Rodeo or Ascot:
Which hat to wear for the crypto race?

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IRTG 1792
High Dimensional Nonstationary Time Series
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Rise of the Cryptocurrency (CC) market?

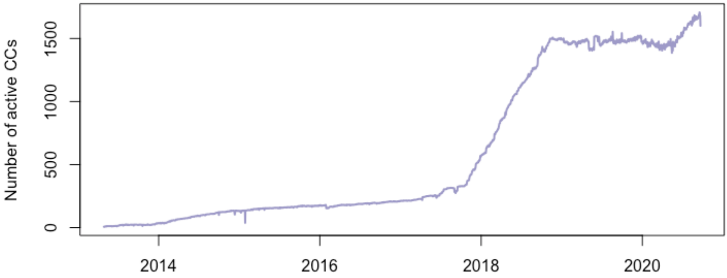

Figure 1: # of active CCs (=market capitalization >0).
SVCJrw_CC_market_caps

Rise of the CC market

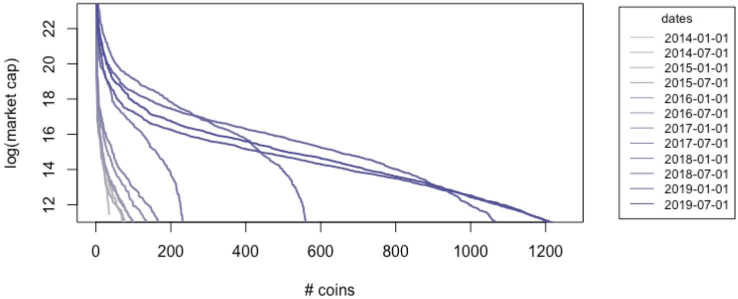

Figure 2: CCs ranked by their log market capitalization over time.
SVCJrw_CC_market_caps

Outline

1. Tracking the Dynamics of the CC Sector
   - Composition of CC indices
   - Evaluation of CC indices
2. Understanding the Dynamics of the CC Sector
   - the SVCJ model
   - Time Series of Parameters
3. Stylized Facts
   - Drift, Volatility & Jumps

Index Funds

- Widely used investment tool
- decentralized indices provided by several research groups and private companies:

  Bloomberg Galaxy crypto index (BGCI), Bitwise 10, CCi30, CRIX, F5 crypto index

CC Indices

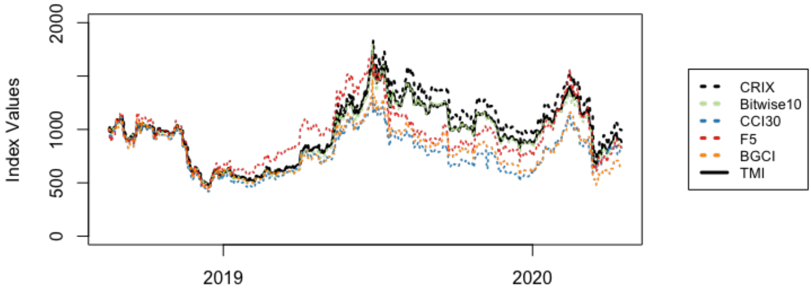

Figure 3: Scaled CCs Indices (dotted lines, starting value 1000) and Total Market Index (black line), period 2018-2020. SVCJrw_Indices_SharpeR

Composition of Indices

Table 1: Overview of existing CC indices and their composition.

	Bitwise 10	CCI30	CRIX	F5	BGCI
Number of Constituents	10	30	depending on AIC	12	$\leq$12

Composition of Indices

	Bitwise 10	CCI30	CRIX	F5	BGCI
Number of Constituents	10	30	depending on AIC	12	≤12
Index & Weights	NA	$I_t = \sum_{j=1}^{30} \frac{\sqrt{M_j^*(t)}}{\sum_{i=1}^{30} \sqrt{M_i^*(t)}} \frac{P_j(t)}{P_j(0)}$	$CRIX_t = \frac{\sum_i P_{it} Q_{i0}}{\sum_i P_{i0} Q_{i0}}$	$momentum_t = price_t - price_{t-n}$	$BGCI_t = \frac{\sum_{i=1}^{x} P_{i,t} \times CS_{i,m} \times CF_{i,m}}{D}$

Overview of existing CC indices and their composition.

M^* = moving-average adjusted Market Capitalization
P_{it} = price of coin i
Q_{it} = number of coins at time t
$CS_{i,m}$ = circulating supply of each constituent
$CF_{i,m}$ =Cap/Floor correction factor: cap maximum weight of each coin to 40%, min floor 1%

Composition of Indices

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Self-image	"industry standard" "like the S& P 500 of crypto"	"industry benchmark"	"the first"	"scientific approach"	

Overview of existing CC indices and their composition.

Performance

	SR	returns	sd	skewness	kurtosis
BGCI	0.005	0.02 %	0.043	-0.224	8.989
Bitwise10	0.013	0.05 %	0.040	-0.166	8.154
CCi30	0.012	0.05 %	0.041	-0.155	8.965
CRIX	0.021	0.02 %	0.040	-1.063	16.191
F5	0.017	0.07 %	0.046	-0.990	13.928
TMI	0.009	0.02 %	0.031	-1.292	17.314

Table 2: Descriptive Statistics on daily level for the CC indices under review. Period of analysis 08/2018-04/2020. SVCJrw_Indices_SharpeR

Probabilistic Sharpe Ratios

- Skewness and Kurtosis impact the confidence intervalls of the Sharpe Ratio estimates and therby their statistical significance
- under the assumption that returns are normally distributed, the estimated variance of the returns is given by Lo (2002)
  $\hat{\sigma}(\widehat{SR}) = \sqrt{\frac{1}{n-1}\left(1 + \frac{1}{2}\widehat{SR}^2\right)}$
- without normality, this variance of the SR extends to
  $\hat{\sigma}(\widehat{SR}) = \sqrt{\frac{1}{n-1}\left(1 + \frac{1}{2}\widehat{SR}^2 - \gamma_3\widehat{SR} + \frac{\gamma_4 - 3}{4}\widehat{SR}^2\right)}$,
  where γ_3 is the skewness and γ_4 the kurtosis

Probabilistic Sharpe Ratios (PSR)

- under non-normality, the asymptotic distribution is

$$(\widehat{SR} - SR) \xrightarrow{\mathcal{L}} N\left(0, \frac{1+\frac{1}{2}SR^2-\gamma_3 SR+\frac{\gamma_4-3}{4}SR^2}{n-1}\right)$$

- Bailey & Lopez de Prado (2012): given a predefined benchmark SR^*, the PSR is defined as

$$PSR\,(SR^*) = \text{Prob}[SR^* \leq \widehat{SR}]$$

- which can be estimated by

$$\widehat{PSR}\,(SR^*) = Z\left[\frac{\left(\widehat{SR}-SR^*\right)}{\hat{\sigma}(\widehat{SR})}\right] = Z\left[\frac{\left(\widehat{SR}-SR^*\right)\sqrt{n-1}}{\sqrt{1+\frac{1}{2}\widehat{SR}^2-\gamma_3\widehat{SR}+\frac{\gamma-3}{4}\widehat{SR}^2}}\right]$$

Probabilistic Sharpe Ratios

	SR	returns	sd	skewness	kurtosis	PSR
BGCI	0.005	0.02 %	0.043	-0.224	8.989	0.549
Bitwise10	0.013	0.05 %	0.040	-0.166	8.154	0.625
CCi30	0.012	0.05 %	0.041	-0.155	8.965	0.614
CRIX	0.021	0.02 %	0.040	-1.063	16.191	0.691
F5	0.017	0.07 %	0.046	-0.990	13.928	0.659
TMI	0.009	0.02 %	0.031	-1.292	17.314	0.587

Table 3: Descriptive Statistics on daily level for the CC indices under review. Period of analysis 08/2018-04/2020. PSR given the benchmark $SR^* = 0$ (probability of positive returns). SVCJrw_Indices_SharpeR

Accurate Representation

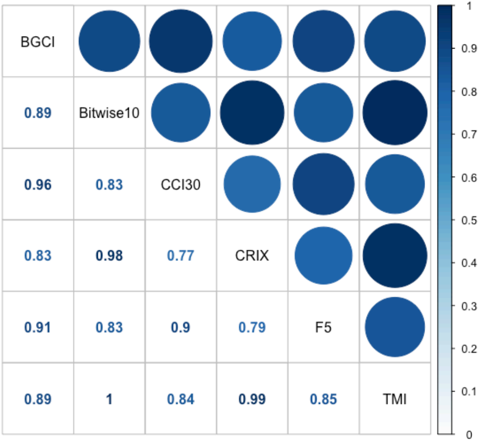


Figure 4: Correlation plot of CC Indices and Total Market Index (TMI).

Who can beat the market?

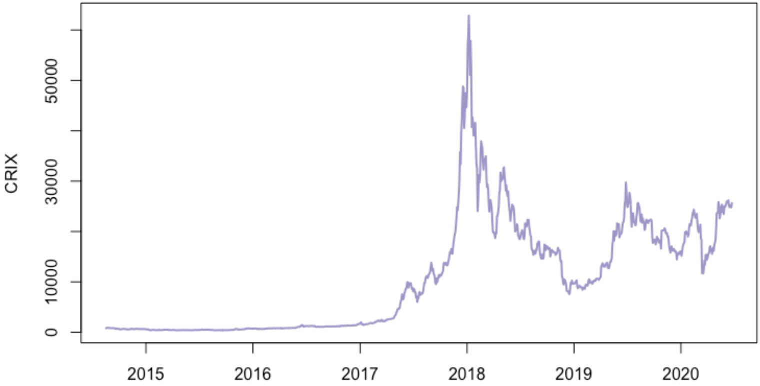
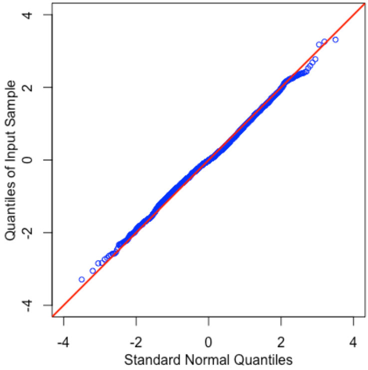


Figure 5: the CRIX, 2014-2020. Data source: thecrix.de

Residuals: SVCJ vs. GARCH



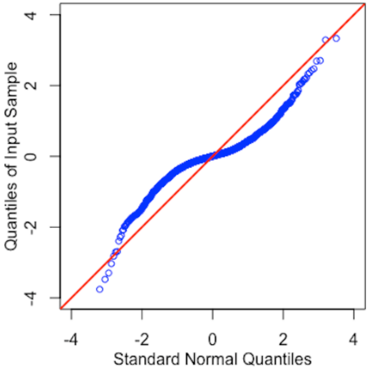


Figure 6: QQ-Plot of SVCJ residuals (left) SVCJOptionApp and GARCH(2,2) residuals(right) econ_garch

SVCJ

$$d\log(S_t) = \mu dt + \sqrt{V_t} dW_t^{(s)} + Z_t^{(y)} dN_t \tag{1a}$$

$$dV_t = \kappa(\theta - V_t)\,dt + \sigma_v \sqrt{V_t} dW_t^{(v)} + Z_t^{(v)} dN_t \tag{1b}$$

Two stochastic processes, for log returns and volatility:

(1a) returns $=$ trend $+$ vola $+$ Jump process

(1b) volatility $=$ Cox model $+$ Jump process

jump process $=$ jump size Z_t $\cdot$ jump frequency dN_t

SVCJ

$$
\begin{aligned}
d\log\left(S_t\right) &= \mu dt + \sqrt{V_t} dW_t^{(s)} + Z_t^{(y)} dN_t \\
dV_t &= \kappa\left(\theta - V_t\right) dt + \sigma_v \sqrt{V_t} dW_t^{(v)} + Z_t^{(v)} dN_t
\end{aligned}
$$

$$
\begin{aligned}
\operatorname{Cov}\left(dW_t^{(s)}, dW_t^{(v)}\right) &= \rho dt \\
P\left(dN_t = 1\right) &= \lambda dt \\
Z_t^y \mid Z_t^v &\sim N\left(\mu_y + \rho_j Z_t^v, \sigma_y^2\right) \\
Z_t^v &\sim \operatorname{Exp}\left(\mu_v\right)
\end{aligned}
$$

SVCJ estimation procedure

- Euler discretization:

$$Y_t = \mu + \sqrt{V_{t-1}}\varepsilon_t^y + Z_t^y J_t$$
$$V_t = \alpha + \beta V_{t-1} + \sigma_v \sqrt{V_{t-1}}\varepsilon_t^v + Z_t^v J_t$$

$\alpha = \kappa\theta, \beta = 1 - \kappa$,
$\varepsilon_t^y, \varepsilon_t^v$ are $N(0,1)$ variables with correlation ρ.
J_t is a Bernoulli random variable with $P\left(J_t = 1\right) = \lambda$

- Metropolis-Hastings-algorithm
- Perez (2018): discussion of prior distribution values
- Rolling windows of several sizes (150, 300, 600days)

Parameter estimates

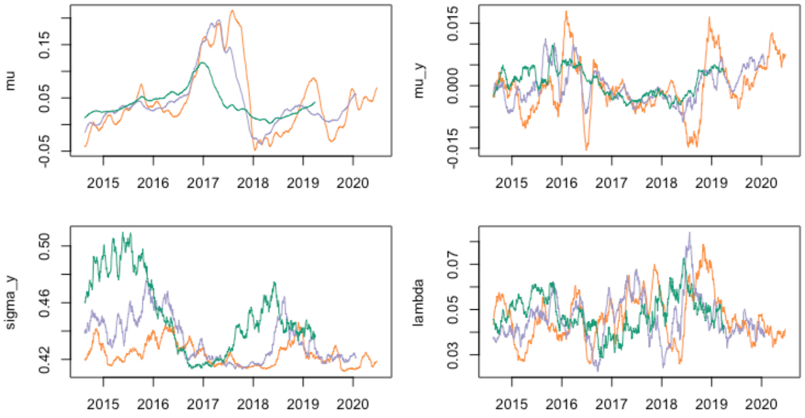
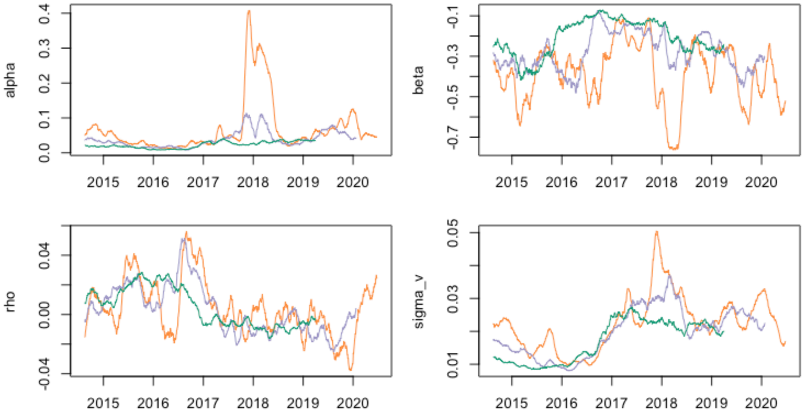


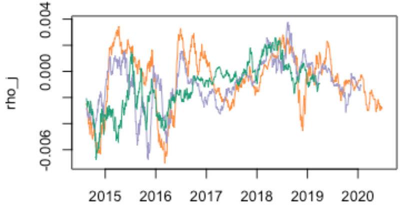
Figure 7: Parameter estimates obtained by rolling windows of size (150, 300 & 600 days). Estimates fluctuate a lot, the lines depict a moving average of 20 days.

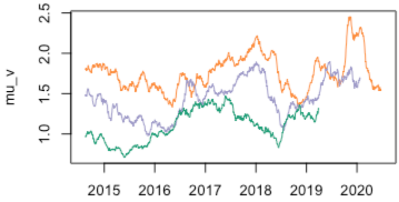
SVCJrw_graph_parameters

Parameter estimates



Parameter estimates





Stylized Facts

overall...

- all paramaters vary over time
- bigger window smooths out temporary fluctuations

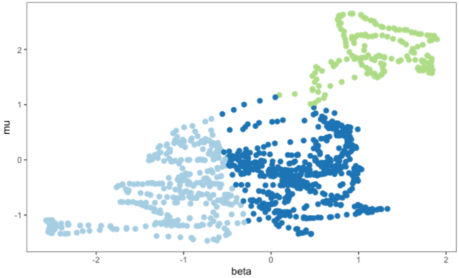
in bullish periods...

- vola remains stable (α, β)
- mean jump size $\mu_y$decreases (as well as its vola σ_y)

in bearish periods...

- volatility is high
- volatility needs time to return to its long-run trend (high κ)

Cluster Analysis of β & μ



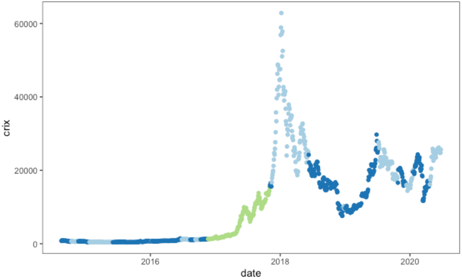
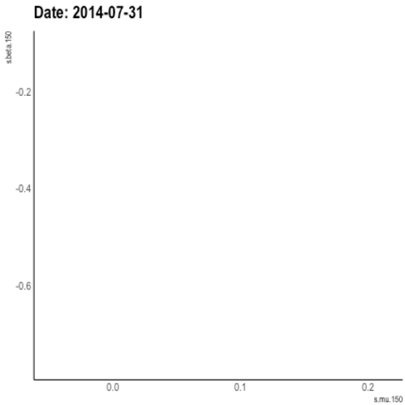
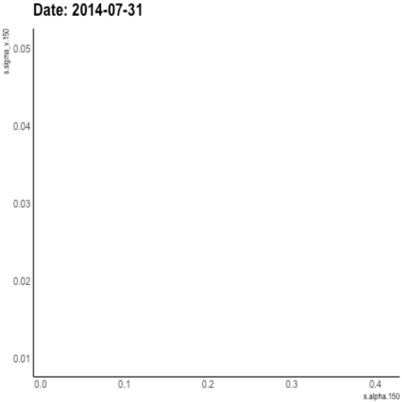


Figure k-means clustering of β and μ. $k = 3$

Interactions among parameters: β & μ



Interactions among parameters α & σ_v



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