Calibrating CAT Bonds for Mexican Earthquakes

Wolfgang Karl Härdle
Brenda López Cabrera

Humboldt-Universität zu Berlin
http://case.wiwi.hu-berlin.de
http://ise.wiwi.hu-berlin.de

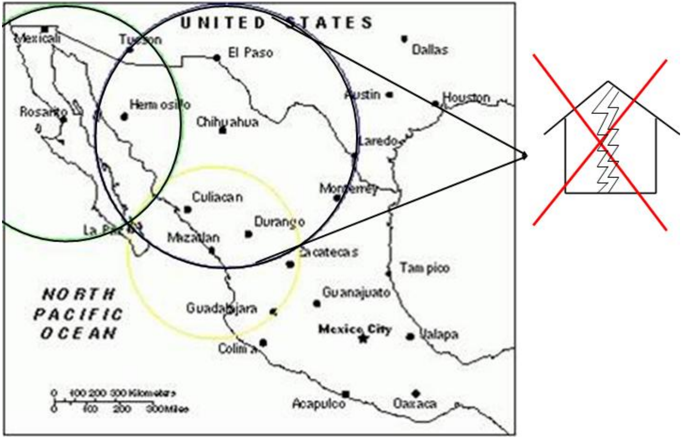

Figure 1: Location of epicenters

Mexico is exposed to earthquake risk (EQ):

- EQ disasters are huge and volatile
- An 8.1 Mw EQ hit Mexico in 1985: estimated payouts of 4 billion dollars

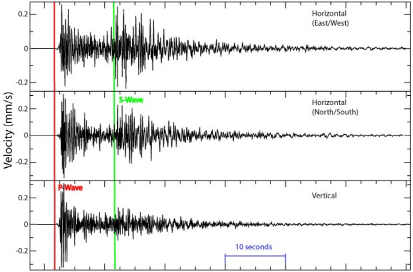

Figure 2: Waves from seismograph

Seismology

- EQ: sudden dislocation of large rock masses along fault lines fractures
- Parameters: location, fault rapture plane, magnitude and depth
- Depth (d): distance between the hypocenter and the epicenter
- Magnitude (Mw): numerical quantity of the total energy released
- Tools: seismograph and the accelerograph

CAT bonds

- Reconstruction can be financed by transfering the risk with CAT bonds
  - From insurers, reinsurance and corporations (sponsors) to capital market investors
- Alternative or complement to traditional reinsurance
- Supply protection against natural catastrophes without credit risk present in reinsurance
- Offer attractive returns and reduce the portfolio risk
- Attractive surplus alternatives

Calibrating CAT bonds

- The intensity rate (λ) describes the flow process of EQ:
  - Reinsurance market (λ_1): Ceding & Reinsurance company
  - Capital market (λ_2): SPV & investors
  - Historical data (λ_3): real intensity of EQ
- Comparative analysis: is $\lambda_1 = \lambda_2 = \lambda_3$? Fair?
- Different variables affect the value of the loss: physical parameters, property value, building material, construction design, impact on main cities, etc.

Outline

1. Motivation ✓
2. What are CAT bonds?
3. Calibrating the parametric Mexican CAT Bond
4. Calibrating a Modeled loss CAT bond

CAT bonds

- Ease the transfer of catastrophic insurance risk
- Coupons and principal depend on the performance of a pool or index of natural catastrophe risks
- Parties: Sponsor, SPV, collateral & investors
- If there is no event: SPV gives the principal back to the investors with the final coupon.
- If there is an event: investors sacrifices fully or partially their principal plus interest and the SPV pays the insured loss

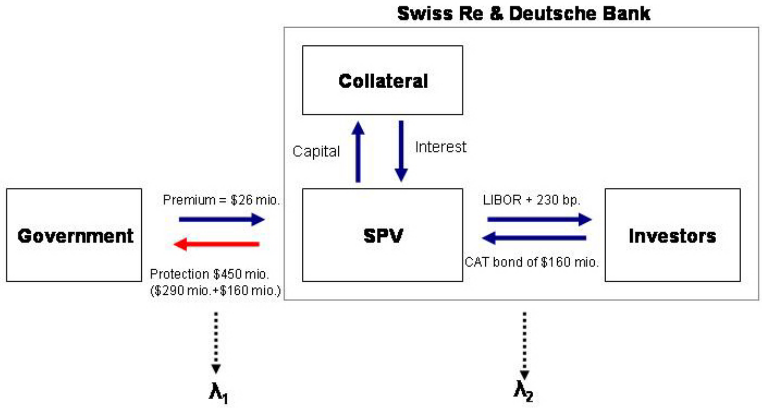

Figure 3: Cash Flows Diagram. Event (red), no event(blue)

Trigger mechanisms

1. Indemnity trigger: Actual loss of the ceding company
2. Industry index trigger: The ceding recovers a % of total industry losses in excess of a predetermined point
3. Pure parametric Index trigger: Richter Scale
4. Parametric index trigger: weighting boxes exposure
   - Hurricane Index value $= K \sum_{i=1}^{I} w_i (v_i - L)^n$
5. Modeled loss trigger: A third party projects the expected losses to the ceding company´s portfolio

Examples

Date	Sponsor	SPV	Total size ($mio)	Term (yrs)	Peril
Jul'97	Swiss Re	SR EQ Fund	$137	2	EQ
Nov'97	Tokyo Mar.	Parametric Re	$100	10	EQ
June'01	Zurich Re	Trinom	$162	3	Multi peril
May'03	USAA Re	Residential Re 2003	$160	3	Multi peril
Jun'03	PIONEER'03 II-B	Swiss Re	$12	3	Wind

Table 1: Examples of CAT bond

CAT-MEX bond

Issue Date	May-06
Sponsor	Mexican government
SPV	CAT-Mex Ltd
Reinsurer	Swiss Re
Total size (P)	$160 million
Risk Period	3 year
Risk	Earthquake
Structure	Parametric
Spread (s)	LIBOR plus 230 basis points
Total coverage	$450 million
Premiums	$26 million

Table 2: Mexican parametric CAT bond

- Air Worldwide Corporation modeled the seismic risk
- Given the federal governmental budget plan: 3 zones

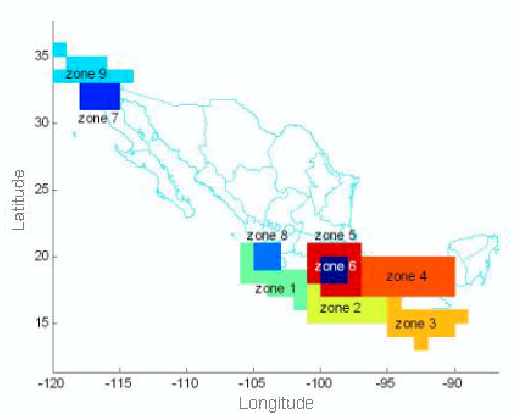

Figure 4: Map of regions

- The CAT bond payment would be triggered if:

Zone	Coverage	Threshold u in $Mw \geq$ to
Zone 1	\$150 mio.	8
Zone 2	\$150 mio.	8
Zone 5	\$150 mio.	7.5

Table 3: Thresholds u's of the Mexican parametric CAT bond

- In case of a trigger event:
  - Swiss Re pays the covered insured amount to the government
  - Investors sacrifices their full principal and coupons
- Premium & proceeds are used to pay coupons to bondholders

Assumptions

- The arrival process of EQ $N_t, t \geq 0$ uses the times between EQ $W_i = T_i - T_{i-1}$:

$$N_t = \sum_{n=1}^{\infty} \mathrm{I}\left(T_n < t\right) \tag{1}$$

- EQ suffer the *loss of memory property*:

$$\mathrm{P}(X > x + y | X > y) = \mathrm{P}(X > x)$$

- N_t can be characterized by a Homogeneous Poisson Process (HPP)

Homogeneous Poisson Process (HPP)

N_t is an HPP with intensity rate $\lambda > 0$ if:

- N_t is a point process governed by the Poisson law
- The waiting times $W_i = T_i - T_{i-1}$ are i.i.d. $\exp(\lambda)$

The probability of occurrence of an EQ in the interval $(0, t]$ is:

$$\mathrm{P}(W_i < t) = 1 - \mathrm{P}(W_i \geq t) = 1 - e^{-\lambda t} \qquad (2)$$

Calibrating Parametric CAT bond

The intensity rate (λ) describes the flow process of EQ:

- Reinsurance market (λ_1): Ceding & Reinsurance company
- Capital market (λ_2): SPV & investors
- Historical data (λ_3): real intensity of EQ

Reinsurance market intensity: λ_1

- Flat term structure of interest rates & an annual continuously compounded discount interest rates equal to the the LIBOR in May 2006 $r = 5.35\%$
- N_t is a HPP with intensity λ_1
- Let H be the annual premium & let J be the Swiss Re's payoff

Let $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathrm{P})$ be a probability space and $\mathcal{F}_t \subset \mathcal{F}$ an increasing filtration, with time $t \in [0, T]$, a compounded discounted *actuarially fair insurance price* that equals the premiums to the expected loss at $t = 0$ is:

$$\mathsf{E}\left[He^{-tr_t}\right] = \mathsf{E}\left[Je^{-tr_t}\right] \tag{3}$$

where:

$$\mathrm{E}\left[He^{-tr_t}\right] = \int_0^T he^{-tr_t}\lambda_1 e^{-\lambda_1 t}dt$$

and

$$\mathrm{E}\left[Je^{-tr_t}\right] = \int_0^T je^{-tr_t}\lambda_1 e^{-\lambda_1 t}dt$$

Then:

$$26 = \int_0^3 450\lambda_1 e^{-t(r_t+\lambda_1)}dt$$

Hence, λ_1=0.0215, i.e. Swiss Re expects 2.15 events in 100 years or a probability of occurrence of an event in 3 years equal to 0.0624.

Capital market intensity: λ_2

- Annual discretely compounded discount interest rate r_t
- CAT bond with coupons every 3 months and payment of the principal P at T
- Coupon bonds pay a fixed spread s=230 bp. over LIBOR
- In case of no event: investor receives principal plus coupons
- In case of event: investor sacrifices principal P & coupons
- Coupons equal to $C = \left(\frac{r+s}{4}\right) P = \$3.06$ mio

Let G be the investors' gain & N_t a HPP with intensity λ_2. A discounted *fair bond price* at time $t = 0$ is given by:

$$P = \sum_{t=1}^{12} \left(\frac{1}{1+r_t}\right)^{\frac{t}{4}} Ce^{-\lambda_2 \frac{t}{4}} + \left(\frac{1}{1+r_t}\right)^{T} Pe^{-\lambda_2 T} \qquad (4)$$

Then,

$$160 = \sum_{t=1}^{12} 3.06 \left(\frac{e^{-\lambda_2}}{1+r_t}\right)^{\frac{t}{4}} + \frac{160e^{-3\lambda_2}}{(1+r_t)^3}$$

Hence, $\lambda_2 = 0.0222$. The capital market estimates a probability of occurrence of an event equal in 3 years to 0.0644, equivalently to 2.22 events in one hundred years.

Historical Intensity: λ_3

Descriptive	time(t)	depth(d)	magnitude(Mw)
Minimum	1900	0	6.5
Maximum	2003	200	8.2
Mean	-	39.54	6.9
Median	-	33	6.9
Sdt. Error	-	39.66	0.37
25% Quantile	-	12	6.6
75% Quantile	-	53	7.1
Excess	-	2.63	0.25
Nr. obs.	192	192	192
Distinct obs.	82	54	18

Table 4: Descriptive statistics of EQ data from 1900 to 2003(SSN)

Intensity model

- Let Y_i be i.i.d rvs. indicating Mw of the i^{th} EQ at time t
- Let $\varepsilon_i = \mathrm{I}\left(Y_i \geq \bar{u}\right)$ characterizing EQ with Mw higher than a defined threshold for a specific location
- N_t be a HPP with intensity $\lambda > 0$

A new process B_t defines the trigger event process:

$$B_t = \sum_{i=1}^{N_t} \mathrm{I}\left(\varepsilon_i > 0\right) \tag{5}$$

- Data contains only 3 events: the calibration of the intensity of B_t is based on 2 W_i

Consider B_t and define p as the probability of occurrence of a trigger event conditional on the occurrence of the earthquake. The probability of no event up to time t:

$$\begin{aligned} \mathrm{P}(B_t = 0) &= \sum_{k=0}^{\infty} \mathrm{P}(N_t = k)(1-p)^k \\ &= \sum_{k=0}^{\infty} \frac{(\lambda t)^k}{k!} e^{(-\lambda t)} (1-p)^k \\ &= e^{-\lambda p t} = e^{-\lambda_3 t} \end{aligned} \tag{6}$$

The annual historical intensity rate for a trigger event is equal to $\lambda_3 = \lambda p = 1.8504 \left(\frac{3}{192}\right) = 0.0289$

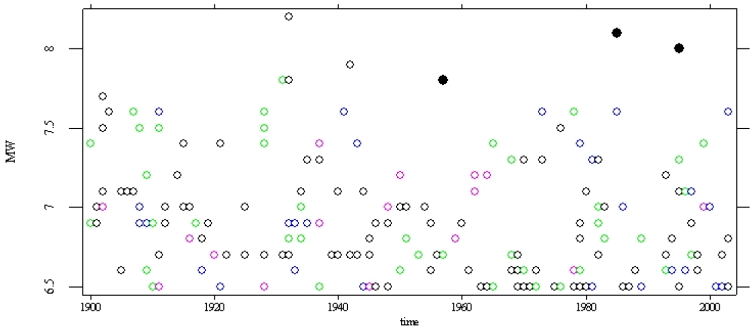

Figure 5: Mw of trigger events (filled circles), EQ in zone 1 (black circles), EQ in zone 2 (green circles), EQ in zone 5 (magenta circles), EQ out of insured zones (blue circles) eq65thMexcase.xpl

Calibration of intensity rates

	λ_1	λ_2	λ_3
Intensity (10^{-2})	2.15	2.22	2.89
Prob. of event in 1 year (10^{-2})	2.12	2.19	2.84
Prob. of event in 3 year (10^{-2})	6.24	6.44	8.30
No. expected events in 100 years	2.15	2.22	2.89

Table 5: Intensity rates

$\lambda_1 \neq \lambda_2$:

- Absence of the public & liquid market of EQ risk in the reinsurance market: limited information is available
- Contracts in the capital market are more expensive than in the reinsurance market: cost of risk capital & risk of default

$\lambda_1 \neq \lambda_2 \neq \lambda_3$:

- λ_3 is based on the time period of the historical data
- If λ_3 would be the "real" intensity rate:
  - The Mexican government paid total premiums of \$26 million that is 0.75 times the real actuarially fair one:

$$\int_0^3 450\lambda_3 e^{-t(r_t+\lambda_3)} dt = 34.49$$

  - Savings of \$8.492 million? NO
  - The mix of the reinsurance contract and the CAT bond: 35% of the total seismic risk to the investors

Modeled Loss CAT bond for earthquakes

- Other variables can affect the value of losses: Richter, depth, location, impact $\mathtt{I}(0,1)$, property value, building materials and construction designs
  - Losses are $\propto$ Mw & time t & inversely $\propto$ d of EQ
- Losses data from EQ during 1900-2003 that López (2003) built
- Losses $\{X_k\}_{k=1}^{\infty}$ adjusted to population, inflation, exchange rate
- Missing data treatment: Expectation-Maximum (EM) algorithm

Losses of EQ in 100 years

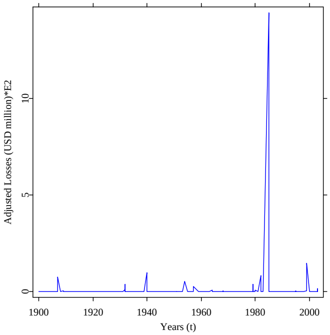

Figure 6: Adjusted Losses - Richter Scale CMX02.xpl

- Modeled loss:

$$\ln(X) = -27.99 + 2.10 Mw + 4.44 d - 0.15 \mathtt{I}(0,1) - 1.11 \ln(Mw) \cdot d$$

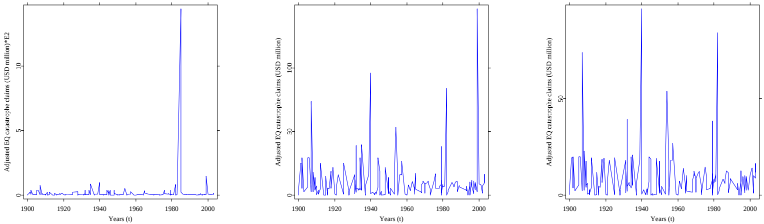

Figure 7: Historical and modeled losses of EQ from 1900-2003 (left panel), without the outlier of the EQ in 1985 (middle panel), without outliers of EQ in 1985 and 1999 (right panel) CMXmyEMalgorithm.xpl

Compound Doubly Stochastic Poisson Pricing Model

Cizek, Härdle & Weron (2005):

- A doubly stochastic Poisson process N_s describing the flow of EQ with an intensity process λ_s, where $s \in [0, T]$
  - HPP with an intensity $\lambda = 1.8504$
  - NHPP with intensity $\lambda_s^1 = 1.8167$
  - Renewal Process: $W_i \sim exp(\lambda)$ with $\lambda_s^2 = 1.88$

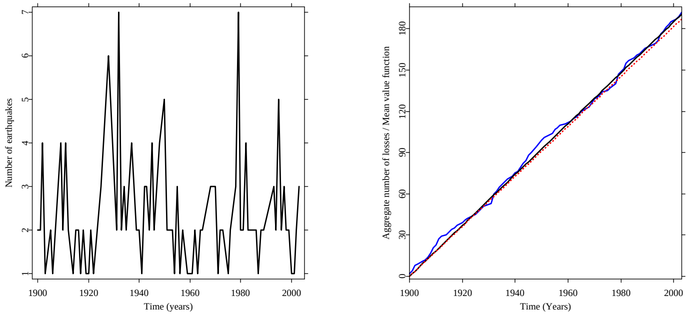

Figure 8: Left panel: Number of EQ occurred in Mexico during 1900-2003. Right panel: The accumulated number of EQ (solid blue line) and mean value functions $\mathrm{E}(N_t)$ of the HPP with intensity $\lambda_s = 1.8504$ (solid black line) and the $\lambda_s^1 = 1.8167$ (dashed red line) CMXrisk03.xpl

- Losses $\{X_k\}_{k=1}^{\infty}$ at t_i are i.i.d with $F(x) = \mathrm{P}(X_i < x)$

Distrib.	Log-normal	Pareto	Burr	Exponential	Gamma	Weibull
Parameter	$\mu = 1.387$ $\sigma = 1.644$	$\alpha = 2.394$ $\lambda = 12.92$	$\alpha = 3.323$ $\lambda = 16.67$ $\tau = 0.919$	$\beta = 0.143$	$\alpha = 0.143$ $\beta = -0.007$	$\beta = 0.220$ $\tau = 0.764$
Kolmogorov Sminorv	0.173	0.131	0.137	0.135	0.295	0.145
(D test)	(< 0.005)	(< 0.005)	(< 0.005)	(< 0.005)	(< 0.005)	(< 0.005)
Kuiper	0.296	0.248	0.260	0.222	0.569	0.282
(V test)	(< 0.005)	(< 0.005)	(< 0.005)	(< 0.005)	(< 0.005)	(< 0.005)
Cramér-von Mises	1.358	0.803	0.884	0.790	7.068	1.051
(W^2 test)	(< 0.005)	(< 0.005)	(< 0.005)	(< 0.005)	(< 0.005)	(< 0.005)
Anderson Darling	10.022	5.635	5.563	9.429	36.076	5.963
(A^2 test)	(< 0.005)	(0.005)	(0.01)	(< 0.005)	(< 0.005)	(< 0.005)

Table 6: Parameter estimates by A^2 minimization procedure and test statistics. In parenthesis, the related p-values based on 1000 simulations

- The countinuous and predictable aggregate loss process is:

$$L_t = \sum_{i=1}^{N_t} X_i \tag{7}$$

- The threshold level D
- A continuously compounded discount interest rate r

$$e^{-R(s,t)} = e^{\int_s^t r(\xi)d\xi}$$

- A threshold time event $\tau = \inf\{t : L_t \geq D\}$. Baryshnikov et al. (1998) defined it as a point of a doubly stochastic Poisson process $M_t = \mathrm{I}(L_t > D)$ with a stochastic intensity:

$$\Lambda_s = \lambda_s \{1 - F(D - L_s)\} \mathrm{I}(L_s < D) \tag{8}$$

Zero Coupon CAT bonds (ZCCB)

- Pays P at T conditional on $\tau > T$
- The payment at maturity is independent from the occurrence and timing of D
- In case of a trigger event P is fully lost

The non arbitrage price of the ZCCB V_t^1:

$$\begin{aligned} V_t^1 &= \mathsf{E}\left[P e^{-R(t,T)} \left(1 - M_T\right) | \mathcal{F}_t \right] \\ &= \mathsf{E}\left[P e^{-R(t,T)} \left\{ 1 - \int_t^T \lambda_s \left\{1 - F(D - L_s)\right\} \mathbb{I}\left(L_s < D\right) d_s \right\} | \mathcal{F}_t \right] \end{aligned} \quad (9)$$

Coupon CAT bonds (CCB)

- Pays P at T & gives coupons C_s until τ
- The payment at maturity is independent from the occurrence and timing of D
- Pays a fixed spread s (bp.+LIBOR)
- In case of a trigger event P is fully lost

The non arbitrage price of the CCB V_t^2:

$$
\begin{aligned}
V_t^2 &= \mathrm{E}\left[Pe^{-R(t,T)}\left(1-M_T\right)+\int_t^T e^{-R(t,s)}C_s\left(1-M_s\right)ds|\mathcal{F}_t\right]\\
&= \mathrm{E}\left[Pe^{-R(t,T)}+\int_t^T e^{-R(t,s)}\left\{C_s\left(1-\int_t^s \lambda_\xi\left\{1-F(D-L_\xi)\right\}\right.\right.\right.\\
&\qquad \left.\left.\left.\mathrm{I}\left(L_\xi<D\right)d\xi\right)-Pe^{-R(s,T)}\lambda_s\left\{1-F(D-L_s)\right\}\mathrm{I}\left(L_s<D\right)\right\}d_s|\mathcal{F}_t\right]
\end{aligned}
$$

Calibration

- r equal to the LIBOR ($r = 5.35\%$)
- P=\$160 mio.
- $T \in [0.25, 3]$ years
- $D \in [\$100, \$135]$ mio. (0.7 & 0.8-quantiles of 3 yearly acc.losses)
- $s = 230$ bp. over LIBOR
- Quarterly $C_t = \left(\frac{LIBOR+230bp}{4}\right)\$160 = \$3.06$ mio.
- N_t is an HPP with intensity $\lambda_s = 1.8504$
- 1000 Monte Carlo simulations

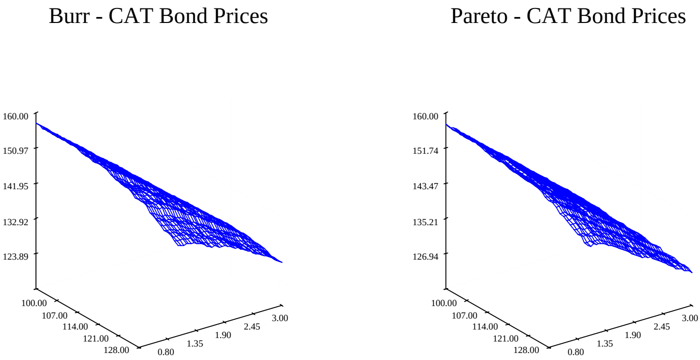

Figure 9: The ZCCB price (vertical axis) with respect to D (horizontal left axis) & T (horizontal right axis) in the Burr-HPP (left panel) & Pareto-HPP (right panel) for the modeled loss CMX05e.xpl

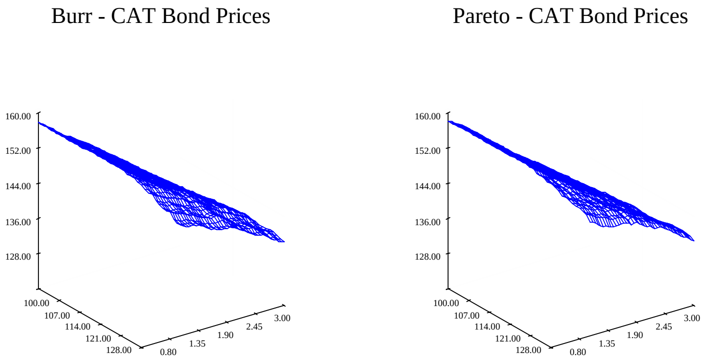

Figure 10: The CCB price (vertical axis) with respect to D (horizontal left axis) & T (horizontal right axis) in the Burr-HPP (left panel) & Pareto-HPP (right panel) for the modeled loss `CMX07e.xpl`

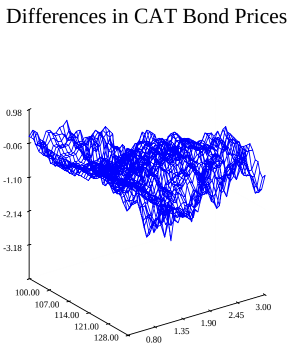

Figure 11: The difference in ZCCB price (left panel) & CCB prices (right panel) in the vertical axis left panel between the Burr & Pareto distributions under a HPP, with respect to the D (horizontal left axis) & T (horizontal right axis)

CMX06f.xpl

	Min. (% Principal)	Max. (% Principal)
Diff. ZCB Burr-Pareto	-2.640	0.614
Diff. CB Burr-Pareto	-1.552	0.809
Diff. ZCB-CB Burr	-6.228	-0.178
Diff. ZCB-CB Pareto	-5.738	-0.375

Table 7: Min. & max. of the diff. in the ZCCB-CCB prices in % of P for the Burr-Pareto distributions of the modeled loss

- V_t^1 & V_t^2 decreases as T increases
- V_t^1 & V_t^2 increases as D increases
- F(x) influences the price of the CAT bond
- $V_t^2 > V_t^1$
- Modeled loss: no significant impact on ZCCB-CCB prices, but more important than the loss distribution

Conclusion

- Seismic risk can be transfered with CAT bonds
- CAT bonds: No credit risk, high returns and better performance of the portfolio
- Calibration of a Mexican CAT bond:
  1. N_t a HPP with intensity λ
  2. Parametric trigger (physical parameters): the intensity rates of EQ in $\neq$ parts of the contract vs. real historical
  3. Modeled loss trigger considers several variables & connected to index trigger
- This analysis prices a CAT bond relative to an *expected level*

References

- P. Cizek, W. Härdle, R. Weron (2005)
  Statistical Tools for Finance and Insurance
  Springer.
- Yu. Baryshnikov, A. Mayo, D.R Taylor (1998)
  Pricing Cat Bonds
  http : //www.cam.wits.ac.za/mfinance/research.html
- K. Burnecki, G. Kukla (2003)
  Pricing of Zero-Coupon and Coupon Cat Bonds, Appl. Math
  (Warsaw) 30(3): 315-324.

References

W. Dubinsky, D. Laster (2005)
SIGMA: Insurance - link securities
Swiss Re publications.

LIBOR Rate
http://www.fanniemae.com/tools/libor/2006.jhtml

B. Lopez (2003)
Valuación de bonos catastróficos para terremotos en México
http://www.mexder.com.mx/inter/info/mexder/avisos/ValuaciondeBonoscatastroficosparaTerremotosenMexico.pdf

References

- B. Lopez (2006)
  Pricing Catastrophic Bonds for earthquakes in Mexico
  http : //www.edoc.hu – berlin.de/docviews/abstract.php?lang = gerid = 27524

- Secretaría de Hacienda y Crédito Público Mexico (SHCP) (2004)
  Administración de Riesgos Catastróficos del FONDEN
  Secretaría de Hacienda y Crédito Público.

- Servicio Sismologico Nacional Instituto de Geosifísica UNAM Mexico (SSN) (2006)
  Earthquakes Data Base
  UNAM.