Dynamic Crypto Networks

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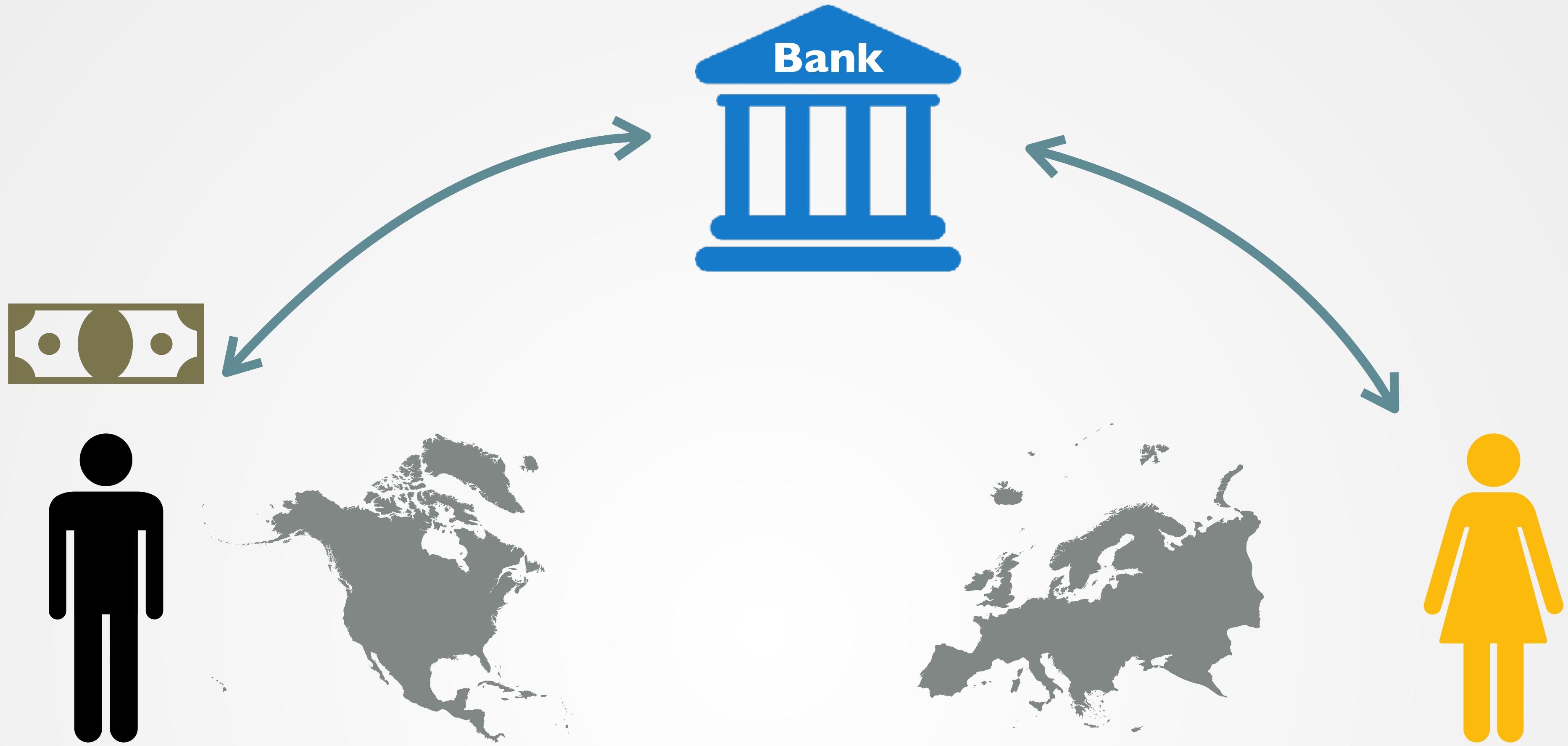
Charles U, WISE, Xiamen U, Fudan U, SMU

Hayek (1976) The Denationalization of Money.

Instead of a national government issuing a specific currency, …, private businesses should be allowed to issue their own forms of money, deciding how to do so on their own. (From Wikipedia)

大元 Yuan Dynasty 1200+

Now



New

	USD	USD/EUR	EUR
USD	350,00 $	1,1372	307,77 €
USD	350,00 $	1,7047	205,31 €
P/L			-102,46 €

Verwendungszweck 1:		Interne Buchung
Verwendungszweck 2:		TRN/Re-Nr. 180607ABIB37267
Verwendungszweck 3:		TRN/MSG 180509ABSE34901
Verwendungszweck 4:	**HU Jerusalem**	USD 350,00
Verwendungszweck 5:		Kurs EUR/USD 1,185100
Verwendungszweck 6:		EUR 295,33
Verwendungszweck 7:		Verwendungszweck:
Verwendungszweck 8:		Gutschrift Ihres zum Inkass
Verwendungszweck 9:		o
Verwendungszweck 10:		eingereichten Auslandsschec
Verwendungszweck 11:		ks-
Verwendungszweck 12:		Nr. 467640275 ueber 350,00
Verwendungszweck 13:		USD
Verwendungszweck 14:		abzueglich Eigen- und Fremd
Verwendungszweck 15:		entgelt
Verwendungszweck 16:		Abrechnung:
Verwendungszweck 17:		Courtage
Verwendungszweck 18:		EUR 2,50
Verwendungszweck 19:		Scheckverkehr/Gutschr. n.E
Verwendungszweck 20:		EUR 25,00
Verwendungszweck 21:		Fremdentgelt Dokumenteninka
Verwendungszweck 22:		EUR 62,52
Verwendungszweck 23:		Endbetrag
Verwendungszweck 24:	**HU Berlin**	EUR 205,31
Verwendungszweck 25:		USt-IdNr.: DE 143589235
Verwendungszweck 26:		USt-befreite Finanzdienstl.

Berlin - Room 77

Hype, fools gold, or fact ?

A scientific dialogue

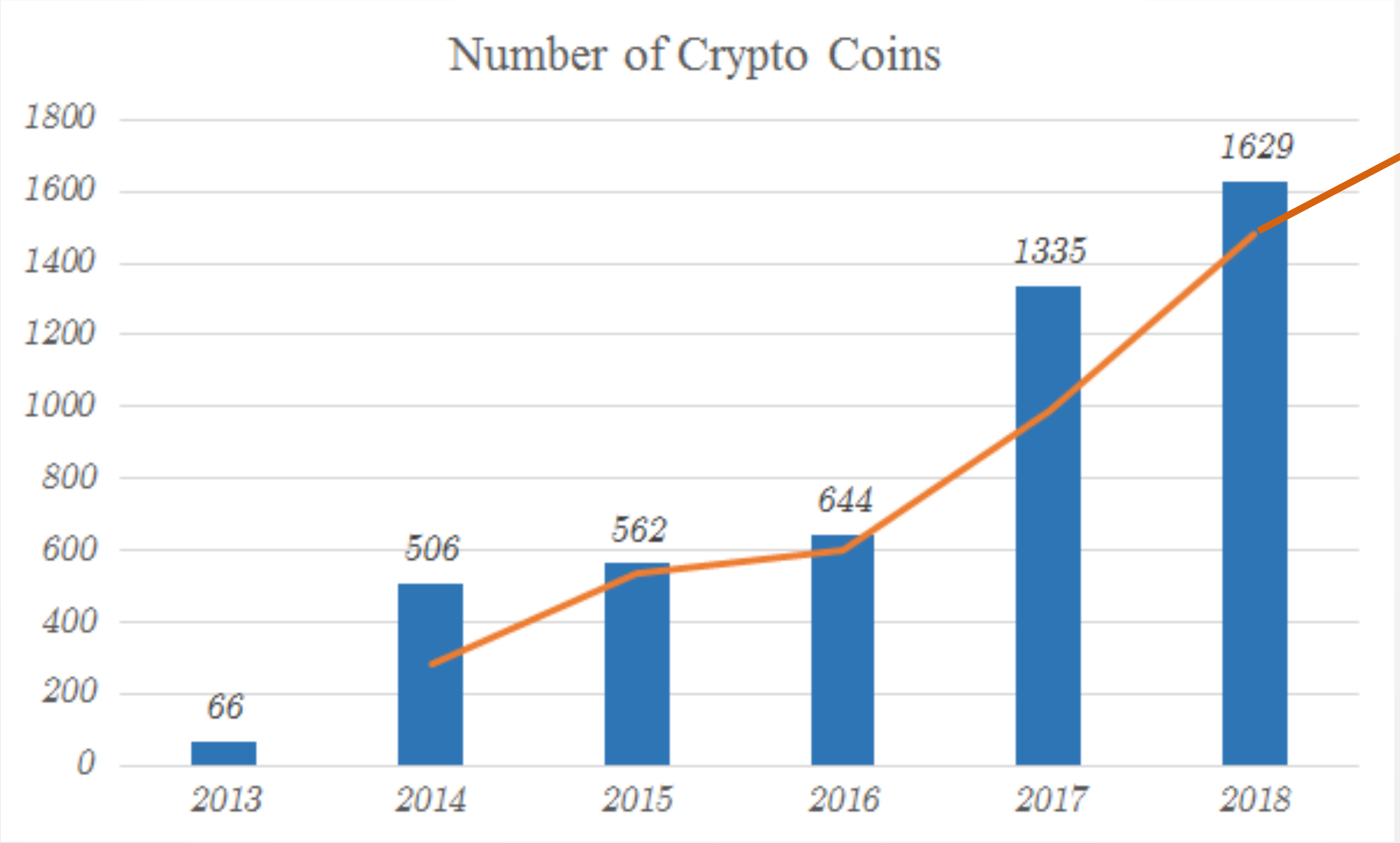
- „Warum machst du so einen Mist?“ (Why do you do such foxtrott?)
- „pecunia non olet.“ (CCs are „money“)
- „Da habe ich nix zu forschen“ (foxtrott oscar)
- „on verra“ (wait and see)

LMS Tech for cryptos

Emerging of Cryptocurrencies

- CoinMarketCap.com : Actively Trading: >1000 CCs
- Market Cap 20181115: $98,151,606,541,
- Market Cap 20190828: $265.167.180.656

20190530: 2213
20190828: 2494
20190930: 2906
20191024: 3006
20191107: 3083



Why Crypto Networks?

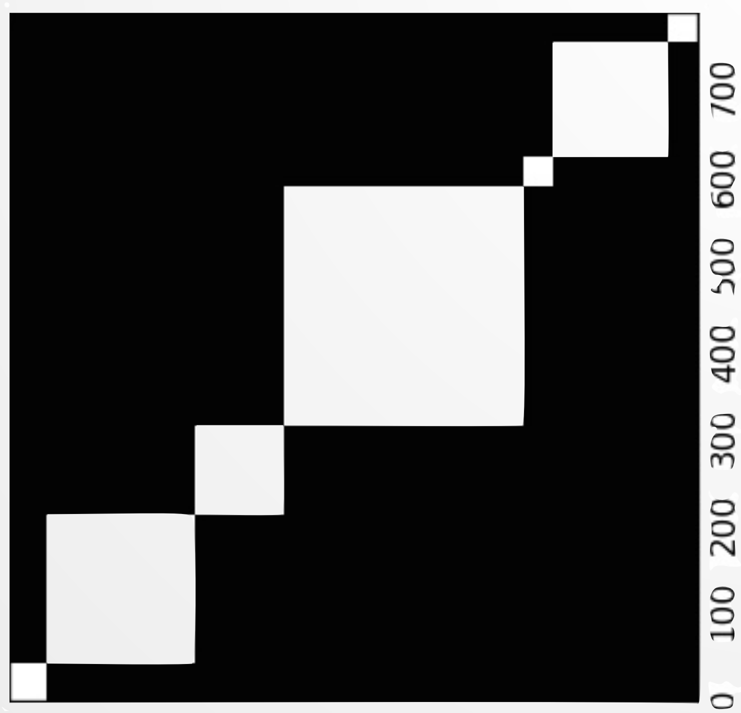
- Peer Effect
  - Open Source of Blockchain — Clonecoins
  - Lack of Fundamental Valuation
- Value of Technology
  - Cryptography: security of the transactions
  - Proof Types: mining activity
  - Comovement or not?
- How fundamental information and return structure jointly determine a market segmentation?

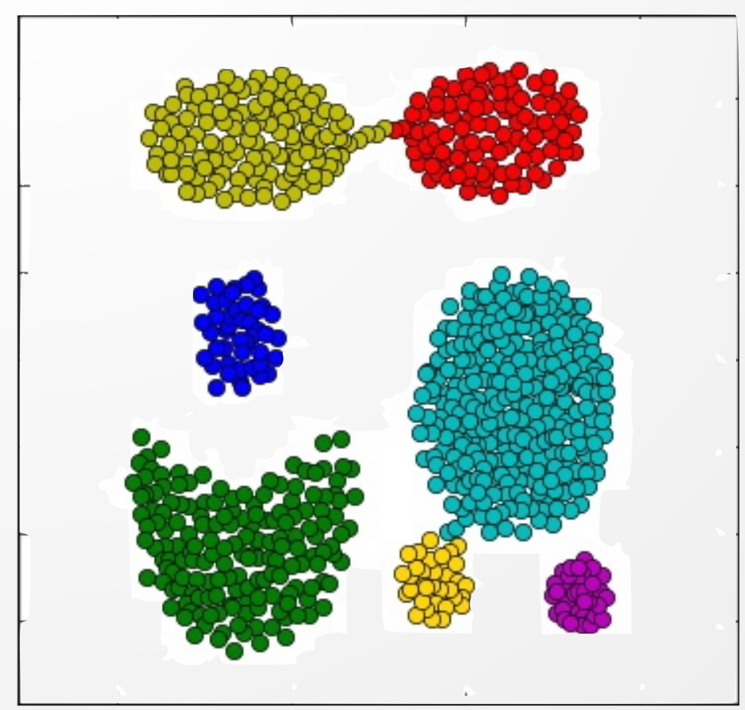
Methods in Equity Markets

- **SIC** Fama&French (1997), Clarke (1989)
- **GICS** firm's operational characteristics & investors' perceptions Bhojraj et al. (2003)
- **Investment Style** Farrell (1974), Elton&Gruber (1970), Brown&Goetzmann (1998)
- **Return Comovement** King (1966), Lessard (1974), Grinold et al (1989), Roll (1992), Connor (1997)
- **Product Similarity** Hoberg&Phillips (2016)

Community Detection

- **Modelling** Stochastic Block Model (Undirected), Stochastic Co Block Model (Directed)
- **Methods** Spectral Clustering, Maximum Likelihood, Bayesian, Modularity Maximisation, …
- **AWC** Adaptive Weights Clustering, Adamyan (2019)





Community Detection

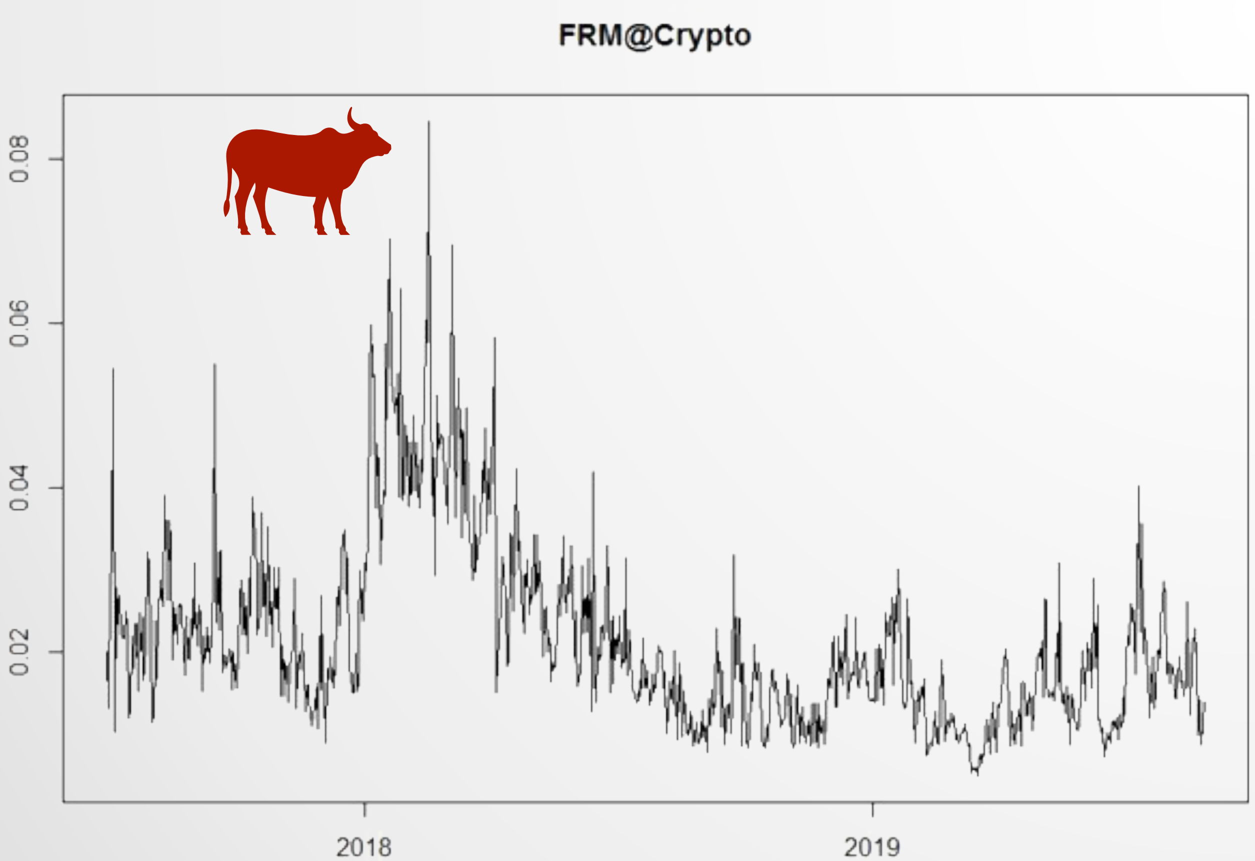
- **Dynamic Structure** Bhattacharyya & Chatterjee (2017), Matias & Miele (2017), Pensky & Zhang (2017), Wilson et al (2016).
- **Node features** Binkiewicz et al (2017), Weng & Feng (2017), Yan & Sarkar (2016), Zhang et al (2017).
- **Sparsity** Amini et al (2013), Qin & Rohe (2013).
- **Degree heterogeneity** Zhao et al (2012)
- **Directionality** Rohe & Yu (2012), Rohe et al (2016).

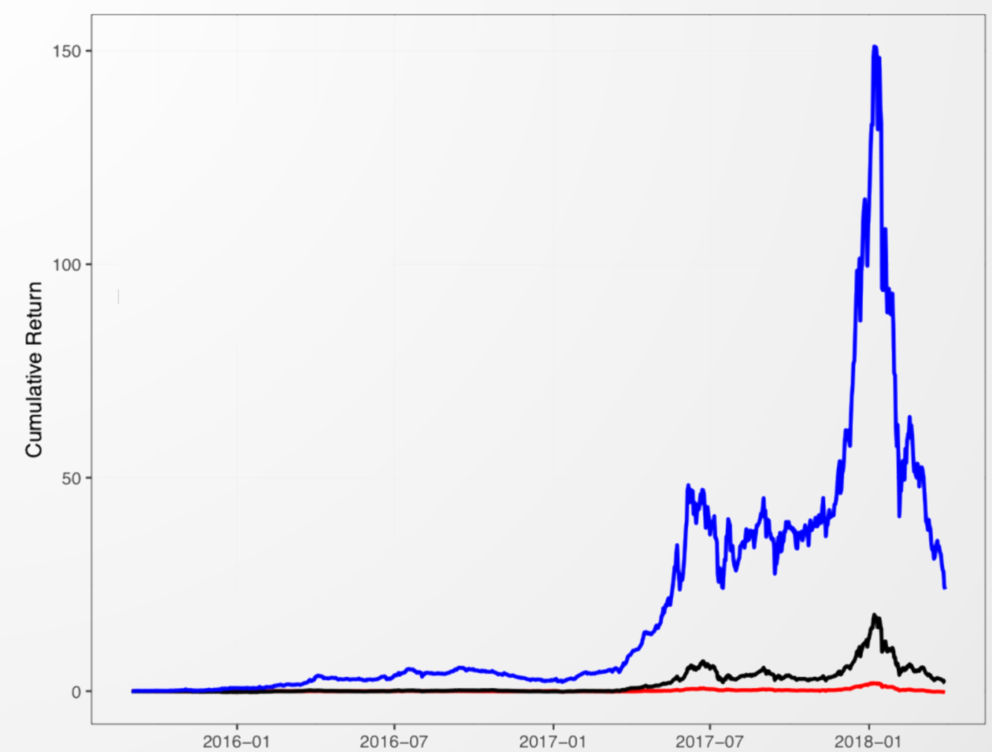
Data

- Sample (CryptoCompare)
  - In-sample Estimation: from 2015-08-31 to 2017-12-31
  - Out-of-Sample Tests: from 2018-01-01 to 2018-03-30
- Cryptocurrency (CC) Daily Return
  - Top 200 Cryptos Sorted on Market Cap, Age, Maximum Price and USD Volume
- Contract Information
  - Algorithm
  - Proof Types

Questions

- LMS Tech on CASC Covariate Assisted Spectral Clustering?
- Discover latent group membership?
- Correspondence with hashing and proof types?
- Segmentation of CC market possible?
- Central CCs to allocate in a portfolio ?





Outline

1. Motivation ✓
2. Model
3. Algorithm
4. Uniform Consistency
5. Simulation
6. Clustering Cryptos
7. Asset Pricing Inference
8. Conclusions

What is a Network?

- A set $\mathbf{V}$ of N nodes, $N = |\mathbf{V}|$, a set $\mathbf{E}$ of edges, indicating relationships between nodes
- Graph $\mathcal{G} = (\mathbf{V}, \mathbf{E})$ consists of $\mathbf{V}$ and $\mathbf{E}$
- Adjacency matrix $\mathbf{A}$: nodes share an edge $(i, j) \in \mathbf{E}$

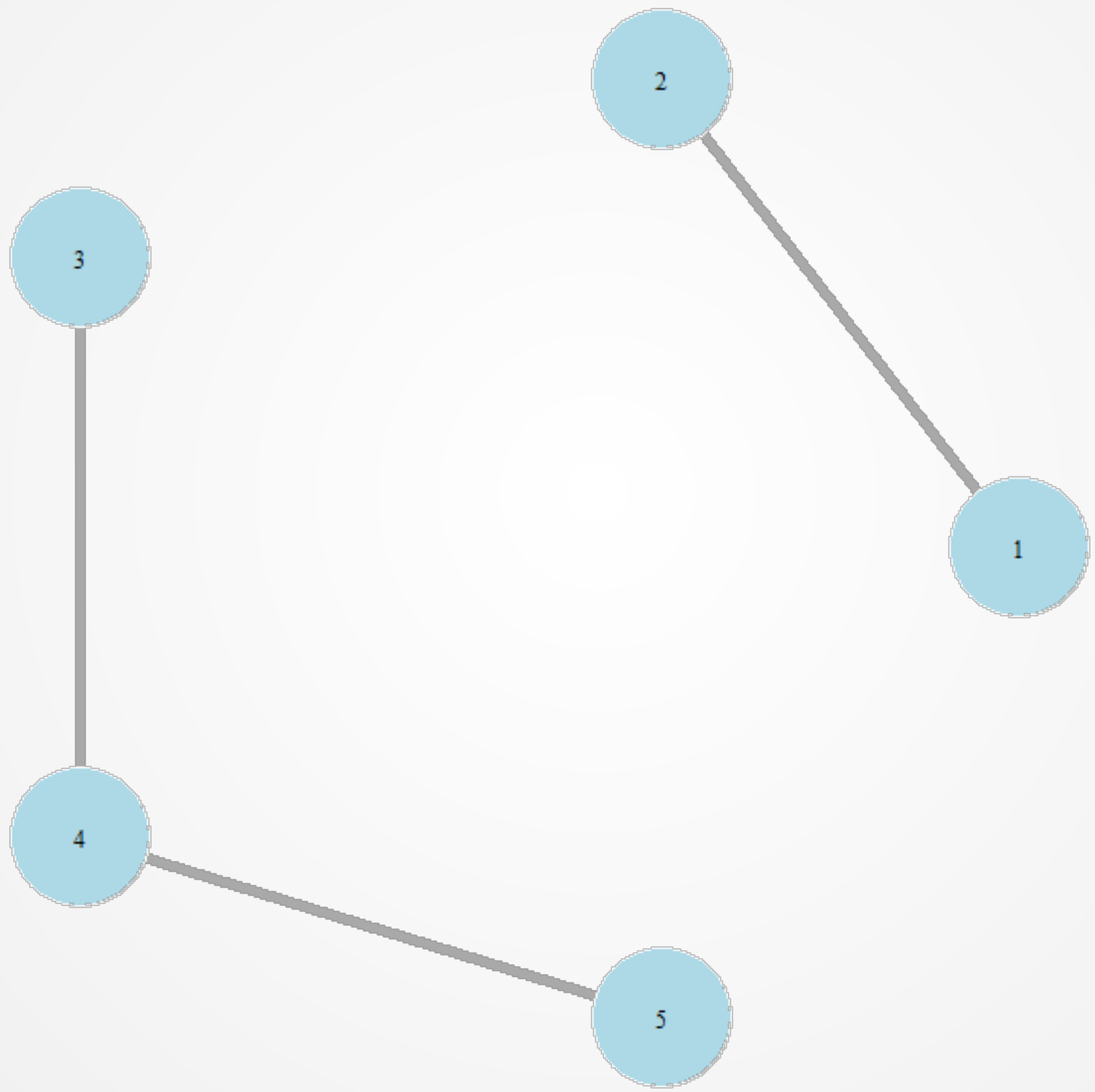
$$A_{ij} = \begin{cases} 1, & (i, j) \in \mathbf{E} \\ 0, & (i, j) \notin \mathbf{E} \end{cases}$$

- Undirected Network $A_{ij} = A_{ji}$, $\forall i, j \in \mathbf{V}$
- Directed Network $A_{ij} \neq A_{ji}$, for some $i, j \in \mathbf{V}$

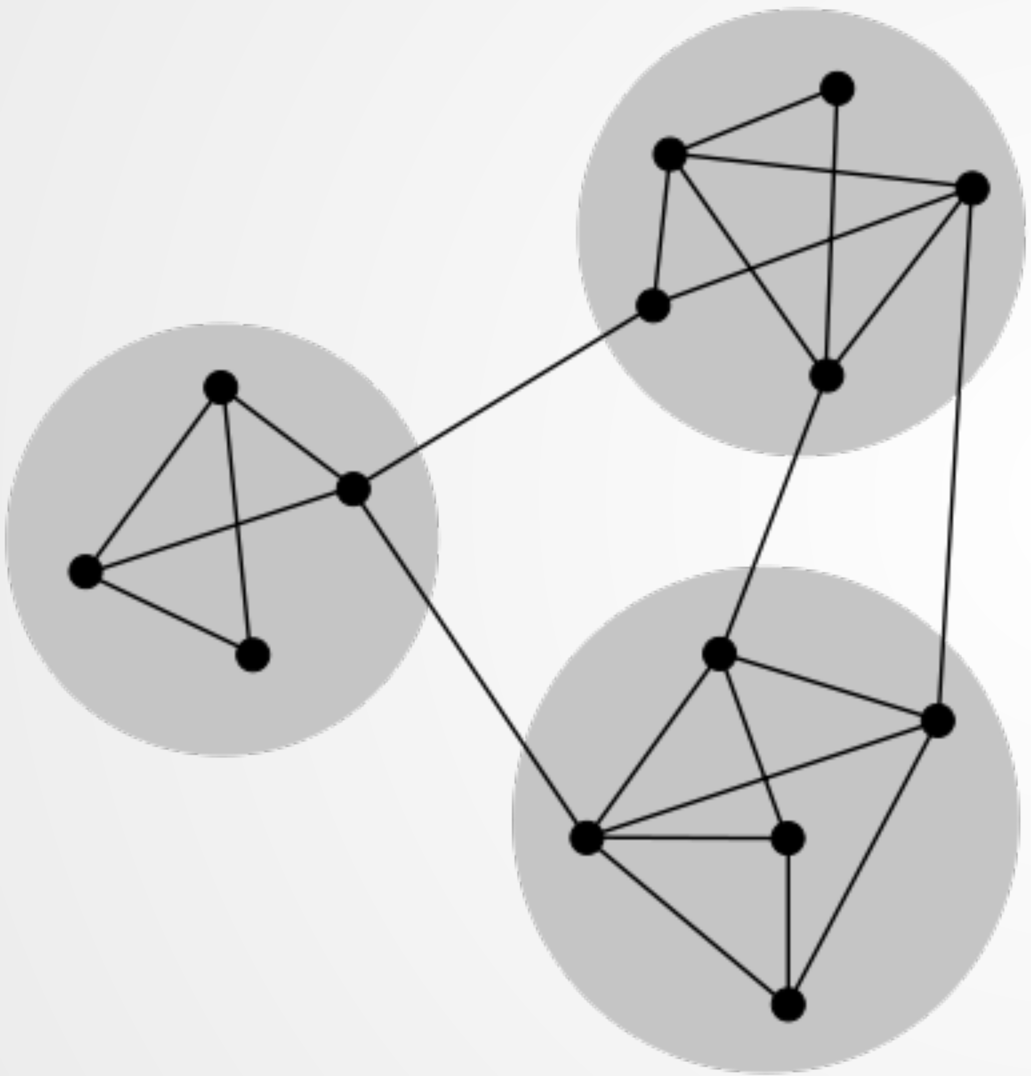
Community (Block) Structure

- **Definition** Groups of nodes more densely connected internally than with the rest
- **Importance**
  - Prediction of missing links and the identification of false links
  - Communities have specific properties /= the average property
  - These may create rumour or epidemic spreading

Dynamic Network



Community (Block) Structure

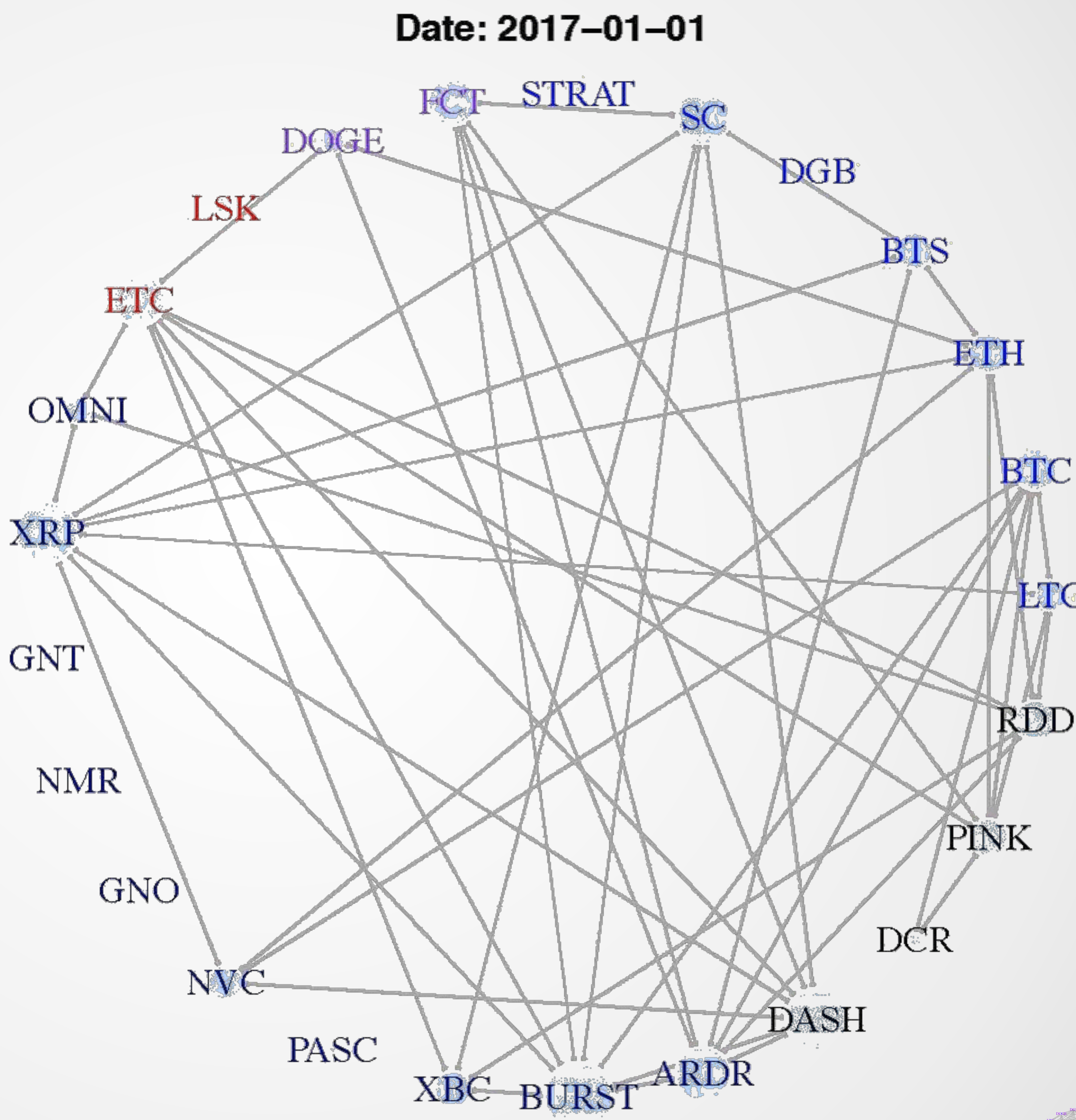


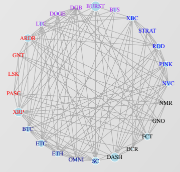
$$\mathscr{A}_t = Z_t B_t Z_t^\top = \begin{pmatrix} a & a & b & b & b \\ a & a & b & b & b \\ b & b & c & c & c \\ b & b & c & c & c \\ b & b & c & c & c \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ b & c \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

Time and Space

- a week in Jan 2017





Dynamic Stochastic Block Model

- Dynamic Stochastic Block Model:

$$A_t(i,j) = \begin{cases} Bernoulli\{P_t(i,j)\}, & i < j \\ 0, & i = j \\ A_t(i,j), & i > j \end{cases}$$

$$\mathscr{A}_t \stackrel{def}{=} \mathrm{E}(A_t \mid Z_t) = Z_t B_t Z_t^\top$$

adaptive Lasso

- ▶ Adjacency matrix based on CC return information: A_t .
- ▶ Connection between *i* and *j* : $P_t(i,j) = P\{A_t(i,j) = 1\}$
- ▶ Clustering Matrix: $Z_t \in \{0,1\}^{N \times K}$
- ▶ Block Probability Matrix: $B_t \in \mathscr{M}^{K \times K}$ and $B_t(k,k') = P_t(i,j)$, $\forall k, k' = \{1, \cdots, K\}$

Dynamic Stochastic Block Model

- Degree Heterogeneity of Groups
  - Degree parameters, Karrer&Newman (2011)

$$\psi = (\psi_1, \ldots, \psi_N)$$

  - Particularly, we can rewrite $P_t(i, j)$ as follow,

$$P_t(i, j) = \psi_i \psi_j B_t(z_{i,t}, z_{j,t})$$

with identifiability restriction

$$\sum_{i \in \mathcal{G}_k} \psi_i = 1, \ \forall k \in \{1, \ldots, K\}$$

where $\mathcal{G}_k$ is the set of nodes that belongs to the k-th group

  - Population adjacency matrices for the dynamic SC-DCBM:

$$\mathcal{A}_t = \Psi Z_t B_t Z_t^\top \Psi$$

where

$$\Psi = Diag(\psi)$$

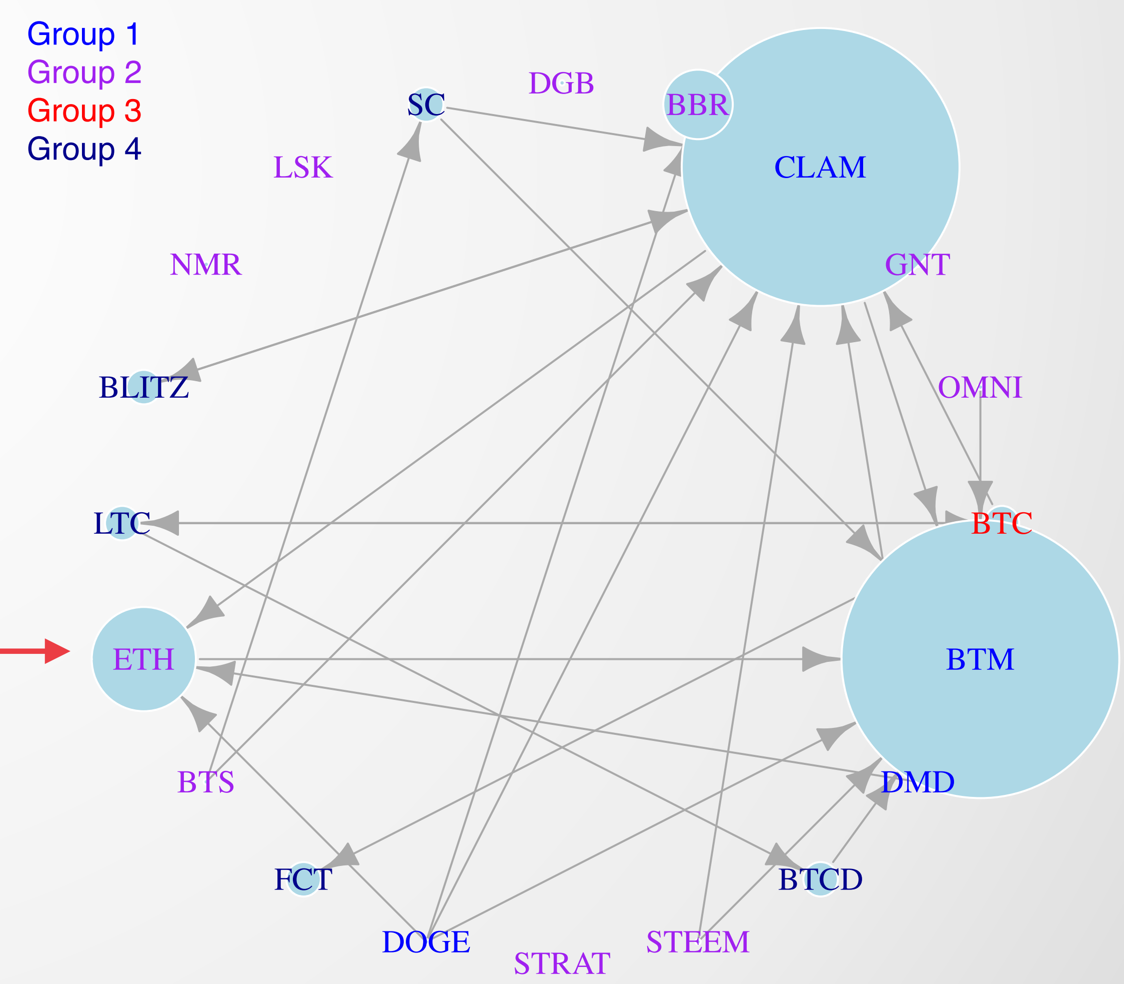
Spectral Contextualised Degree Corrected stochastic Block Model

Return Network Structure from Adaptive LASSO

- Calculate CC returns
- Connections between returns *(t, t-1)* of top 200 cryptos

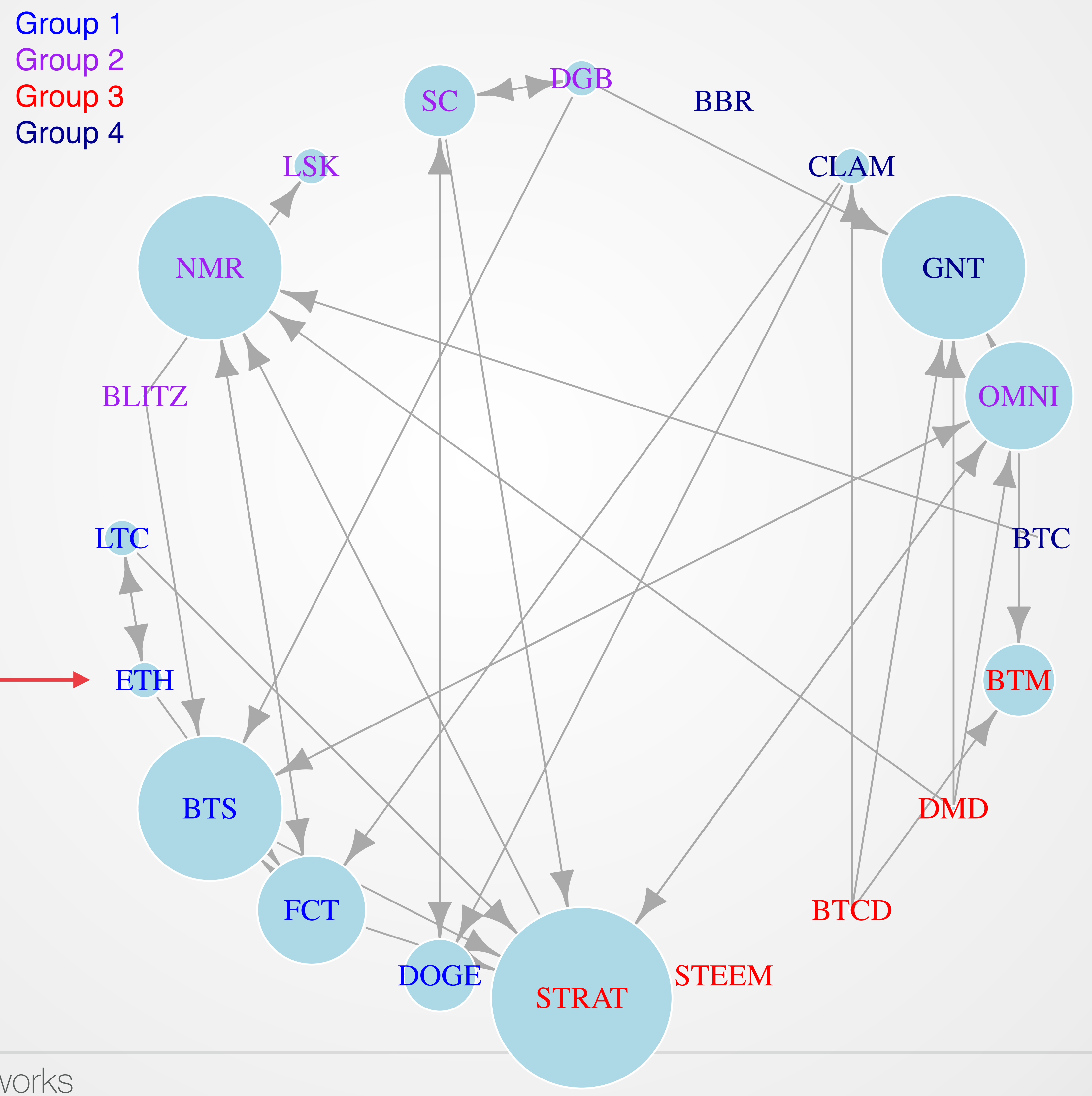
$$Ret_{eth} = \beta_1 Ret_{btc} + \beta_2 Ret_{xrp} + \beta_3 Ret_{qtum} + \ldots$$

- e.g. ETH Lasso: 20 cryptos on RHS

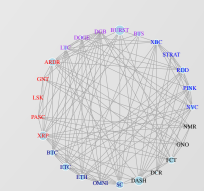


Return Network Structure from Adaptive LASSO

Sparsity 20191107 # 3000 +



A_t



Dealing with Sparsity

- Regularized Graph Laplacian

$$L_{\tau,t} = D_{\tau,t}^{-1/2} A_t D_{\tau,t}^{-1/2}$$

where $D_{\tau,t} = D_t + \tau_t I$ and D is a diagonal matrix with

$D_t(i,i) = \sum_{j=1}^{N} A_t(i,j)$ and $\tau_t = N^{-1} \sum_{i=1}^{N} D_t(i,i)$ average node degree

- Intuition of Regularisation
  - Add a weak edge on every node pair with edge weight τ_t / N
  - Spectral Clustering (SC): Sparse graphs create small trees that sparsely connect to the „core“ of the graph
  - Regularized Spectral Clustering: leads to a “deeper cut”
- Improve SC classification by covariates X_t

Incorporating Covariates

- Covariates X_t : algorithms and proof types
- X_t $N \times M$ matrix, where $M = n_1 + n_2$, n_1 # of algos, n_2 # proof types
- Each column of X_t = 1 if CC has this this feature, otherwise 0

Härdle et al (2019) Understanding CryptoCurrencies, JFEC

Incorporating Covariates

- Similarity Matrices (Covariate-assisted Graph Laplacian):

$$S_t = L_{\tau,t} + \alpha_t C_t^w$$

where $C_t^w = XW_tX^\top$ and $\alpha_t \in [0,\infty)$ is a tuning parameter

- Example

$$X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \succ XX^\top = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Dealing with Dynamics

- Discrete Kernel Function

$$\mathscr{F}_{r,1} = \{0,\cdots,r\} \qquad \mathscr{D}_{r,1} = \{1,\cdots,r\}$$

$$\mathscr{F}_{r,2} = \{-r,\cdots,r\} \qquad \mathscr{D}_{r,2} = \{r+1,\cdots,T-r\}$$

$$\mathscr{F}_{r,3} = \{-r,\cdots,0\} \qquad \mathscr{D}_{r,3} = \{T-r+1,\cdots,T\}$$

$$\frac{1}{|\mathscr{F}_{r,j}|}\sum_{i\in\mathscr{F}_{r,j}} i^k W^j_{r,\ell}(i) = \begin{cases} 1, & k=0 \\ 0, & k=1,2,\ldots,l \end{cases}$$

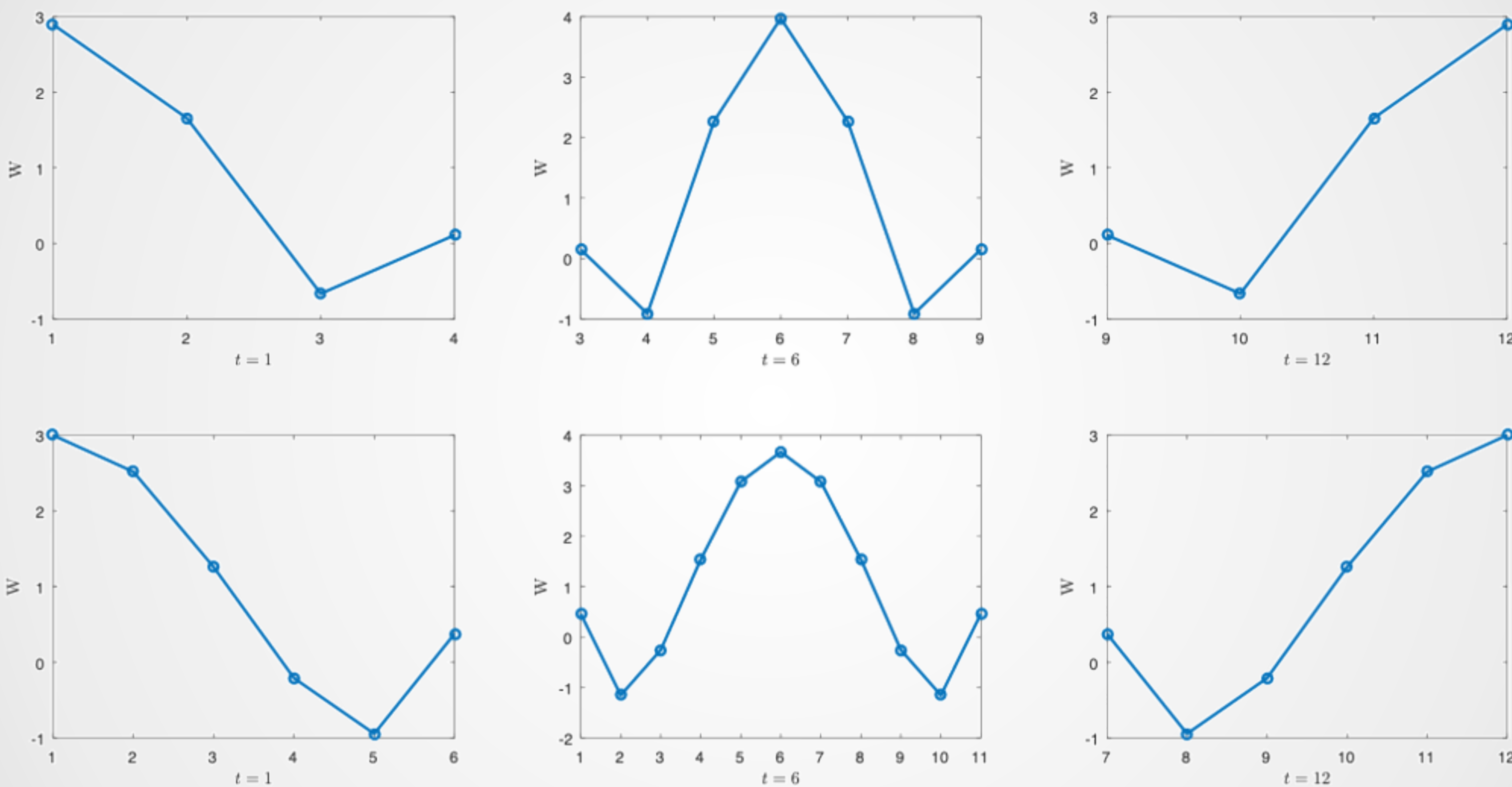
with $W^j_{r,\ell}(i) = \sum_{j=1}^{m} a_j (i/r)^{2j}$ and $\ell = 2m$. Solve

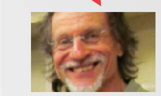
- Discrete Kernel Estimator

$$\widehat{\mathscr{S}}_{t,r} = \sum_{j=1}^{3} \mathbf{I}_{\{t\in\mathscr{D}_{r,j}\}} \left\{ \frac{1}{|\mathscr{F}_{r,j}|} \sum_{i\in\mathscr{F}_{r,j}} W^j_{r,\ell}(i) S_{t+i} \right\}$$

Shape of Kernel Function

boundary kernel





- **Figure**: Discrete kernel functions with bandwidth $r = 3$ and $r = 5$. The horizon is $T=12$, and the kernel polynomial $L= 4$

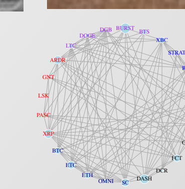
Choice of Tuning Parameters

- Choice of r:

$$r^* = \arg \min_{0 \le r \le T/2} \left(\| \widehat{\mathcal{S}}_{t,r} - \mathcal{S}_{t,r} \| + \| \mathcal{S}_{t,r} - \mathcal{S}_t \| \right)$$

$$\hat{r} = \max_{r} \left\{ \| \widehat{\mathcal{S}}_{t,r} - \widehat{\mathcal{S}}_{t,\rho} \| \le 4 W_{\max} \sqrt{\frac{N \| S_t \|_\infty}{\rho \vee 1}}, \forall \rho < r \right\}$$

LMS Tech for Cryptos



SFB 373

The Annals of Statistics
1997, Vol. 25, No. 3, 929–947

OPTIMAL SPATIAL ADAPTATION TO INHOMOGENEOUS SMOOTHNESS: AN APPROACH BASED ON KERNEL ESTIMATES WITH VARIABLE BANDWIDTH SELECTORS[1]

BY O. V. LEPSKI,[2] E. MAMMEN AND V. G. SPOKOINY

Institute for System Analysis and Humboldt-Universität; Universität Heidelberg; and Institute for Information Transmission Problems and Institute of Applied Analysis and Stochastics

A new variable bandwidth selector for kernel estimation is proposed. The application of this bandwidth selector leads to kernel estimates that achieve optimal rates of convergence over Besov classes. This implies that the procedure adapts to spatially inhomogeneous smoothness. In particular, the estimates share optimality properties with wavelet estimates based on thresholding of empirical wavelet coefficients.

Algorithm for Undirected Graphs

Algorithm 1: CASC in the Dynamic DCBM

Input : Adjacency matrices A_t for $t = 1, \cdots, T$, Covariates matrix X
Number of communities K, Approximation parameter ε

Output: Membership matrices Z_t for any $t = 1, \cdots, T$

1. Calculate regularized graph Laplacian $L_{\tau,t}$ and estimate $\mathcal{S}_t$ by $\widehat{\mathcal{S}}_{t,r}$ defined in (7).
2. Let $\widehat{U}_t \in \mathbb{R}^{N \times K}$ be a matrix representing the first K eigenvectors of $\widehat{\mathcal{S}}_{t,r}$.
3. Let N_+ be the number of nonzero rows of $\widehat{U}_t$, then obtain $\widehat{U}^+ \in \mathbb{R}^{N_+ \times K}$ consisting of normalized nonzero rows of $\widehat{U}_t$, i.e. $\widehat{U}_t^+(i,*) = \widehat{U}_t(i,*) / \left\| \widehat{U}_t(i,*) \right\|$ for i such that $\left\| \widehat{U}_t(i,*) \right\| > 0$.
4. Apply the $(1+\varepsilon)$-approximate k-medians algorithm to the row vectors of $\widehat{U}_t^+$ to obtain $\widehat{Z}_t^+ \in \mathcal{M}_{N_+,K}$.
5. Extend $\widehat{Z}_t^+$ to obtain $\widehat{Z}_t$ by arbitrarily adding $N - N_+$ many canonical unit row vectors at the end, such as, $\widehat{Z}_t(i) = (1, 0, \cdots, 0)$ for i such that $\left\| \widehat{U}_t(i,*) \right\| = 0$.
6. Output $\widehat{Z}_t$.

Algorithm for Directed Graphs

Algorithm 2: CASC in the Dynamic DCcBM

Input : Adjacency matrices A_t for $t = 1, \cdots, T$; Covariates matrix X; Number of row clusters K_R and number of column clusters K_C; Approximation parameter ε.
Output: Membership matrices of rows and columes $Z_{R,t}$ and $Z_{C,t}$.

Calculate regularized graph Laplacian $L_{\tau,t}$.

Estimate $\mathcal{S}_t$ by $\widehat{\mathcal{S}}_{t,r}$ defined in (7).

Compute the singular value decomposition of $\widehat{\mathcal{S}}_{t,r} = U_t \Sigma_t V_t^\top$ for $t = 1, \cdots, T$.

Extract the first K columns of U_t and V_t that correspond to the K largest singular values in Σ_t, where $K = \min\{K_R, K_C\}$. Denote the resulting matrices $U_t^K \in \mathbb{R}^{N \times K}$ and $V_t^K \in \mathbb{R}^{N \times K}$.

Let N_+^R be the number of nonzero rows of U_t^K, then obtain $U_{t+}^K \in \mathbb{R}^{N_+^R \times K}$ consisting of normalized nonzero rows of U_{t+}^K, i.e., $U_{t+}^K(i,*) = U_t^K(i,*)/\left\|U_t^K(i,*)\right\|$ for i such that $\left\|U_t^K(i,*)\right\| > 0$.

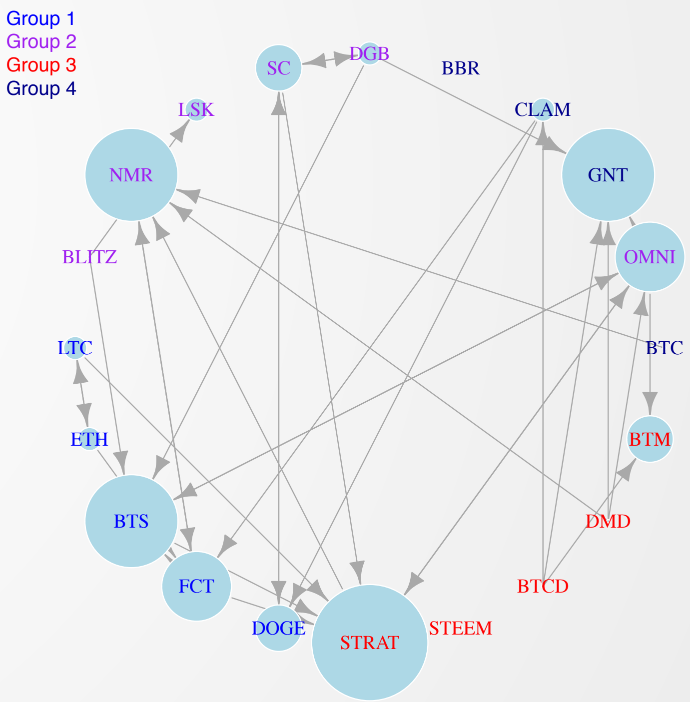
Similarly, let N_+^C be the number of nonzero rows of V_t^K, then obtain $V_{t+}^K \in \mathbb{R}^{N_+^C \times K}$ consisting of normalized nonzero rows of V_{t+}^K, i.e., $V_{t+}^K(i,*) = V_t^K(i,*)/\left\|V_t^K(i,*)\right\|$ for i such that $\left\|V_t^K(i,*)\right\| > 0$.

Apply the $(1+\varepsilon)$-approximate k-means algorithm to cluster the rows (columns) of $\widehat{\mathcal{S}}_t$ into K_R (K_C) clusters by treating each row of U_{t+}^K (V_{t+}^K) as a point in $\mathbb{R}^K$ to obtain $\widehat{Z}_{R,t}^+$ ($\widehat{Z}_{C,t}^+$).

Extend $Z_{R,t}^+$ ($Z_{C,t}^+$) to obtain $Z_{R,t}$ ($Z_{C,t}$) by arbitrarily adding $N - N_+^R$ ($N - N_+^C$) many cano unit row vectors at the end, such as, $\widehat{Z}_{R,t}(i) = (1, 0, \cdots, 0)$ ($\widehat{Z}_{C,t}(i) = (1, 0, \cdots, 0)$) for i that $\|U_t(i,*)\| = 0$ ($\|V_t(i,*)\| = 0$).

Mis-clustering

- Binkiewicz (2018) employ cluster centroids $\mathscr{C}_{i,t}$ $C_{i,t}$
- $\mathscr{O}_t^{\top}$ rotation matrix minimising $\|U_t\mathscr{O}_t^{\top} - \mathscr{U}_t\|$
- CASC does row normalisation of U; no degree correction needed
- Centroids: $\mathscr{C}_{i,t}$
- Assortativity: share edges in same cluster



Misclassification rate

$$\mathbb{M}_t = \{i : \|C_{i,t}\mathscr{O}_t^{\top} - \mathscr{C}_{i,t}\| > \|C_{i,t}\mathscr{O}_t^{\top} - \mathscr{C}_{j,t}\|, \text{for any } j \neq i\}$$

CASC in Dynamic SC-DCBM

- Use Algorithm 1 with $\widehat{\mathcal{S}_{t,r}}$. Let $Z_t \in \mathscr{M}_{N,K}$ and $P_{max} = \max_{i,t}(Z_t^\top Z_t)_{ii}$ be the size of the largest block. Mis-clustering rate:

$$\sup_t \frac{|\mathbb{M}_t|}{N} \leq \frac{c(\varepsilon) K W_{\max}^2}{m_z^2 N \lambda_{K,\max}^2} \left\{ (4 + 2c_w) \frac{b}{\underline{\delta}^{1/2}} + \frac{2K}{b} (\sqrt{2P_{\max} rs} + 2P_{\max}) + \frac{NL}{b^2 \cdot l!} \left(\frac{r}{T} \right)^{\beta} \right\}^2$$

- With probability $\geq 1 - \epsilon$, $\lambda_{K,\max} = \max_t \{\lambda_{K,t}\}$
  where $c(\epsilon) = 2^9(2+\epsilon)^2$,here $\lambda_{K,\max} = \max_t\{\lambda_{K,t}\}$ $b = \sqrt{3\log(8NT/\epsilon)}$

CASC in Dynamic SC-DCBM

- Use Algorithm 2 with $\widehat{\mathcal{S}}_{t,r}$. Let $Z_t \in \mathcal{M}_{N,K}$ and $P_{max} = \max_{i,t}(Z_t^\top Z_t)_{ii}$ denote the size of the largest block. Then the mis-clustering rate satisfies

$$\sup_t \frac{|\mathbb{M}_t|}{N} \leq \frac{c(\varepsilon)KW_{\max}^2}{m_z^2 N \lambda_{K,\max}^2} \left\{ (4+2c_w)\frac{b}{\underline{\delta}^{1/2}} + \frac{2K}{b}\left(\sqrt{2P_{\max} rs} + 2P_{\max}\right) + \frac{NL}{b^2 \cdot l!}\left(\frac{r}{T}\right)^{\beta} \right\}^2$$

- With probability $\geq 1-\epsilon$, where $\lambda_{K,\max} = \max_t\{\lambda_{K,t}\}$
- and $c(\epsilon) = 2^9(2+\epsilon)^2$, $b = \sqrt{3\log(8NT/\epsilon)}$

Simulation Settings

- Mis-clustering Rate with number of nodes:
  - Block Probability: $B_t = \frac{t}{T}\begin{bmatrix} 0.9 & 0.6 & 0.3 \\ 0.6 & 0.3 & 0.4 \\ 0.3 & 0.4 & 0.8 \end{bmatrix}$
  - $R = \lfloor \log(N) \rfloor$ $X(i,j) \overset{i.i.d}{\sim}$ U(0,10)
  - $N = \{10, 15, \ldots, 100\}$.
  - $T = 10$, $s = N^{1/2}$, # of Replication: 100

Dynamic Spectral Clustering

- Compare to DSDC Bhattacharyya & Chatterjee (2018)
- Compare to DSPZ Pensky & Zhang (2017)

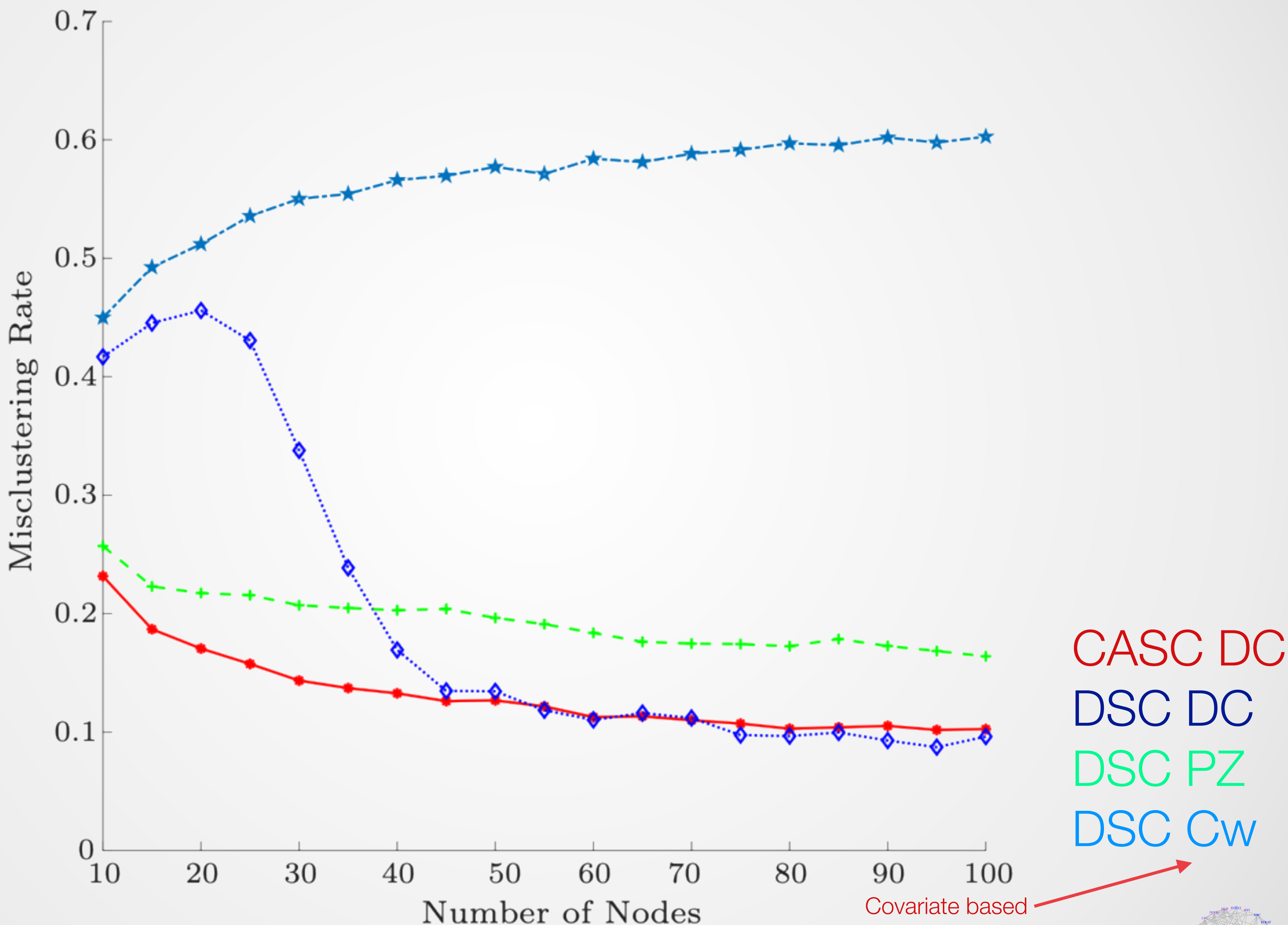
Simulation Settings

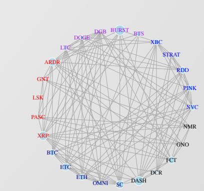
- Mis-clustering Rate with number of membership changes:
  - Block Probability:
  - Maximum number of membership changes:

$$S = [0,\ 2,\ 4,\ 5,\ 10,\ 20,\ 25,\ 50,\ 100]$$

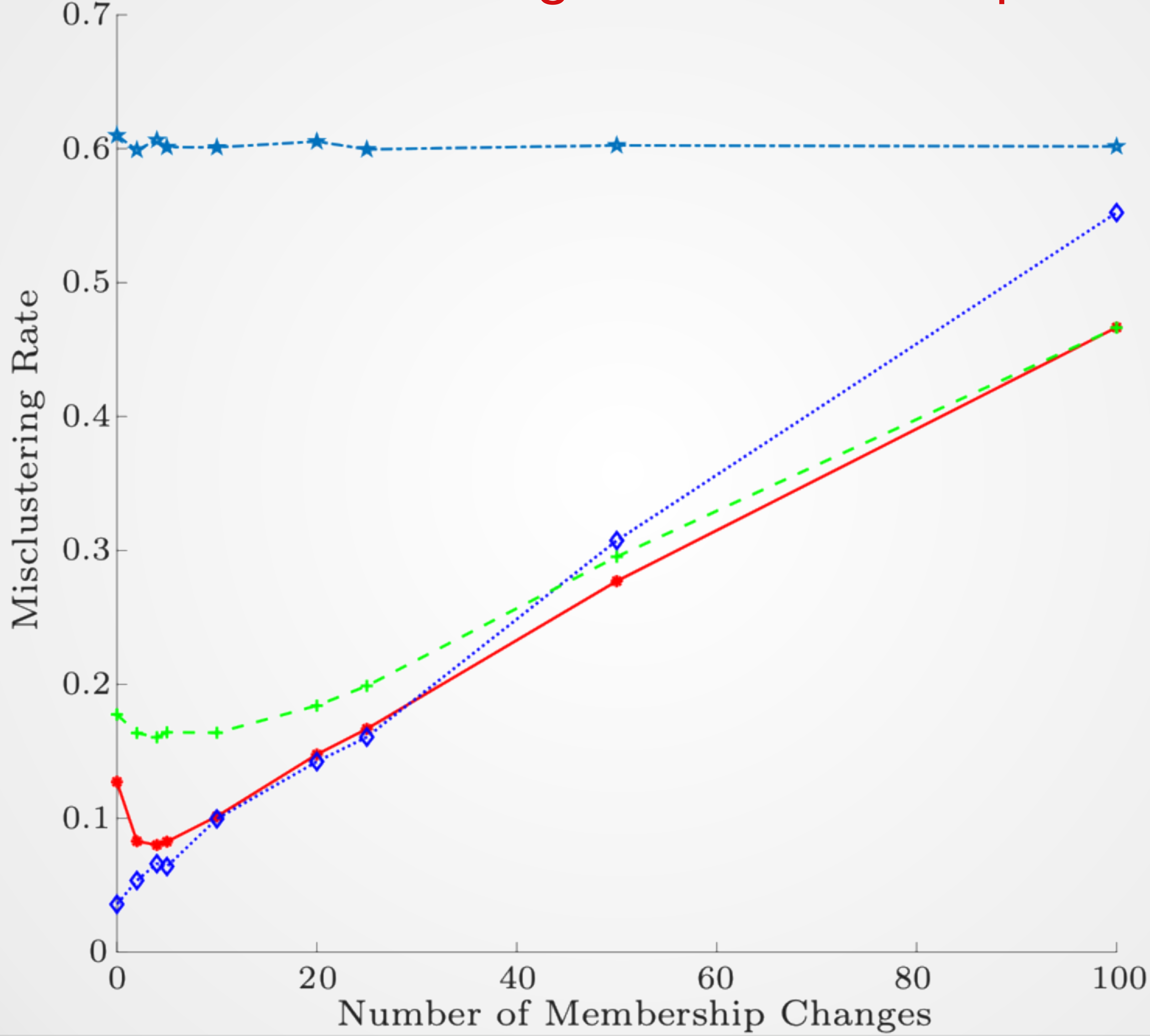
  - $R = \lfloor \log(N) \rfloor, \quad X(i,j) \overset{i.i.d}{\sim}$ U(0,10)
  - $N = 100$, $T = 10$,# of Replication 100

Performance with Growing # of Vertices





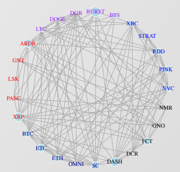
Performance with Growing # of Membership Changes



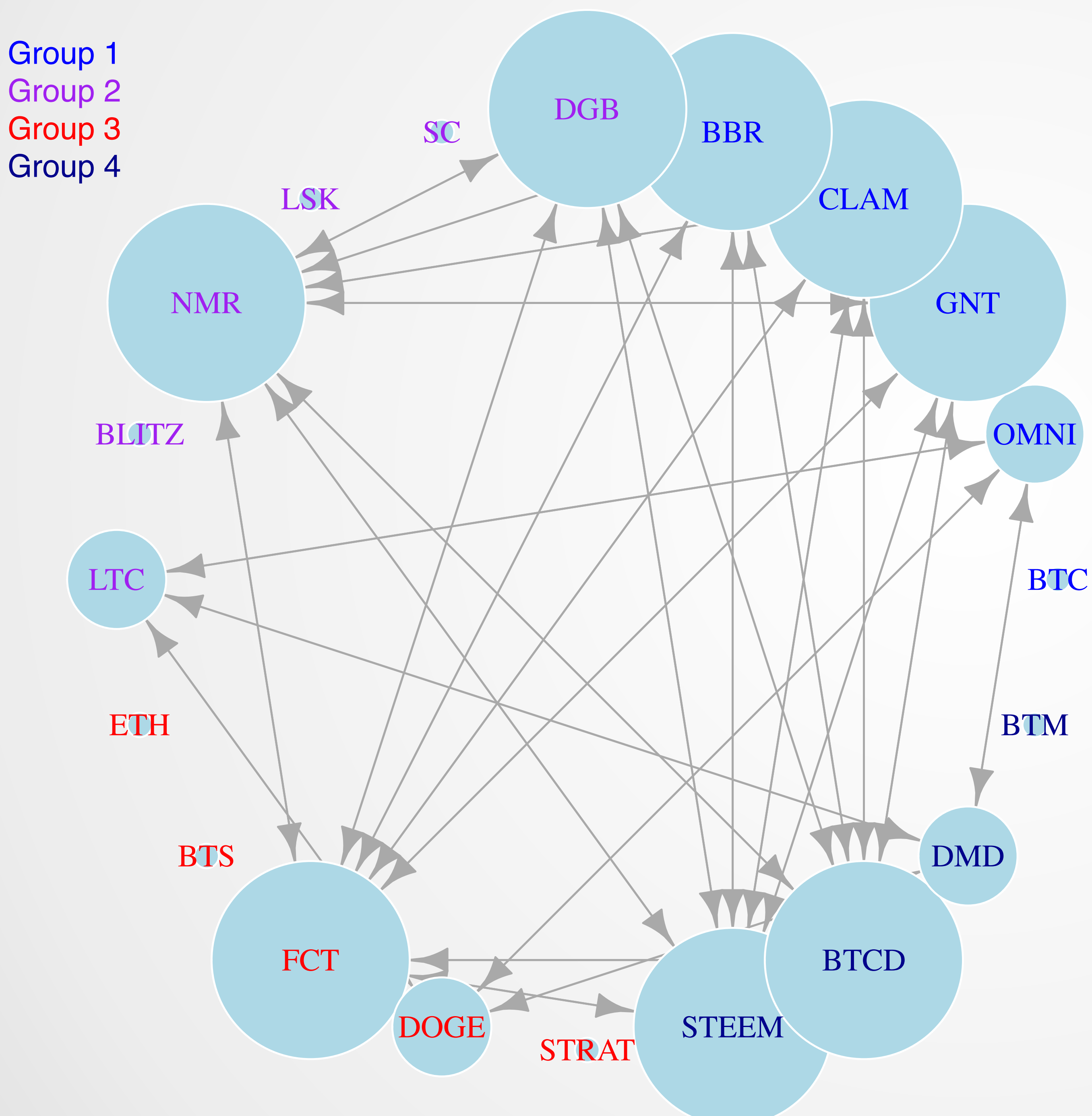
CASC DC
DSC DC
DSC PZ
DSC Cw

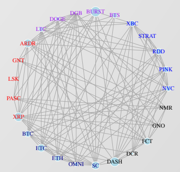
Crypto Currency Data

- 20150831 - 20180331, API cryptocompare.com # CCs 200
- In sample period: 20150831- 20181231community detection
- out of sample: 20180101 - 20180331 portfolio construction
- time invariant attributes: **Algo** type (SHA256, Scrypt, …)
- **Proof** type: PoW, PoS, DMD (Diamond)

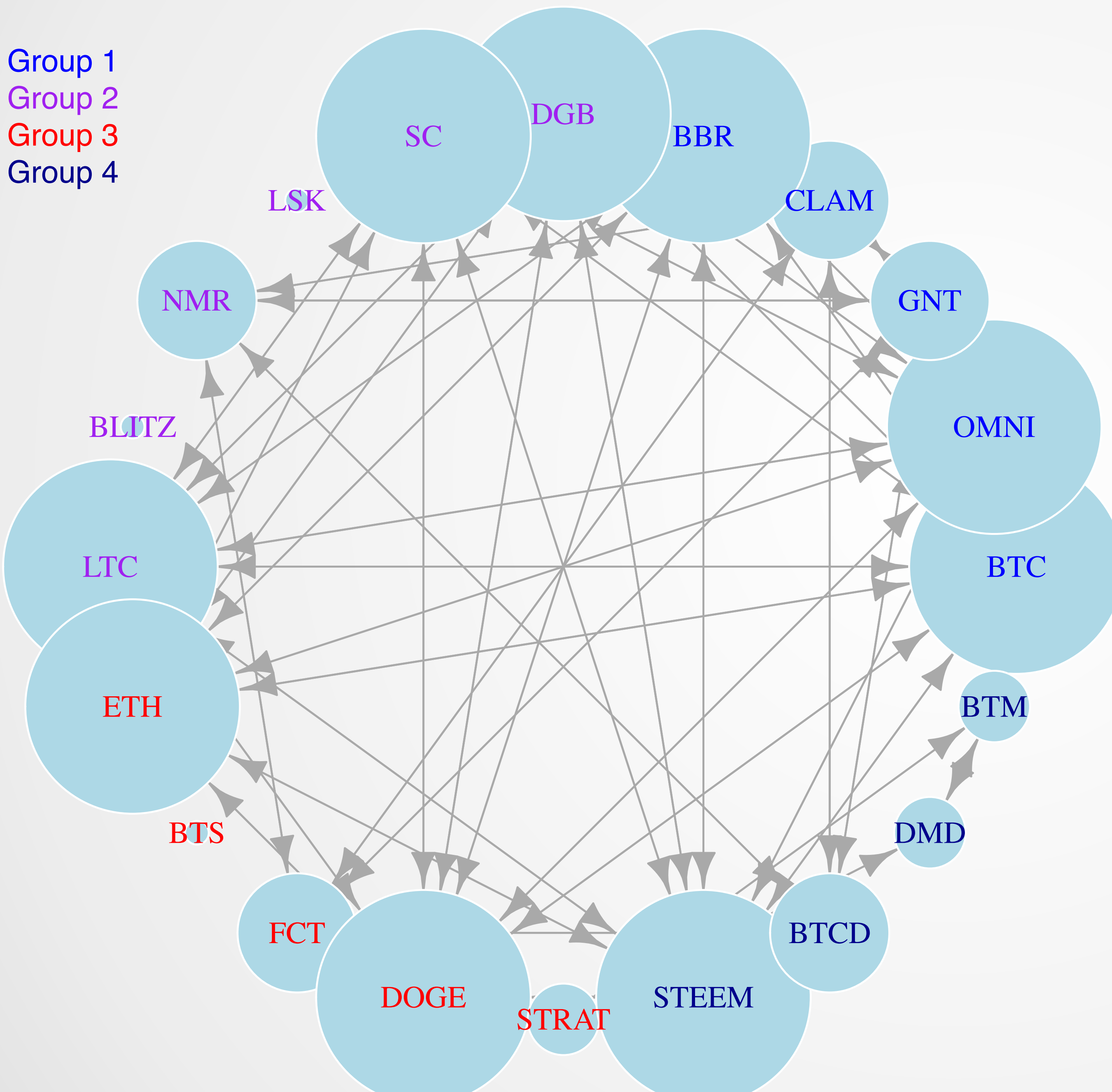


Node Features (Algorithm)

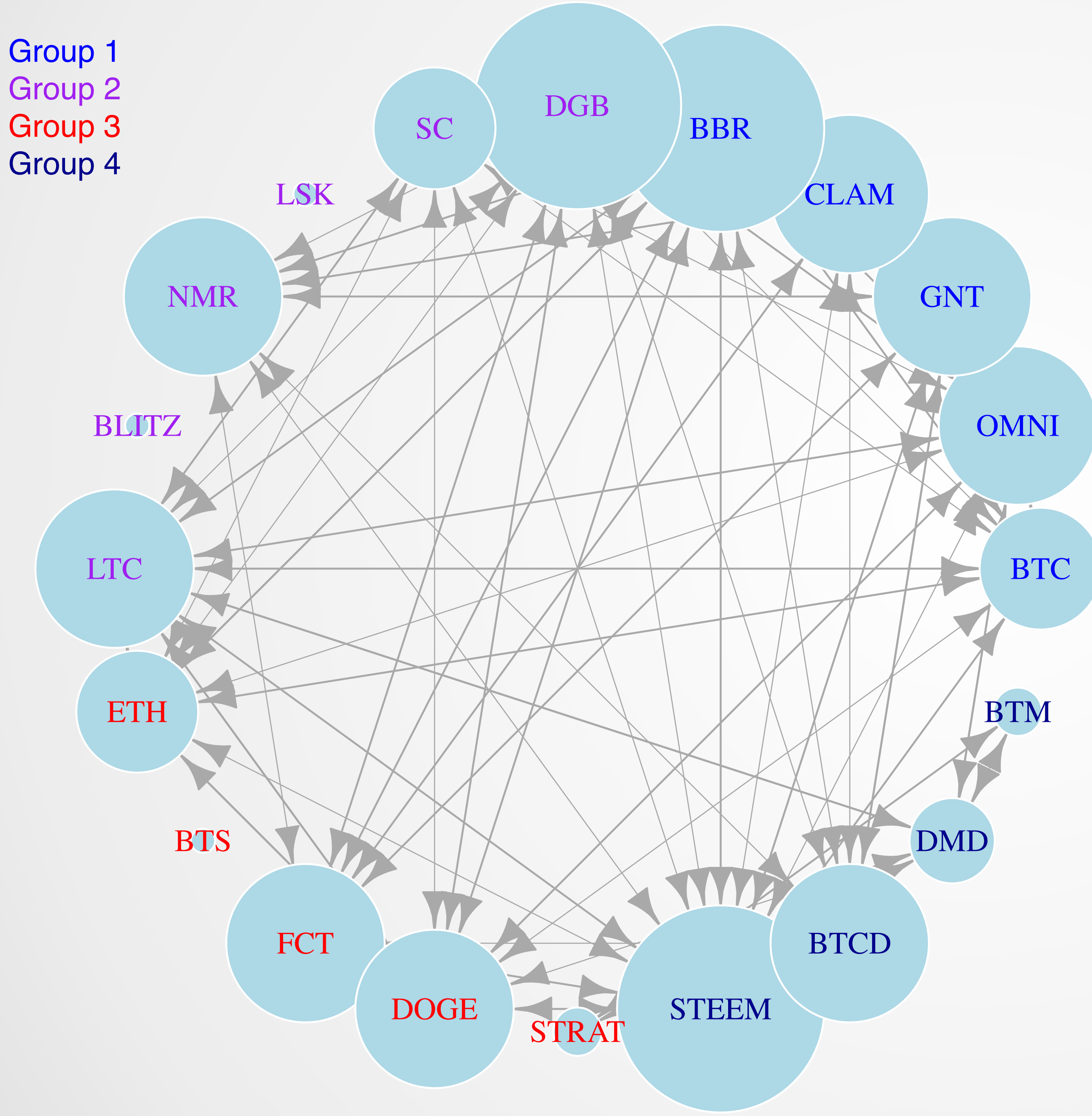




Node Features (Proof type)

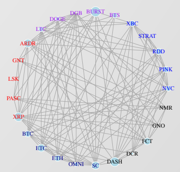


Node Features (Combined Attribution Network Structure)



algo type gives sparse connections
proof type has directed edges
combined features connect more links

node size is degree centrality
Group Colors



Group Results: 2017-12-31

Group ID	Group 1	Group 2	Group 3	Group 4
	BBR	BLITZ	BTS	BTCD
	BTC	DGB	DOGE	BTM
Cryptocurrencies	CLAM	LSK	ETH	DMD
	GNT	NMR	FCT	STEEM
	OMNI	SC	LTC	STRAT

Table:Top 4 Group Members

DISIM (Rohe et al., 2016)
Evaluation I: Return

- $\textit{Within-Group}_g = \dfrac{\text{\# of Degrees within Group } g}{N_g}$
- $\textit{Cross-Group}_g = \dfrac{\text{\# of Degrees within Group } g \text{ and other Groups}}{N_g}$

Group ID	Within-Group	Cross-Group	Diff(W-C)
Group 1	0.033	0.051	
Group 2	0.084	0.074	
Group 3	0.086	0.075	
Group 4	0.084	0.073	
All	0.072	0.068	

Statistical significance is indicated by 1%, 5%, 10% for positive sign, and 1%, 5%, 10% for negative sign

DISIM (Rohe et al., 2016)
Evaluation II: Algorithm

- $\textit{Within-Group}_g = \dfrac{\text{\# of Degrees within Group } g}{N_g}$
- $\textit{Cross-Group}_g = \dfrac{\text{\# of Degrees within Group } g \text{ and other Groups}}{N_g}$

Group ID	Within-Group	Cross-Group	Diff(W-C)
Group 1	0.252	0.204	
Group 2	0.216	0.198	
Group 3	0.215	0.196	
Group 4	0.216	0.197	
All	0.211	0.209	

DISIM (Rohe et al., 2016) Evaluation III: Proof Types

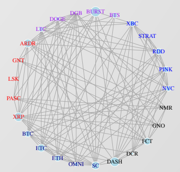
- $\textit{Within-Group}_g = \dfrac{\text{\# of Degrees within Group } g}{N_g}$
- $\textit{Cross-Group}_g = \dfrac{\text{\# of Degrees within Group } g \text{ and other Groups}}{N_g}$

Group ID	Within-Group	Cross-Group	Diff(W-C)
Group 1	0.252	0.233	
Group 2	0.282	0.242	
Group 3	0.284	0.242	
Group 4	0.283	0.242	
All	0.275	0.239	

CASC Evaluation I: Return

- $\textit{Within-Group}_g = \dfrac{\text{\# of Degrees within Group } g}{N_g}$
- $\textit{Cross-Group}_g = \dfrac{\text{\# of Degrees within Group } g \text{ and other Groups}}{N_g}$

Group ID	Within-Group	Cross-Group	Diff(W-C)
Group 1	0.064	0.058	
Group 2	0.063	0.057	
Group 3	0.065	0.057	
Group 4	0.065	0.057	
All	0.065	0.057	



CASC Evaluation II: Algorithm

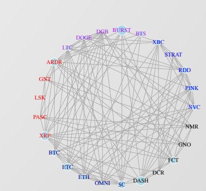
- $\textit{Within-Group}_g = \dfrac{\text{\# of Degrees within Group } g}{N_g}$
- $\textit{Cross-Group}_g = \dfrac{\text{\# of Degrees within Group } g \text{ and other Groups}}{N_g}$

Group ID	Within-Group	Cross-Group	Diff(W-C)
Group 1	0.232	0.202	
Group 2	0.243	0.203	
Group 3	0.240	0.202	
Group 4	0.240	0.202	
All	0.239	0.202	

CASC Evaluation III: Proof Types

- $\textit{Within-Group}_g = \dfrac{\text{\# of Degrees within Group } g}{N_g}$
- $\textit{Cross-Group}_g = \dfrac{\text{\# of Degrees within Group } g \text{ and other Groups}}{N_g}$

Group ID	Within-Group	Cross-Group	Diff(W-C)
Group 1	0.270	0.236	
Group 2	0.277	0.236	
Group 3	0.277	0.236	
Group 4	0.277	0.236	
All	0.275	0.236	

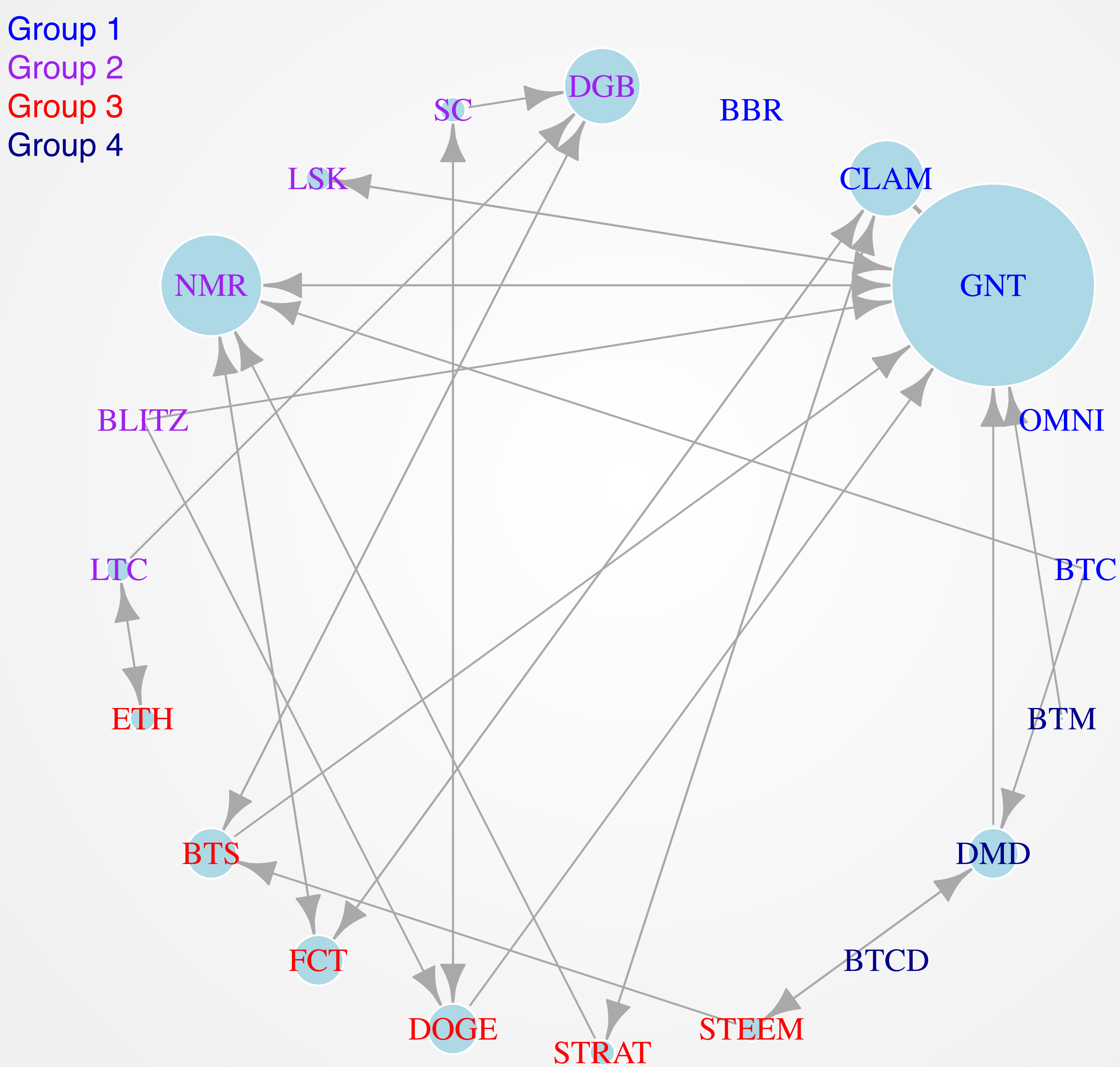


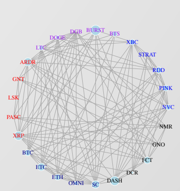
Combined Centrality and Return Predictability

Ahern (2013) Network centrality and the cross section of stock returns, J Finance

- Industries that are more central in the network of intersectoral trade earn higher stock returns than industries that are less central.
- Macroeconomic fluctuations are the aggregation of sector-specific shocks and the systematic risk originates from idiosyncratic shocks.
- Stocks in more central industries have greater systematic risk and earn higher returns because they have greater exposure to idiosyncratic shocks that transmit from one industry to another through intersectoral trade.

Directed Graph (Network Structure from Adaptive Lasso)





Within- and Cross-group Cryptos' Average Return Correlations by Dynamic CASC

	Within Group	Cross Group	Diff.
Group 1	0.169	0.154	
	(7.626)	(7.423)	(6.856)
Group 2	0.179	0.154	
	(8.077)	(7.423)	(6.077)
Group 3	0.181	0.157	
	(8.191)	(7.506)	(10.374)
Group 4	0.188	0.157	
	(8.114)	(7.416)	(5.607)
All	0.188	0.157	
	(7.697)	(7.381)	(6.331)

17% reduction when investing across groups

Statistical significance is indicated by 1%, 5%, 10% for positive sign, and 1%, 5%, 10% for negative sign

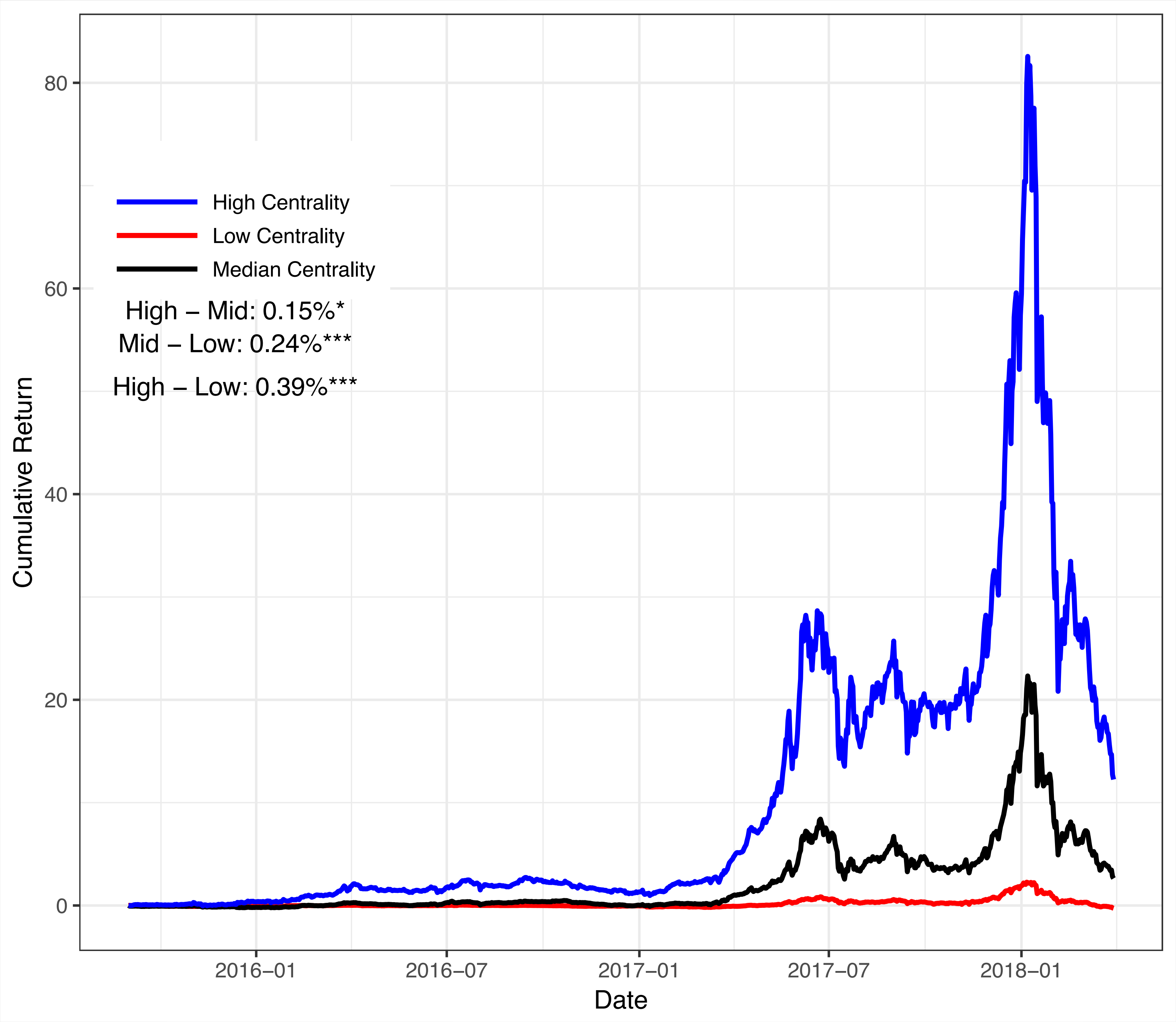
Combined Centrality and Return Predictability

Centrality	Ret_{t+1}	Ret_{t+2}	Ret_{t+3}	Ret_{t+4}	Ret_{t+5}	Ret_{t+6}	Ret_{t+7}
Low	0.00%	0.04%	-0.03%	-0.02%	0.03%	0.04%	0.08%
2	0.13%	0.15%	0.18%	0.19%	0.16%	0.17%	0.13%
3	0.36%	0.34%	0.28%	0.36%	0.37%	0.28%	0.30%
High	0.39%	0.36%	0.47%	0.38%	0.35%	0.42%.	0.39%
High-Low							
T-stats	3.47	3.10	4.23	3.40	2.77	3.37	2.73

EV centrality quartiles

Statistical significance is indicated by 1%, 5%, 10% for positive sign, and 1%, 5%, 10% for negative sign

Portofolio Return Sorted on Combined Centrality



Network CASC

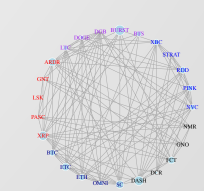
Conclusions

What we do

- Extend regularised spectral clustering methods to analysing dynamic networks (both directed and undirected), especially when there are membership changes.
- Incorporate node covariates into the network to assist community detection in dynamic networks.

Takeaways

1. Attribution Matrix provides valuable information to connect within group members.
2. Return-based Adjacency Matrix reveal connections across different groups.
3. Risk premium is higher for cryptos with higher combined centrality scores.



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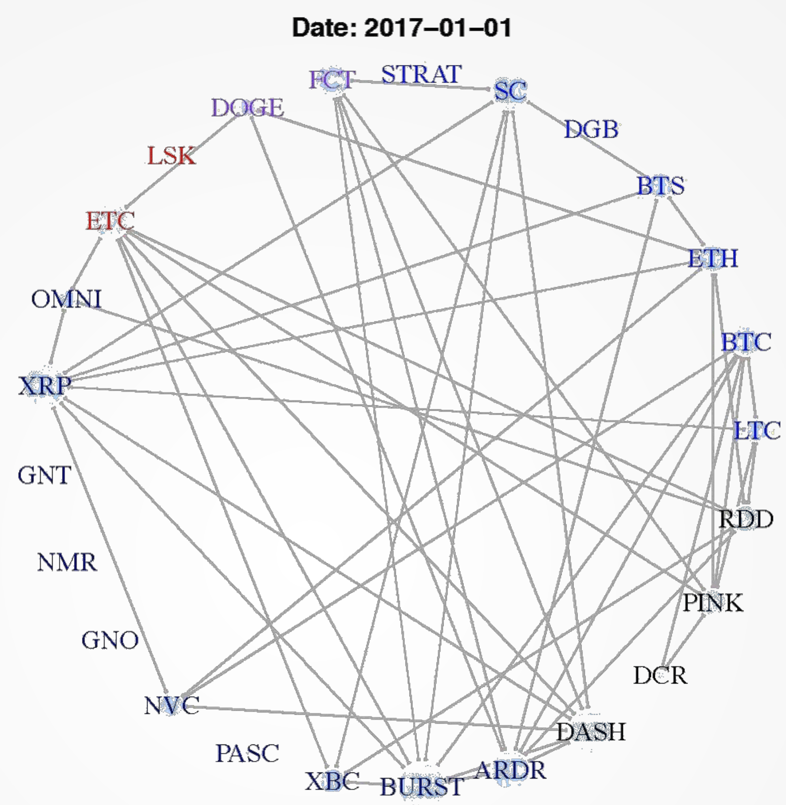
Thank you Volodia!

Dynamic Networks like ours may even catch fish for lunch and advance science

Dynamic Crypto Networks



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Assumptions

- Assumption 1
- The dynamic network is composed of a series of assortative graphs generated with the stochastic block model with covariates whose block probability matrix for all *t*.

- Assumption 2 $B_t \geq 0$
- There are at most number of nodes can switch their memberships between any consecutive time instances.

$s < \infty$

Assumptions

- Assumption 3
- For $1 \leq k \leq k' \leq K$, there exists a function $f(\,\cdot\,; k, k')$ such that
- $B_t(k, k') = f(\varsigma_t; k, k')$ and $f(\,\cdot\,; k, k') \in \Sigma(\beta, L)$, where $\Sigma(\beta, L)$ is a Hölder class of functions $f(\,.\,)$ on $[0,1]$ such that $f(\,.\,)$ are ℓ times differentiable and

$$|f^{(\ell)}(x) - f^{(\ell)}(x')| \leq L|x - x'|^{\beta-\ell}, \forall x, x' \in [0,1]$$

- with ℓ being the largest integer smaller than β .

Assumptions

- Assumptions 4
- Let $\lambda_{1,t} \geq \lambda_{2,t} \geq \cdots \geq \lambda_{K,t} > 0$ be the K largest eigenvalues of S_t for each $t = 1,\cdots,T$. Assume:

$$\underline{\delta} = \inf_t \{\min_i \mathscr{D}_{\tau,t}(i,i)\} > 3\log(8NT/\epsilon)$$

$$\alpha_{\max} = \sup_t \left\{ \alpha_t \leq \frac{a}{NRJ^2\xi} \right\}$$

- with
- where $\xi = \max(\sigma^2 \|L_\tau\|_F \sqrt{\log(TR)}, \sigma^2 \|L_\tau\| \log(TR), NRJ^2/\underline{\delta})$
- $\sigma = \max_{i,j} \|X_{ij} - \mathscr{X}_{ij}\|_{\phi_2} \quad L_\tau = \sup_t L_{\tau,t}$

Choice of Tuning Parameters

- Choice of α_t

$$\alpha_{\min} = \frac{\lambda_K(L_{\tau,t}) - \lambda_{K+1}(L_{\tau,t})}{\lambda_1(C_t^w)}$$

$$\alpha_{\max} = \frac{\lambda_1(L_{\tau,t})}{\lambda_K(C_t^w) - \lambda_{K+1}(C_t^w)}$$

$$\alpha_t = (\alpha_{\min} + \alpha_{\max})/2$$

- Determination of K: Network Cross-Validation Chen & Lei (2017), Edge Cross-Validation, Li et al (2016)

Combined Network Structure