Independent Component Analysis and VaR

Ying Chen
Wolfgang Härdle

Institut für Statistik and Ökonometrie
CASE - Center for Applied Statistics and Economics
Humboldt-Universität zu Berlin
http://ise.wiwi.hu-berlin.de
http://www.case.hu-berlin.de

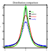
Value at Risk (VaR) calculation

$$R_t = b_t^\top x(t) = b_t^\top \Sigma_t^{1/2} \varepsilon_t$$

R_t the portfolio return
b_t the portfolio weight
$x(t) \in \mathbb{R}^d$ the individual returns with cov Σ_t
ε_t the stochastic term.

VaR at level $100a\%$: $VaR_{a,t} = F_t^{-1}(a)$,
F_t^{-1} equals the inverse of cdf of R_t.

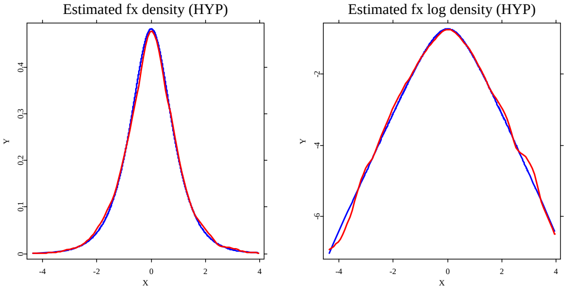
Critical point: pdf of $x(t)$.



A univariate story

Chen, Härdle and Jeong (2005), GHADA:
Generalized **h**yperbolic distribution + **ada**ptive volatility estimation

The estimated density (left) and log density (right) of the standardized returns of DEM/USD rates from 1979-12-01 to 1994-04-01 (red: nonparametric kernel density, blue: GH density).



Extension to the high-dimensional case

Independent component analysis (ICA)

Given $x(t) = \{x_1(t), \cdots, x_d(t)\}^\top$, find ICs $y(t)$:

$$y(t) = Wx(t),$$

where W is a $d \times d$ nonsingular matrix.

$$x(t) = W^{-1}y(t) = Ay(t)$$

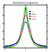
What will ICA bring us?

1. Marginal pdf f_{y_j} and volatility σ_{t,y_j} of IC y_j, $j = 1, \cdots, d$ can be estimated univariately.
2. The pdf of $y(t)$ and $x(t)$ are:

$$
\begin{aligned}
f_y &= \prod_{j=1}^{d} f_{y_j} \\
f_x &= \text{abs}(|W|) f_y(Wx)
\end{aligned}
$$

3. Covariances of $y(t)$ and $x(t)$ are::

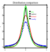
$$
\begin{aligned}
\Sigma_{t,y} &= \text{diag}(\sigma^2_{t,y_1}, \cdots, \sigma^2_{t,y_d}) \\
\Sigma_{t,x} &= A \Sigma_{t,y} A^\top
\end{aligned}
$$



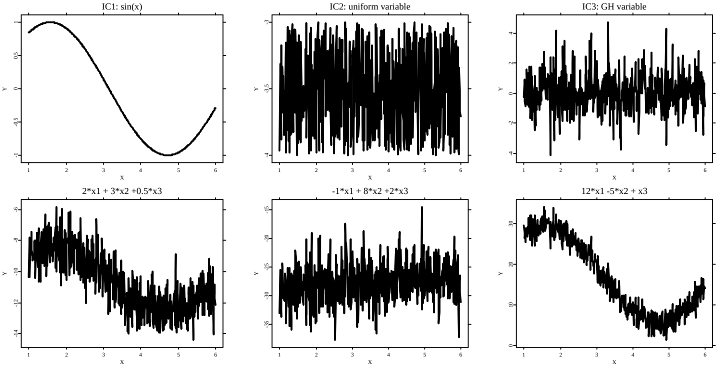
A simple example

s1(t): sin(t), $t = \text{aseq}(1, 500, 0.01)$
s2(t): uniform variable $U[0,1]$
s3(t): generalized hyperbolic variable $GH(x; 1, 2, 0, 1, 0)$

$$
\begin{aligned}
A &= \begin{pmatrix} 2 & -3 & 0.5 \\ -1 & -8 & 2 \\ 12 & -5 & 1 \end{pmatrix} \\
x(t) &= A * \begin{pmatrix} s1(t) \\ s2(t) \\ s3(t) \end{pmatrix}
\end{aligned}
$$

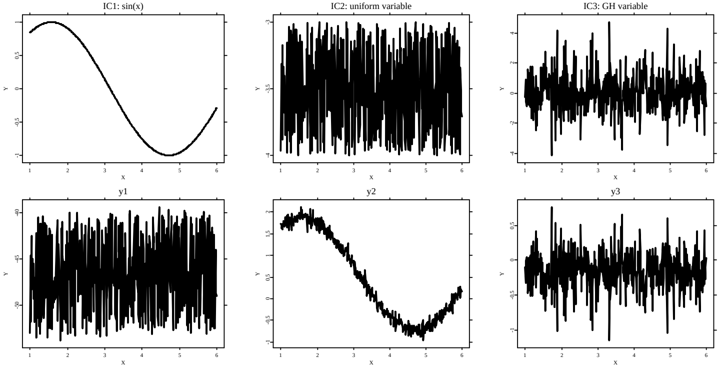


Simulated independent vector (top) and the mixed vector (bottom).



ICAsim.xpl

Simulated independent vector (top) and the estimated independent vector (bottom).



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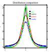
A practical example

s1(t): generalized hyperbolic variable $GH(x; 1, 2, 0, 1, 0)$
s2(t): $GH(1, 1.71, -0.17, 0.55, 0.13)$
s3(t): $GH(1, 1.78, 0.03, 0.72, -0.02)$

$$
\begin{aligned}
A &= \begin{pmatrix} 2 & -3 & 0.5 \\ -1 & -8 & 2 \\ 12 & -5 & 1 \end{pmatrix} \\
x(t) &= A * \begin{pmatrix} s1(t) \\ s2(t) \\ s3(t) \end{pmatrix}
\end{aligned}
$$



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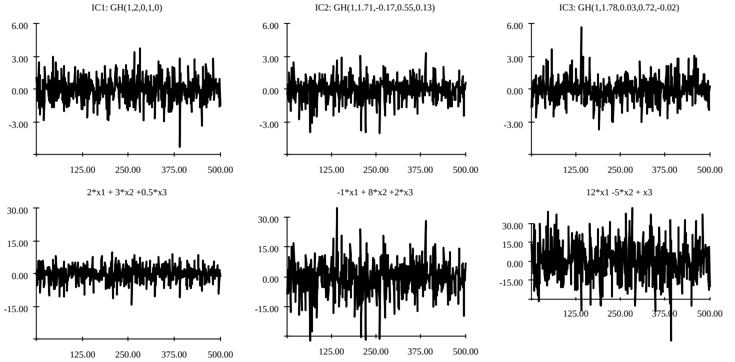

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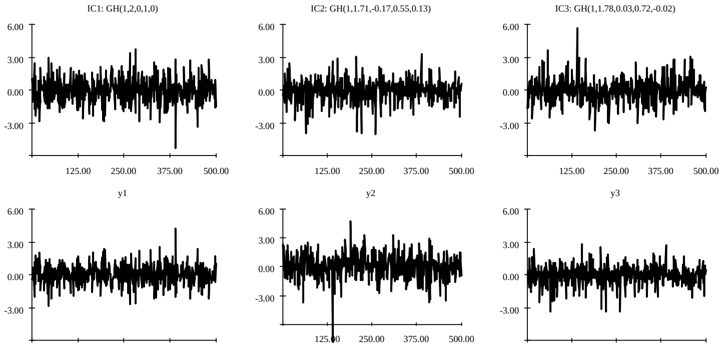
Simulated independent vector (top) and the estimated independent vector (bottom).



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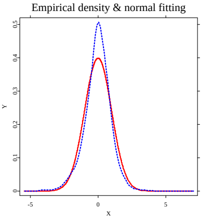
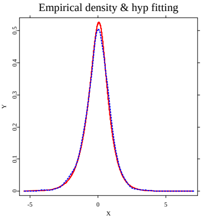
Important factors in VaR

Distribution family: Normal or heavy-tailed distribution?

Densities of the first IC for the daily DEM/USD and GBP/USD FX rates from 1979-12-01 to 1994-04-01 (3720 observations).





ICAfx.xpl

Volatility estimation: Parametric or nonparametric estimation?

VaR estimation procedure: GHICA

1. ICA to get independent components.
2. Apply the GHADA to fit the marginal pdf.
3. Determine VaR from the joint pdf via MC.
4. Compute VaR and perform backtesting.

Outline

1. Motivation: ICA + GHADA = GHICA ✓
2. ICA: properties and estimation
3. Empirical study
4. Conclusion

Definition

ICA model:

$$
\begin{pmatrix} x_1(t) \\ \vdots \\ x_d(t) \end{pmatrix} = \begin{pmatrix} a_{11} & \cdots & a_{1d} \\ \cdot & \cdots & \cdot \\ a_{d1} & \cdots & a_{dd} \end{pmatrix} \begin{pmatrix} s_1(t) \\ \vdots \\ s_d(t) \end{pmatrix}
$$

$$
{\color{blue} x(t)} = {\color{blue} As(t)}
$$

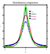
where $x(t) = \{x_1(t), \cdots, x_d(t)\}^\top$ is the observed vector, $s(t) = \{s_1(t), \cdots, s_d(t)\}^\top$ is the unknown IC and A is an unknown nonsingular matrix.

Remark

Alternatively, the ICA model can be formulated as:

$$\begin{aligned}\begin{pmatrix} y_1(t) \\ \vdots \\ y_d(t) \end{pmatrix} &= \begin{pmatrix} w_{11} & \cdots & w_{1d} \\ \cdot & \cdots & \cdot \\ w_{d1} & \cdots & w_{dd} \end{pmatrix} \begin{pmatrix} x_1(t) \\ \vdots \\ x_d(t) \end{pmatrix} \\ y(t) &= Wx(t)\end{aligned}$$

where $y(t) = \{y_1(t), \cdots, y_d(t)\}^\top$ is the IC estimated and W is the inverse of A.

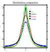


Properties of ICA

Property 1: the scales of the ICs are not identifiable since both $y(t)$ and W ($s(t)$ and A) are unknown.

$$x_1(t) = \sum_{j=1}^{d} a_{1j} s_j(t) = \sum_{j=1}^{d} \{\frac{1}{c_j} a_{1j}\}\{c_j s_j(t)\}$$

where c_j are constants.
Hence: prewhiten $x(t)$ and assume that each IC has unit variance: $\mathrm{E}[s_j^2(t)] = 1$. The matrix W is then an orthogonal matrix.



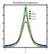
Properties of ICA

Property 2: order of the ICs is undetermined.

$$x(t) = As(t) = AP^{-1}Ps(t)$$

where P is a permutation matrix and $Ps(t)$ are the original ICs but in a different order.

Property 3: ICs are necessarily non-Gaussian.



Gaussian variables and ICA

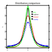
Consider two prewhitened Gaussian ICs s_1 and s_2 with pdf:

$$f(s_1, s_2) = |2\pi \mathrm{I}|^{-\frac{1}{2}} \exp(-\frac{s_1^2 + s_2^2}{2}) = \frac{1}{2\pi} \exp(-\frac{||s||^2}{2})$$

where $||s||$ is the norm of the vector $(s_1, s_2)^\top$.
The joint density of the observation x_1 and x_2 is given by:

$$f(x_1, x_2) = |2\pi \mathrm{I}|^{-\frac{1}{2}} \exp(-\frac{||A^\top x||^2}{2})|\det A^\top| = \frac{1}{2\pi} \exp(-\frac{||x||^2}{2}).$$

Since A is an orthogonal matrix after prewhitening.

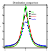


Maximize nongaussianity and Fast ICA

CLT: the distribution of a sum of independent r.v. is closer to Gaussian than the original ones.

$$
\begin{aligned}
x(t) &= As(t) \\
y(t) &= Wx(t) = WAs(t) \\
&= Qs(t) \\
y_i(t) &= \sum_{j=1}^{d} q_{ij} s_j(t) \quad i = 1, \cdots, d
\end{aligned}
$$

IC estimate $y_i(t)$ is less Gaussian if it equals one of the ICs, $s_l(t)$.



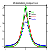
Find the maximal nongaussianity of $y_i(t) = \sum_j^d w_{ij} x_j$ under the constraint that the variance of $y_i(t)$ is 1.

How to measure nongaussianity?

$$\mathrm{kurt}(y) = \frac{\mathrm{E}[(y-\mu_y)^4]}{\sigma^4} \tag{1}$$

$$\mathrm{exkurt}(y) = \frac{\mathrm{E}[(y-\mu_y)^4]}{\sigma^4} - 3 \tag{2}$$

However Kurtosis is sensitive to outliers. Negentropy is more robust and is invariant for invertible linear transformations.



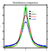
Negentropy

$$
\begin{aligned}
J(y) &= H(y_{gauss}) - H(y) \\
\text{entropy: } H(y) &= -\int f_y(\eta) \log f_y(\eta) d\eta.
\end{aligned}
$$

Here y_{gauss} is a Gaussian variable with $\mathrm{cov}(y_{gauss}) = \mathrm{cov}(y)$.

Negentropy calculation requires the knowledge of the pdf of $y(t)$.
Recall that Gaussian variable has the maximal entropy given the same variance:

$$
H(y_{gauss}) = \frac{1}{2} \log |\det \Sigma| + \frac{d}{2}(1 + \log 2\pi)
$$

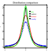


Negenentropy is scale-invariant

Given $h = My$ we have $\mathrm{E}\, hh^\top = M\Sigma M^\top$:

$$
\begin{aligned}
J(My) &= H(My_{gauss}) - H(My) \\
&= \frac{1}{2}\log|\det\Sigma| + 2\frac{1}{2}\log|\det M| + \frac{d}{2}(1+\log 2\pi) \\
&\quad -\{H(y) + \log|\det M|\} \\
&= \frac{1}{2}\log|\det\Sigma| + \frac{d}{2}(1+\log 2\pi) - H(y) \\
&= H(y_{gauss}) - H(y) = J(y)
\end{aligned}
$$

Negentropy requires the knowledge of the pdf in advance.
FastICA based on the approximations of negentropy.

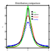


Negentropy approximations and FastICA

Find approximations of negentropy that are computationally feasible and apply a fast gradient method, see Hyvärinen (1998).

$$J(y) \approx k_1[\mathrm{E}\{G_1(y)\}]^2 + k_2[\mathrm{E}\{G_2(y)\} - \mathrm{E}\{G_2(y_{gauss})\}]^2$$

where G_1 is an odd function and G_2 an even function.



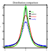
Negentropy approximation

Approximation a: $k_1 = 36/(8\sqrt{3} - 9)$ and $k_2^a = 1/(2 - 6/\pi)$

$$
\begin{aligned}
J(y) &\approx k_1[\mathrm{E}\{x \exp(-y^2/2)\}]^2 + k_2^a[\mathrm{E}\{\exp(-y^2/2)\} - \sqrt{1/2}]^2 \\
G_1^a(y) &= y \exp(-y^2/2) \\
G_2^a(y) &= \exp(-y^2/2)
\end{aligned}
$$

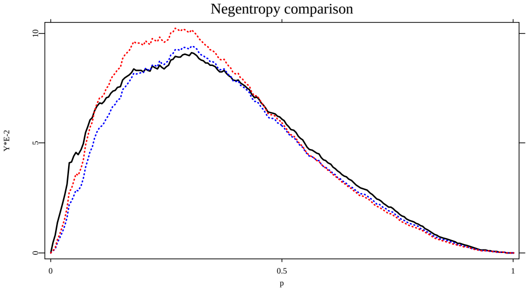
Approximation b: $k_1 = 36/(8\sqrt{3} - 9)$ and $k_2^b = 24/(16\sqrt{3} - 27)$

$$
\begin{aligned}
J(y) &\approx k_1[\mathrm{E}\{x \exp(-y^2/2)\}]^2 + k_2^b[\mathrm{E}\{|y|\} - \sqrt{2/\pi}]^2 \\
G_1^b(y) &= y \exp(-y^2/2) \\
G_2^b(y) &= |y|
\end{aligned}
$$



Negentropy approximation example

Comparison of the true negentropy (black) and its approximations (a: red, b: blue) of simulated Gaussian mixture variable: $pN(0,1) + (1-p)N(1,4)$ *for* $p \in [0,1]$.



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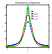


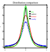
FastICA algorithm

$$J(y) \approx k_1[\mathrm{E}\{G_1(y)\}]^2 + k_2[\mathrm{E}\{G_2(y)\} - \mathrm{E}\{G_2(y_{gauss})\}]^2$$

Assume that y is symmetrically distributed, we can obtain:

$$\begin{aligned} J(y|W) &\approx k_2[\mathrm{E}\{G(y)\} - \mathrm{E}\{G(y_{gauss})\}]^2 \\ &\propto [\mathrm{E}\{G(Wx)\} - \mathrm{E}\{G(\mathrm{N}(0,1))\}]^2 \end{aligned} \tag{3}$$

where $G(\cdot)$ is an even function (denoted as G_2 before).

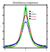


Proposed negentropy approximations

$$
\begin{aligned}
G(y) &= \frac{1}{a}\log\cosh ay, \quad 1 \le a \le 2 && (4)\\
g(y) \stackrel{\text{def}}{=} G'(y) &= \tanh(ay) && (5)\\
g'(y) &= a\{1-\tanh^2(ay)\} && (6)
\end{aligned}
$$

very often, $a = 1$ is taken in this approximation.

$$
\begin{aligned}
G(y) &= -\exp(-y^2/2) && (7)\\
g(y) \stackrel{\text{def}}{=} G'(y) &= y\exp(-y^2/2) && (8)\\
g'(y) &= (1-y^2)\exp(-y^2/2) && (9)
\end{aligned}
$$



Objective function:

$$\{\mathrm{E}\{G(Wx)\} - \mathrm{E}[G\{\mathrm{N}(0,1)\}]\}\,\mathrm{E}\{xg(Wx)\} = 0 \tag{10}$$

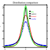
$$\omega = G(Wx) - \mathrm{E}[G\{\mathrm{N}(0,1)\}]. \tag{11}$$

For example, if g is the tanh function, ω is -1 for leptokurtic ICs. A faster gradient method can be formulated as:

$$\mathrm{E}\{xg(Wx)\} + \chi W = 0 \tag{12}$$

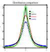
The iteration of w_i with respect to y_i:

$$w_i^{(n+1)} = \mathrm{E}[xg(w_i^{(n)}x) - \mathrm{E}\{g'(w_i^{(n)}x)\}w_i^{(n)}] \tag{13}$$



A fast fixed-point algorithm (Gram-Schmidt-like decorrelation):

1. Set the number of ICs as d and $j = 1$.
2. Set $i = j$ and choose an initial vector w_i of unit norm.
3. Let $\tilde{w}_i = \mathrm{E}\{xg(w_i x)\} - \mathrm{E}\{g'(w_i x)\} w_i$, where g is the first derivative of G, g' the second derivative.
4. Orthogonalization 1 (decorrelated):
   $\tilde{w}_i = w_i - \sum_{k \neq i} (w_i^\top w_k) w_k$
5. Orthogonalization 2: $w_i = \tilde{w}_i / ||\tilde{w}_i||$
6. If not converged, go back to 3.
7. Set $j = j + 1$. For $j \leq d$, go back to step 2.

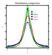


A fast fixed-point algorithm (symmetric orthogonalization):

1. Set the number of ICs as d.
2. Set $i = 1, \cdots, d$ and choose d initial vectors w_i of unit norm.
3. For each i, $\tilde{w}_i = \mathrm{E}\{xg(w_i x)\} - \mathrm{E}\{g'(w_i x)\}w_i$.
4. Symmetric othogonalization of $W = (w_1, \cdots, w_d)^\top$:

$$W = (WW^\top)^{-1/2} W$$

5. If not converged, go back to 3.



Foreign exchange rate portfolio

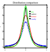
Data: (DEM/USD, GBP/USD) daily rates from 1979-12-01 to 1994-04-01 (3720 observations), available at http://www.quantlet.org/mdbase

ICA:

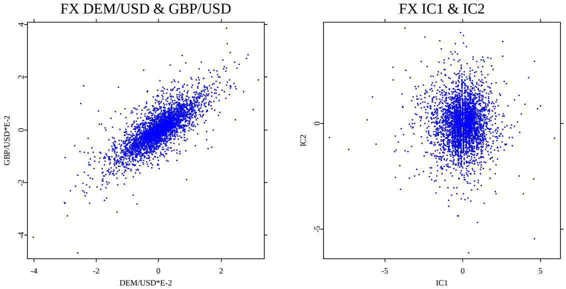
$$\hat{W} = \begin{pmatrix} 207.93 & -213.63 \\ 77.72 & 73.29 \end{pmatrix} \qquad \hat{A} = \begin{pmatrix} 2.30e-3 & 6.71e-3 \\ -2.44e-3 & 6.53 \end{pmatrix}$$

GH parameters estimation:

$$\begin{aligned} \text{IC1:} \quad & HYP(1.71, -0.17, 0.55, 0.13) \\ & NIG(1.22, -0.18, 1.11, 0.13) \\ \text{IC2:} \quad & HYP(1.78, 0.03, 0.72, -0.02) \\ & NIG(1.37, 0.03, 1.28, -0.03) \end{aligned}$$



Scatterplots of the FX returns (left) and ICs (right) of the FX portfolio.



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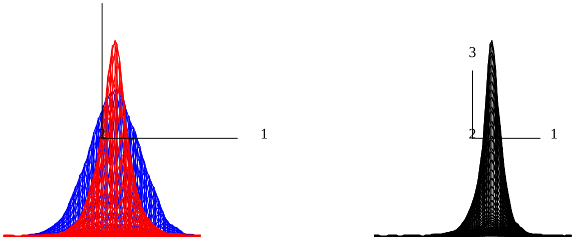
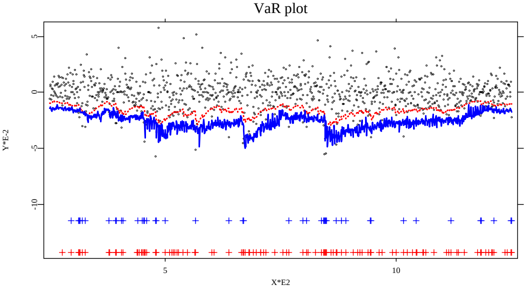
The estimated joint densities of the returns and ICs of DEM/USD rates from 1979-12-01 to 1994-04-01 (black: nonparametric kernel density, red: Gaussian density, blue: density of ICs.



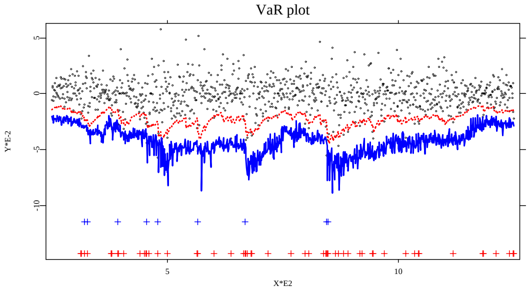
ICAfxjointdensitycomp.xpl

VaR forecasts at 95% level of exchange rate portfolio (DEM/USD and GBP/USD) for 1000 days to 1994-04-01, $\beta = (1,1)^\top$. VaR forecasts and exceedances of the proposed methodology are marked in blue while those based on the RiskMetrics are marked in red.



ICAfxVaRplot.xpl

VaR forecasts at 99% level of exchange rate portfolio (DEM/USD and GBP/USD) for 1000 days to 1994-04-01, $\beta = (1, 1)^\top$.



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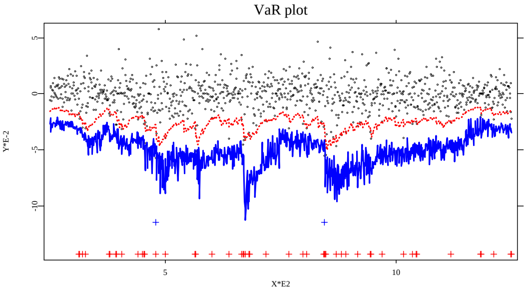
VaR forecasts at 99.5% level of exchange rate portfolio (DEM/USD and GBP/USD) for 1000 days to 1994-04-01, $\beta = (1,1)^\top$.



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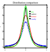
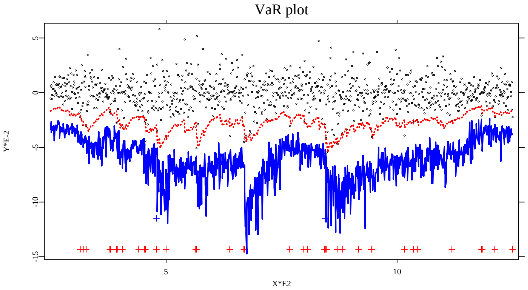


VaR forecasts at 99.75% level of exchange rate portfolio (DEM/USD and GBP/USD) for 1000 days to 1994-04-01, $\beta = (1,1)^\top$.



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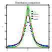
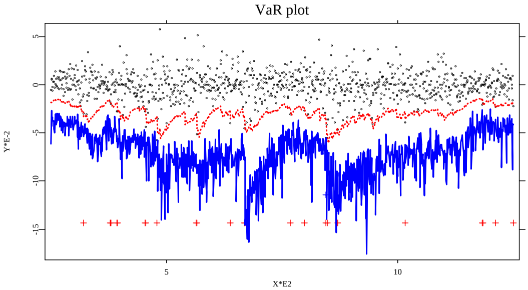


VaR forecasts at 99.9% level of exchange rate portfolio (DEM/USD and GBP/USD) for 1000 days to 1994-04-01, $\beta = (1,1)^\top$.

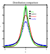


ICAfxVaRplot.xpl

Backtesting

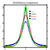
	(10^{-2})	Proposed Methodology			RiskMetrics Method		
$b^\top$	a	(n/τ)%	LR1	LR2	(n/τ)%	LR1	LR2
(1,1)	5.00	4.5	0.54	31.91	10.8	53.96	19.85
(1,1)	1.00	0.9	0.10	0.00	5.5	99.60	21.71
(1,1)	0.50	0.2	2.34	0.00	5.0	142.33	23.44
(1,1)	0.25	0.2	0.11	0.00	3.8	137.10	21.05
(1,1)	0.10	0.1	0.00	0.00	2.3	100.72	27.74
(1,2)	5.00	4.1	1.81	27.22	10.9	55.64	24.79
(1,2)	1.00	0.9	0.10	0.00	5.3	92.67	22.38
(1,2)	0.50	0.4	0.22	0.00	4.7	128.43	21.78
(1,2)	0.25	0.2	0.11	0.00	4.0	148.23	24.41
(1,2)	0.10	0.0	2.00	-NAN	2.5	113.53	18.06

LR1 is the likelihood ratio w.r.t. the risk level test and LR2 w.r.t. the independence test.

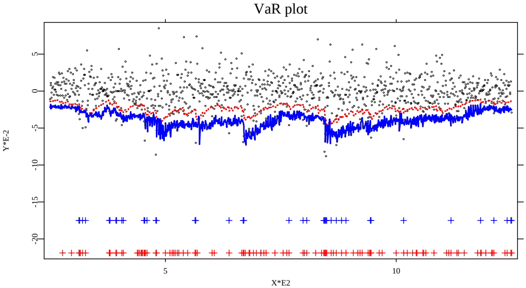


Backtesting

	(10^{-2})	Proposed Methodology			RiskMetrics Method		
$b^\top$	a	$(n/\tau)\%$	LR1	LR2	$(n/\tau)\%$	LR1	LR2
(-1,2)	5.00	3.9	2.75	29.63	12.0	75.40	35.67
(-1,2)	1.00	1.0	0.00	5.35	6.0	117.58	25.86
(-1,2)	0.50	0.6	0.19	9.11	4.4	114.93	18.33
(-1,2)	0.25	0.2	0.11	0.00	3.1	99.92	21.27
(-1,2)	0.10	0.0	2.00	-NAN	2.5	113.53	23.82
(-2,1)	5.00	3.6	4.55	11.89	12.7	89.19	18.41
(-2,1)	1.00	0.8	0.43	0.00	5.6	103.12	14.79
(-2,1)	0.50	0.4	0.22	0.00	4.2	106.16	2.41
(-2,1)	0.25	0.1	1.17	0.00	3.2	105.05	5.54
(-2,1)	0.10	0.0	2.00	-NAN	2.3	100.72	6.22



VaR forecasts at 95% level of exchange rate portfolio (DEM/USD and GBP/USD) for 1000 days to 1994-04-01, $\beta = (1,2)^\top$.



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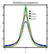
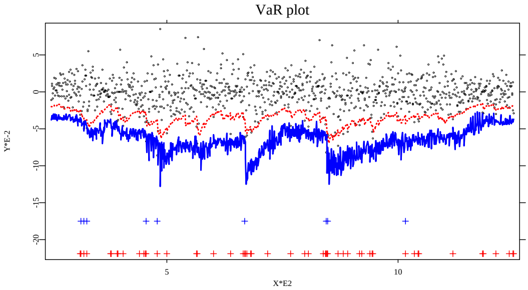


VaR forecasts at 99% level of exchange rate portfolio (DEM/USD and GBP/USD) for 1000 days to 1994-04-01, $\beta = (1,2)^\top$.



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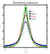
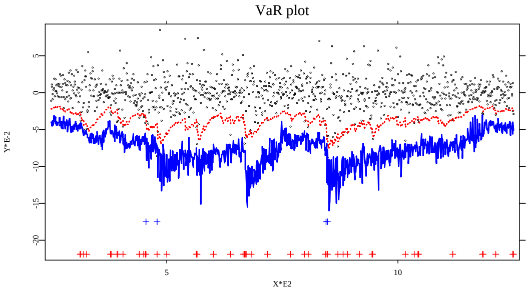


VaR forecasts at 99.5% level of exchange rate portfolio (DEM/USD and GBP/USD) for 1000 days to 1994-04-01, $\beta = (1,2)^\top$.



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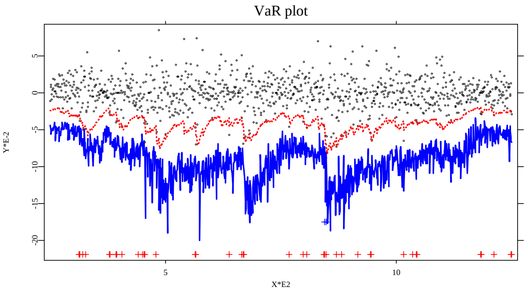

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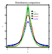
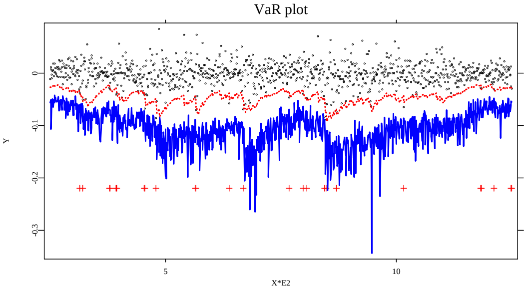


VaR forecasts at 99.9% level of exchange rate portfolio (DEM/USD and GBP/USD) for 1000 days to 1994-04-01, $\beta = (1,2)^\top$.



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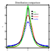
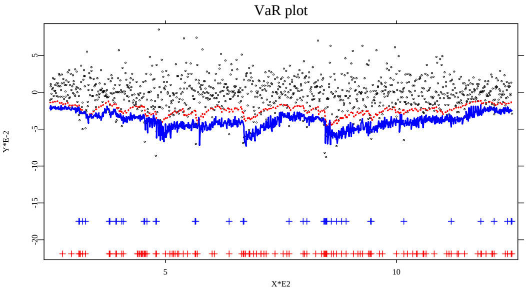


VaR forecasts at 95% level of exchange rate portfolio (DEM/USD and GBP/USD) for 1000 days to 1994-04-01, $\beta = (-1, 2)^\top$.



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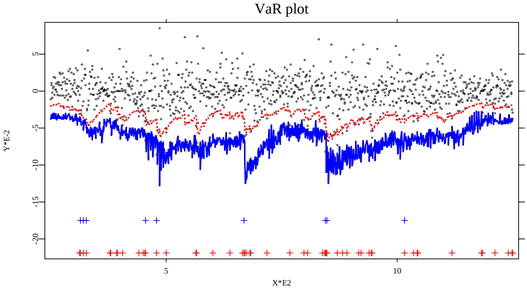
VaR forecasts at 99% level of exchange rate portfolio (DEM/USD and GBP/USD) for 1000 days to 1994-04-01, $\beta = (-1, 2)^\top$.



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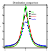

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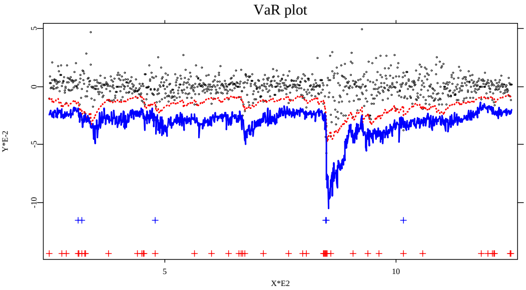

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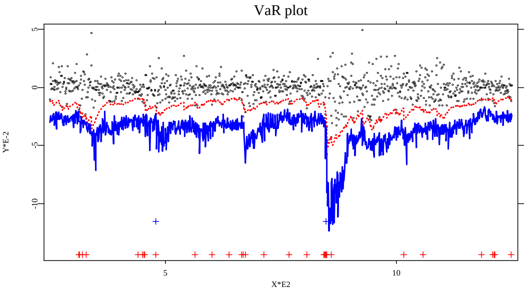

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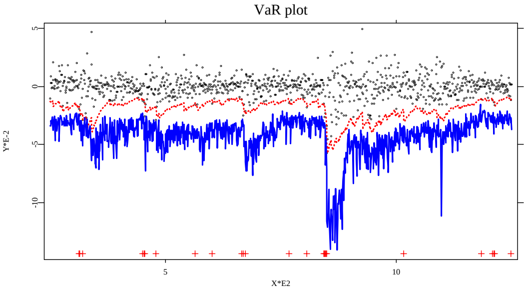

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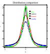
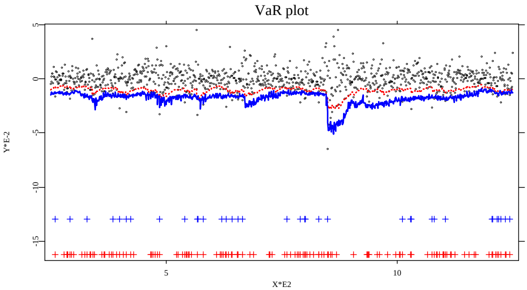


VaR forecasts at 95% level of exchange rate portfolio (DEM/USD and GBP/USD) for 1000 days to 1994-04-01, $\beta = (-2, 1)^\top$.



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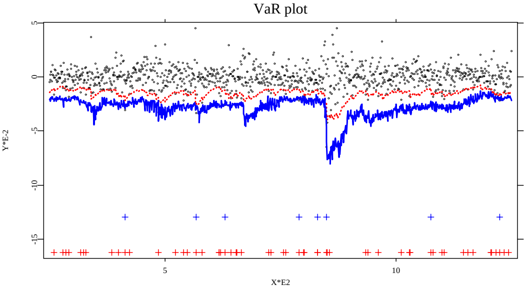
VaR forecasts at 99% level of exchange rate portfolio (DEM/USD and GBP/USD) for 1000 days to 1994-04-01, $\beta = (-2, 1)^\top$.



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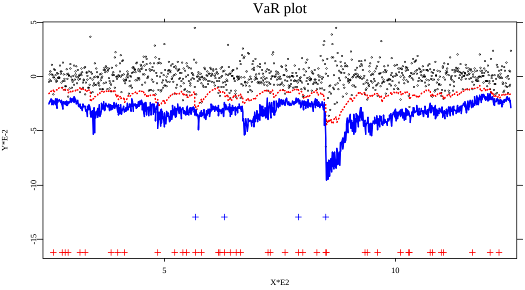
VaR forecasts at 99.5% level of exchange rate portfolio (DEM/USD and GBP/USD) for 1000 days to 1994-04-01, $\beta = (-2, 1)^\top$.



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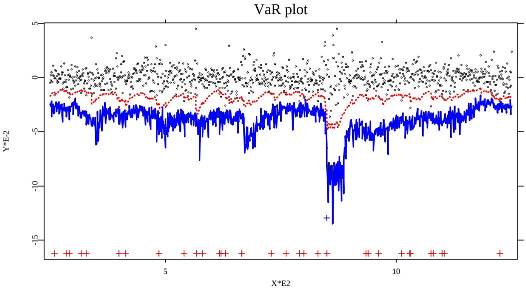

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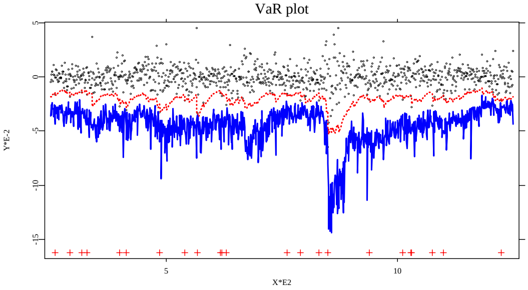

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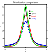


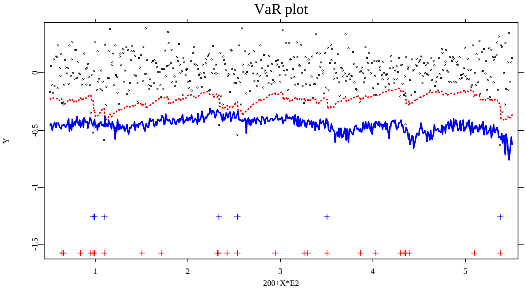
Study on German stock portfolio

Data: 20 components (allianz, basf, bayer, bmw, cobank, daimler, deutsche bank, degussa, dresdner, hoechst, karstadt, linde, man, mannesmann, preussag, rwe, schering, siemens, thyssen, volksw) 5748 observations from 1974-01-02 to 1996-12-30. Downloaded from MD*Base.

Backtesting

$(10^{-}2)$	Proposed Methodology			RiskMetrics Method		
a	$(n/\tau)\%$	LR1	LR2	$(n/\tau)\%$	LR1	LR2
0.10	0.0	1.00	-NAN	2.0	41.10	0.15
0.25	0.4	0.38	0.00	2.6	37.67	0.28
0.50	0.8	0.76	0.00	3.4	36.60	1.52
1.00	1.4	0.72	0.05	5.0	41.29	3.52
5.00	2.4	8.74	2.73	10.0	20.65	3.97

VaR forecasts at 99% level of the 20 German stocks portfolio for 500 days to 1996-12-30, β is a unit vector.



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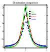
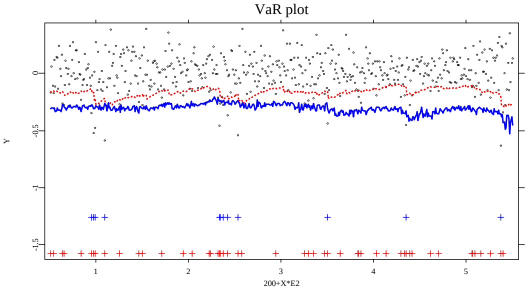


VaR forecasts at 95% level of the 20 German stocks portfolio for 500 days to 1996-12-30, β is a unit vector.



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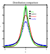
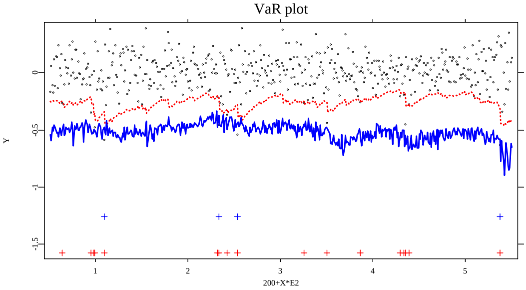


VaR forecasts at 99.5% level of the 20 German stocks portfolio for 500 days to 1996-12-30, β is a unit vector.



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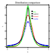

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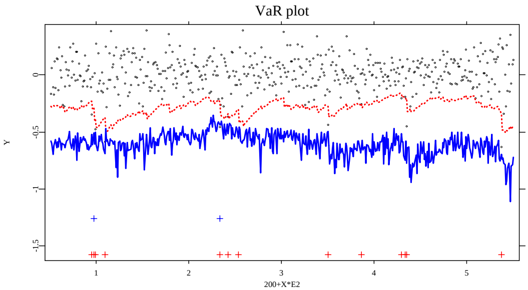

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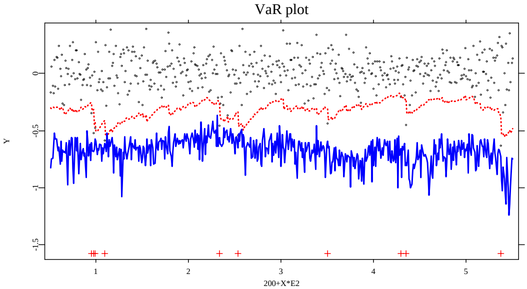
VaR forecasts at 99.9% level of the 20 German stocks portfolio for 500 days to 1996-12-30, β is a unit vector.



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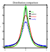

Conclusion

1. CHICA
2. Dimension reduction
3. Portfolio including bond and derivatives

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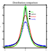

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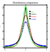

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