F}_{t-1}$ and a fixed value of j-th shock on time t.

GI Analysis

$GI(l, \delta_{jt}, \mathcal{F}_{t-1})$

$\delta_{jt} : (\delta_{1t}, \delta_{2t}, \ldots, \delta_{Kt})^\top \sim e_j$
measure the persistent effect of a shock on the behavior of a series

$\delta_{jt} : (\delta_{1t}, \delta_{2t}, \ldots, \delta_{Kt})^\top \sim \hat{u}^\star_{jt} e_j$
use *GI* to construct connectedness measure

- *GI* with unit shock, Koop (1996). ▸ Measuring price direction
- *GI* with shock coming from system, Lanne and Nyberg (2016).
- Conditional mean forecast for more than one period ahead with bootstrap-based method. ▸ Bootstrap-based multistep forecast method

Fashionable Connectedness Measures

- Construct connectedness measures with generalized forecast error variance decomposition (GFEVD).
- DY-connectedness of Diebold and Yilmaz (2014): use GFEVD of Koop et al. (1996), Pesaran and Shin (1998)
  - The relative contributions to *h*-period impact of the shocks do not sum to unity.
  - GFEVD is constructed by orthogonalized impulse response function based on MA representation of VAR
- In this paper, we use the LN-type GFEVD,
  - LN-GFEVD of Lanne and Nyberg (2016) is a modification of GFEVD
  - The relative contributions to *h*-period impact of the shocks sum to unity, facilitates convenient interpretation.
  - The LN-GFEVD can be obtained in nonlinear models.

New Connectedness Measure

- The LN-GFEVD is defined by j-th shock hitting i-th variable at time t with h forecast horizon,

$$\lambda_{ij}(h) = \frac{\sum_{l=0}^{h} GI(l, \delta_{jt}, \mathcal{F}_{t-1})_i^2}{\sum_{j=1}^{K} \sum_{l=0}^{h} GI(l, \delta_{jt}, \mathcal{F}_{t-1})_i^2}, \quad i, j = 1, \ldots, K$$

  - measuring the relative contribution of a shock δ_{jt} to the j-th equation in relation to the total impact of all K shocks on the i-th variable in y_t after h periods.

- Conditional mean forecast for more than one period ahead with bootstrap yields $\lambda_{ij}^{b}(h)$.
- The pairwise directional connectedness is $C_{i\leftarrow j} = \lambda_{ij}^{b}(h)$

Connectedness Table

	x_1	x_2	$\dots$	x_K	From others
x_1	$\lambda_{11}^b(h)$	$\lambda_{12}^b(h)$	$\dots$	$\lambda_{1K}^b(h)$	$\sum_{j=1}^K \lambda_{1j}^b(h) = 1, j \neq 1$
x_2	$\lambda_{21}^b(h)$	$\lambda_{22}^b(h)$	$\dots$	$\lambda_{2K}^b(h)$	$\sum_{j=1}^K \lambda_{2j}^b(h) = 1, j \neq 2$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
x_K	$\lambda_{K1}^b(h)$	$\lambda_{K2}^b(h)$	$\dots$	$\lambda_{KK}^b(h)$	$\sum_{j=1}^K \lambda_{Kj}^b(h) = 1, j \neq K$
To others	$\sum_{i=1}^K \lambda_{i1}^b(h)$ $i \neq 1$	$\sum_{i=1}^K \lambda_{i2}^b(h)$ $i \neq 2$	$\dots$	$\sum_{i=1}^K \lambda_{iK}^b(h)$ $i \neq K$	$\frac{1}{K} \sum_{i=1,j=1}^K \lambda_{ij}^b(h)$ $i \neq j$

Table 2: Connectedness table of interest, estimated by bootstrap.

- LN-GFEVD is economically interpretable since $\sum_{j=1}^K \lambda_{ij}^b(h) = 1$.
- The bootstrapped $C_{i \leftarrow j} = \lambda_{ij}^b(h)$ relies neither on the ordering of the variables nor on the distribution of the innovations.

Dataset

Industry	Stock	Company	MktCap (billion $)
Technology	IBM	International Business Machines Corp.	171.72
	MSFT	Microsoft Corporation	499.35
	T	AT&T Inc.	257.53
Healthcare	JNJ	Johnson & Johnson	328.91
	PFE	Pfizer Inc.	206.69
	MRK	Merck & Co. Inc.	181.56
Finance	JPM	JP Morgan Chase & Co.	326.04
	WFC	Wells Fargo & Company	293.39
	C	Citigroup Inc.	168.06

- Sample data from 01.06 to 29.07.2016, MktCap is the market capitalization by Feb 23rd, 2018.

Dataset -ctd

- Divide the trading period into 1-minute intervals, i.e. $\Delta T = 1$ min.
  - Calculate mid price on the first level, obtain $\Delta \tilde{p}_t^{(n)}$
  - Calculate corresponding bid and ask sizes (i.e. size intensity) on the first three levels, i.e. market order, best limit order and 2nd best limit order, i.e. $\Delta \tilde{s}_t^{a1(n)}, \Delta \tilde{s}_t^{a2(n)}, \Delta \tilde{s}_t^{a3(n)}, \Delta \tilde{s}_t^{b1(n)}, \Delta \tilde{s}_t^{b2(n)}, \Delta \tilde{s}_t^{b3(n)}$
- Then pre-average the above values over smooth-window of 15 minutes.
- Upon the estimates of the sparse HD VAR model, calculate the bootstrapped LN-GFEVD and corresponding connectedness at horizon $h = 30$ for every trading day.

Individual Stock Network

- A network is a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with,
  - nodes (or vertices) $\mathcal{V}$: entities we are evaluating
  - edges $\mathcal{E}$: connections between entities
- Cross-stock network $\mathcal{G}_p = (\mathcal{V}_p, \mathcal{E}_p)$ with only price factors $p^{(n)}$,

$$\mathcal{V}_p = p^{(n)}, \quad n = 1, \ldots, N \quad \text{and} \quad \mathcal{E}_p = C_{i \leftarrow j}, \quad i, j \in \mathcal{V}_p \quad (3)$$

Pairwise Stock Connectedness

- To evaluate the node importance, use
  - degree centrality $deg(\mathcal{V})$: interpreted as a form of popularity.
  - closeness centrality $Clos(\mathcal{V})$: distinguish influencers in the network.
  - betweenness centrality $Bet(\mathcal{V})$: help to decide which nodes act as "bridges" between nodes in a network

Pairwise Stock Connectedness - ctd

	MSFT	T	IBM	JNJ	PFE	MRK	JPM	WFC	C
$\mu_{outdeg}(\mathcal{V}_p)$	3.26	3.33	2.52	3.33	3.07	2.81	2.83	2.50	3.40
$\mu_{indeg}(\mathcal{V}_p)$	3.88	2.86	3.43	3.57	2.38	1.69	3.21	2.60	3.45
$\mu_{Clos}(\mathcal{V}_p)$	167.70	163.99	173.45	159.09	159.43	154.56	175.89	171.95	157.81
$\mu_{Bet}(\mathcal{V}_p)$	6.00	7.55	7.98	4.33	4.43	3.02	5.98	5.24	6.07

Table 3: Period: 06-07.2016. $\mu.$ is the mean.

- Citigroup, AT&T and Johnson&Johnson are central in the network, they are choice maker. Meanwhile JNJ is a choice receiver with high "in-degree".
- Conventional centrality measure works well for probing certain phenomena, it fails to capture the node's spreading potential, e.g. JNJ

Including Order Flows

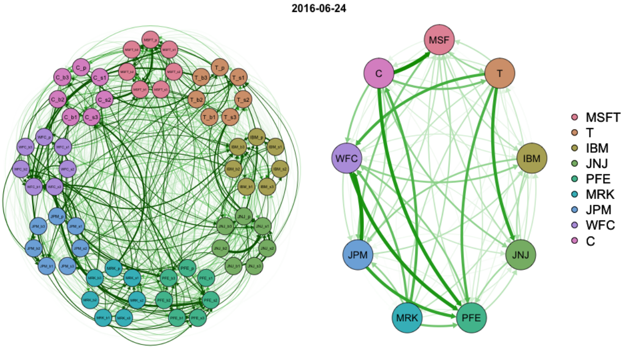


Figure 1: Left panel: the full sample network plot. Right panel: the aggregated network plot of nine stocks, on 24.06.2016. hfhd_cirnet

Including Order Flows - graph

- Aggregated individual stock network $\mathcal{G}_g = (\mathcal{V}_g, \mathcal{E}_g)$:

$$
\begin{aligned}
\mathcal{V}_g &= v_g^{(n)} && (4)\\
v_g^{(n)} &= p^{(n)} + \sum_r bs_r^{(n)} + \sum_r as_r^{(n)}, \quad n = 1, \dots, N && (5)\\
\mathcal{E}_g &= C_{i\leftarrow j} = \lambda_{ij}, \quad i, j \in \mathcal{V}_g && (6)
\end{aligned}
$$

- $as_r^{(n)}$ and $bs_r^{(n)}$ are the r-th level ask/bid size factors for stock n.
- By including size factors from LOB, we are able to investigate how the network is affected by the presence of liquidity effects.

Including Order Flows - ctd

	MSFT	T	IBM	JNJ	PFE	MRK	JPM	WFC	C
$\mu_{outdeg}(\mathcal{V}_g)$	128.83	147.02	132.71	129.76	127.95	125.48	120.31	113.50	123.00
$\mu_{indeg}(\mathcal{V}_g)$	136.29	122.50	121.24	118.31	121.88	121.00	136.79	133.98	136.60
$\mu_{Clos}(\mathcal{V}_g)$	0.13	0.13	0.12	0.13	0.13	0.13	0.13	0.13	0.13
$\mu_{Bet}(\mathcal{V}_g)$	4.17	3.26	2.36	2.07	2.79	2.19	3.12	3.57	3.69

Table 4: Period: 06-07.2016. $\mu.$ is the mean.

- Produces different results compared to those obtained for pairwise stock network in Table 2.
- The impacts caused by less important nodes may be neglected by conventional centrality measures, this will potentially cause inaccuracy and thus result in the poor performance.

Including Order Flows

- Conventional centrality measures are rarely accurate when the majority of nodes are not highly influential in the network.
- Use the quantiles to measure the time-varying net spillover effects for each stock,

$$
\begin{aligned}
C_i &= C_{\bullet\leftarrow i} - 2r - 1 = \sum_j C_{j\leftarrow i} - 2r - 1, \quad i,j \in \mathcal{V}_g \\
Q_{C_i}(\alpha) &= F^{-1}(\alpha) = \inf\{C_i : F(C_i) \geq \alpha\} \quad (7) \\
\alpha &= 5\%, 15\%, 50\%, 85\%, 95\% \quad (8)
\end{aligned}
$$

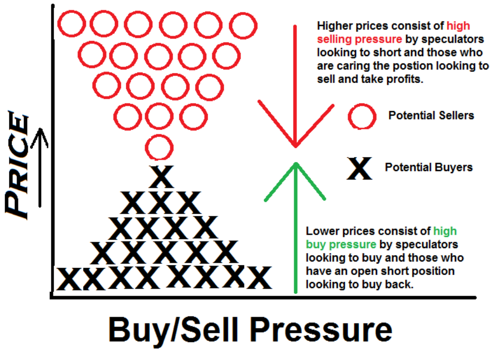
- Quantiles of net connectedness performs better than conventional centrality measures.

Including Order Flows

	MSFT	T	IBM	JNJ	PFE	MRK	JPM	WFC	C
$Q_{C_i}(0.05)$	-3.19	-3.51	-3.72	-3.79	-2.84	-3.38	-2.57	-2.80	-3.35
$Q_{C_i}(0.15)$	-2.15	-3.01	-3.24	-2.99	-2.16	-2.62	-1.97	-2.30	-2.61
$Q_{C_i}(0.50)$	0.17	-0.70	-0.91	-0.50	0.14	-0.71	0.97	0.34	0.35
$Q_{C_i}(0.85)$	2.01	1.47	1.31	2.27	1.78	2.47	3.68	3.21	3.04
$Q_{C_i}(0.95)$	3.84	2.54	2.99	2.70	3.81	3.28	5.11	4.63	4.74

- JP Morgan is most influential in the network, while IBM and AT&T are main risk receivers in the aggregated system.
- Financial companies are dominant stocks driving the networks
- The aggregated individual stock network is a better measure of how central a stock is within the network since it takes into consideration the trading volumes.

Market Interaction



Asymmetric Market Sell/Buy Pressure

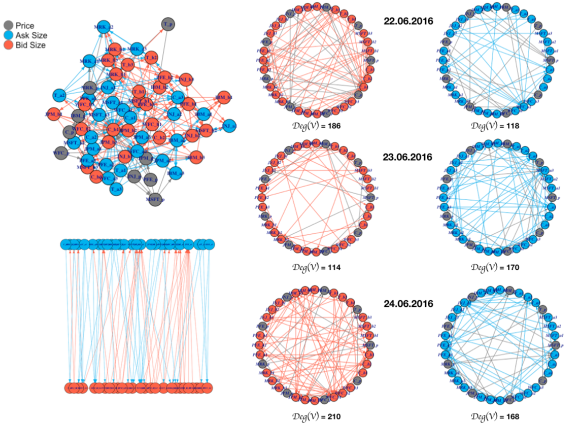


Figure 2: LOB networks from 22.06.2016-24.06.2016. 23.06: Brexit

Asymmetric Market Sell/Buy Pressure - formula

- The LOB network during Brexit announcement: The impacts on returns respond to ask and bid limit orders are not symmetric.
- Construct a graph $\mathcal{G}_s = (\mathcal{V}_s, \mathcal{E}_s)$ to study the asymmetric impact from aggregated size factors to price factors,

$$\mathcal{V}_s = \left(p^{(n)}, \sum_n bs_r^{(n)}, \sum_n as_r^{(n)}\right) \quad n = 1, \ldots, N \tag{9}$$

$$\mathcal{E}_s = C_{i \leftarrow j} \quad i \in \{p^{(n)}\}, \quad j \in \left\{\sum_n bs_r^{(n)}, \sum_n as_r^{(n)}\right\} \tag{10}$$

Asymmetric Market Sell/Buy Pressure - ctd

	MSFT	T	IBM	JNJ	PFE	MRK	JPM	WFC	C
$\mu C_{p(n) \leftarrow \sum as\mathbf{1}}$	0.44	0.43	0.45	0.34	0.45	0.68	0.86	0.53	0.83
$\mu C_{p(n) \leftarrow \sum as\mathbf{2}}$	0.53	0.41	0.59	0.48	0.60	0.65	0.33	0.50	0.64
$\mu C_{p(n) \leftarrow \sum as\mathbf{3}}$	0.90	0.69	0.54	0.33	0.58	0.89	1.03	0.54	0.39
$\mu C_{p(n) \leftarrow \sum bs\mathbf{1}}$	0.12	0.44	0.47	0.40	0.32	0.41	0.46	0.61	0.40
$\mu C_{p(n) \leftarrow \sum bs\mathbf{2}}$	0.50	0.35	0.38	0.20	0.37	0.22	0.48	0.20	0.28
$\mu C_{p(n) \leftarrow \sum bs\mathbf{3}}$	0.63	0.37	0.24	0.39	0.51	0.38	0.43	0.73	0.47

Table 5: Summary of the aggregated impacts from size factors to the stock price factor from 06.2016-07.2017.

- The higher are the values, the stronger are the stocks affected by trading activities over time.
- Financial stocks: price patterns are highly related to market trading activity.

Asymmetric Market Sell/Buy Pressure - ctd

	$Q_C(0.25)$	$Q_C(0.50)$	$Q_C(0.75)$	μ_C
$C_{\sum_N p \leftarrow \sum_N as_1^{(n)}}$	0.12	0.27	0.67	0.50
$C_{\sum_N p \leftarrow \sum_N as_2^{(n)}}$	0.19	0.30	0.55	0.47
$C_{\sum_N p \leftarrow \sum_N as_3^{(n)}}$	0.17	0.35	0.83	0.59
$C_{\sum_N p \leftarrow \sum_N bs_1^{(n)}}$	0.09	0.18	0.39	0.36
$C_{\sum_N p \leftarrow \sum_N bs_2^{(n)}}$	0.08	0.16	0.41	0.30
$C_{\sum_N p \leftarrow \sum_N bs_3^{(n)}}$	0.13	0.29	0.63	0.42

Table 6: Summary of the impacts from aggregated size factors to the aggregated price factor from 06.2016-07.2017.

- Results are consistent with Table 5. We can observe stronger impacts on prices caused by market sell pressure.

Asymmetric Market Sell/Buy Pressure - test

- let μ_1 be the mean of the overall impacts from selling orders over the sample period ($T = 42$), and μ_2 the corresponding mean of the overall impacts from buying orders,

$$
\begin{aligned}
\mu_1 &= \frac{1}{3T}\left(C_{t,\sum_N p \leftarrow \sum_N as_{\mathbf{1}}^{(n)}} + C_{t,\sum_N p \leftarrow \sum_N as_{\mathbf{2}}^{(n)}} + C_{t,\sum_N p \leftarrow \sum_N as_{\mathbf{3}}^{(n)}}\right) \\
\mu_2 &= \frac{1}{3T}\left(C_{t,\sum_N p \leftarrow \sum_N bs_{\mathbf{1}}^{(n)}} + C_{t,\sum_N p \leftarrow \sum_N bs_{\mathbf{2}}^{(n)}} + C_{t,\sum_N p \leftarrow \sum_N bs_{\mathbf{3}}^{(n)}}\right)
\end{aligned}
$$

- The hypothesis of interest can be expressed as,

$$
\begin{aligned}
H_0 &: \mu_1 - \mu_2 = 0 \\
H_a &: \mu_1 - \mu_2 > 0
\end{aligned}
$$

Asymmetric Market Sell/Buy Pressure - test

	t-statistics	p-value
Pooled t-test	2.7557	0.003144
Welsh t-test	2.7557	0.003168

Table 7: Comparison of two hypothesis tests, when assuming/not assuming equal standard deviation.

- Both the pooled t-test and the Welsh t-test give roughly the same results.
- Reject the null hypothesis, indicating that there is strong evidence of a significant larger impact from selling orders in the market.

Own-price Market Impact

	MSFT	T	IBM	JNJ	PFE	MRK	JPM	WFC	C
$\mu_{C_{p^{(n)} \leftarrow as\mathbf{1}^{(n)}}}$	0.47	0.90	1.28	0.06	0.31	2.95	3.58	2.47	0.26
$\mu_{C_{p^{(n)} \leftarrow as\mathbf{2}^{(n)}}}$	0.34	0.31	0.47	0.13	0.07	1.42	0.38	2.17	1.43
$\mu_{C_{p^{(n)} \leftarrow as\mathbf{3}^{(n)}}}$	2.57	0.26	1.30	0.25	1.69	0.90	5.26	0.74	0.32
$\sum \mu_{C_{p^{(n)} \leftarrow as^{(n)}}}$	3.38	1.47	3.05	0.44	2.07	5.27	9.22	5.38	2.01
$\mu_{C_{p^{(n)} \leftarrow bs\mathbf{1}^{(n)}}}$	0.09	0.59	0.14	0.05	0.60	1.14	1.12	2.47	0.70
$\mu_{C_{p^{(n)} \leftarrow bs\mathbf{2}^{(n)}}}$	1.35	0.18	0.29	0.05	0.43	0.08	0.83	0.89	0.42
$\mu_{C_{p^{(n)} \leftarrow bs\mathbf{3}^{(n)}}}$	2.58	0.07	0.23	1.20	0.11	2.16	1.37	2.14	1.42
$\sum \mu_{C_{p^{(n)} \leftarrow bs^{(n)}}}$	4.02	0.84	0.66	1.30	1.14	3.38	3.32	5.50	2.54

Table 8: The mean of own-price market impacts caused by market orders $\{as1^{(n)}, bs1^{(n)}\}$, limit orders $\{as2^{(n)}, bs2^{(n)}\}$ and $\{as3^{(n)}, bs3^{(n)}\}$ for each stock n. All numbers are multiplied by 100.

Cross-price Market Impact

- We measure the cross-price market impacts by adding up the impacts from all ask/bid orders for each stock.
- Denote the graph as $\mathcal{G}_{cross} = (\mathcal{V}_c, \mathcal{E}_c)$, with cross-price market impacts from the aggregated size factors to the price factor given by,

$$
\begin{aligned}
\mathcal{V}_c &= \left(p^{(m)}, \sum_r bs_r^{(n)}, \sum_r as_r^{(n)}\right) \\
\mathcal{E}_c &= C_{i\leftarrow j} \quad i \in \{p^{(m)}\}, \quad j \in \left\{\sum_r bs_r^{(n)}, \sum_r as_r^{(n)}\right\} \\
m, n &\in \{1, \dots, N\} \quad r = 1, 2, 3 \quad m \neq n
\end{aligned}
$$

Cross-price Market Impact - Ask Side

	MSFT	T	IBM	JNJ	PFE	MRK	JPM	WFC	C
$\mu^{C}_{p(MSFT)\leftarrow\sum_r as_r^{(n)}}$	3.38	0.68	1.82	0.65	1.76	0.73	5.46	0.54	3.66
$\mu^{C}_{p(T)\leftarrow\sum_r as_r^{(n)}}$	3.08	1.47	1.00	0.62	1.86	2.58	1.08	1.29	2.38
$\mu^{C}_{p(IBM)\leftarrow\sum_r as_r^{(n)}}$	1.52	0.38	3.06	1.33	0.91	1.57	1.41	3.58	2.05
$\mu^{C}_{p(JNJ)\leftarrow\sum_r as_r^{(n)}}$	1.69	0.62	1.04	0.45	1.47	1.05	1.37	0.31	3.49
$\mu^{C}_{p(PFE)\leftarrow\sum_r as_r^{(n)}}$	1.07	0.96	0.44	0.13	2.06	4.83	1.86	2.89	2.12
$\mu^{C}_{p(MRK)\leftarrow\sum_r as_r^{(n)}}$	3.18	1.17	0.43	0.83	2.44	5.27	4.15	2.25	2.57
$\mu^{C}_{p(JPM)\leftarrow\sum_r as_r^{(n)}}$	2.09	1.10	1.81	0.72	2.68	1.13	9.22	1.34	2.16
$\mu^{C}_{p(WFC)\leftarrow\sum_r as_r^{(n)}}$	1.22	2.38	1.70	0.55	1.93	1.22	0.79	5.37	0.60
$\mu^{C}_{p(C)\leftarrow\sum_r as_r^{(n)}}$	2.55	1.11	2.37	0.84	2.57	1.33	4.51	1.23	2.01

Table 9: The mean of the market impacts caused by ask orders of stock m for each stock n. All numbers are multiplied by 100.

- The stock price can be affected not only by their own ask order flows, but also by the ask order flows of financial stocks

Cross-price Market Impact - Bid Side

	MSFT	T	IBM	JNJ	PFE	MRK	JPM	WFC	C
$\mu C_{p(MSFT) \leftarrow \sum_r bs_r^{(n)}}$	4.02	2.26	0.62	0.53	0.59	1.67	0.41	0.89	1.61
$\mu C_{p(T) \leftarrow \sum_r bs_r^{(n)}}$	1.36	0.84	0.22	1.04	1.41	3.67	0.92	1.10	1.03
$\mu C_{p(IBM) \leftarrow \sum_r bs_r^{(n)}}$	0.79	1.29	0.66	0.13	0.58	0.97	3.47	1.85	1.15
$\mu C_{p(JNJ) \leftarrow \sum_r bs_r^{(n)}}$	0.63	0.85	0.30	1.30	0.86	0.99	0.50	1.90	2.59
$\mu C_{p(PFE) \leftarrow \sum_r bs_r^{(n)}}$	2.12	0.36	1.10	0.19	1.13	0.37	1.43	4.08	1.23
$\mu C_{p(MRK) \leftarrow \sum_r bs_r^{(n)}}$	0.72	0.49	0.25	0.25	1.35	3.37	1.59	0.84	1.27
$\mu C_{p(JPM) \leftarrow \sum_r bs_r^{(n)}}$	1.66	0.47	1.25	0.97	1.39	0.59	3.32	1.87	2.16
$\mu C_{p(WFC) \leftarrow \sum_r bs_r^{(n)}}$	1.99	1.29	0.30	0.73	1.37	0.83	1.75	5.50	1.67
$\mu C_{p(C) \leftarrow \sum_r bs_r^{(n)}}$	1.02	1.80	0.42	1.24	0.84	1.08	1.41	1.12	2.54

Table 10: The mean of the market impacts caused by bid orders of stock m for each stock n. All numbers are multiplied by 100.

- Financial stocks have stronger cross-price market impacts compared with healthcare and technology stocks.

Price and Order Flows

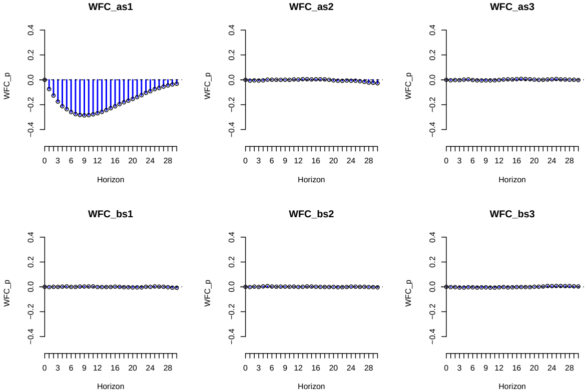
- Even though sufficiently large market order immediately affects the price direction, the bid/ask sizes alone do not give enough information on price direction.
- To measure the evolution of the market/limit order, recall

▸ Return to GI analysis

$$
\begin{aligned}
\delta_{jt} &: (\delta_{1t}, \delta_{2t}, \ldots, \delta_{Kt})^\top \sim e_j \\
GI(l, \delta_{jt}, \mathcal{F}_{t-1}) &= \mathsf{E}(y_{t+l} \mid u_{jt} = \delta_{jt}, \mathcal{F}_{t-1}) - \mathsf{E}(y_{t+l} \mid \mathcal{F}_{t-1})
\end{aligned}
$$

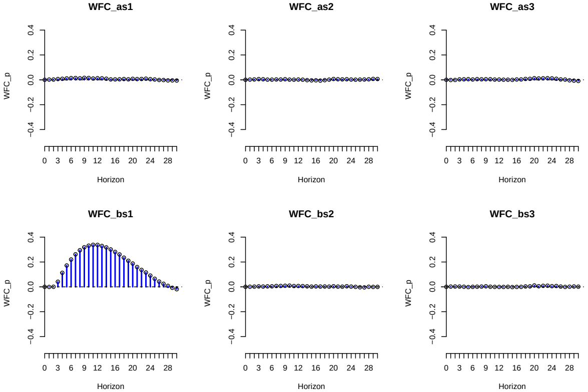
when the shock δ_{jt} is treated as one of the size factors $(\Delta\tilde{s}_t^{a1(n)}, \Delta\tilde{s}_t^{a2(n)}, \Delta\tilde{s}_t^{a3(n)}, \Delta\tilde{s}_t^{b1(n)}, \Delta\tilde{s}_t^{b2(n)}, \Delta\tilde{s}_t^{b3(n)})$ hitting the system. hfhd_marketimpact

Example - Wells Fargo



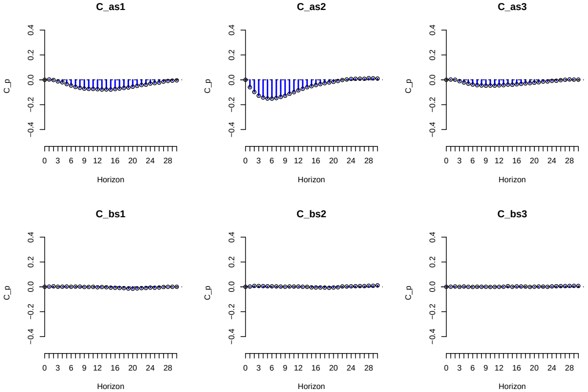
- Own-price market impact on 25.07.2016.
- The investors will start marking down their bid price when there is a wave of sell orders coming into the order book.

Example - Wells Fargo



- On 19.07.2016, the argument also holds for bid market order.
- Both impacts last for almost 10 min before the price shifts back, HF investors have enough reaction time to arbitrage.

Example - Citigroup



- The market impacts of orders posted deeper in the book.
- Positive pile-on effect where larger ask order may further perpetuating a price decrease, last for 20 min. (01.06.0216)

r_{size} Ratio

- For each trading day, we use $\ominus$ and $\oplus$ to represent the significant negative and positive response of price after the arrival of a market/limit order.
- Define a ratio denoted as r_{size} to measure the price direction of market impacts,

$$
\begin{aligned}
r_{size} &= \frac{|\operatorname{sgn}(GI_t)|}{42} \\
\operatorname{sgn}(GI_t) &= \begin{cases} -1 & -GI_t(h) > Q_{0.05}(GI_t(h)) \\ 0 & |GI_t(h)| \leq Q_{0.05}(GI_t(h)) \\ 1 & GI_t(h) > Q_{0.05}(GI_t(h)) \end{cases} \\
t &= 1 \ldots T, \quad h = 1, \ldots, 30
\end{aligned}
$$

Summary

	MSFT	T	IBM	JNJ	PFE	MRK	JPM	WFC	C
$as1$		$\ominus\ \ominus\ \ominus\ \ominus\ \ominus$		$\oplus\ \oplus\ \oplus$		$\oplus\ominus$	$\ominus\ \ominus\ \ominus$	$\ominus\ \ominus\ \ominus$	$\ominus$
$as2$	$\ominus\ \ominus\ \ominus$	$\oplus$	$\oplus\oplus$		$\oplus$			$\ominus\ominus$	$\ominus\ \ominus\ \ominus\ \ominus\ \ominus\ominus$
$as3$	$\oplus\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus\ \oplus\ \oplus$		$\ominus$	$\oplus\oplus$	$\ominus$
$bs1$	$\oplus\ \ominus\ \ominus$	$\oplus\ \ominus\ \ominus\ominus$		$\oplus$	$\oplus$	$\oplus\oplus$	$\oplus\ \oplus\ \oplus\ominus$	$\oplus\oplus$	$\oplus$
$bs2$	$\ominus$	$\ominus$			$\oplus\ominus$	$\oplus$	$\oplus$		$\oplus$
$bs3$		$\ominus$	$\ominus\ominus$	$\ominus\ominus$	$\ominus$	$\ominus\ominus$	$\ominus$		
r_{size}	44%	46%	0%	14%	25%	57%	80%	78%	100%

- The group of financial stocks is of higher r_{size} values, finance sector is leading the market, the history information indicates that their response of price to trading volumes is stable and thus robust for statistical arbitrage.
- Healthcare and technology stocks are price-dominated, i.e., they have multiple risk sources except for their own trading activity. This is consistent with our main findings in LOB network analysis.

Conclusion

- LOB network is constructed in a MN context and non-synchronous trading.
- Network relies neither on the ordering of the variables nor on the distribution of the innovations, the resulting connectedness measures is economically interpretable.
- Main findings:
  - Network involving trading volumes is a better measure of the stock connectedness.
  - Significant market impact caused by the arrival of a large limit order is identified.
  - Order imbalance generally exists across stocks
  - Bootstrapped market impacts can be quantified.
  - The financial institutions are connected more closely compared with the firms come from other industry

Time-varying Limit Order Book Networks

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Wolfgang Karl Härdle

Chong Liang

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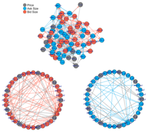

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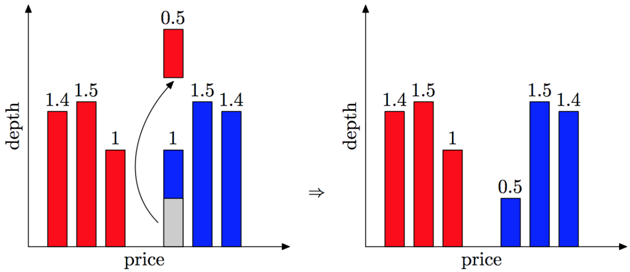
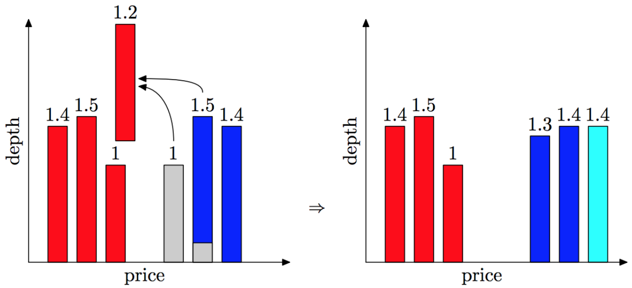
Normal Order Book



- An incoming bid (buy) market order with price 1002 and size 0.5 which results in a buy transaction.

Aggressive Order Book



- An incoming bid (buy) market order with price 1003 and size 1.2 "walking up" the order book.
- Passive limit order: arrival of a buy limit order with price 999 and size 0.5 to be posted behind the market. Back

Setting

- To construct the estimator, choose a sequence k_n of integers that satisfy,

$$k_n\sqrt{\Delta_n} = \theta + \mathcal{O}\left(\Delta_n^{\frac{1}{4}}\right), \text{for some } \theta > 0$$

  - TRTS (transaction time sampling): i indexes time points associated with Δ_n-th trade.
  - CTS (calendar time sampling): i indexes equal-spaced time intervals of length Δ_t with $n = \dfrac{t}{\Delta_n}$.
- Consider a continuous function $g : [0,1] \to \mathbb{R}$ which is piecewise continuously differentiable such that,
  - $g^{'}$ is a piecewise Lipschitz derivative
  - $g(0) = g(1) = 0$ and $\int_0^1 g^2(s)ds > 0$

Setting - ctd

- Choose the simple form of $g(x) = x \wedge (1-x)$, Podolskij et al. (2009), Christensen et al. (2010).

$$\psi_1 = 1, \quad \psi_2 = \frac{1}{12}, \quad \Phi_{11} = \frac{1}{6}, \quad \Phi_{12} = \frac{1}{96}, \quad \Phi_{22} = \frac{151}{80640}$$

- g is associated with the following real-valued numbers,

$$\begin{aligned} \psi_1 &= \int_0^1 \left\{g^\top(s)\right\}^2 ds, \quad \psi_2 = \int_0^1 \left\{g(s)\right\}^2 ds, \quad s \in [0,1] \\ \Phi_1(s) &= \int_s^1 g^\top(u) g^\top(u-s) du, \quad \Phi_2(s) = \int_s^1 g(u) g(u-s) du \\ \Phi_{ij} &= \int_0^1 \Phi_i(s) \Phi_j(s) du, \quad i,j = 1,2 \end{aligned}$$

Back

Bootstrap-based multistep forecast method

The computation steps to yield bootstrapped GFEVD, Lanne and Nyberg (2016); Terasvirta et al. (2010),

1. $\mathcal{F}_{t-1}$ information prior to Y_t; forecast horizon h.
2. Randomly sample B vectors of shocks $(\delta_{1t}, \delta_{2t}, \dots, \delta_{Kt})^\top$ from the residuals,

$$\delta_{jt} : (\delta_{1t}, \delta_{2t}, \dots, \delta_{Kt})^\top \sim u^\star_{jt} e_j$$

$u^\star_{jt}$ coms from $\hat{u}_{jt}$,

$$\begin{aligned} \hat{u}_{jt} &= Y_t - \left(\hat{A}_1, \hat{A}_2, \dots, \hat{A}_p\right) \left(Y_{t-1}^\top, Y_{t-2}^\top, \dots, Y_{t-p}^\top\right)^\top \\ &= Y_t - g(Y_{t-1}) \end{aligned}$$

Bootstrap-based multistep forecast method

3. Compute conditional multistep forecast $\mathsf{E}(y_{t+l}|\mathcal{F}_{t-1})$,

$$
\begin{aligned}
f_{t,0} &= g(Y_{t-1}) \\
f_{t,1} &= \mathsf{E}^{\star}[Y_{t+1}|\mathcal{F}_{t-1}] = \mathsf{E}^{\star}[g(f_{t,0} + u^{\star}_{t})|\mathcal{F}_{t-1}] \\
f_{t,2} &= \mathsf{E}^{\star}[Y_{t+2}|\mathcal{F}_{t-1}] = \mathsf{E}^{\star}[g(f_{t,1} + u^{\star}_{t+1})|\mathcal{F}_{t-1}] \\
&\ldots
\end{aligned}
$$

with $u^{\star}_{t+l}, l = 1, \ldots, h$ Bootstrap sample from residuals $\{\hat{u}_{t+l}\}_{t=1}^{T}$ over the sample period.

Bootstrap-based multistep forecast method

4. Repeat steps 3 for all N_B vectors of estimated innovations with bootstrap methods, iterating on the estimated model,

$$
\begin{aligned}
fb_{t,1} &= \frac{1}{B}\sum_{b=1}^{B} g(f_{t,0} + u_t^{\star(b)}) \\
fb_{t,2} &= \frac{1}{B}\sum_{b=1}^{B} g(g(f_{t,0} + u_t^{\star(b)}) + u_{t+1}^{\star(b)}) \\
&\dots
\end{aligned}
$$

Bootstrap-based multistep forecast method

5. By the same logic, compute $\mathsf{E}(y_{t+l} \mid u_{jt} = \delta_{jt}, \mathcal{F}_{t-1})$ when the shock is given as $\delta_{jt} = u^{\star}_{jt} e_j$,

$$
\begin{aligned}
f_{t,0} &= g(Y_{t-1}) \\
f_{t,1} &= \mathsf{E}^{\star}[Y_{t+1} \mid \mathcal{F}_{t-1}] = \mathsf{E}^{\star}[g(f_{t,0} + u^{\star}_{jt} e_j) \mid \mathcal{F}_{t-1}] \\
f_{t,2} &= \mathsf{E}^{\star}[Y_{t+2} \mid \mathcal{F}_{t-1}] = \mathsf{E}^{\star}[g(f_{t,1} + u^{\star}_{j,t+1} e_j) \mid \mathcal{F}_{t-1}] \\
&\cdots
\end{aligned}
$$

with $u^{\star}_{j,t+l}, l = 1, \ldots, h$ Bootstrap sample from residuals $\{\hat{u}_{j,t+l}\}_{t=1}^{T}$ over the sample period.

Bootstrap-based multistep forecast method

6. Repeat steps 5 for all N_B vectors of estimated innovations with bootstrap methods, iterating on the estimated model,

$$
\begin{aligned}
fb_{t,1} &= \frac{1}{B}\sum_{b=1}^{B} g(f_{t,0} + u_{jt}^{\star(b)} e_j) \\
fb_{t,2} &= \frac{1}{B}\sum_{b=1}^{B} g\{g(f_{t,0} + u_{jt}^{\star(b)} e_j) + u_{j,t+1}^{\star(b)} e_j\} \\
&\dots
\end{aligned}
$$

Bootstrap-based multistep forecast method

7. Plug in the GI function

$$\widehat{GI}(l, \delta_{jt}, \mathcal{F}_{t-1}) = \mathsf{E}^{\star}(y_{t+l}|u_{jt} = \delta_{jt}, \mathcal{F}_{t-1}) - \mathsf{E}^{\star}(y_{t+l}|\mathcal{F}_{t-1})$$

to obtain the relative contribution of a shock δ_{jt} to the i-th variable with horizon h at time t,

$$\hat{\lambda}_{ij,\mathcal{F}_{t-1}}(h) = \frac{\sum_{l=0}^{h} \widehat{GI}(l, \delta_{jt}, \mathcal{F}_{t-1})_i^2}{\sum_{j=1}^{K} \sum_{l=0}^{h} \widehat{GI}(l, \delta_{jt}, \mathcal{F}_{t-1})_i^2}, \quad i, j = 1, \ldots, K$$

8. Repeat steps 2-6 for all histories.
9. Construct connectedness table using averaged $\hat{\lambda}_{ij,\mathcal{F}_{t-1}}(h)$ generated from step 7. Return to LN-GFEVD

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