Hedging Cryptos with futures

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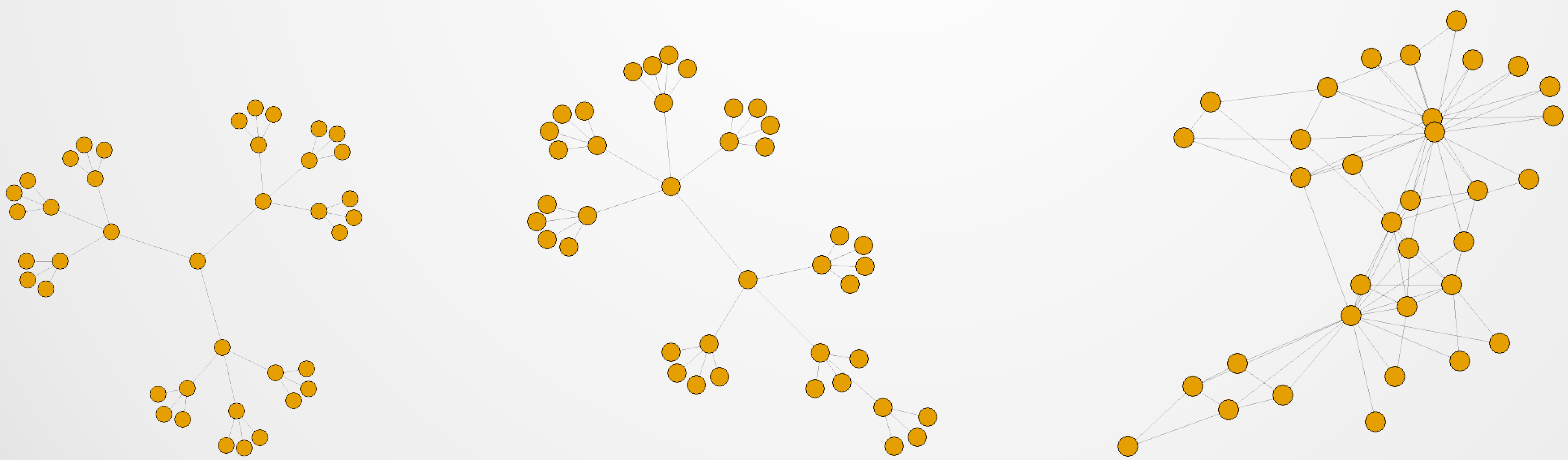
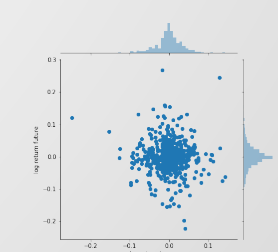
20210430 Lu Hae Liu Pac CopulaBasedHedging Kopie.key

Digital assets are here to stay

- Markets for cryptocurrencies are maturing
  - Institutional investors are buying into it
  - Regulators are working hard to make stablecoins "safe" (e.g. resolve issues of jurisdiction, financial stability)
  - Exchanges (e.g. CME) are issuing futures and options

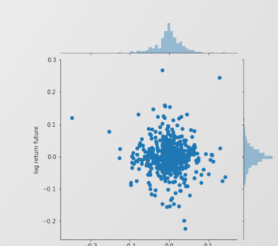
We are in the middle of a rapid decentralisation of financial markets!





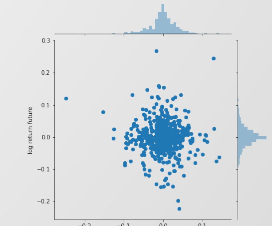
Digital assets are here to stay

https://www.garp.org/#!/risk-intelligence/market/investment-management/a1Z1W000005kZDGUA2



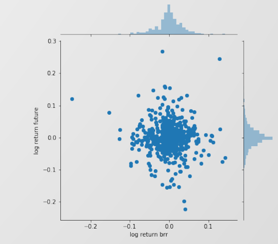
Berlin - Room 77

Beer for Bitcoin, 2011



Bitcoin futures

- CME launched BTC Futures in December 2017 and options on futures in January 2020
- Bitcoin Future
  - Underlying: BTC Reference Rate (BRR), based on relevant bitcoin transaction on certain exchanges
  - Maturities: nearest two Decembers and nearest six consecutive months
  - Settlement: cash
- https://www.cmegroup.com/trading/equity-index/us-index/bitcoin.html /trading/equity-index/us-index/bitcoin.html



Hedging cryptos

- Hedging Bitcoin exposure with Bitcoin futures
  - Basis risk
  - BRR not traded
  - Ability of futures to hedge tail risks?

- Hedge other cryptos with Bitcoin?
  - High correlation,
  - Tail risks, extreme events?

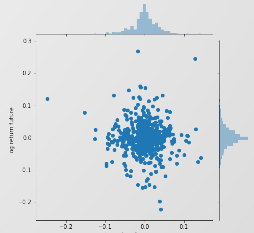
- Two directions
  - Copulae
  - Risk measures

skew.

1M Correlation Matrix

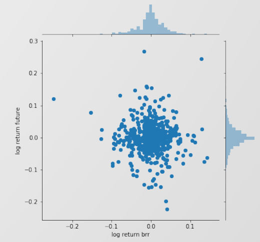
index	BTC	ETH	XRP	USDT	BCH	LTC	EOS	BNB
BTC	100.00%	91.17%	81.77%	-15.59%	88.69%	86.85%	90.70%	84.47%
ETH	91.17%	100.00%	78.50%	-24.51%	90.18%	93.10%	92.84%	88.67%
XRP	81.77%	78.50%	100.00%	-8.21%	81.90%	81.68%	84.96%	75.23%
USDT	-15.59%	-24.51%	-8.21%	100.00%	-16.64%	-18.96%	-15.91%	-20.18%
BCH	88.69%	90.18%	81.90%	-16.64%	100.00%	87.79%	87.02%	88.45%
LTC	86.85%	93.10%	81.68%	-18.96%	87.79%	100.00%	95.78%	78.61%
EOS	90.70%	92.84%	84.96%	-15.91%	87.02%	95.78%	100.00%	84.92%
BNB	84.47%	88.67%	75.23%	-20.18%	88.45%	78.61%	84.92%	100.00%
BSV	64.46%	69.87%	59.06%	-10.90%	72.47%	74.45%	72.54%	70.57%
XTZ	27.30%	21.79%	43.02%	13.80%	26.99%	17.18%	24.13%	34.09%

Source: skew.com, December 2019

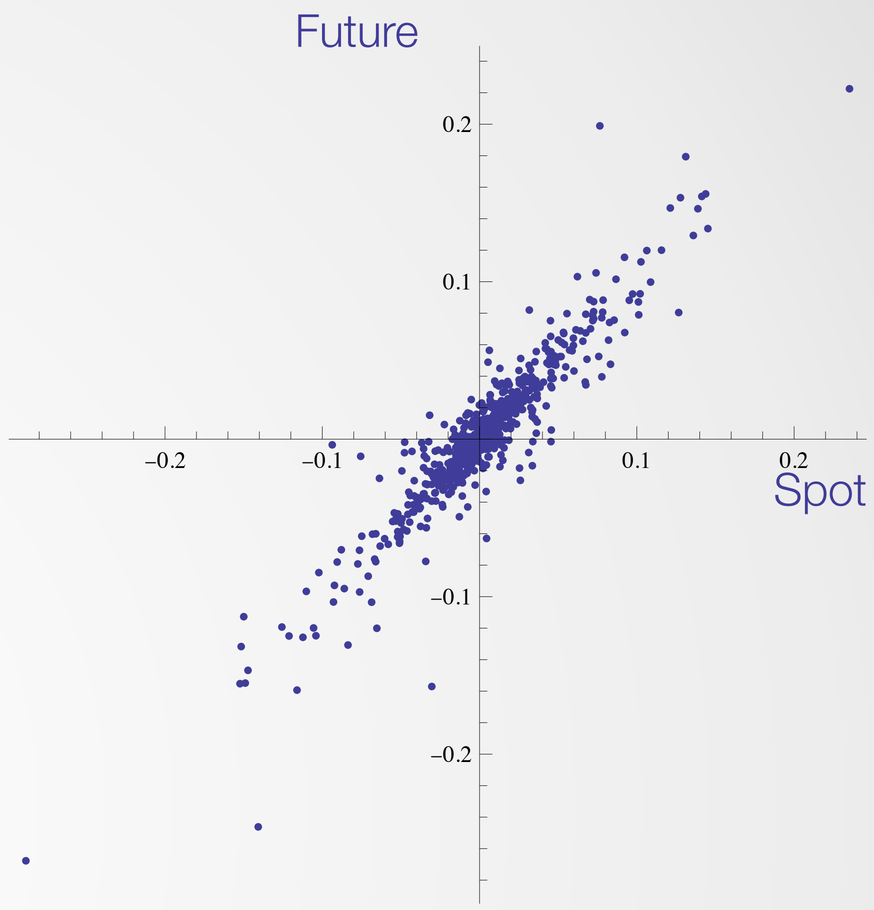


Outline

- Motivation ✓
- Copula-based hedging
- Data
- Results



Hedging spot with futures

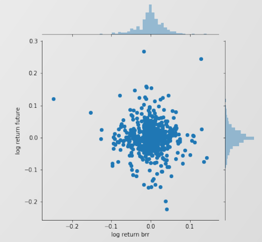


- Hedge portfolio return: $R_t^h = R_t^S - h\,R_t^F$,
  - R_t^S: spot return
  - R_t^F: futures return
  - h : hedge ratio

- Hedge portfolio return: $R_t^h = R_t^S - h\,R_t^F$,

- Goal: Find optimal hedge ratio $h*$
- Minimum-variance hedge ratio, e.g., variance as risk measure and elliptical return distribution
- Extensions: risk measures, copulae,

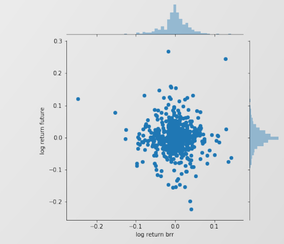
Ederington (1979), Harris and Shen (2006), Barbi and Romagnoli (2014)



Copulae

A (bivariate) copula is a distribution function on $[0,1]^2$ with standard uniform marginals, i.e. $C : [0,1]^2 \mapsto [0,1]$.

- Copulae differ only through the dependence between the marginals.
- Hoeffding's Theorem captures that copulae allow to separate
  - modelling of the marginals
  - modelling of the dependence structure



Hoeffding's Theorem

Let $F_{X,Y}$ be a joint distribution function with margins F_X and F_Y. Then there exists a copula C such that for all $x, y \in \mathbb{R}$

$$F_{X,Y}(x,y) = C\{F_X(x), F_Y(y)\}$$

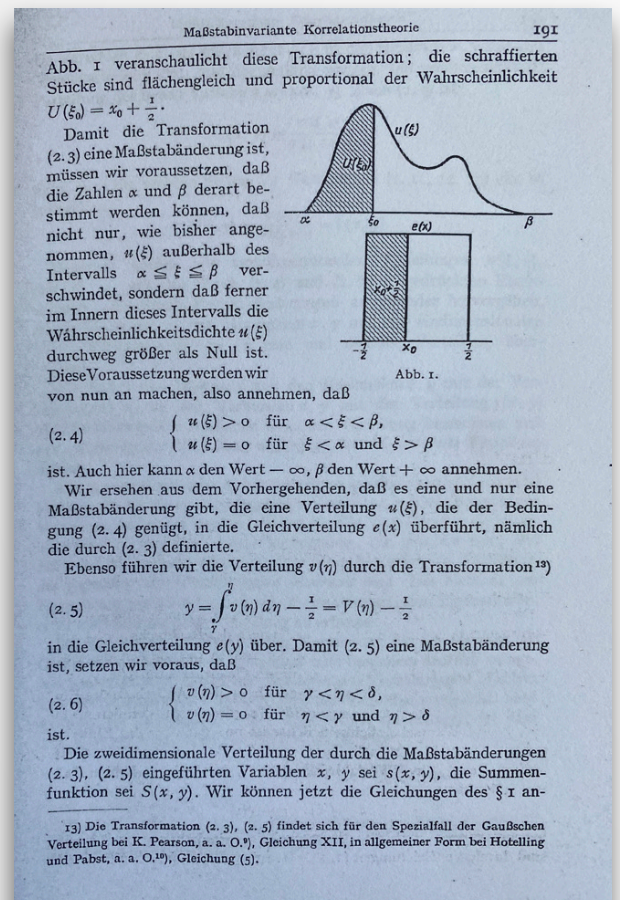

Maßstabinvariante Korrelationstheorie 191

Abb. I veranschaulicht diese Transformation; die schraffierten Stücke sind flächengleich und proportional der Wahrscheinlichkeit $U(\xi_0) = x_0 + \frac{1}{2}$.

Damit die Transformation (2. 3) eine Maßstabänderung ist, müssen wir voraussetzen, daß die Zahlen α und β derart bestimmt werden können, daß nicht nur, wie bisher angenommen, $u(\xi)$ außerhalb des Intervalls $\alpha \leqq \xi \leqq \beta$ verschwindet, sondern daß ferner im Innern dieses Intervalls die Wahrscheinlichkeitsdichte $u(\xi)$ durchweg größer als Null ist. Diese Voraussetzung werden wir von nun an machen, also annehmen, daß

Abb. I.

$$(2.\,4) \qquad \begin{cases} u(\xi) > 0 & \text{für} \quad \alpha < \xi < \beta, \\ u(\xi) = 0 & \text{für} \quad \xi < \alpha \text{ und } \xi > \beta \end{cases}$$

ist. Auch hier kann α den Wert $-\infty$, β den Wert $+\infty$ annehmen.

Wir ersehen aus dem Vorhergehenden, daß es eine und nur eine Maßstabänderung gibt, die eine Verteilung $u(\xi)$, die der Bedingung (2. 4) genügt, in die Gleichverteilung $e(x)$ überführt, nämlich die durch (2. 3) definierte.

Ebenso führen wir die Verteilung $v(\eta)$ durch die Transformation[13])

$$(2.\,5) \qquad y = \int_\gamma^\eta v(\eta)\, d\eta - \frac{1}{2} = V(\eta) - \frac{1}{2}$$

in die Gleichverteilung $e(y)$ über. Damit (2. 5) eine Maßstabänderung ist, setzen wir voraus, daß

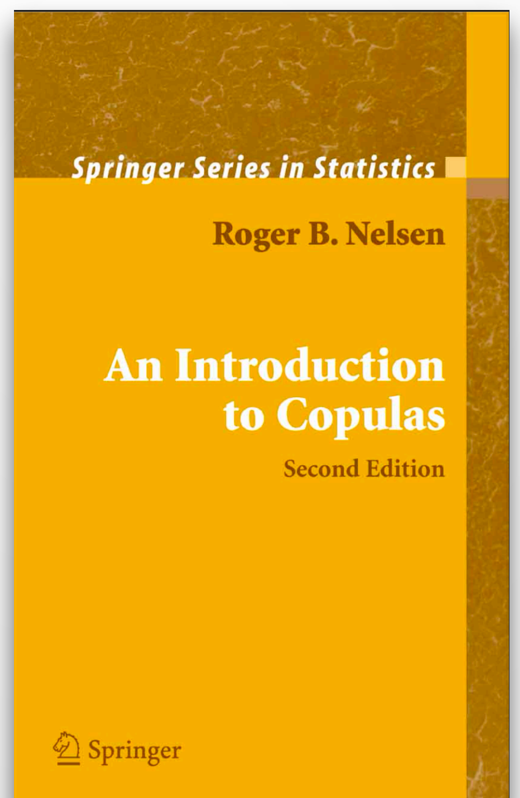
$$(2.\,6) \qquad \begin{cases} v(\eta) > 0 & \text{für} \quad \gamma < \eta < \delta, \\ v(\eta) = 0 & \text{für} \quad \eta < \gamma \text{ und } \eta > \delta \end{cases}$$

ist.

Die zweidimensionale Verteilung der durch die Maßstabänderungen (2. 3), (2. 5) eingeführten Variablen x, y sei $s(x, y)$, die Summenfunktion sei $S(x, y)$. Wir können jetzt die Gleichungen des § I an-

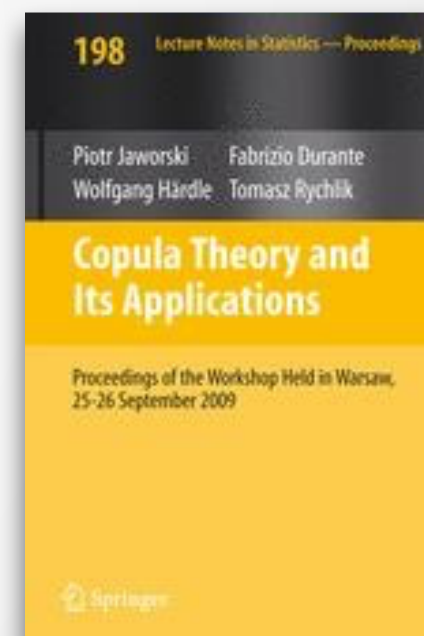
13) Die Transformation (2. 3), (2. 5) findet sich für den Spezialfall der Gaußschen Verteilung bei K. Pearson, a. a. O.[9]), Gleichung XII, in allgemeiner Form bei Hotelling und Pabst, a. a. O.[10]), Gleichung (5).

Wassili Hoeffding (1940)



An Introduction to Copulas

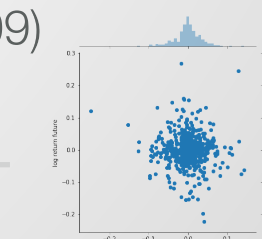
Nelsen (2005)



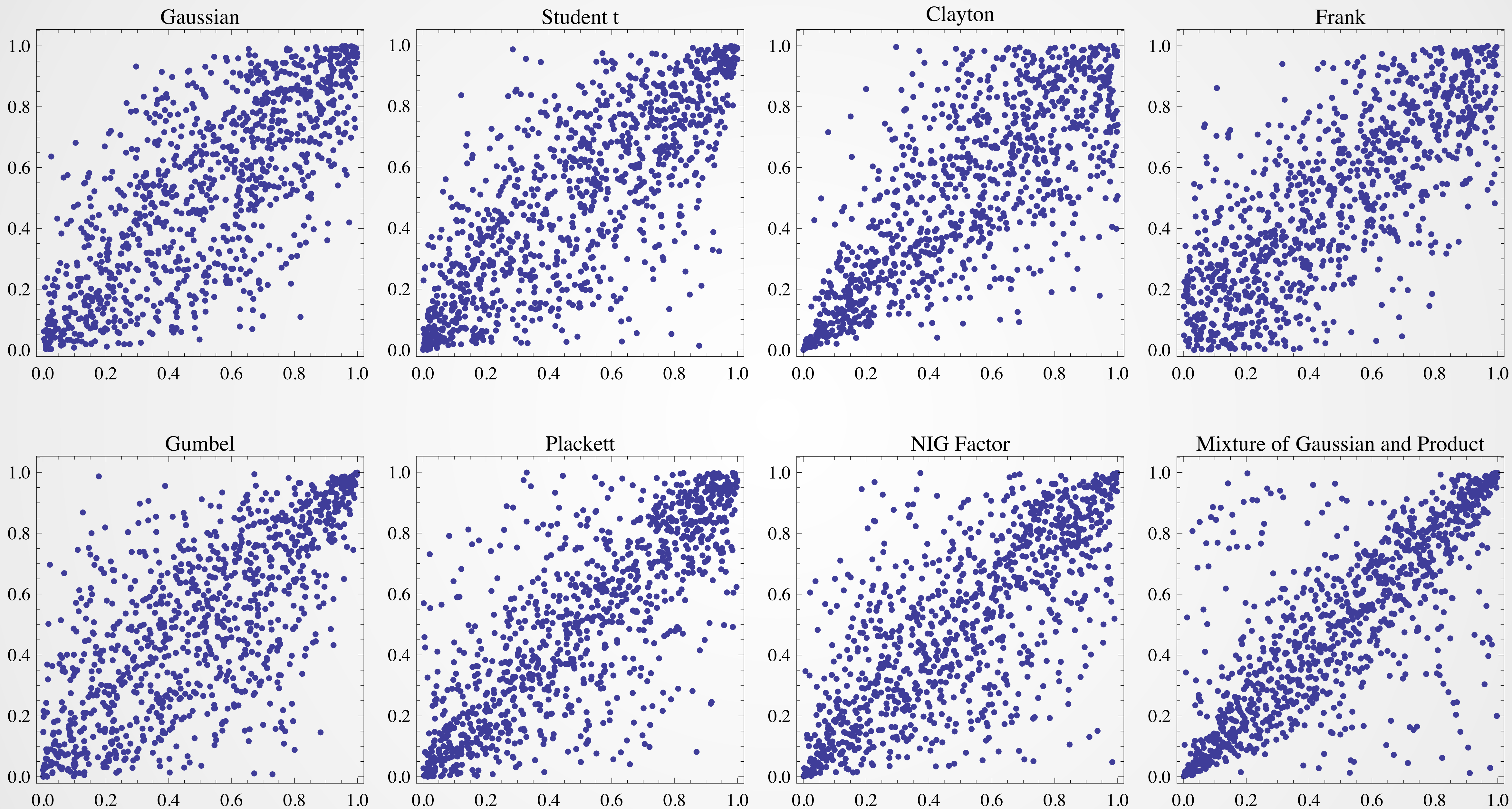
Copula Theory

Jaworski et al (2009)

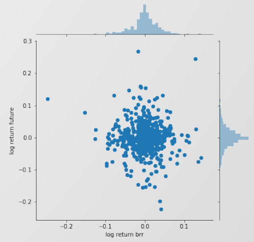
Hoeffding in BBI



Some copulae



All copulae are calibrated to a Spearman's Rho of 0.75.



Examples of Copulae

Copula is nothing more than parameterising the dependency structure by a function, e.g.

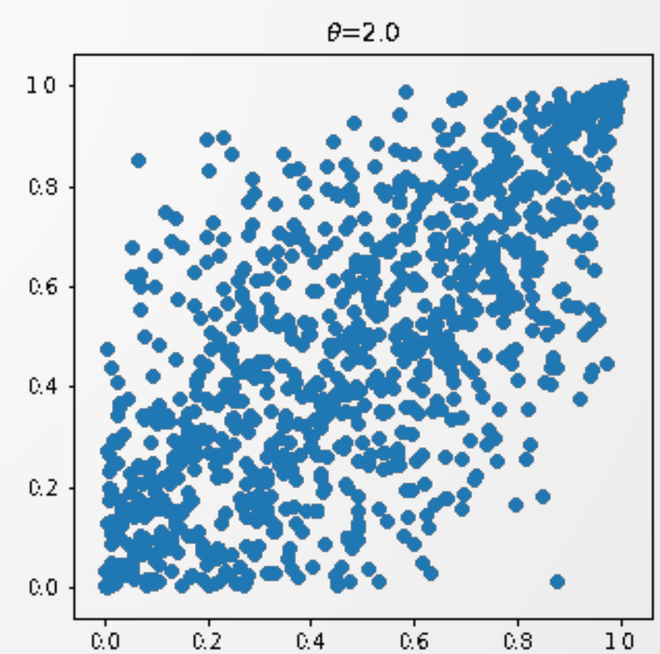
Gaussian Copula

$$C(u_1, u_2) = \Phi_\rho \left\{\Phi^{-1}(u_1), \Phi^{-1}(u_2)\right\}$$

$$= \int_{-\infty}^{\Phi^{-1}(u_1} \int_{-\infty}^{\Phi^{-1}(u_2} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left\{-\frac{x^2 - 2\rho xy + y^2}{2(1-\rho^2)}\right\} dxdy,$$

Gumbel

$$C(u_1, u_2) = \exp\left[-\left\{(-\log u_1)^\theta + (-\log u_2)^\theta\right\}^{\frac{1}{\theta}}\right],$$



where u_1 and u_2 are $F_X(X)$ and $F_Y(Y)$ respectively.

Distribution of the hedge

Let X and Y be two rv's with corresponding abs cts copula C and marginals F_X and F_Y.

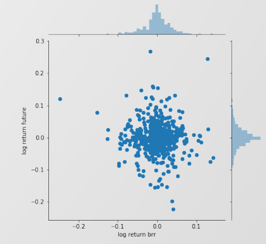
Then, the distribution of $Z = X - hY$ is given by

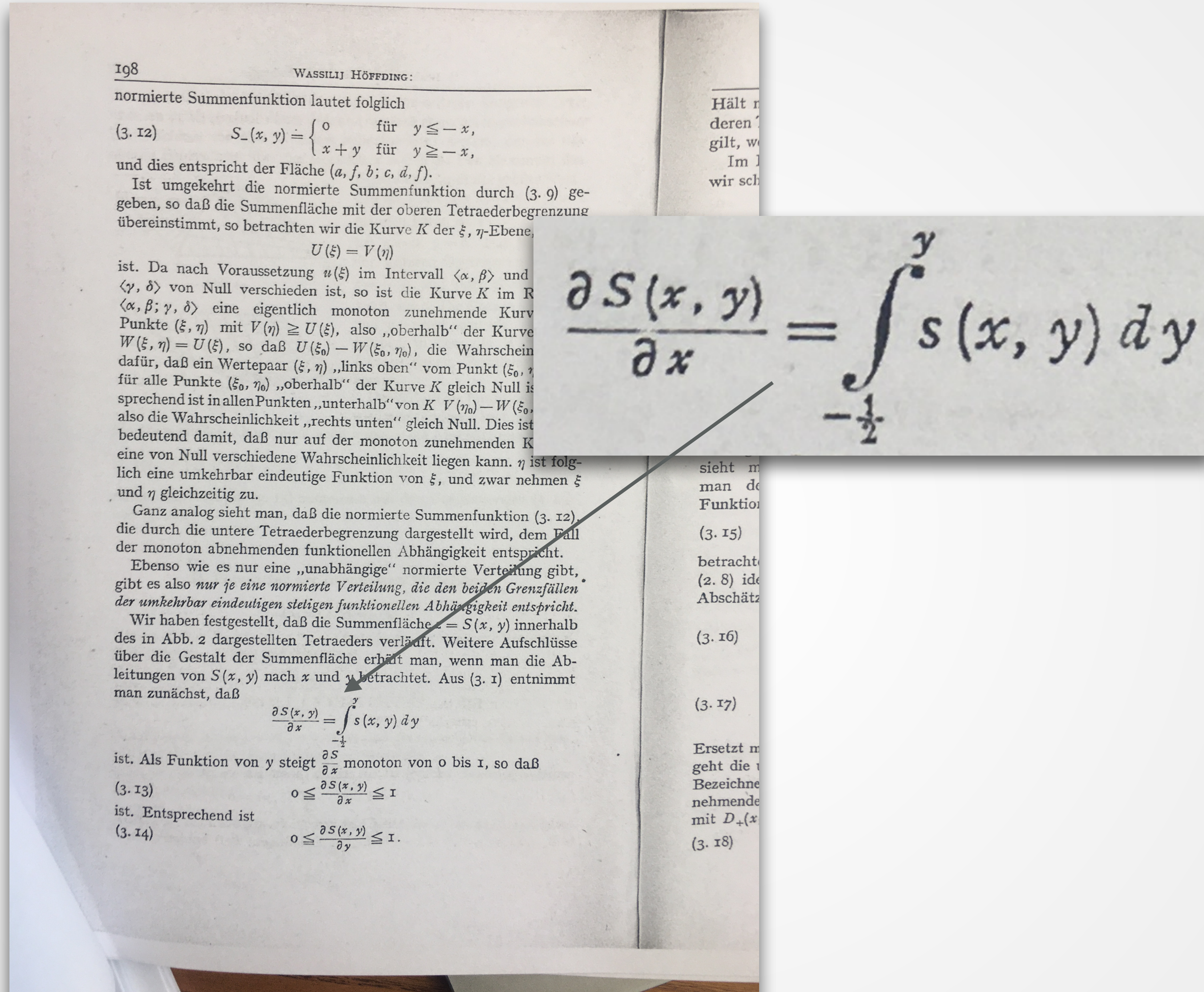
$$F_Z(x) = 1 - \int_0^1 D_1 C \left[u, F_Y \left\{ \frac{F_X^{-1}(u) - x}{h} \right\} \right] du$$

Barbi and Romagnoli (2014)

Easy to show Hoeffding (1940), McNeil et al. (2005)

$$D_1 C\{F_X(x), F_Y(y)\} = \frac{\partial}{\partial u} C(u, v) = \mathbf{P}(Y \leq y \,|\, X = x)$$





198 WASSILIJ HÖFFDING:

normierte Summenfunktion lautet folglich

(3. 12) $$S_-(x, y) = \begin{cases} 0 & \text{für } y \leqq -x, \\ x + y & \text{für } y \geqq -x, \end{cases}$$

und dies entspricht der Fläche $(a, f, b; c, d, f)$.

Ist umgekehrt die normierte Summenfunktion durch (3. 9) gegeben, so daß die Summenfläche mit der oberen Tetraederbegrenzung übereinstimmt, so betrachten wir die Kurve K der ξ, η-Ebene,

$$U(\xi) = V(\eta)$$

ist. Da nach Voraussetzung $u(\xi)$ im Intervall $\langle\alpha, \beta\rangle$ und $\langle\gamma, \delta\rangle$ von Null verschieden ist, so ist die Kurve K im R $\langle\alpha, \beta; \gamma, \delta\rangle$ eine eigentlich monoton zunehmende Kurv Punkte (ξ, η) mit $V(\eta) \geqq U(\xi)$, also „oberhalb" der Kurve $W(\xi, \eta) = U(\xi)$, so daß $U(\xi_0) - W(\xi_0, \eta_0)$, die Wahrschein dafür, daß ein Wertepaar (ξ, η) „links oben" vom Punkt $(\xi_0,$ für alle Punkte (ξ_0, η_0) „oberhalb" der Kurve K gleich Null is sprechend ist in allen Punkten „unterhalb" von K $V(\eta_0) - W(\xi_0,$ also die Wahrscheinlichkeit „rechts unten" gleich Null. Dies ist bedeutend damit, daß nur auf der monoton zunehmenden K eine von Null verschiedene Wahrscheinlichkeit liegen kann. η ist folglich eine umkehrbar eindeutige Funktion von ξ, und zwar nehmen ξ und η gleichzeitig zu.

Ganz analog sieht man, daß die normierte Summenfunktion (3. 12), die durch die untere Tetraederbegrenzung dargestellt wird, dem Fall der monoton abnehmenden funktionellen Abhängigkeit entspricht.

Ebenso wie es nur eine „unabhängige" normierte Verteilung gibt, gibt es also *nur je eine normierte Verteilung, die den beiden Grenzfällen der umkehrbar eindeutigen stetigen funktionellen Abhängigkeit entspricht.*

Wir haben festgestellt, daß die Summenfläche $z = S(x, y)$ innerhalb des in Abb. 2 dargestellten Tetraeders verläuft. Weitere Aufschlüsse über die Gestalt der Summenfläche erhält man, wenn man die Ableitungen von $S(x, y)$ nach x und y betrachtet. Aus (3. 1) entnimmt man zunächst, daß

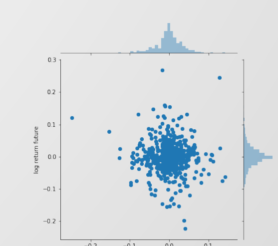
$$\frac{\partial S(x, y)}{\partial x} = \int_{-\frac{1}{2}}^{y} s(x, y)\, dy$$

ist. Als Funktion von y steigt $\frac{\partial S}{\partial x}$ monoton von 0 bis 1, so daß

(3. 13) $$0 \leqq \frac{\partial S(x, y)}{\partial x} \leqq 1$$

ist. Entsprechend ist

(3. 14) $$0 \leqq \frac{\partial S(x, y)}{\partial y} \leqq 1.$$



Risk measures

Variance: Var(Z)

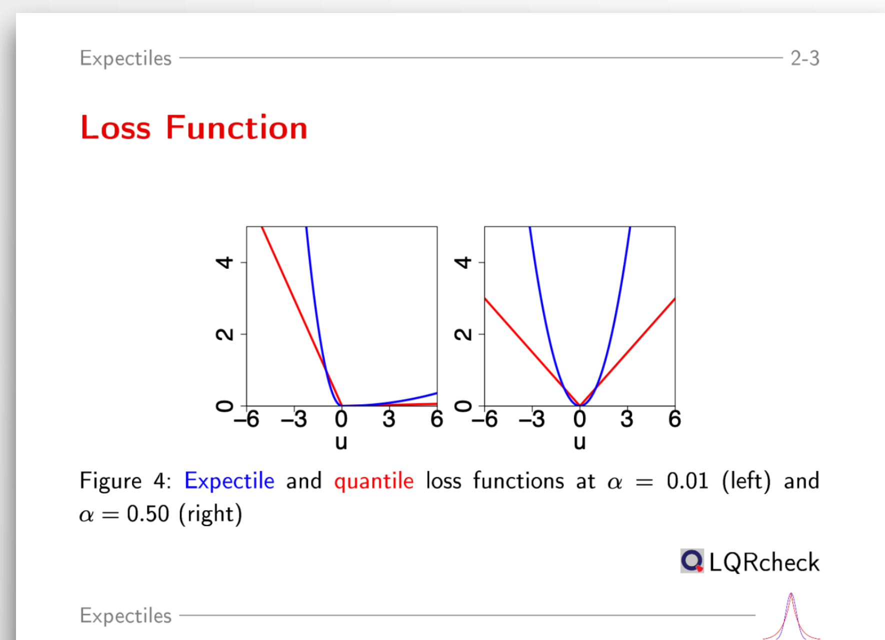
$$\text{Var}(Z)$$

Value-at-risk (VaR)

$$\text{VaR}_\alpha = -F_Z^{-1}(1-\alpha)$$

Expected Shortfall (ES)

$$\text{ES}_\alpha = -\frac{1}{1-\alpha}\int_0^{1-\alpha} F_Z^{-1}(p)\text{d}p$$

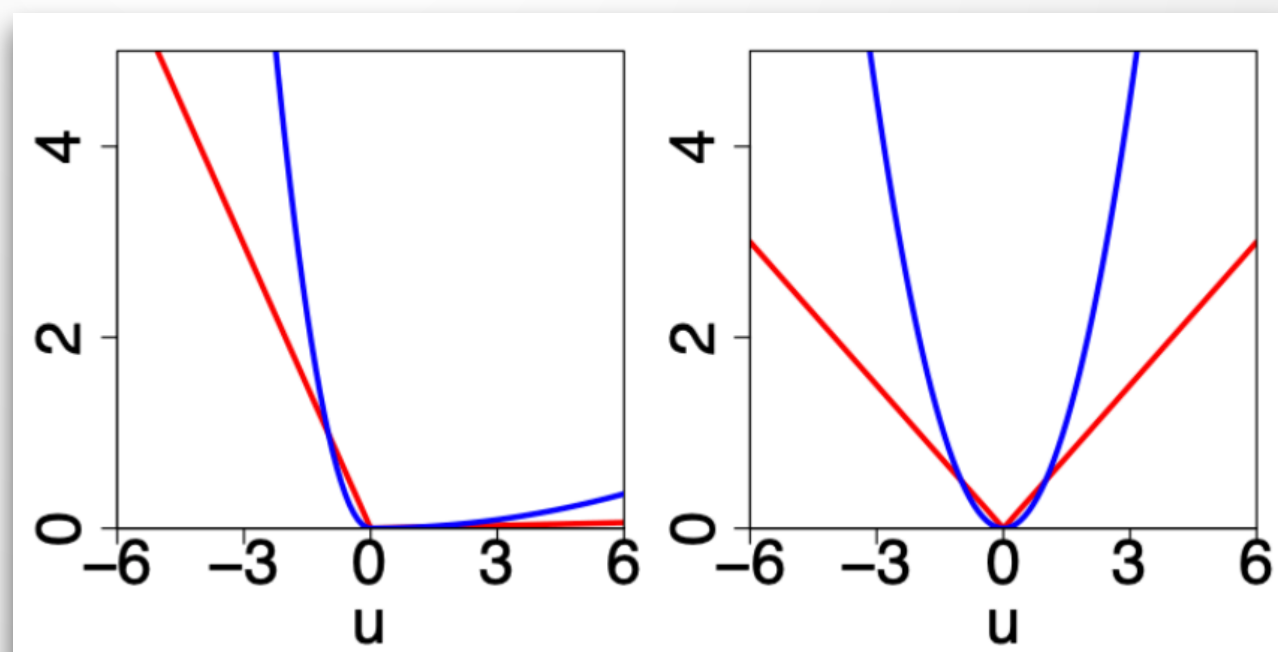

Expectiles 2-3

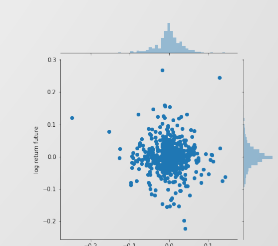
Loss Function

Figure 4: Expectile and quantile loss functions at $\alpha = 0.01$ (left) and $\alpha = 0.50$ (right)

LQRcheck

Expectiles





Spectral Risk Measures

Acerbi (2002), Cotter and Dowd (2006):

$$\rho_\phi = -\int_0^1 \phi(p)\, F_Z^{-1}(p)\, dp,$$

$\phi(s)$, $s \in [0,1]$, is the risk aversion function, a weighting function such that

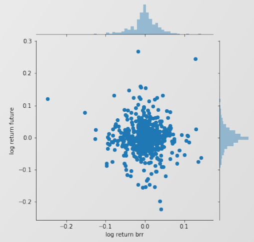
(i) $\phi \geq 0$

(ii) $\int_0^1 \phi(p)dp = 1$

(iii) $\phi' \leq 0$

With *(iii)* SRM's are coherent risk measures.

Value-at-risk(VaR): $\text{VaR}_\alpha = -F_Z^{-1}(1-\alpha)$ ← with a delta fct $\phi(p)$



Spectral Risk Measures

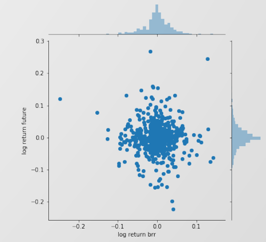
Expected Shortfall (ES): $\mathrm{ES}_\alpha = -\dfrac{1}{1-\alpha}\displaystyle\int_0^{1-\alpha} F_Z^{-1}(p)\mathrm{d}p$

Exponential spectral risk measures: weighting function

$\phi(p) = \lambda e^{-k(1-p)}$, λ a normalising constant, derived from exp utility fct:

$$\rho_\phi = \int_0^1 \phi(p)\, F_Z^{-1}(p)\, dp = \frac{k}{1-e^{-k}} \int_0^1 e^{-k(1-p)}\, F_Z^{-1}(p)\, dp,$$

e.g. „spectral 10“ uses *k=10*

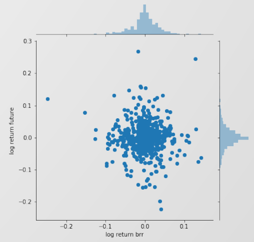


Optimal hedge ratio

- Hedge portfolio return: $R_t^h = R_t^S - h\,R_t^F$, with h hedge ratio
- Optimal hedge ratio:

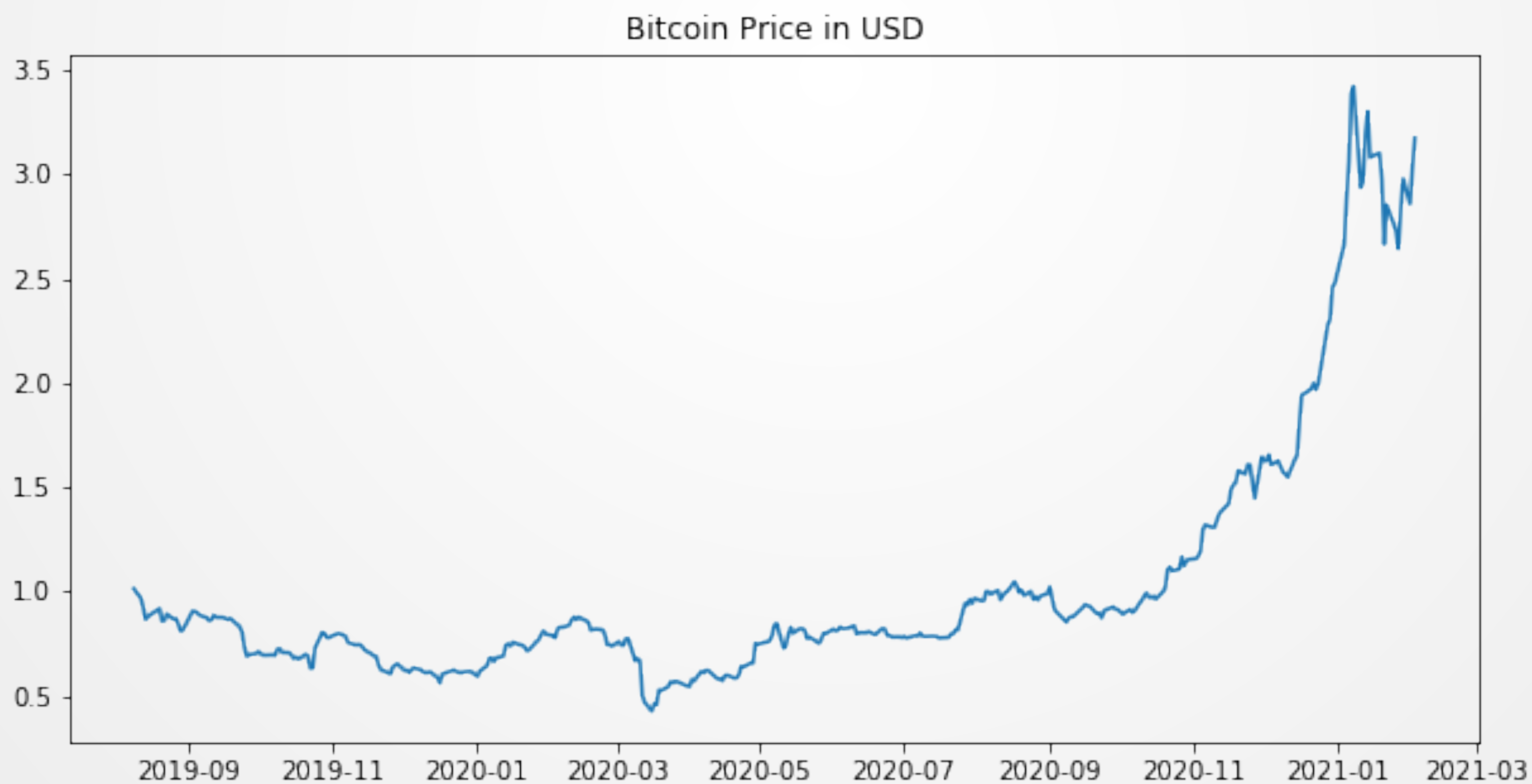
$$h^* = \text{argmin}_h \rho(h)$$

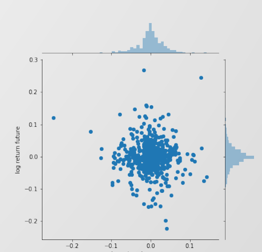
where $\rho(h)$ is the risk of the hedge portfolio with hedge ratio h.



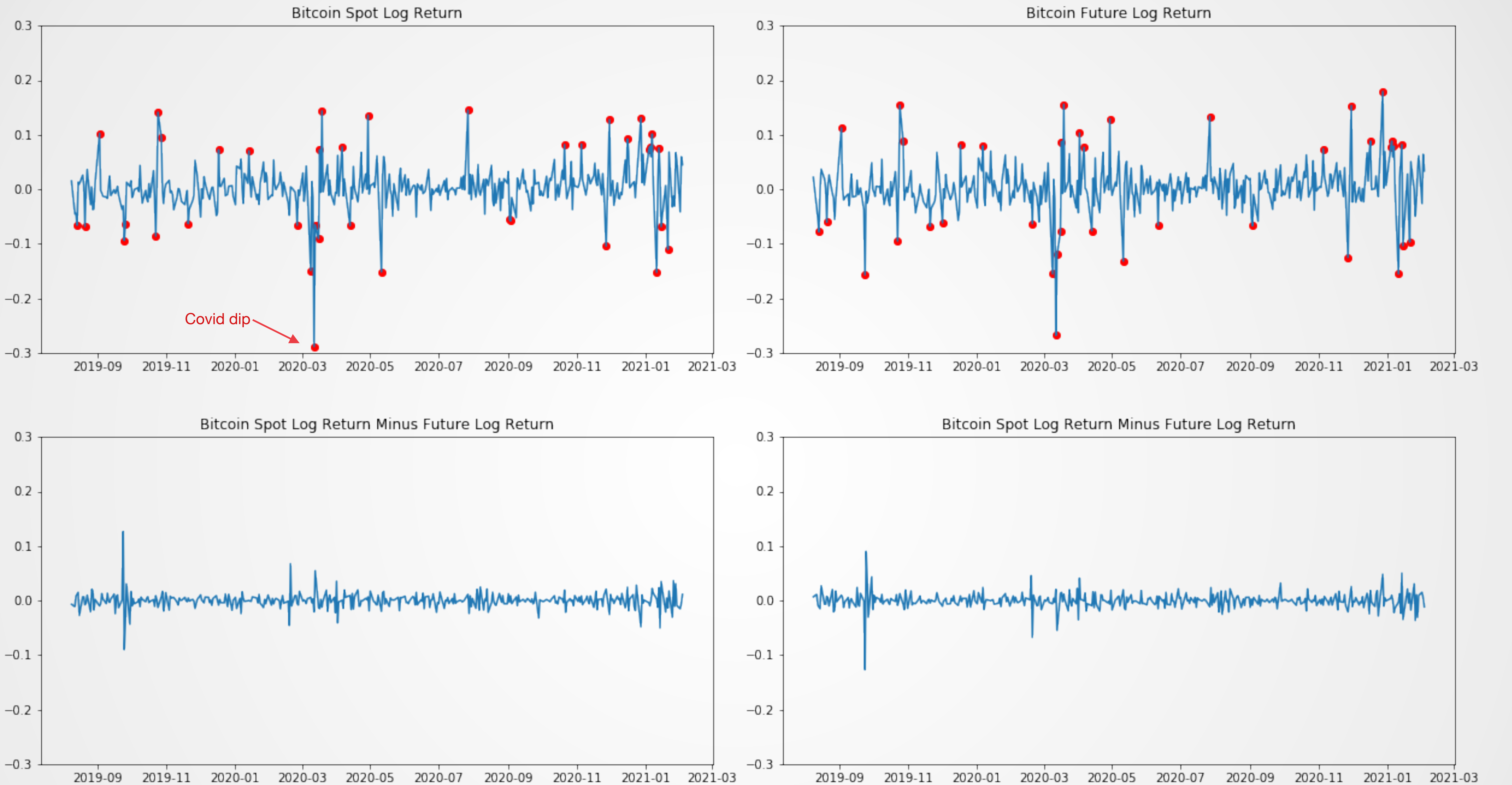
BTC and its „future“

- Daily log returns, 23pm CET
- 29 May 2018 through 3 Feb 2021
- Spot: Coingecko Bitcoin /USD
- Future: CME BTC Future
- Sources: Bloomberg, Coingecko

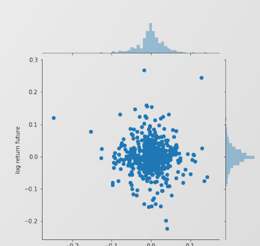




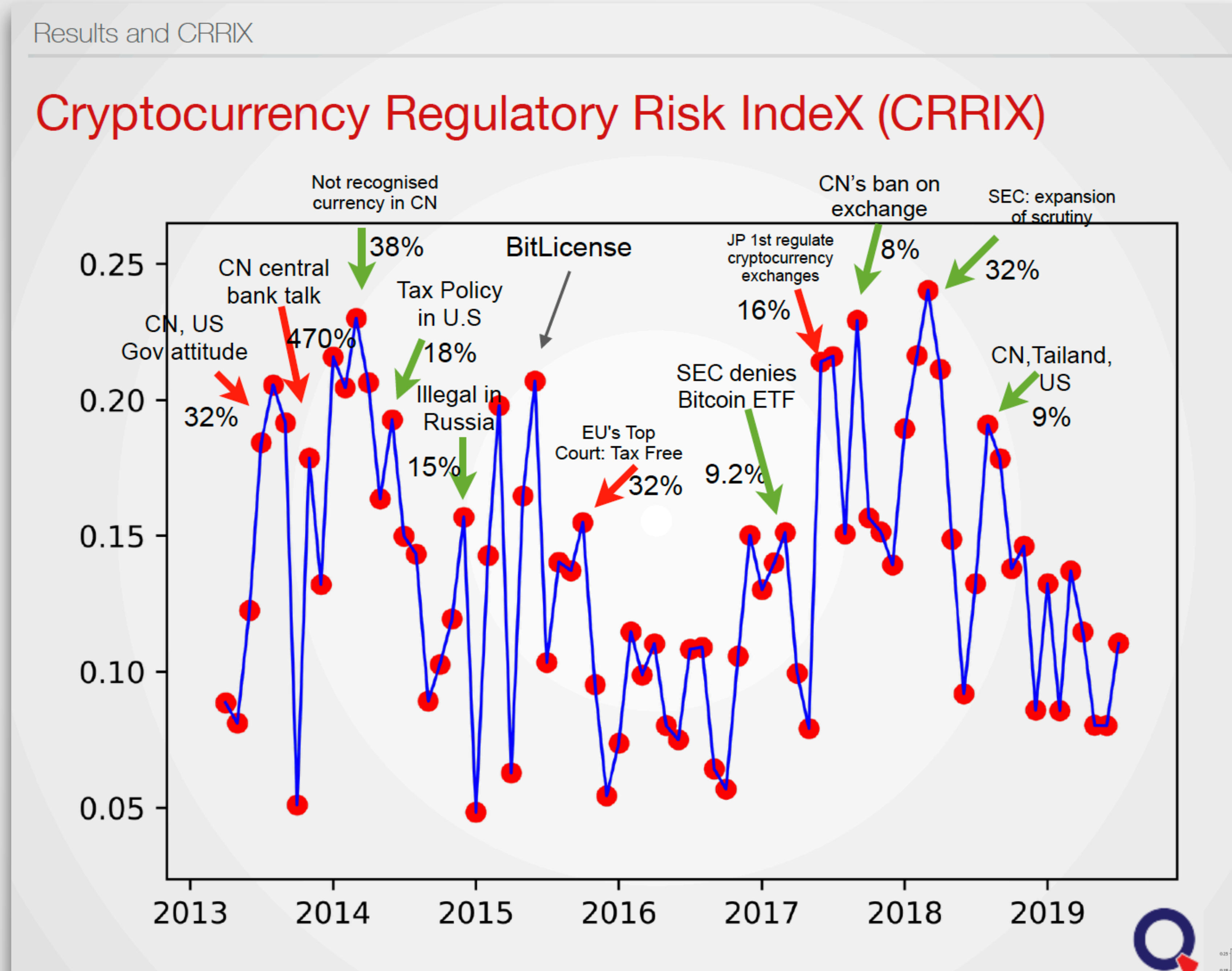
Time series



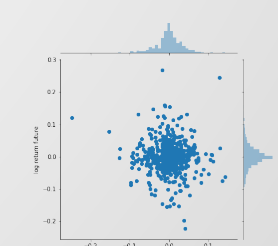
- Log return: $r = \log(P_t/P_{t-1})$
- Red dots are 10% extremes (upper and lower)
- Naive Hedge: Spot - Future, i.e. $h = 1$



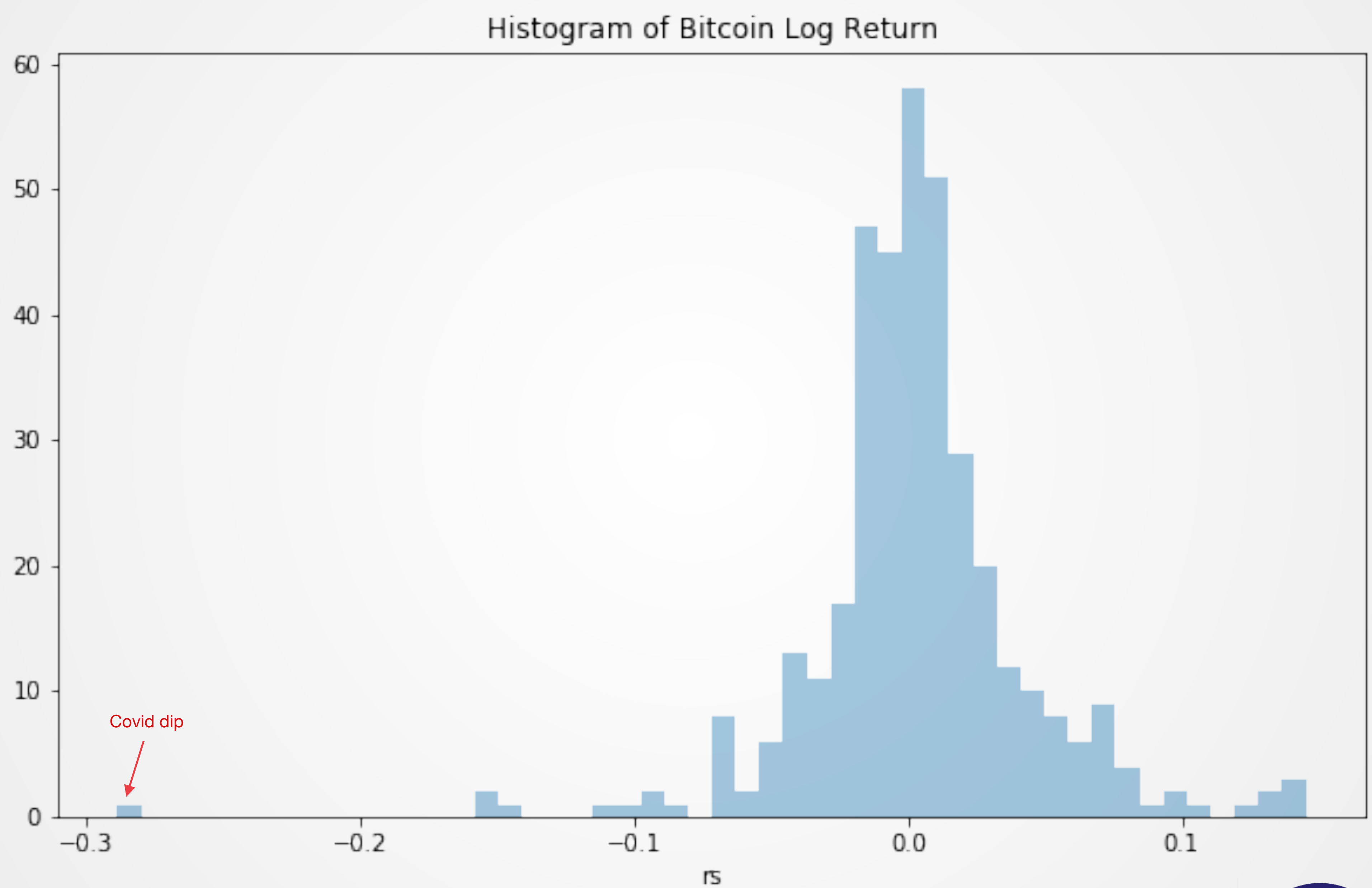
Time series



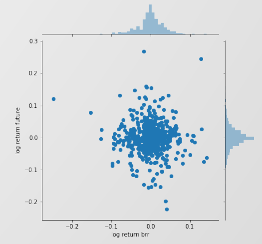
Coming soon



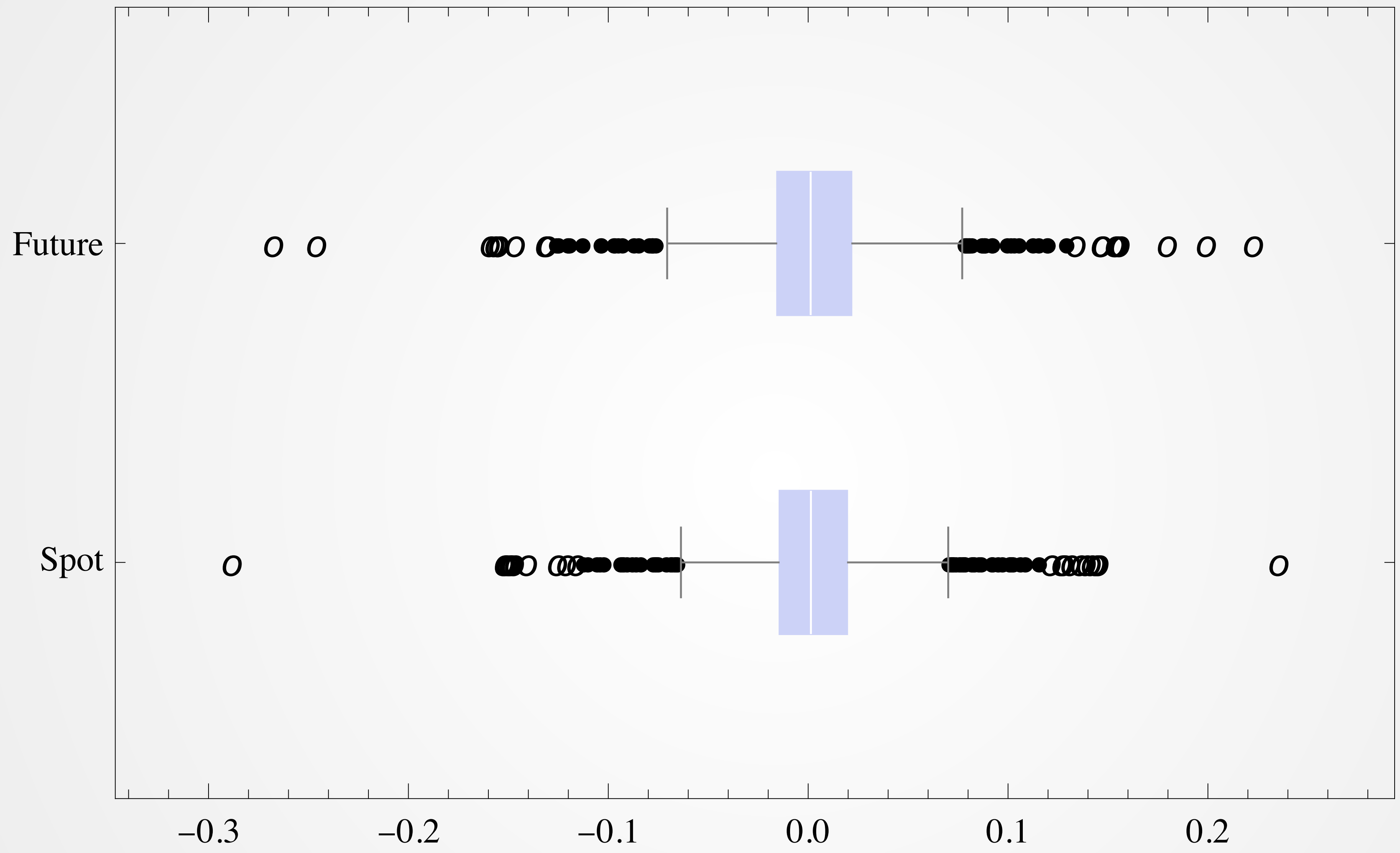
Distribution

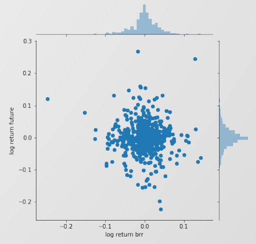


- Student t distribution: ν =7.95
- Generalised Pareto distribution (EVT): tail index $1/\xi = 4.92$

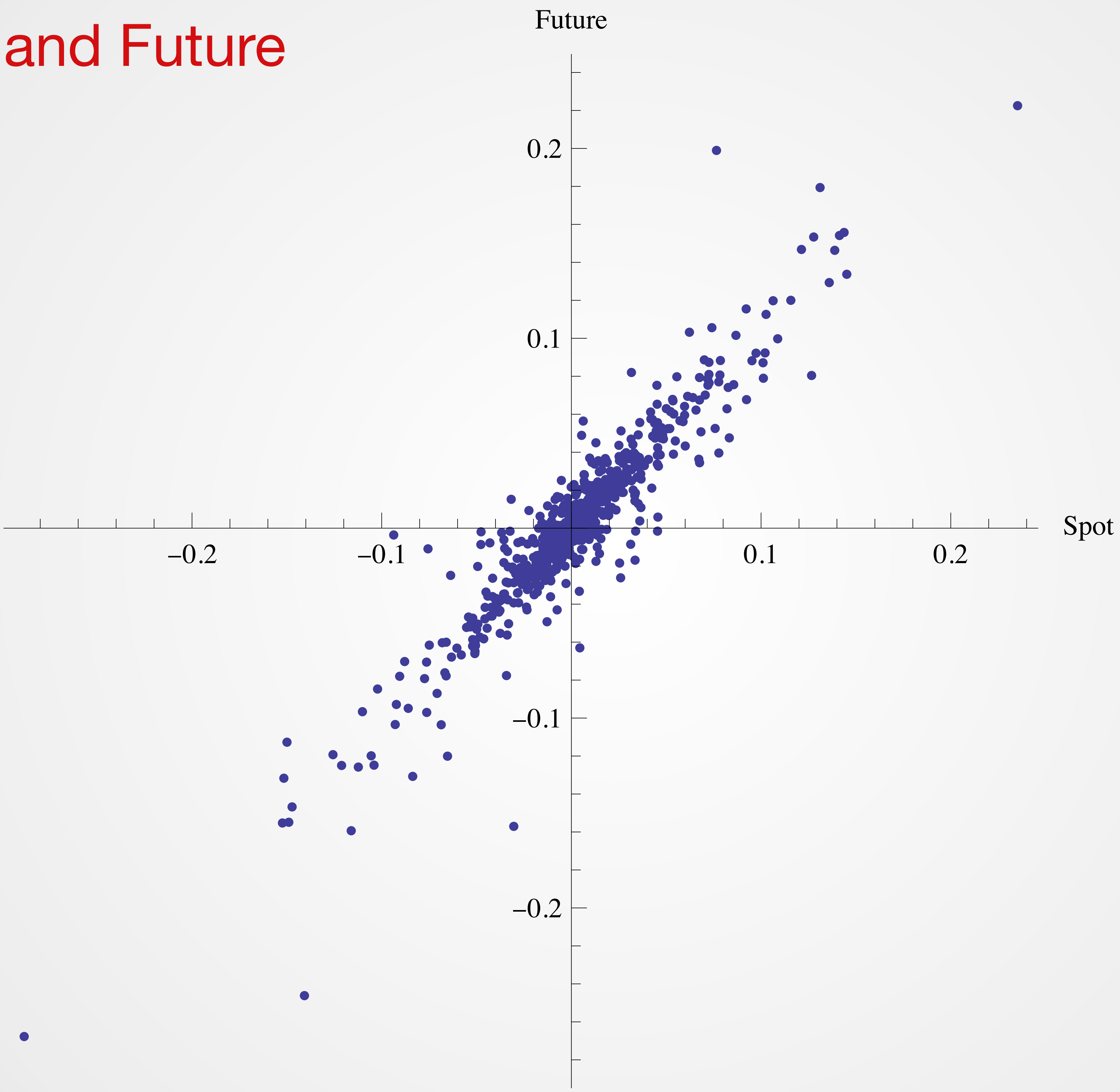


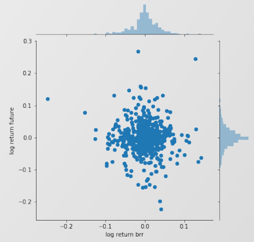
Spot and Future



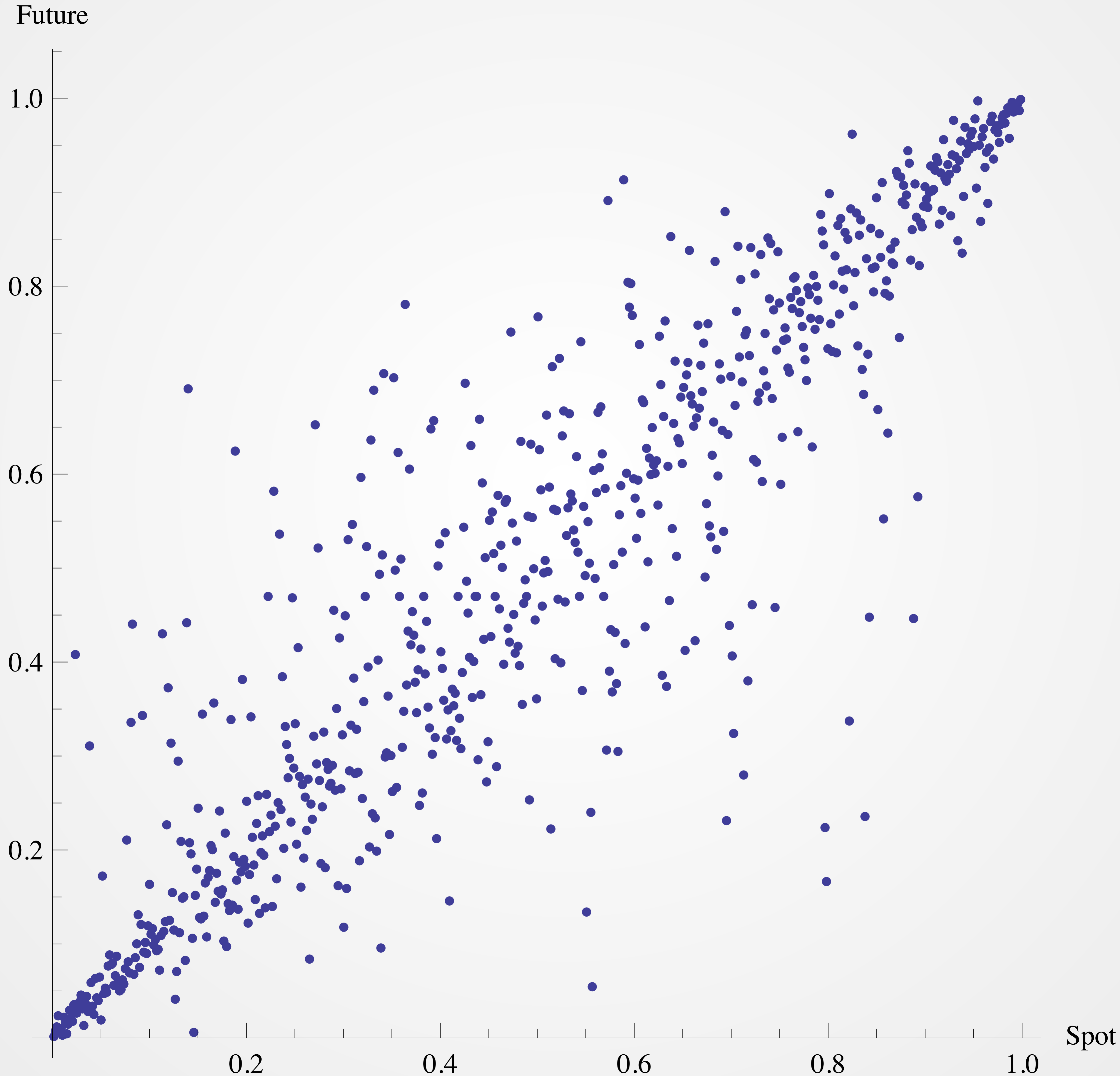


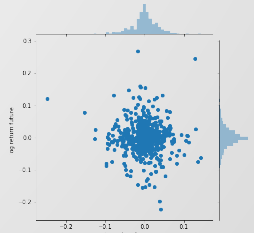
Spot and Future





Spot and Future empirical copula





Calibration

Copulae

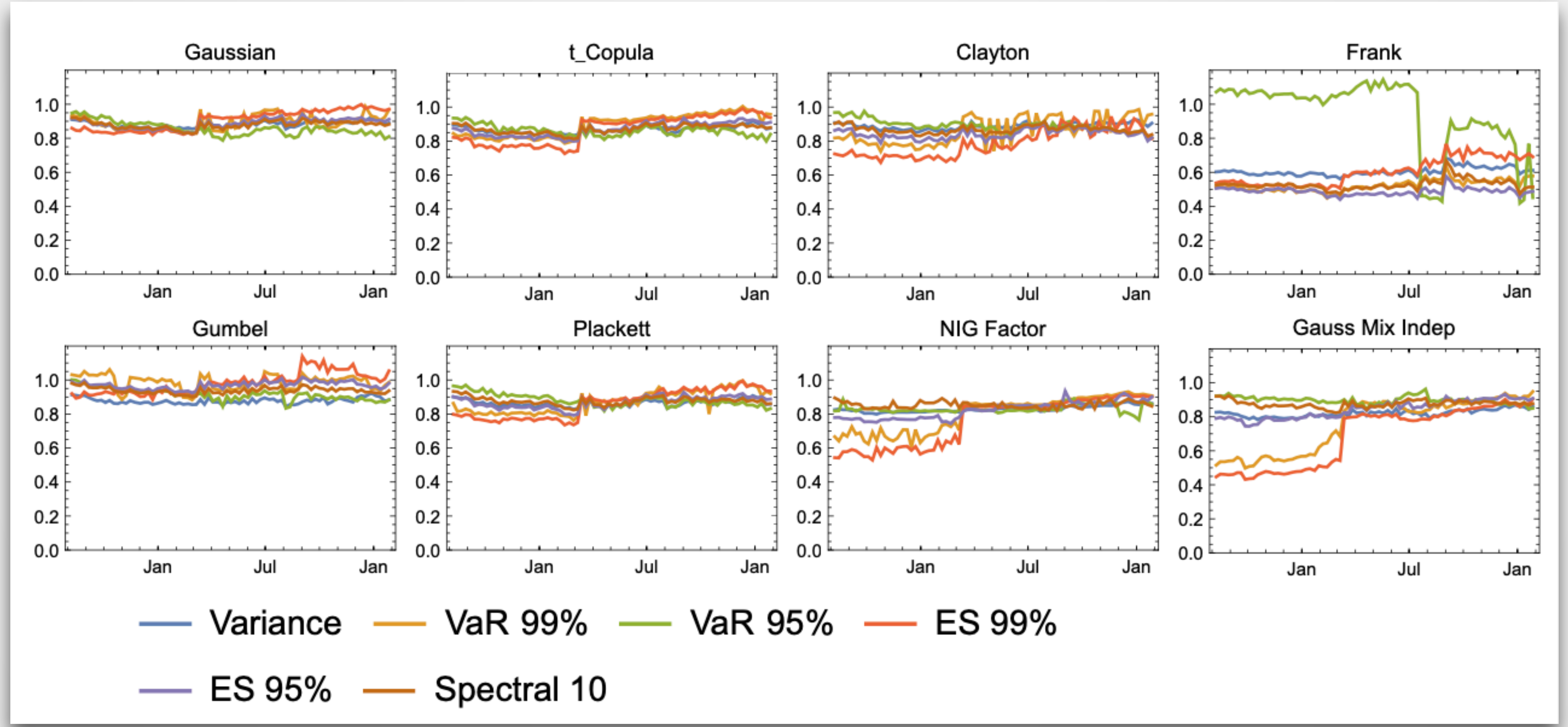
- Method of moments for copulae (Genest and Rivest, 1993; Oh and Patton, 2013)
- „Moments“:
  - Spearman’s Rho
  - Quantile dependence at 0.05, 0.1, 0.9, 0.95 quantiles
- Margins distribution: pdf by kernel density estimator (Gaussian kernel)

Optimal Hedge Ratio

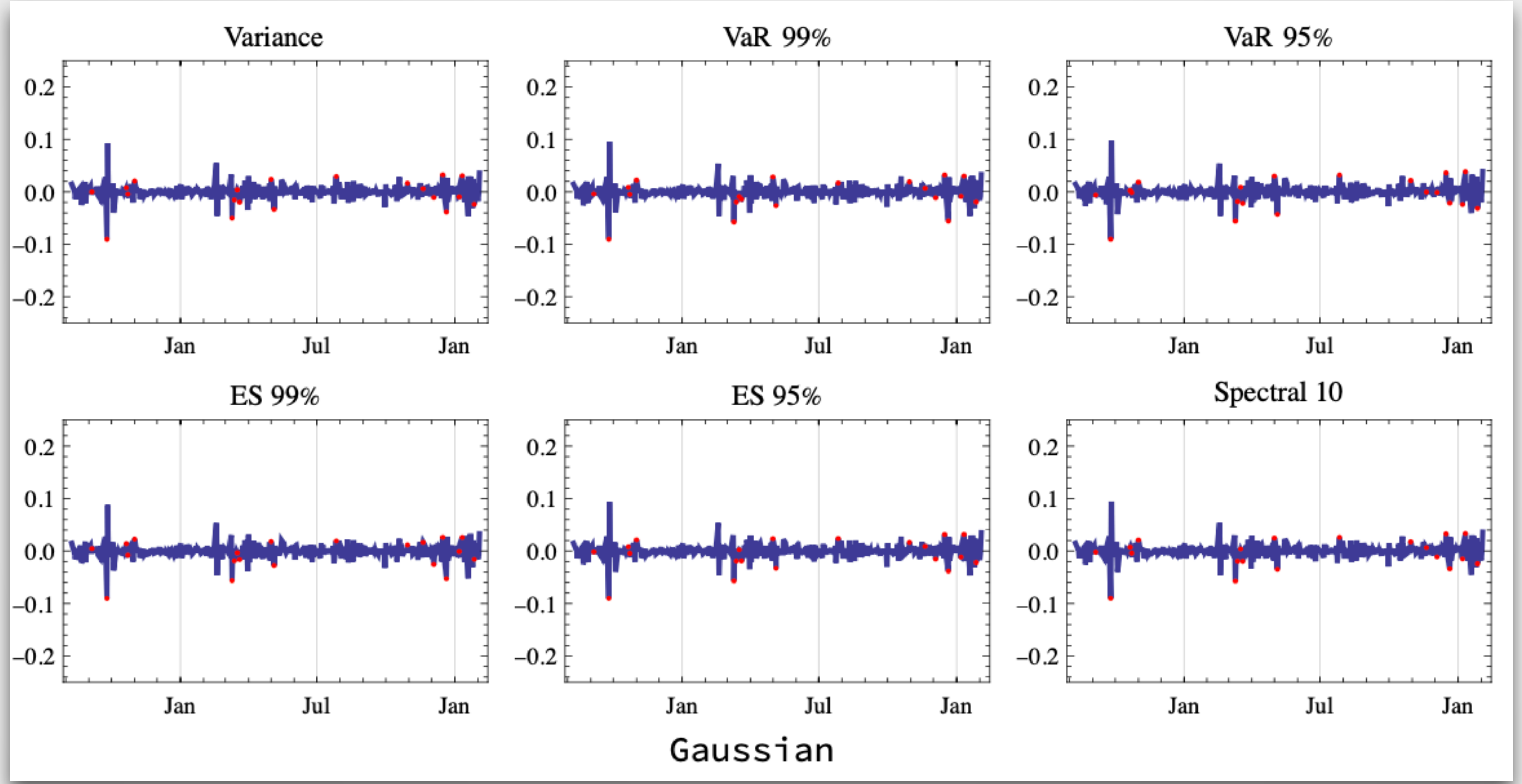
- Draw samples from copulae, calculate risk measure w.r.t. h
- Find h which minimise risk measure
- By Nelder-Mead simplex method by Python package Scipy

A First Glance to the Results

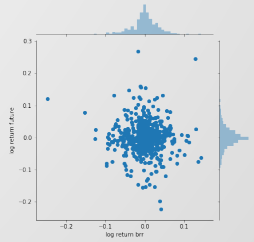
- Out-of-sample optimal h from 2019 Sept to 2021 Mar



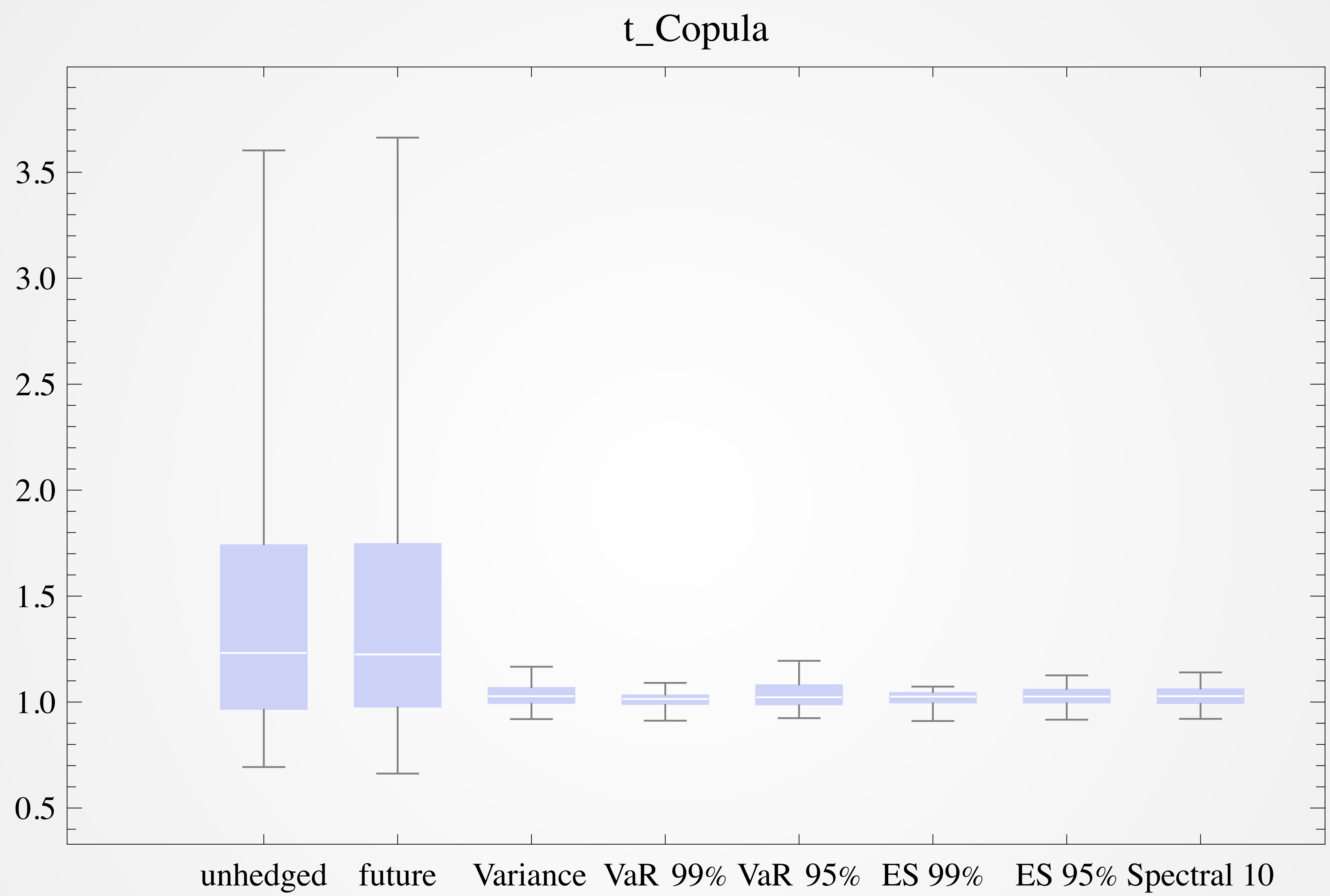
P&L



- Daily return from hedge, out-of-sample
- Recalibration every 5 days

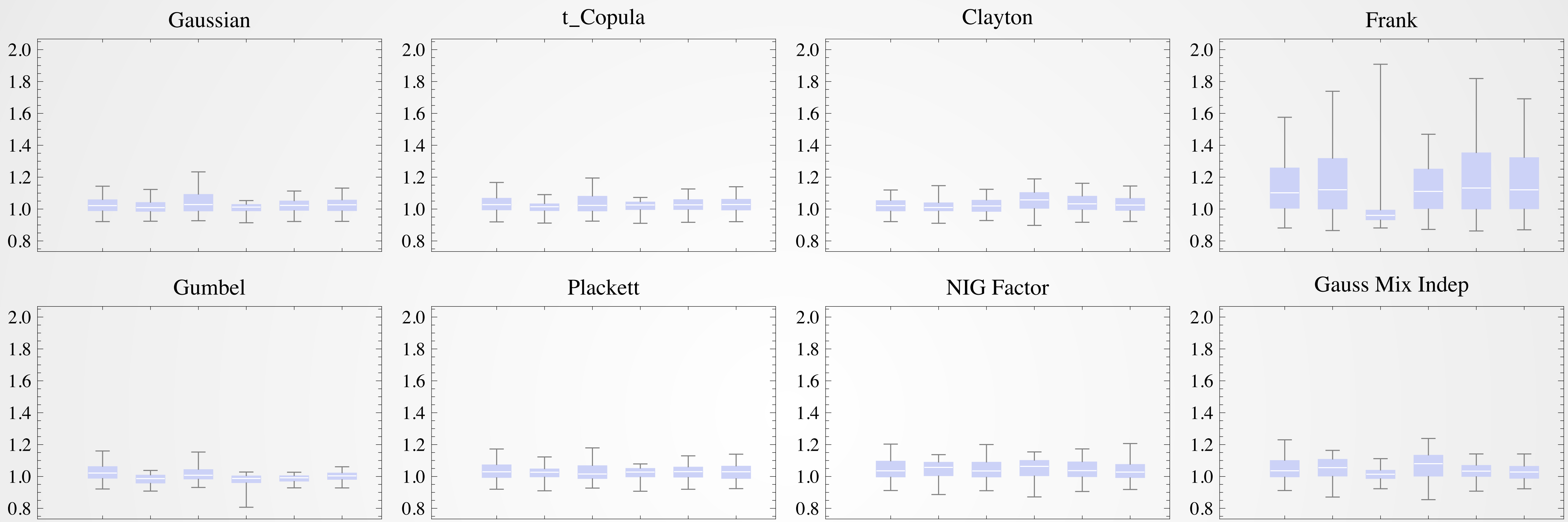


P&L



- P&L from static hedge, out-of-sample, 100 days, rolling every 5 days

P&L

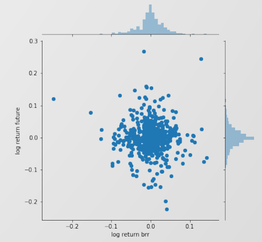


- P&L from static hedge, out-of-sample, rolling every 5 days
- From left to right: Variance, VaR 99%, VaR 95%, ES 99%, ES 95%, Spectral 10

Interactive Reports

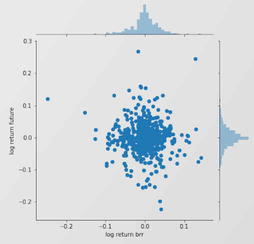
- https://francisliu2.github.io/francis.github.io/***Copula name_risk measure***.html
  - e.g. https://francisliu2.github.io/francis.github.io/Clayton_ERM k=10.html

- Available ***copula names***:
  - Gaussian, t_Copula Frank, Clayton, Gumbel, Plackett, NIG_factor
- Available ***risk measure names***:
  - ERM k=10, ES q=0.01, ES q=0.05, VaR q=0.01, VaR q=0.05, Variance



Quality of Hedge

- Hedging Effectiveness (Ederington, 1979)
- Mean square difference
- Downside Semi Variance
- Robustness: sensitivity of the procedure w.r.t. „outliers“



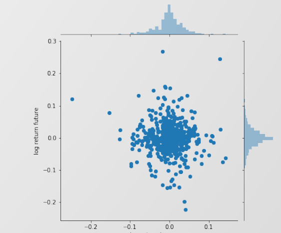
Hedge Effectiveness

- Hedge effectiveness (Ederington, 1979) captures percentage reduction in risk:

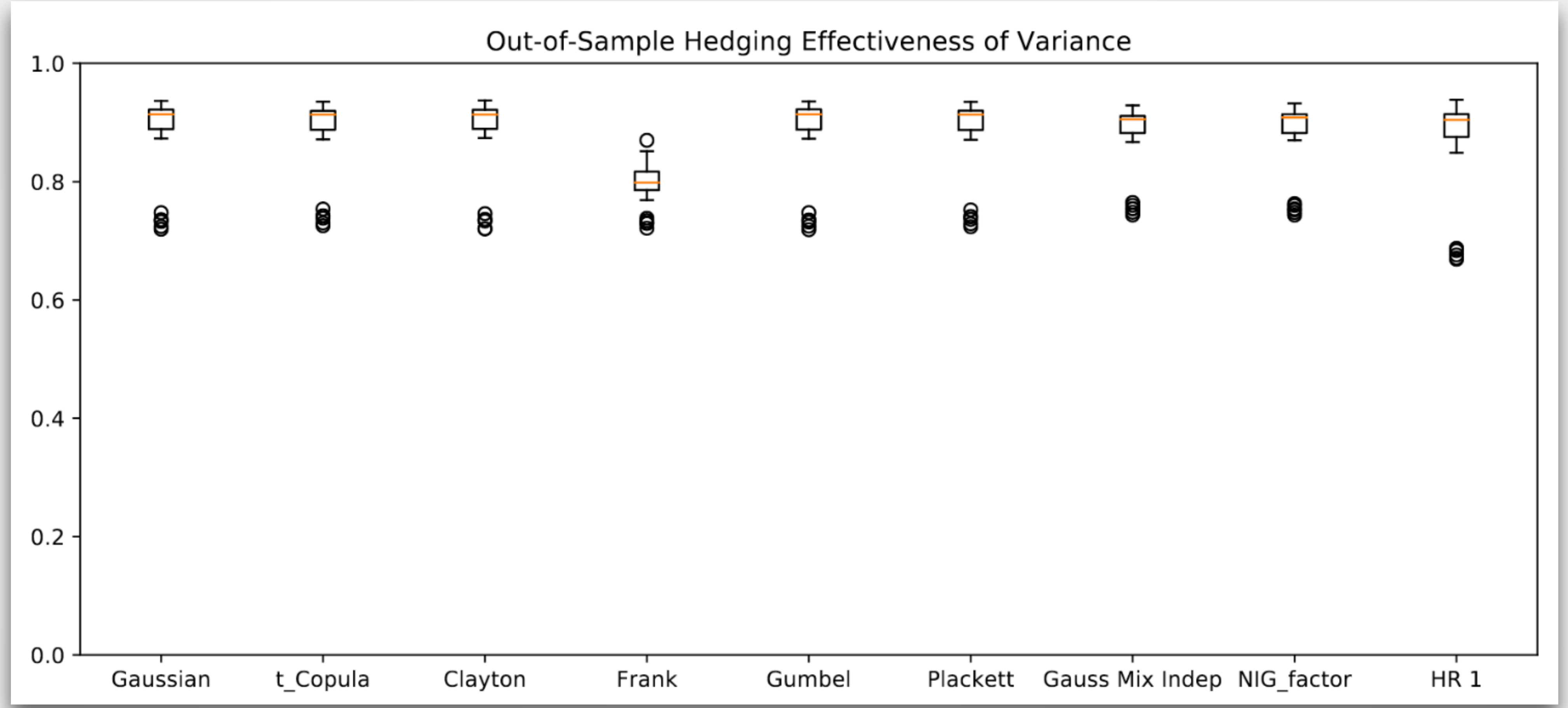
$$1 - \frac{\rho(r^h)}{\rho(r^S)}.$$

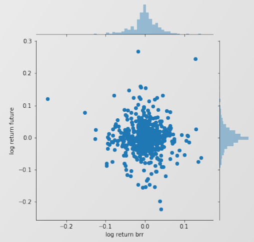
- Compare the hedging effectiveness among copulae under different risk measures
- Measure the risk reduction by loss function, e.g. for the pair of Gaussian and Expected Shortfall 99%, we measure the HE

$$1 - \frac{\text{ES } 99\%(r^h)}{\text{ES } 99\%(r^S)}$$

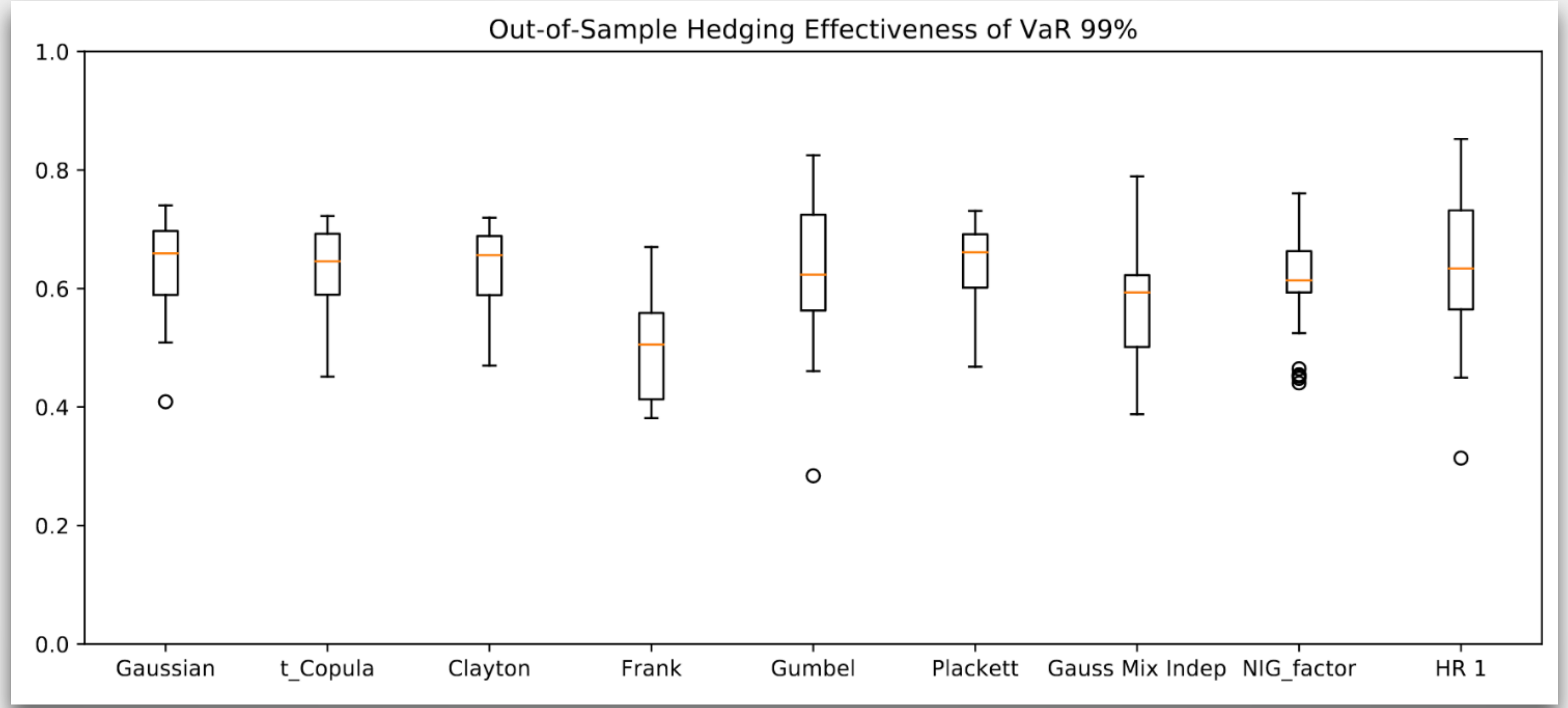


Hedge Effectiveness

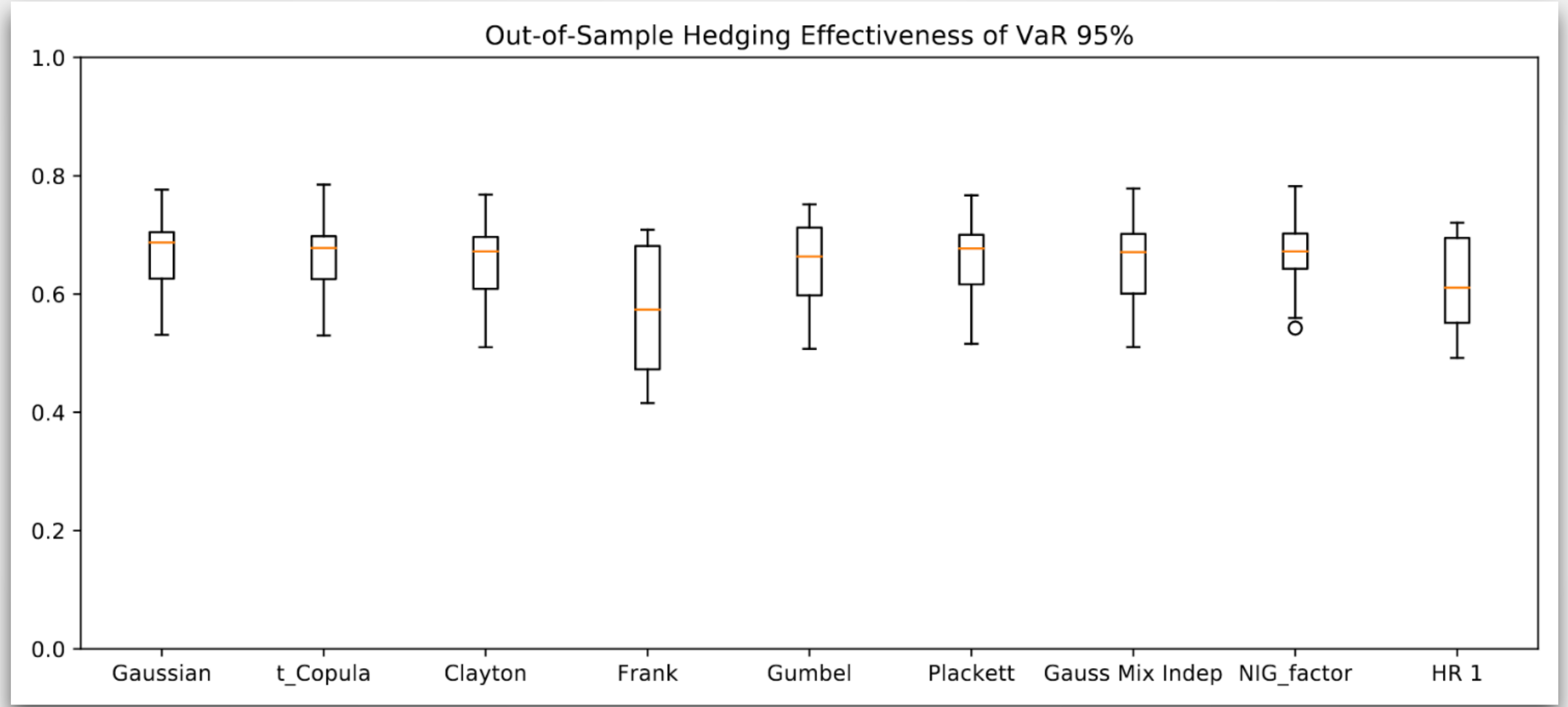


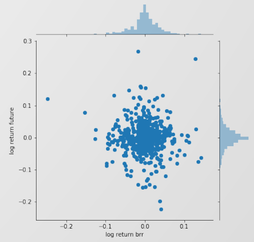


Hedge Effectiveness

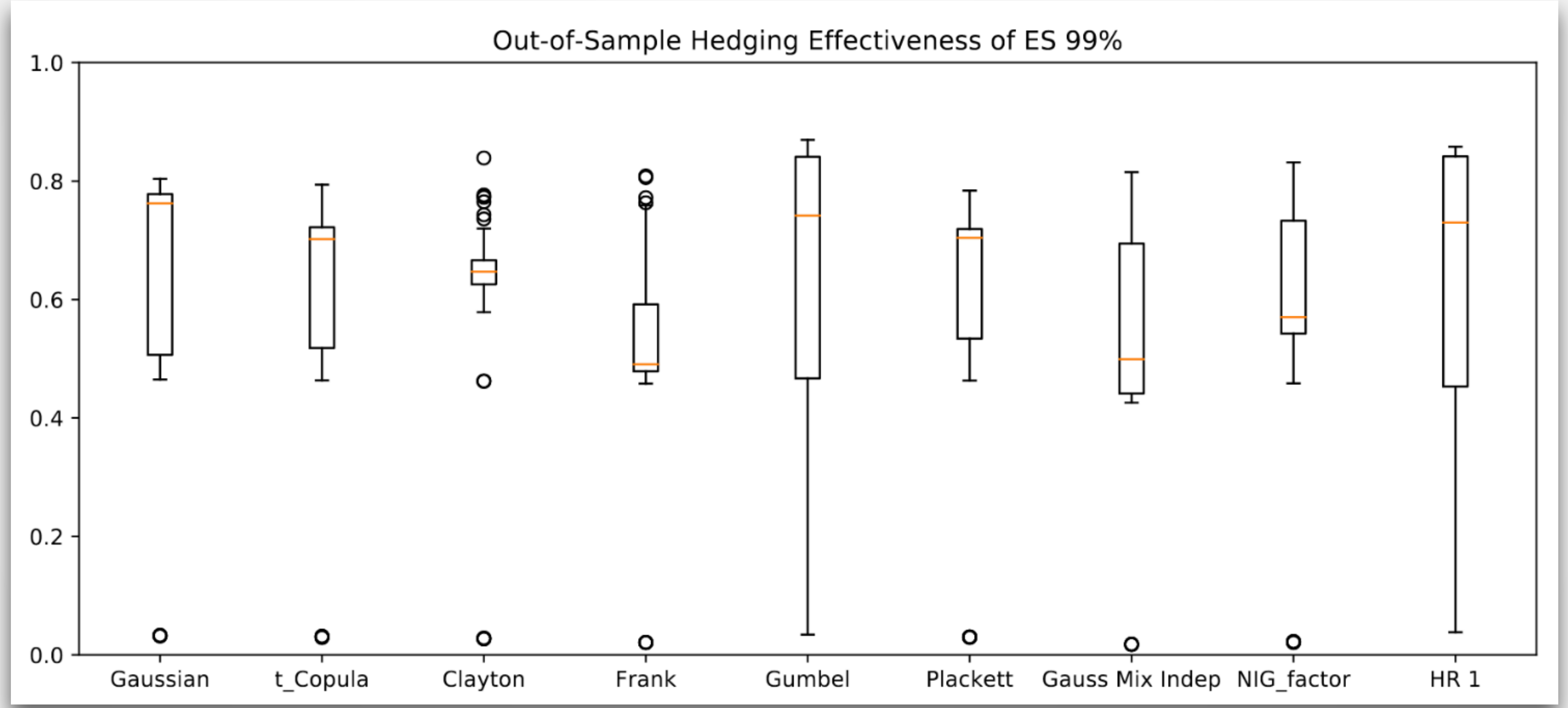


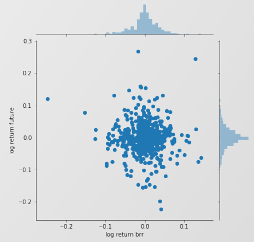
Hedge Effectiveness



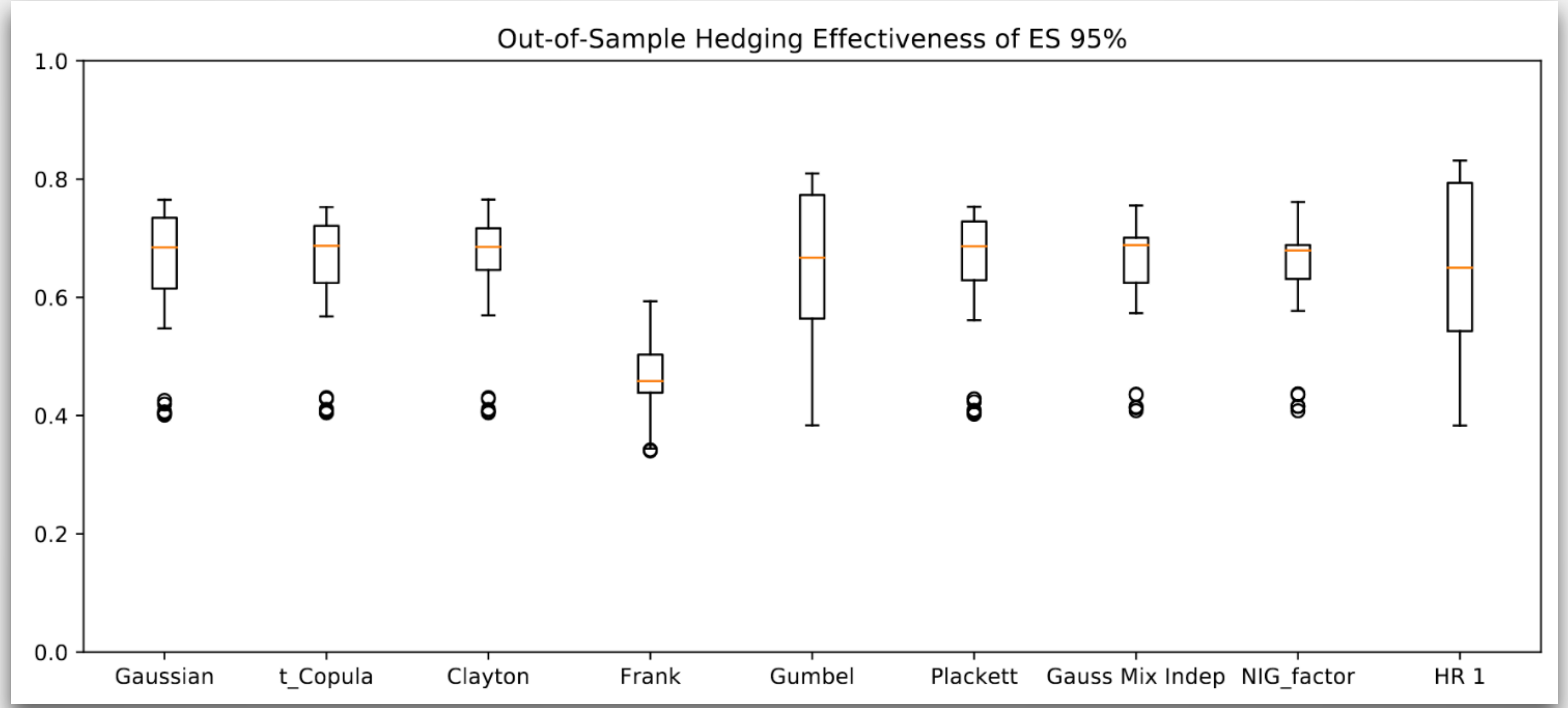


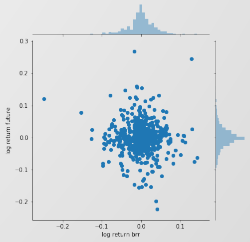
Hedge Effectiveness



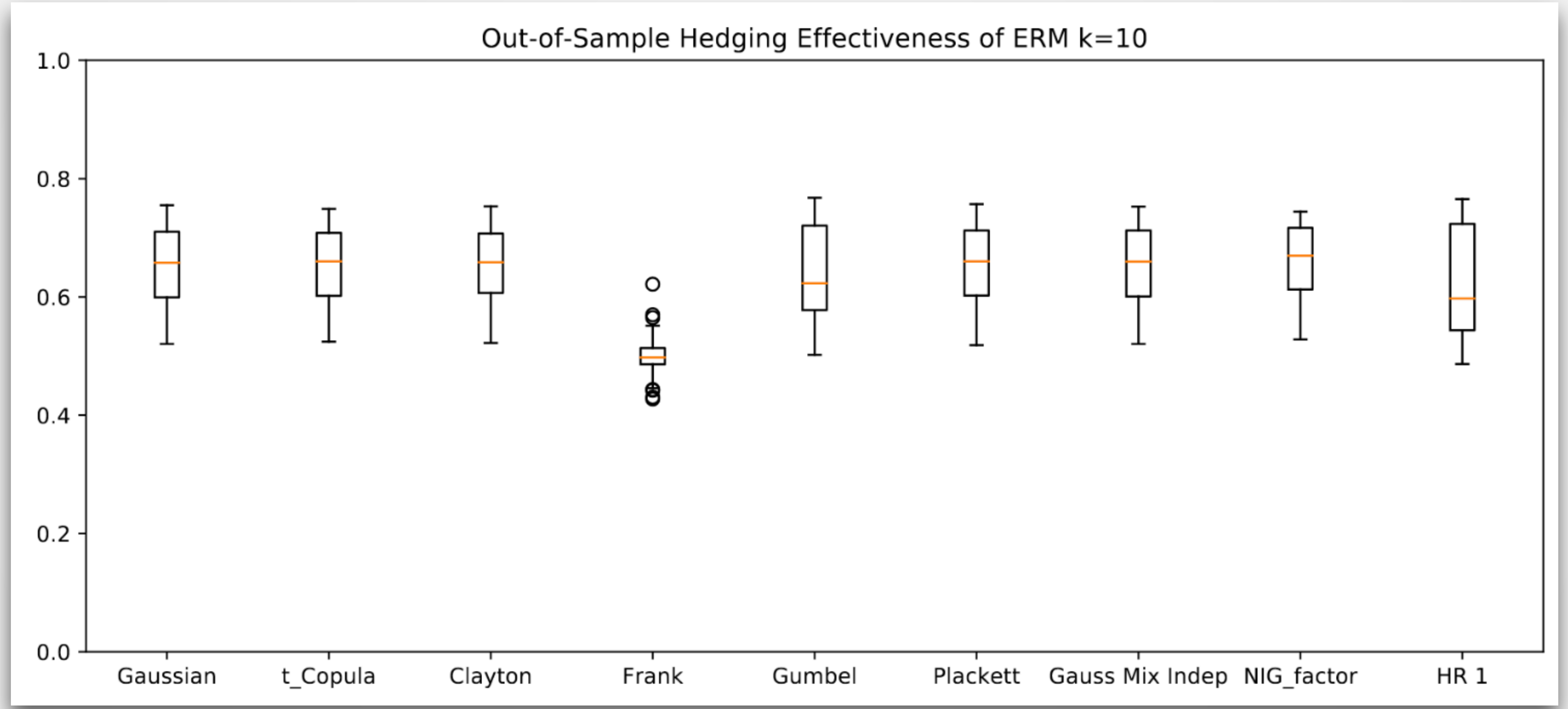


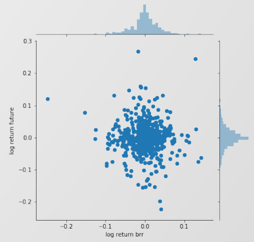
Hedge Effectiveness





Hedge Effectiveness



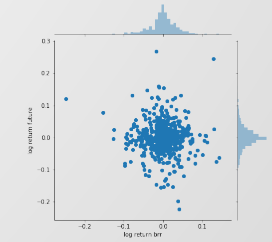


Root Mean Square Error

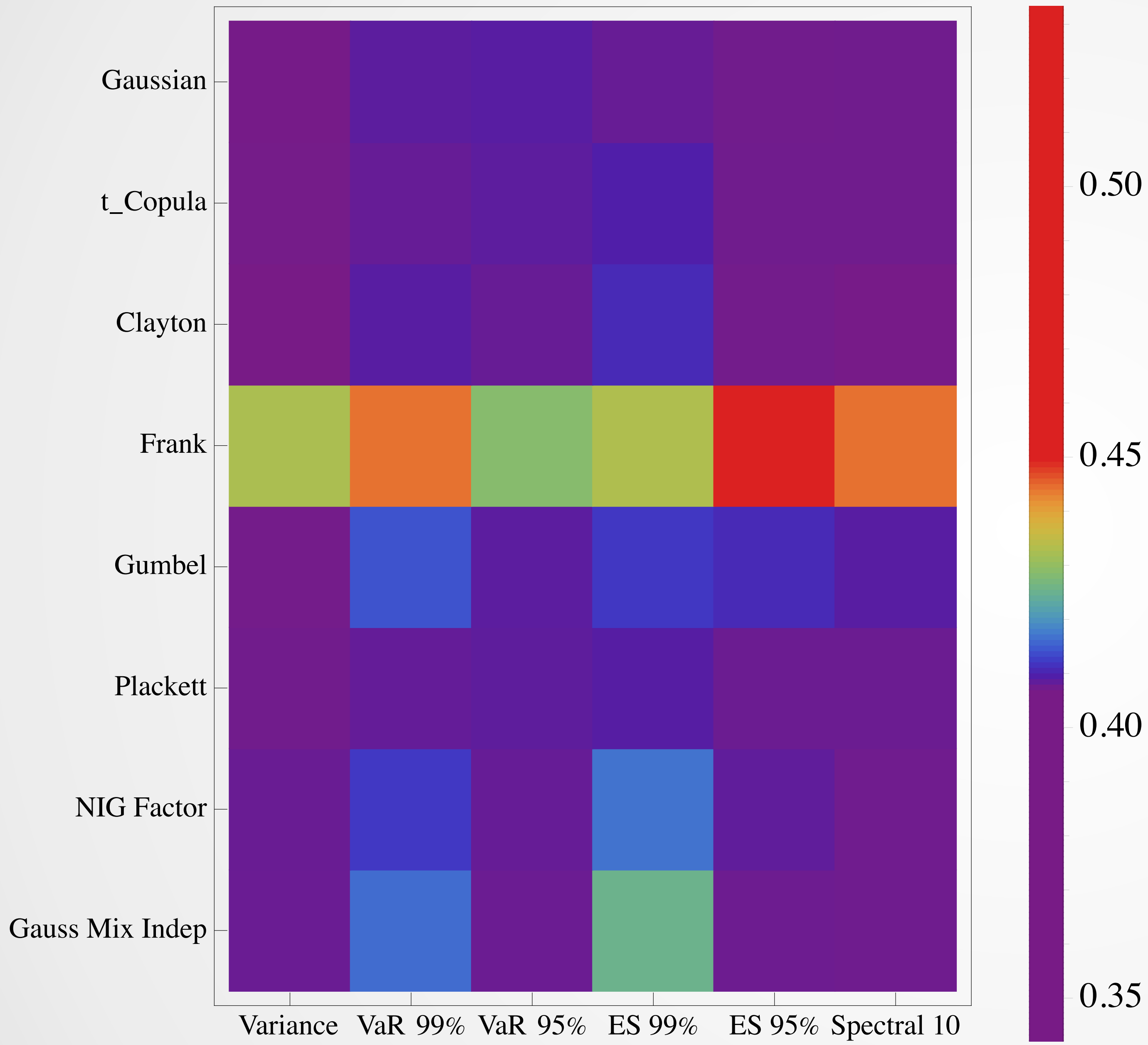
- Recall our final goal: to form a portfolio with spot and future such that the P&L is zero
- A perfect hedge $r^h_{\text{ideal}} = 0$
- Root Mean Square Error is intuitive measure to assess the quality of hedge
- For portfolio return:

$$\text{RMSE}(r^h) = \sqrt{(r^h - 0)^2}$$

- Simply the standard deviation

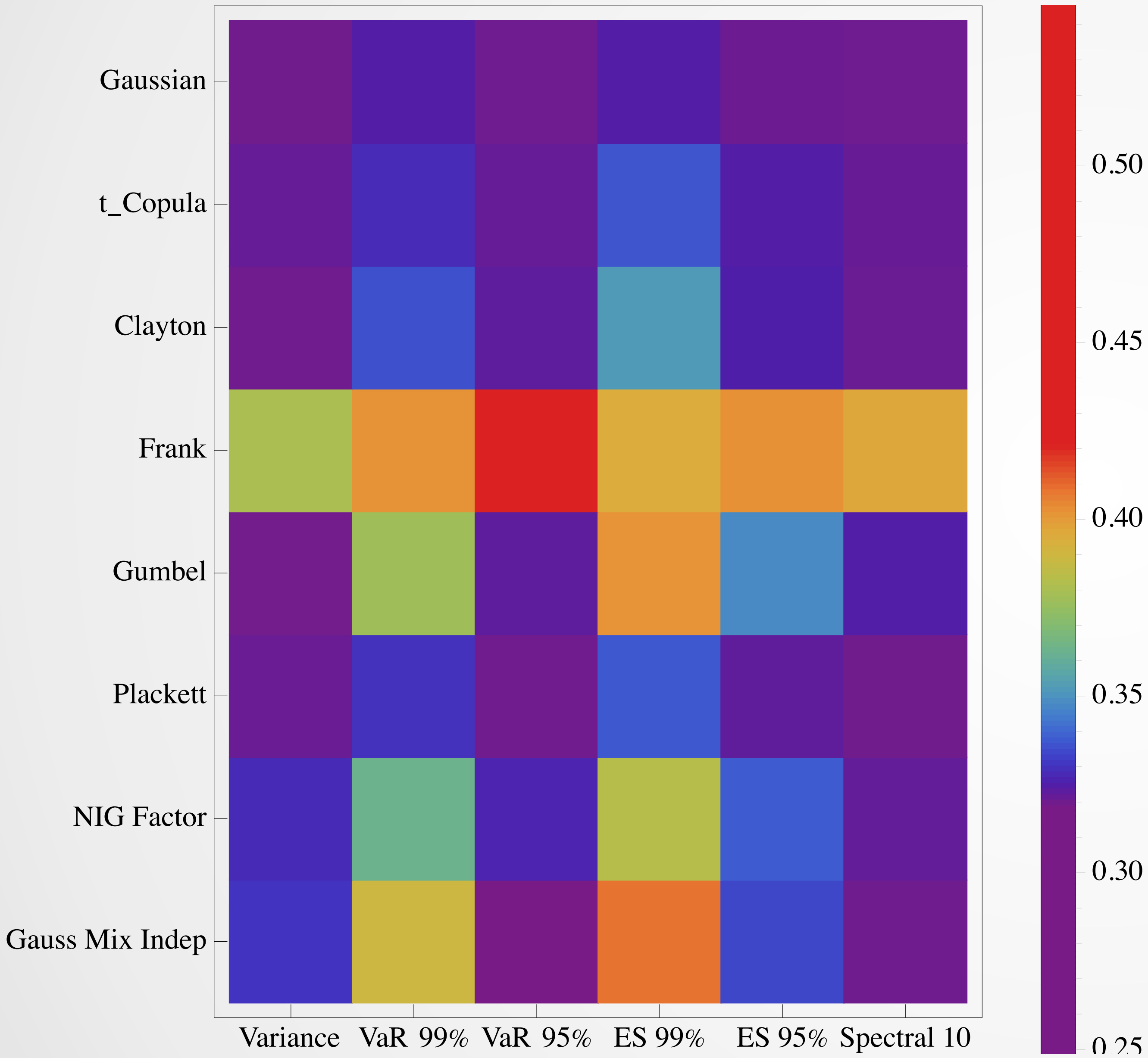


Root Mean Square Error



- Out-of-sample RMSE
- Rule out Frank Copula
- VaR 99% and ES 99% are slightly underperforming
- Observation: Minimising Variance in-sample data does not necessary minimise variance out-of-sample
- Reflects the highly unstable nature of the correlation between spot and future

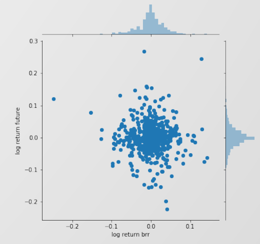
Semi Variance



- Lower semi variance

  $$\mathsf{E}\left[X - \mathsf{E}(X)\}^2 \cdot 1\{X \leq \mathsf{E}(X)\}\right]^{\frac{1}{2}}$$

- See also McNeil (2005) and Markowitz (1991)
- Rule out Frank, VaR 99%, and ES 99%



Robustness

- Intense discussions about „what is robust?“ Hampel vs. Huber
- Our case: jumps and correlation distress
- Market dynamics always exist, but do we want the optimal hedge ratio to react to extreme market changes?
  - Elon Musk tweets
  - A sudden large order from institutional investor
  - An incident of system failure in crypto exchanges
- Hampel’s infinitesimal approach: influence function

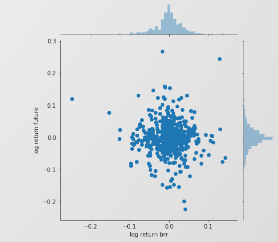
$$\text{IF} = \frac{\hat{h}_\rho(\mathbf{X_1}, \ldots, \mathbf{X_n}, \mathbf{z}) - \hat{\mathbf{h}}_\rho(\mathbf{X_1}, \ldots, \mathbf{X_n})}{\frac{1}{n+1}}$$

n data points

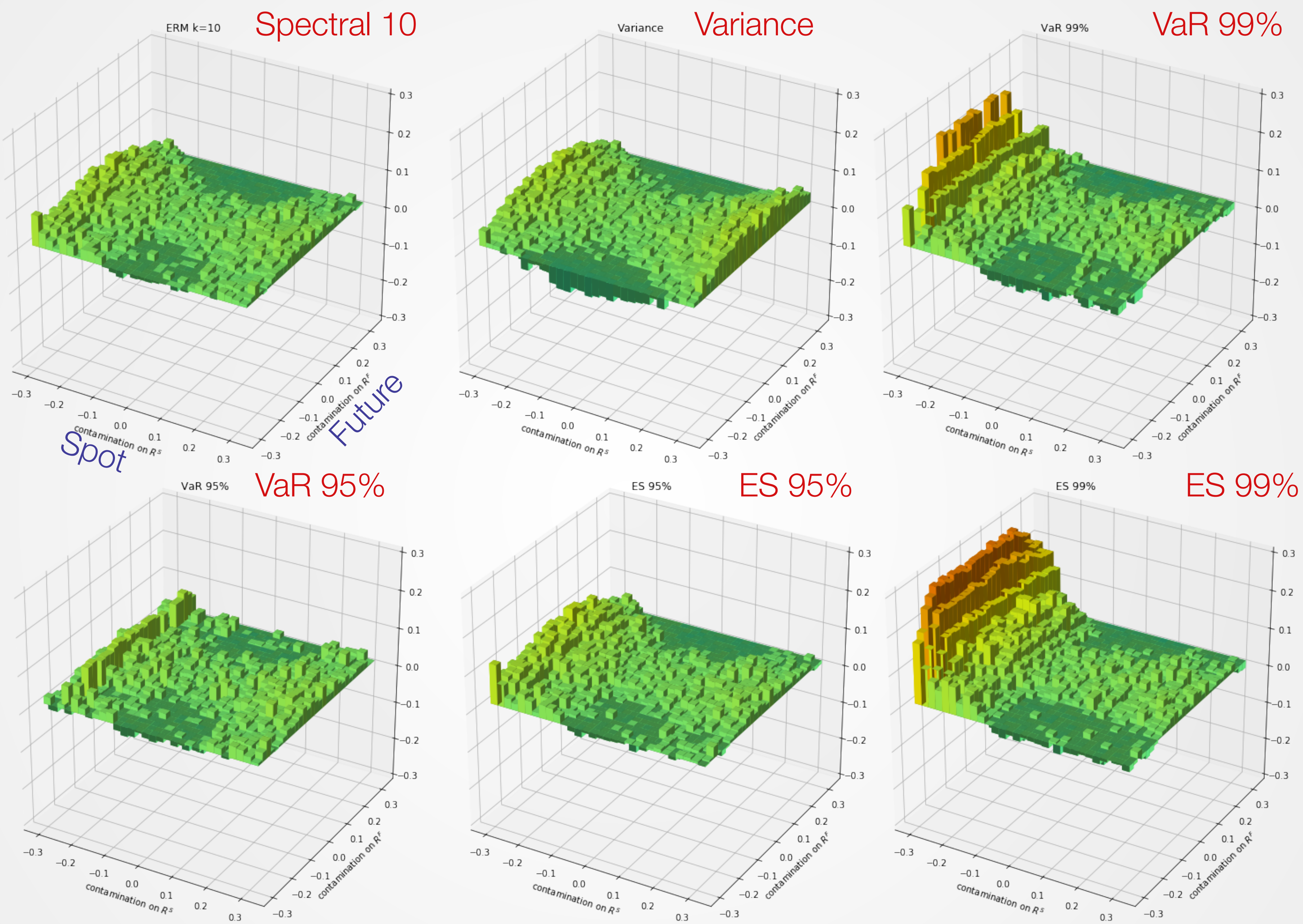
Procedure of getting h

Artificial shock

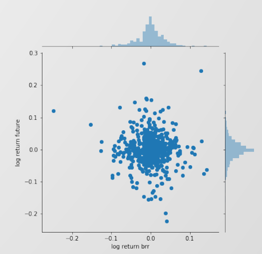
- Read as the change of optimal hedge ratio if we add one shock (two dimensional, r^s and r^f) to the training data



Robustness



- *t*-Copula with different risk measures
- 300 data points from Dec 2018 to Feb 2020
- ES 99% and VaR 99% are very sensitive to outliers

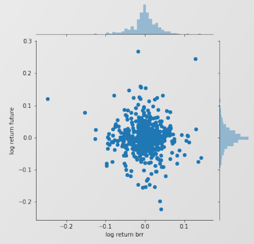


Conclusion

- Hedging with different copulae and risk measure produces mixed results:
  - Frank copula underperforms consistently in hedging effectiveness and robustness
  - NIG and Gaussian Mix produce small hedge ratios pre-Covid-19 pandemic
  - NIG factor produces good hedge effectiveness
  - Gumbel produces good results in P&L

- Risk measures as loss function
  - Min variance in-sample does not necessary min that of out-of-sample data
  - ES 99% and VaR 99% are sensitive to outliers

- Next step: Hedge other cryptos (e.g. CRIX index) with BTC futures



Blockchain Research Center

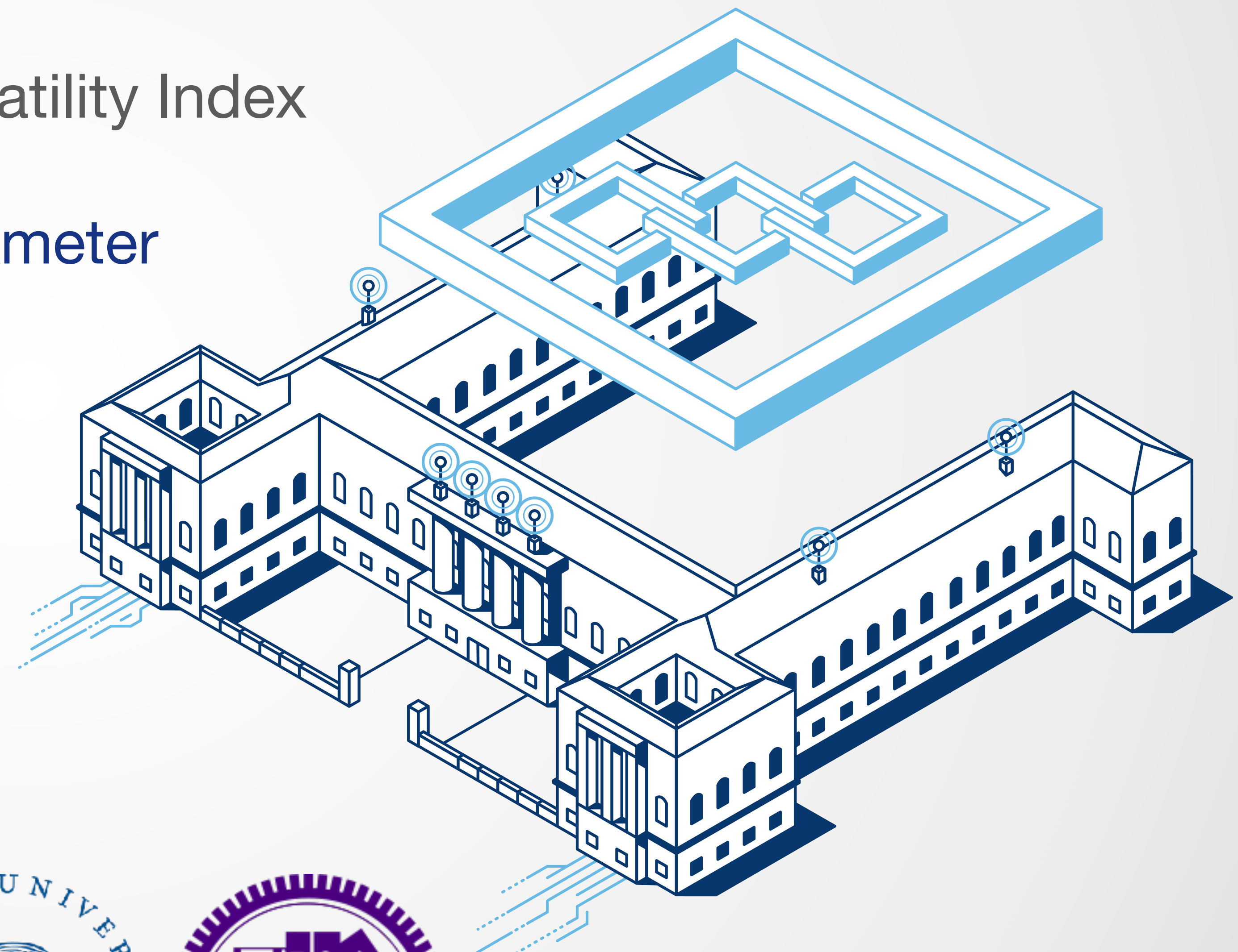
1. The BRC data pool

CRIX Cryptocurrency - Index - Data - Derivatives

VCRIX VCRIX - Volatility Index

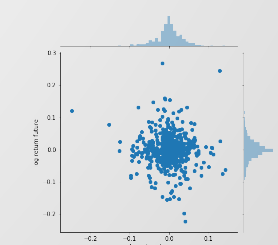
FRM financialriskmeter

Quantlet



2. Joint BRCs

blockchain-research-center.de



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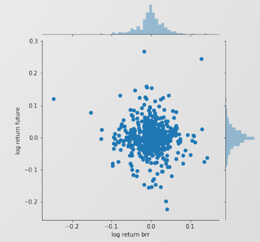

Hedging Cryptos with futures

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