High Dimensional Nonstationary Time Series Modeling

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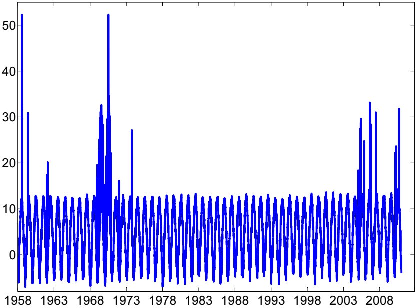
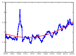
Temperatures and Climate Change

- China Meteorological Administration, $J = 159$ weather stations in China from 19570101 - 20091231
- Daily observations (averaged over stations), $T = 19358$





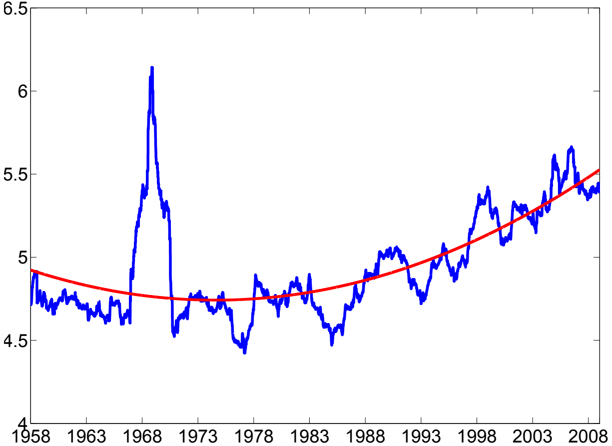


Figure 1: The moving average (of 730 nearby days) temperatures of China of 19570101 - 20091231

Weather Derivatives

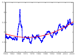
Econometrics

$$T_t$$
$$\downarrow$$
$$X_t = T_t - \Lambda_t$$
$$\downarrow$$
$$X_{t+3} = a^\top X_t + \sigma_t \varepsilon_t$$
$$\downarrow$$
$$\hat{\varepsilon}_t = \frac{\hat{X}_t}{\hat{\sigma}_t} \sim N(0,1)$$

Fin. Mathematics

$$CAR(3)$$
$$\downarrow$$
$$F_{CAT(t,\tau_1,\tau_2)}$$
$$\downarrow$$
$$MPR$$

- Detect complex trends, evaluate "non priced" places



Risk Perception

functional Magnetic Resonance Imaging

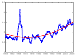
measures the oxygen level (BOLD) in the blood every 2-3 sec

Data Set

Series of 3-dim images

- each scan transformed on the resolution $2 \times 2 \times 2mm^3$
- 91 slices
- observed every 2.5 seconds
- data set: series of $T = 1360$ images with $91 \times 109 \times 91$ voxels

High-dimensional, high frequency & large data set.



Functional Magnetic Resonance Imaging

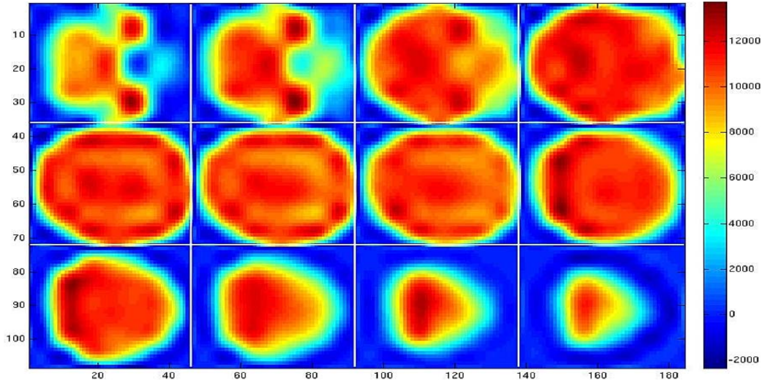
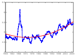


Figure 2: Typical example of fMRI image in a particular time point, 12 different horizontal slices of the brain's scan. fMRI

Risk Patterns and Brain Activities

- Which part is activated during *risk related decisions* ?
- Can statistical analysis help to detect this area?
- Can we provide an *integrated* dynamic analysis?
- Response curve (to stimuli)? classify "risky people"?



Implied Volatility Surface

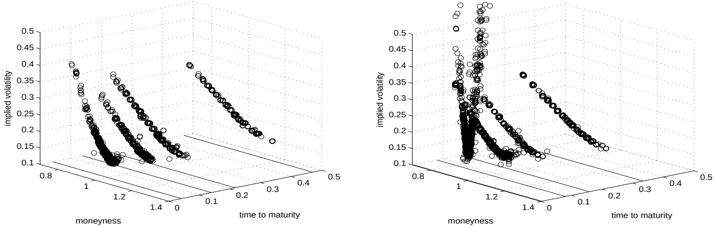
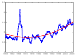


Figure 3: *Typical IV data design on two different days. Bottom solid lines indicate the observed maturities, which move towards the expiry. Left panel: observations on 20040701. Right panel: observations on 20040819.*

IVS

IVS

- "Strings", "Smile", "Skewness", Dimensionality not fixed
- Trading, hedging and risk management of option portfolios
- IVS reflects perception of market risk, Bakshi et al. (2000)...



Order Book

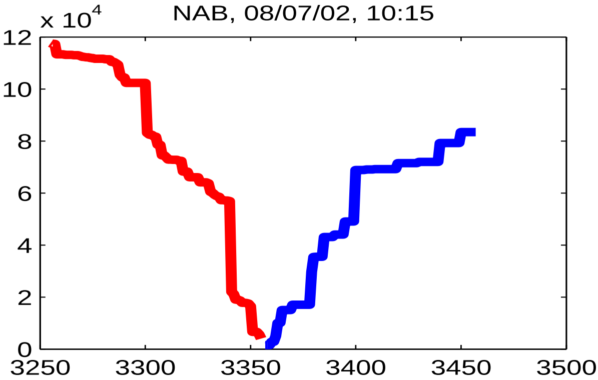


Figure 4: Bid and ask curves constructed from the order book of National Australian Bank stock prices on 20020801. LOB

Collateralized Debt Obligation

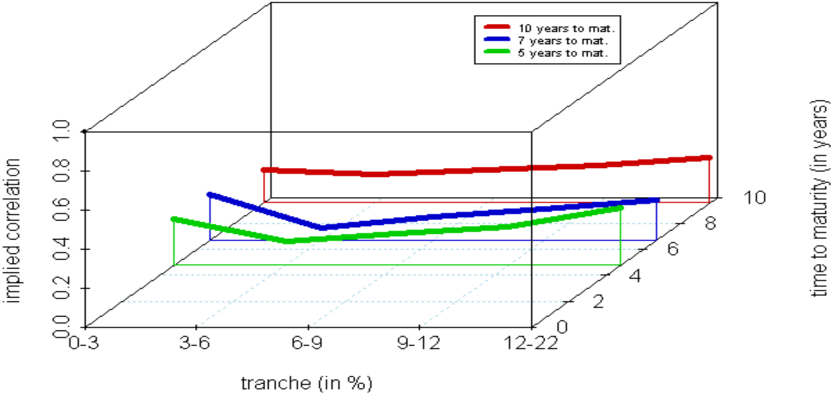
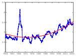


Figure 5: Compound correlations on 070321 w.r.t. time to maturity (in years), implied correlation and tranche (in %).  CDO



CO_2 Emission Allowance

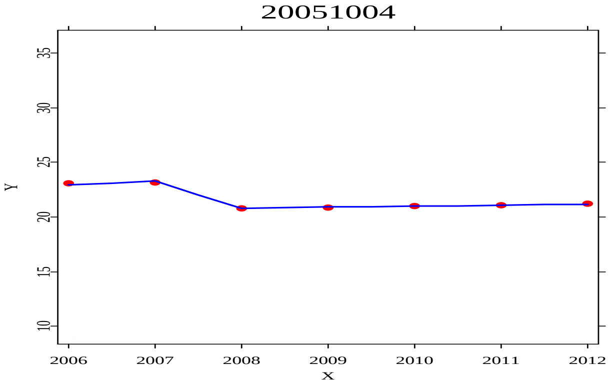
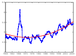


Figure 6: Term structure for CO_2 emission allowance's spot and futures prices, trading on 20051004 in the EEX market. CO2



Empirical Pricing Kernel

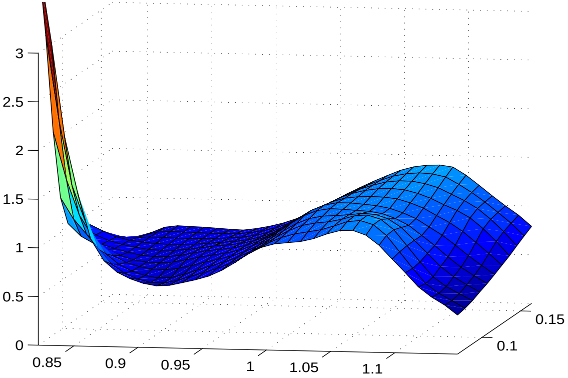


Figure 7: Estimated PK across moneyness κ and maturity τ at t = 20010710. EPK

Electricity Forward Prices

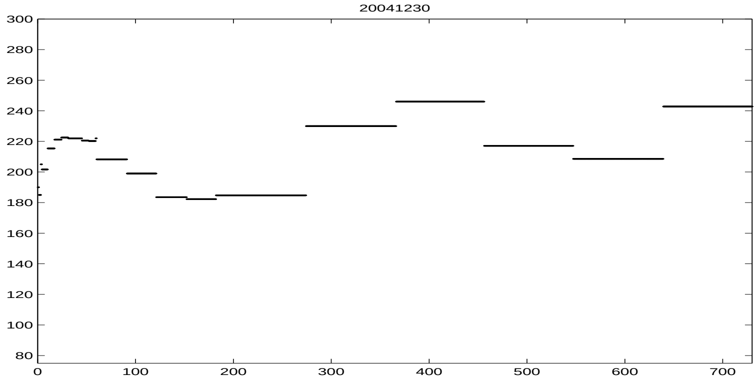


Figure 8: Term structure of the electricity prices (NOK/MWh) from the Nord Pool on 20041230. EFP

High Dimensional Nonstationary Time Series

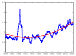
- Medicine
  - fMRI data
- Meteorology
  - Temperatures, rainfall etc. from many stations
- Finance
  - Implied Volatility Surface
  - Order Book
  - Collateralized Debt Obligation
  - CO_2 Emission Allowance
  - Empirical Pricing Kernel
  - Electricity Forward Price
- “eBird”, tree rings, . . .

Formal Setting

$$\underbrace{(X_{1,1}, Y_{1,1}), \ldots, (X_{J_1,1}, Y_{J_1,1})}_{t=1} \underbrace{(X_{1,2}, Y_{1,2}), \ldots}_{t=2} \ldots \underbrace{\ldots (X_{J_T,T}, Y_{1,T})}_{t=T}$$

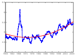
where:
$X_{j,t} \in \mathbb{R}^d, \; Y_{j,t} \in \mathbb{R}$
T - the number of observed time periods (days)
J_t - the number of the observations in (day) t
$\mathsf{E}(Y_t | X_t) = F_t(X_t)$.

What is $F_t(X_t)$? How it moves?



Basic Idea

- Use a "time & space" dynamic approach
- Separate time dynamics from space functions
- Low dim time series dynamics
- High dim (time invariant) space functions
- $\#$ of factors $\nearrow J$, fitting $\nearrow$



Dynamic Semiparametric Factor Model

$$E(Y_t|X_t) = \sum_{l=1}^{L} Z_{0,t,l} m_l(X_t) = Z_{0,t}^\top m(X_t) = Z_{0,t}^\top A\psi(X)$$

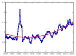
- $Z_t = (Z_{0,t,1}, \dots, Z_{0,t,L})^\top$ low dim (stationary) time series
- $m(\cdot)$ tuple of functions $(m_0, m_1, \dots, m_L)^\top$
- $\psi(x) = (\psi_1, \dots, \psi_K)^\top(x)$ vector of known basis functions
- A: $L \times K$ coefficient matrix.

Deterministic Model

$$Y_{t,j} = \sum_{l=1}^{L}\sum_{r=1}^{R} u_r(t)\gamma_{rl}\sum_{k=1}^{K} a_{lk}\psi_k(X_{t,j}) + \varepsilon_{tj} \quad (1)$$

$$Y_t^\top = \underbrace{U_t^\top \Gamma^*}_{Z_t^\top}\underbrace{A^*\Psi_t}_{m} + \varepsilon_t \stackrel{\text{def}}{=} U_t^\top \beta^{*\top}\Psi_t + \varepsilon_t. \quad (2)$$

- $U_t^\top = (u_1(t), \ldots, u_R(t))$, $u_r(t)$ time basis
- $\Psi_t = (\psi_1(X_t), \ldots, \psi_K(X_t))^\top$, $\psi_k(x)$ space basis
- $\beta^{*\top}$ $R \times K$ matrix consisting of β_{rk}s
  $\|\beta\|_{2,1} = \sum_{r=1}^{R}\sqrt{\sum_{k=1}^{K}\beta_{rk}^2}$ (group Lasso)

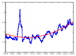


Generalized Dynamic Semiparametric Factor Model

$$
\begin{aligned}
Y_t^\top &= (Z_{0,t}^\top + U_t^\top \Gamma) A \Psi_t + \varepsilon_t' = U_t^\top \Gamma A \Psi_t + (Z_{0,t}^\top A \Psi_t + \varepsilon_t') \\
&\stackrel{\text{def}}{=} U_t^\top \Gamma A \Psi_t + \varepsilon_t, \quad \textit{with} \quad \mathsf{E}(Z_{0,t} | X_t) = 0.
\end{aligned}
$$

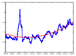
2 Step Estimation Procedure

- Find the trend based on $Y_t^\top = U_t^\top \Gamma A \Psi_t + \varepsilon_t$
- Based on $\widehat{\widehat{Y}}_t^\top \stackrel{\text{def}}{=} Y_t^\top - U_t^\top \widehat{\beta} \Psi_t$, $\widehat{A}$ and Ψ_t, obtain $\widehat{Z}_{0,t}$



Questions

- How to fit GDSFM?
- What risk is involved?
- How to select basis?

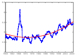


Overview

1. Data & Motivation✓
2. Estimation
3. Its Properties
4. Generalized Dynamic Semiparametric Factor Model
5. Weather, fMRI and IVS
6. How well are we doing?

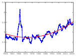
Time Basis

- Global trend: 1, t, $(3t^2 - 1)/2, \dots$ Legendre Polynomial
- Local variation: $\sin\{it/(p2\pi)\}$, $\cos\{it/(p2\pi)\}$, $i = 1, \dots$ Fourier Series
- Period $p = 11.8$ (fMRI), $365, 3650$ (weather)



Lasso & Group Lasso

- Lasso (least absolute shrinkage and selection operator), Tibshirani (1996)
- Group Lasso: Yuan and Lin (2006)
- Shrink some coefficients to 0
- Advantages over subset and ridge regressions



Space Basis

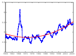
- FPCA (data dependent basis), motivated by Hall et. al (2006)
- ψ_k: eigenfunctions of the smoothed covariance operator

$$\widehat{\psi}(u,v) = \widehat{a}_0(u,v) - \widehat{a}(u)\widehat{a}(v)$$

$$\sum_{t=1}^{T}\sum_{j=1}^{J_t}\{Y_{tj} - a - \sum_{c=1}^{d} b^c(u^c - X_{tj}^c)\}^2 K\left(\frac{X_{tj}-u}{h_\mu}\right)$$

$$\sum_{t=1}^{T}\sum_{1\leqslant j\neq k\leqslant J_t}\{Y_{tj}Y_{tk} - a_0 - \sum_{c=1}^{d} b_1^c(u^c - X_{tj}^c) - \sum_{c=1}^{d} b_2^c(v^c - X_{tk}^c)\}^2$$

$$\times K\left(\frac{X_{tj}-u}{h_\phi}\right)K\left(\frac{X_{tj}-v}{h_\phi}\right).$$



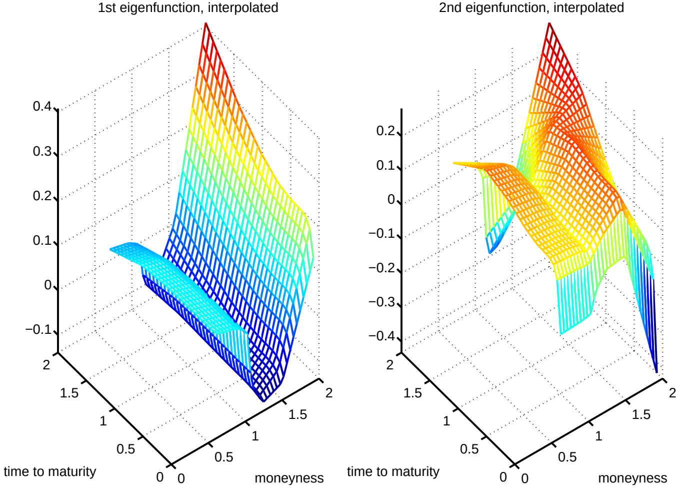
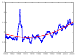


Figure 9: IVS Modeling using FPCA for space basis.



Estimation Procedure

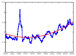
0 Space basis Ψ_t via FPCA

1 Time basis U_t selection via group Lasso; $\widehat{\beta}$ $(R \times K)$

$$\min_{\beta} (TJ)^{-1} \sum_{t=1}^{T} \left(Y_t^\top - U_t^\top \beta^\top \Psi_t \right) \left(Y_t^\top - U_t^\top \beta^\top \Psi_t \right)^\top + 2\lambda \|\beta\|_{2,1} \quad (3)$$

2 Split $\widehat{\beta}$ into $\widehat{\Gamma}$, $\widehat{A}$

$\widehat{\Gamma}$: first "L" eigenvectors of $\widehat{\beta}\widehat{\beta}^\top$; $\quad \widehat{A} = \widehat{\Gamma}^\top \widehat{\beta}$

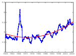


Tuning parameter λ

- Take 100 equally spaced $\lambda \in [0, \max_r \| \sum_t \Psi_t Y_t U_{tr} \| / \sqrt{K}]$
- Evaluate $C_p(\lambda)$ where

$$
\begin{aligned}
C_p(\lambda) &= \frac{\sum_t \| Y_t^\top - U_t^\top \widehat{\beta}^\top \Psi_t \|^2}{\tilde{\sigma}^2} - JT + 2df \\
\tilde{\sigma}^2 &= \frac{\sum_t \| Y_t^\top - U_t^\top \widehat{\beta}_{OLS}^\top \Psi_t \|^2}{JT - df} \\
df &= \sum_r \mathbf{1}\{\| \widehat{\beta}_r \| > 0\} + \sum_r \frac{\| \widehat{\beta}_r \|}{\| \widehat{\beta}^{OLS} \|}(K-1)
\end{aligned}
$$

- Choose the minimal $C_p(\lambda)$



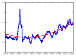
Theorem 1: Risk Bound (Gaussian)

Under Assumptions (A1, A2, A3, A4, 1), let $\lambda = 2\sigma/\sqrt{JT}\sqrt{1 + A\log R/\sqrt{T}}$ with $A > 8$, and then with probability at least $1 - R^{1-q}$ with $q = \min(A\log R, \sqrt{T})$, for all $\widehat{\beta}$:

$$(JT)^{-1}\sum_{t=1}^{T} \| \Psi_t^\top(\widehat{\beta} - \beta^*)U_t \|^2 \leqslant 64\sigma^2 s(1 + A\log R/\sqrt{T})/(\kappa^2 J),$$

$$T^{-1/2}\| \widehat{\beta} - \beta^* \|_{2,1} \leqslant 32\sigma s\sqrt{1 + A\log R/\sqrt{T}}/(\kappa^2\sqrt{J}),$$

$$M(\widehat{\beta}) \leqslant 64\phi^2_{max} s/\kappa^2$$



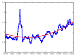
Theorem 2: Risk Bound (Non-Gaussian)

Under Assumptions (A1, A2, A3, A5, 1, 2), let $\lambda = \sigma\sqrt{(\log R)^{1+\delta}/JT}$, $\delta > 0$, and then with probability at least $1 - (2e\log R - e)C/(\log R)^{1+\delta}$, for all $\widehat{\beta}$:

$$(JT)^{-1}\sum_{t=1}^{T} \| \Psi_t^\top(\widehat{\beta} - \beta^*)U_t \|^2 \leqslant 16\sigma^2 s(\log R)^{1+\delta}/(\kappa^2 J)$$

$$T^{-1/2}\| \widehat{\beta} - \beta^* \|_{2,1} \leqslant 16\sigma s\sqrt{(\log R)^{1+\delta}}/(\kappa^2\sqrt{J})$$

$$M(\widehat{\beta}) \leqslant 64\phi_{max}^2 s/\kappa^2$$



Theorem 3: Risk Bound (Dependent)

Under Assumptions (A1, A2, A3, 1, 3), let

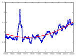
$$\lambda = \frac{C'}{\sqrt{T}} + \sqrt{\frac{\mathcal{X}^*(\mathcal{T}) \sum_t b_t^2}{(\log R)^{1-\delta'} T^2}}, \quad \delta' > 0,$$

and then with probability at least $p(1 - R^{-\delta'})$, for $\forall\ \widehat{\beta}$:

$$(JT)^{-1} \sum_{t=1}^{T} \| \Psi_t^\top (\widehat{\beta} - \beta^*) U_t \|^2 \leqslant 16 \left(C' + \sqrt{\frac{\mathcal{X}^*(\mathcal{T}) \sum_t b_t^2}{(\log R)^{1-\delta'} T}} \right)^2 s/\kappa^2$$

$$T^{-1/2} \| \widehat{\beta} - \beta^* \|_{2,1} \leqslant 16 \left(C' + \sqrt{\frac{\mathcal{X}^*(\mathcal{T}) \sum_t b_t^2}{(\log R)^{1-\delta'} T}} \right) s/\kappa^2$$

$$M(\widehat{\beta}) \leqslant 64 \phi_{max}^2 s/\kappa^2$$



Comparison

i.i.d. Gaussian

- Dependence on R is negligible for large T
- Low sparsity, s/κ^2 large, bounds large, $M(\widehat{\beta})$ large

Independent, bounded 2nd moment

- Dependence on R not made negligible for large T

Dependent

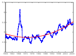
- Dependence level $\nearrow$, bound $\nearrow$

Theorem 4: $\widehat{Z}_{0,t}$ not get affected

Suppose that all assumptions in Theorem 3 and Assumptions (B1 - B7) hold. Then we have

$$\frac{1}{T}\sum_{1\leq t\leq T}\left\|\widehat{Z}_{0,t}^{\top}\widehat{A}-Z_{0,t}^{\top}A^{*}\right\|^{2}=\mathcal{O}_{P}(\rho^{2}+\delta_{K}^{2}). \quad (4)$$

since $\widehat{\beta}$ is close enough to β



Definitions

$$B \stackrel{\text{def}}{=} (\sum_{t=1}^{T} Z_{0,t}\widehat{Z}_{0,t})^{-1} \sum_{t=1}^{T} Z_{0,t} Z_{0,t}^{\top}$$

$$\widetilde{Z}_{0,t} \stackrel{\text{def}}{=} B^{\top}\widehat{Z}_{0,t}$$

$$\widetilde{Z}_{n,t} \stackrel{\text{def}}{=} (T^{-1}\sum_{s=1}^{T} \widetilde{Z}_{0,s}\widetilde{Z}_{0,s}^{\top})^{-1/2}\widetilde{Z}_{0,t}$$

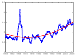
$$Z_{n,t} \stackrel{\text{def}}{=} (T^{-1}\sum_{s=1}^{T} Z_{0,s}Z_{0,s}^{\top})^{-1/2} Z_{0,t}$$

Theorem 5: Covariance Equivalence

Suppose that all assumptions in Theorem 3 and Assumptions (B1 - B7, C1 - C2) hold. Then we have for $h \geq 0$

$$T^{-1} \sum_{t=\max[1,-h+1]}^{\min[T,T-h]} \tilde{Z}_{0,t} \left(\tilde{Z}_{0,t+h} - \tilde{Z}_{0,t} \right)^\top - Z_{0,t} \left(Z_{0,t+h} - Z_{0,t} \right)^\top = \mathcal{O}_P(T^{-1/2})$$

$$T^{-1} \sum_{t=\max[1,-h+1]}^{\min[T,T-h]} \tilde{Z}_{n,t} \tilde{Z}_{n,t+h}^\top - Z_{n,t} Z_{n,t+h}^\top = \mathcal{O}_P(T^{-1/2})$$



Weather: space functions

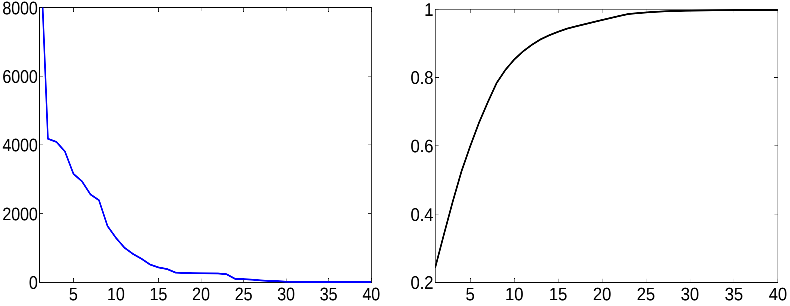
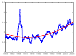
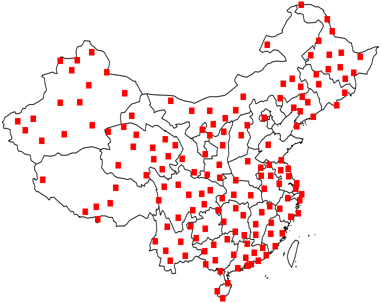


Figure 10: Distribution of the eigenvalues and the relative proportion of variance explained by the first K basis.



China Climate Types



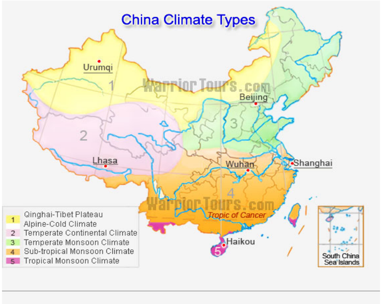
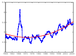


Figure 11: Weather stations & China Climate Types

Time Basis

	Factors		Factors
Trend	1	Large	$\sin\{2\pi t/(365 \cdot 10)\}$
(Year by Year)	t	Period	$\cos\{2\pi t/(365 \cdot 10)\}$
	$3t^2 - 1$		$\sin\{4\pi t/(365 \cdot 10)\}$
Seasonal	$\sin\{2\pi t/365\}$		$\cos\{4\pi t/(365 \cdot 10)\}$
Effect	$\cos\{2\pi t/365\}$		$\sin\{6\pi t/(365 \cdot 10)\}$
	$\ldots$		$\ldots$
	$\cos\{20\pi t/365\}$		$\cos\{20\pi t/(365 \cdot 10)\}$

Table 1: Initial choice of $53 \cdot 3 + 20 = 179$ time basis.



Time Basis Coefficients

- Long term: quadratic trend, warming effect
- Yearly variation ($p = 365$): earth rotation
- 10-year variation ($p = 3650$): solar activity

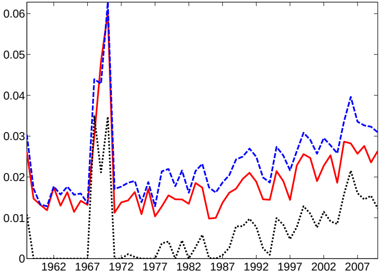
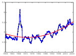


Figure 12: Estimated coefficients of the 1st factor $\widehat{\Gamma}_{r1}$ w.r.t. the yearly polynomial time basis (constant, linear, quadratic).



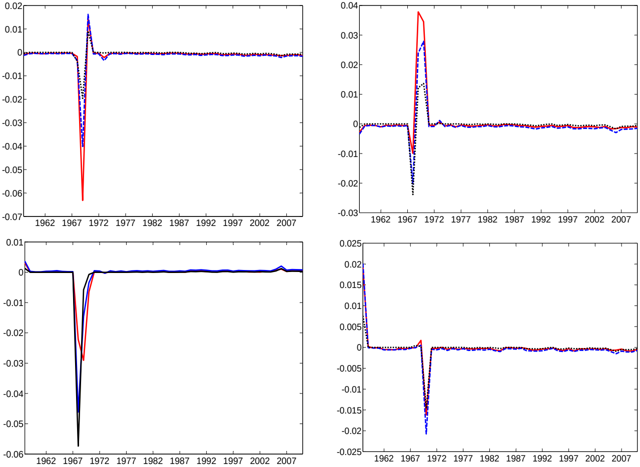
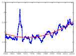
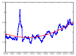


Figure 13: Estimated coefficients of the 2nd - 5th factors $\widehat{\Gamma}_{r2}, \ldots, \widehat{\Gamma}_{r5}$ w.r.t. the $54 \cdot 3$ yearly polynomial time basis.



Basis			Estimates		
$\sin 2\pi t/365$	-0.1777	0.0076	0.0177	-0.0136	0.0084
$\cos 2\pi t/365$	-0.6081	0.0126	0.0366	-0.0369	0.0114
$\sin 4\pi t/365$	0.0000	0.0000	0.0000	0.0000	0.0000
$\cos 4\pi t/365$	-0.0145	0.0028	0.0021	-0.0022	0.0029
...	0.0000	...			
$\cos 20\pi t/365$	0.0000	...			
$\sin 2\pi t/(365 \cdot 10)$	0.0025	-0.0006	0.0009	-0.0008	-0.0001
$\cos 2\pi t/(365 \cdot 10)$	0.0000	...			
...	0.0000	...			
$\cos 20\pi t/(365 \cdot 10)$	0.0000	...			

Table 2: Estimated coefficients of the 5 factors w.r.t. the 20 Fourier series time basis.



Extracted Trends

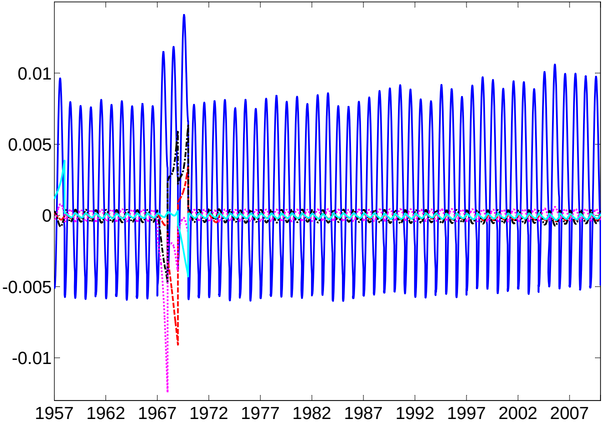
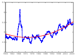


Figure 14: Extracted trends based on $U_t^\top \widehat{\beta}$.



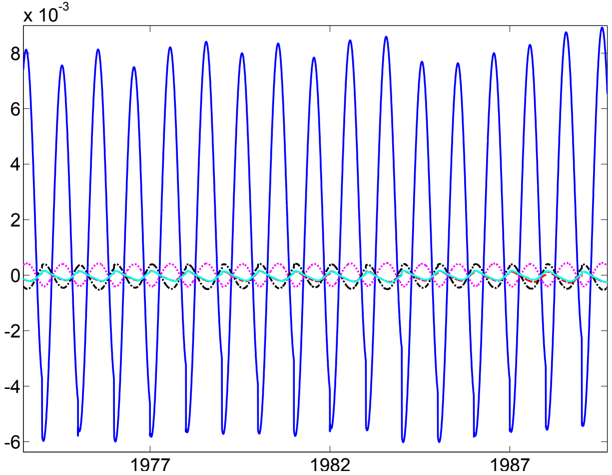
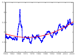


Figure 15: Extracted trends based on $U_t^\top \widehat{\beta}$.



Estimated Stochastic Process $\widehat{Z}_{0,t,1}$

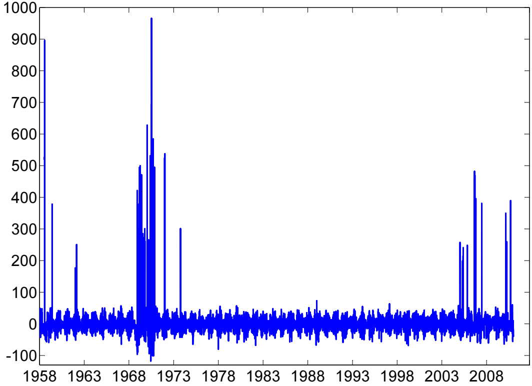
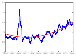


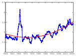
Figure 16: Estimated Stochastic Process $\widehat{Z}_{0,t,1}$



Econometric Modeling of $\widehat{Z}_{0,t}$

$\widehat{Z}_{0,t} = \mathcal{R}\widehat{Z}_{0,t-1} + \varepsilon_{0,t}$ with random vector $\varepsilon_{0,t}$ and estimated coefficient matrix:

$$\begin{pmatrix} 0.9732 & -0.0135 & -0.0002 & -0.0006 & -0.0002 \\ 0.0127 & 0.1766 & -0.1824 & -0.0682 & -0.0009 \\ 0.0358 & -0.2867 & 0.4493 & -0.1138 & 0.0053 \\ -0.0001 & -0.1967 & -0.1962 & 0.8010 & -0.0052 \\ 0.0790 & 0.0492 & 0.0690 & -0.0225 & 0.8418 \end{pmatrix}.$$



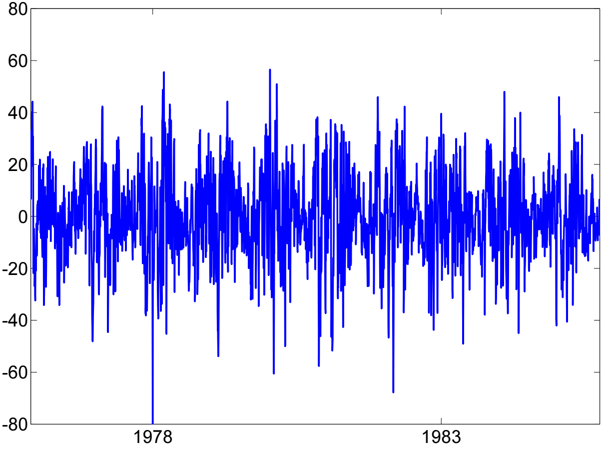


Figure 17: Estimated Stochastic Process $\widehat{Z}_{0,t,1}$

Improvements

- Periodic behavior in $\widehat{Z}_{0,t}$
- Step 1 (detrending, both weather + fMRI):
  Variance of the noise $\sim$ mean of the measurements!
- Group Lasso under heteroscedasticity with high J (Poisson - like model)

$$Y_t^\top = U_t^\top \Gamma A \Psi_t + \varepsilon_t, \quad \text{Cov}(\varepsilon_t) = diag(|U_t^\top \Gamma A \Psi_t|)$$

- Lasso under heteroscedasticity, Jia et. al. (2009)

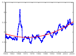
Current calibration

- 2 Steps: Fourier truncated series + GARCH(p,q) $\hat{\sigma}^2_{t,FTSG}$

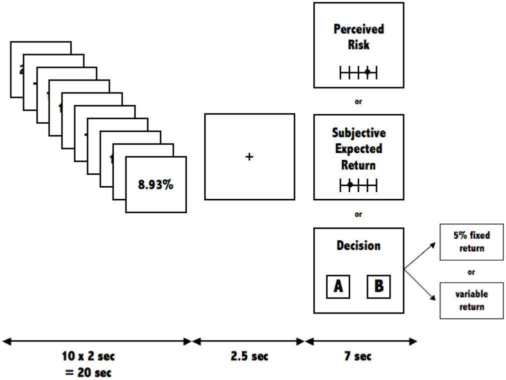
$$\begin{aligned}\sigma_t^2 &= c_1 + \sum_{i=1}^{16}\left\{c_{2i}\cos\left(\frac{2i\pi t}{365}\right) + c_{2i+1}\sin\left(\frac{2i\pi t}{365}\right)\right\}\\ &+ \alpha_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2\end{aligned}$$

- 1 Step: Local linear Regression (LLR) $\hat{\sigma}^2_{t,LLR}$, $Y_i = \hat{\varepsilon}^2_{t_i}$

$$\min_{a,b}\sum_{i=1}^{n}\left\{Y_i - a(t) - b(t)(t_i - t)\right\}^2 K\left(\frac{t_i - t}{h}\right)$$



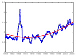
Risk Perception and Investment Decision



Returns Pause Decision

fMRI methods (Panel)

- existing methods to analyze these data: voxel-wise GLM
  - strong a priori hypothesis necessary
- new statistical method: DSFM
  - dimension reduction keeping the data structure
  - exploratory analysis



Panel Version Model with Multi Subjects $1 \leq i \leq I$

$$Y^i_{t,j} = \sum_{l=1}^{L} (\alpha^i_{t,l} + U_t^\top \Gamma^i_l) m_l(X_{t,j}) + \varepsilon_{t,j}, \;\; 1 \leq j \leq J_t, \; 1 \leq t \leq T,$$

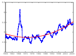
with fixed effect $\alpha^i_{t,l}$ and

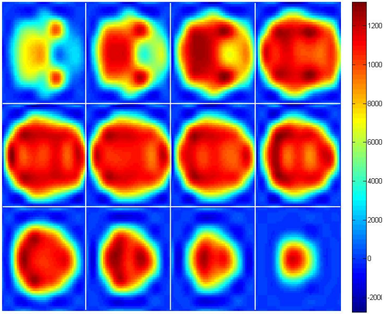
$$\sum_{i=1}^{I} \left(\sum_{l=1}^{L} \alpha^i_{t,l} m_l(X_{t,j}) | X_{t,j} \right) = 0.$$

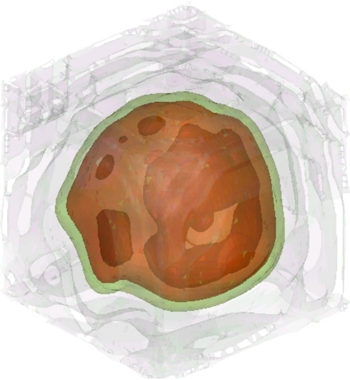
2-step estimation procedure

1. Average $Y^i_{t,j}$ over i, and estimate common m_l as in the original approach.
2. Given the common m_l, for i, estimate their specific factors in time $Z^i_{t,l}$.

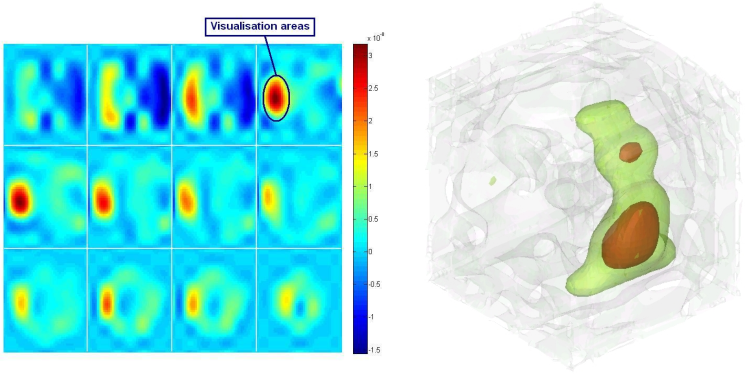
$$Y^i_{t,j} = \sum_{l=1}^{L} U_t^\top \Gamma^i_l \overline{m}_l(X_{t,j}) + \varepsilon^i_{t,j}$$



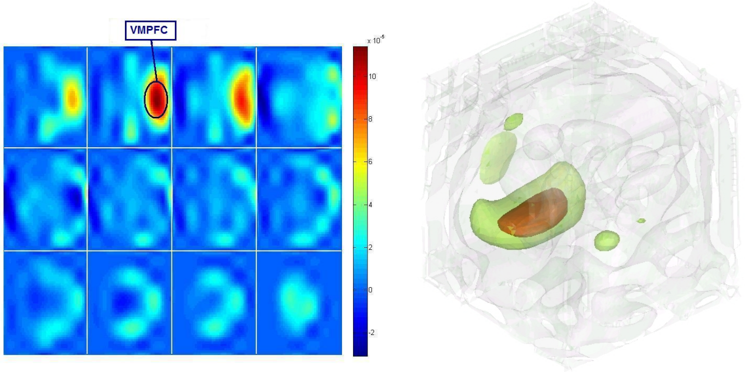




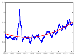
Estimated factor loading $\hat{m}_1$ with $L = 5$.

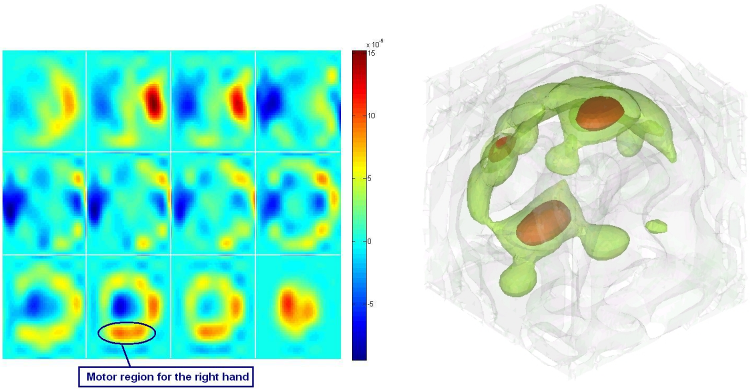


Estimated factor loading $\hat{m}_2$ with $L = 5$.

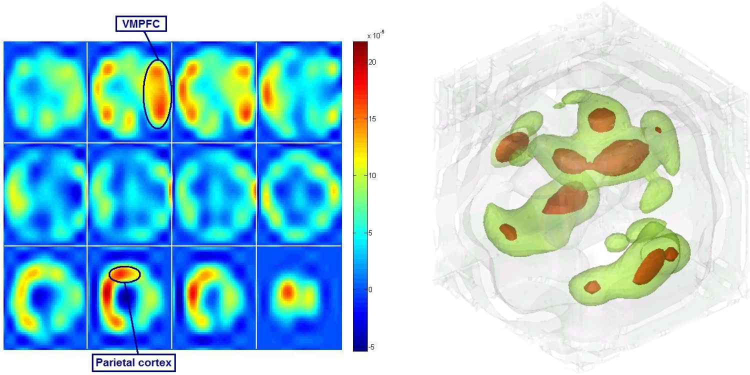


Estimated factor loading $\hat{m}_3$ with $L = 5$.
(VMPFC = Ventromedial prefrontal cortex)





Estimated factor loading $\hat{m}_4$ with $L = 5$.



Estimated factor loading $\hat{m}_5$ with $L = 5$.

Response to Stimuli

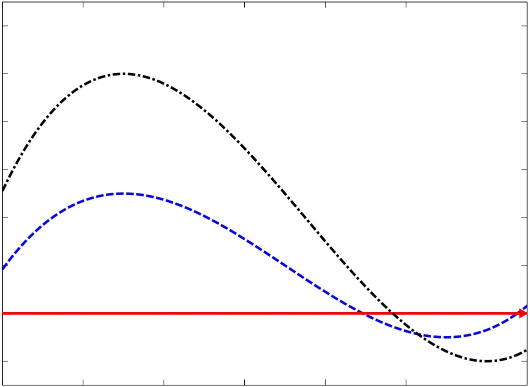
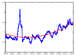


Figure 18: Response curves (to stimuli) $U_t^\top \widehat{\Gamma}_2^i$ for probands $i = 9$ & $i = 19$ with periodic cubic polynomial as time basis.



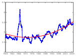
SVM Analysis (Risk)

- Different subjects' response curves have different shapes
- SVM based on the $\widehat{\beta}$

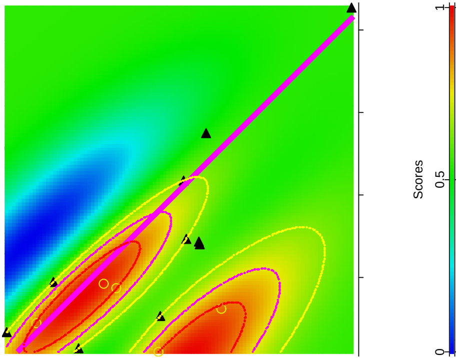
MEAN		Estimated	
Data	Strongly	0.85	0.14
	Weakly	0.59	0.40

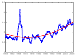
Table 3: Classification rates of the SVM method.

The rates hold over a wide range of parameters



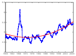
SVM Classification





Related & Future Research

- Risk Patterns and Correlated Brain Activities (with A. Myšičková)
- Tell what the subject see
- Emotions recognition from face (FOX: “Don’t lie to me”) + fMRI analysis



Implied Volatility Surface Modeling

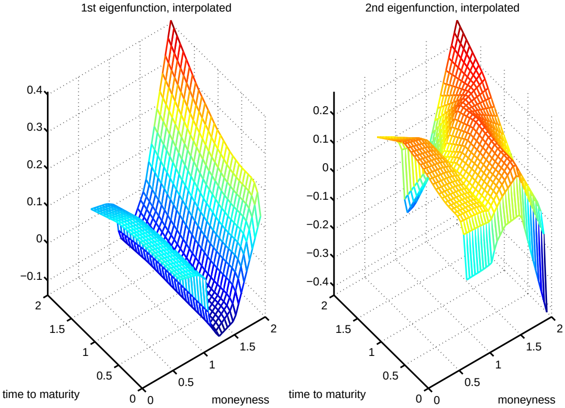
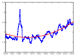


Figure 19: IVS Modeling using FPCA for space basis.



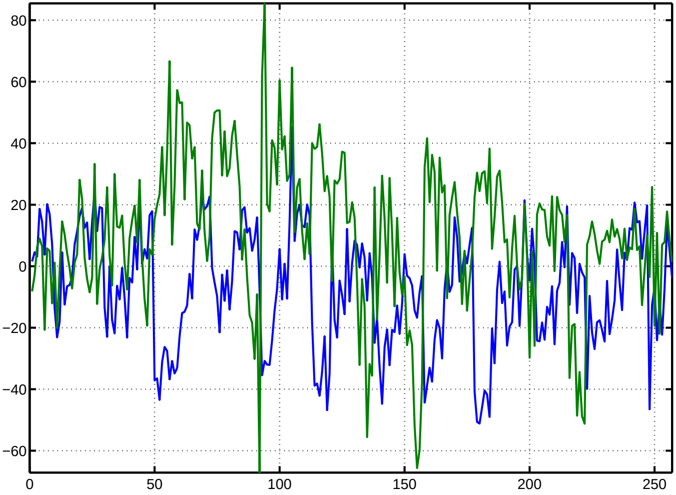


Figure 20: Estimated time series of factors $\widehat{Z}_{t,1}$, $\widehat{Z}_{t,2}$ (20070102 - 20081230)

High Dimensional Nonstationary Time Series Modeling

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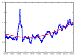

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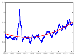

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Assumptions

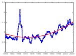
- A1 Normalization of U_t & Ψ_t: $\Psi_t \Psi_t^\top / J = I_K$, $\sum_{t=1}^{T} U_t^\top U_t / R = 1$
- A2 The number of nonzero β_r^*s: $M(\beta^*) \leq s$
- A3 ϕ_{max} is the maximum eigenvalue of $\sum_{t=1}^{T} U_t U_t^\top$
- A4 The error terms $\varepsilon_1, \ldots, \varepsilon_T$ are i.i.d. **Gaussian** with mean 0 and variance $\sigma^2 I_{J \times J}$
- A5 The error terms $\varepsilon_1, \ldots, \varepsilon_T$ are independent with mean 0 and **finite** variance $\mathrm{E}(\varepsilon_{tj}^2) \leqslant \sigma^2$

Assumption 1

There exists a positive number $\kappa = \kappa(s)$ *such that*

$$\min \left\{ \frac{\sum_t \|\Psi_t^\top \Delta U_t\|}{\sqrt{J} \parallel \Delta_{\mathcal{R}} \parallel} : |\mathcal{R}| \leqslant s, \Delta \in \mathbb{R}^{K \times R} \backslash \{0\}, \right.$$
$$\left. \parallel \Delta_{\mathcal{R}^c} \parallel_{2,1} \leqslant 3 \parallel \Delta_{\mathcal{R}} \parallel_{2,1} \right\} \geqslant \kappa.$$

- Restriction on the eigenvalues of U_t as a function of sparsity s
- Low sparsity, s big, κ small

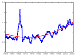


Assumption 2

The matrices Ψ_t and U_t are such that

$$(JT)^{-1} \sum_{t=1}^{T} \sum_{j=1}^{J} \left(\max_r \left| \sum_{k=1}^{K} \Psi_{tkj} U_{tr} \right| \right)^2 \leqslant C,$$

for a constant $C > 0$.

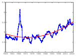


Measure of Dependence, Jason (2004)

Given a set $\mathcal{T}$ and random variables V_t, $t \in \mathcal{T}$, we say:

- A subset $\mathcal{T}'$ of $\mathcal{T}$ is *independent* if the corresponding random variables $\{V_t\}_{t\in\mathcal{T}'}$ are independent.
- A family $\{\mathcal{T}_j\}_j$ of subsets of $\mathcal{T}$ is a *cover* of $\mathcal{T}$ if $\bigcup_j \mathcal{T}_j = \mathcal{T}$.
- A family $\{(\mathcal{T}_j, w_j)\}_j$ of pairs $(\mathcal{T}_j, w_j)$, where $\mathcal{T}_j \subseteq \mathcal{T}$ and $w_j \in [0,1]$ is a *fractional cover* of $\mathcal{T}$ if $\sum_j w_j \mathbf{1}_{\mathcal{T}_j} \geqslant \mathbf{1}_{\mathcal{T}}$, i.e. $\sum_{j:t\in\mathcal{T}_j} w_j \geqslant 1$ for each $t \in \mathcal{T}$.
- A (fractional) cover is *proper* if each set $\mathcal{T}_j$ in it is independent.
- $\mathcal{X}(\mathcal{T})$ is the size of the smallest proper cover of $\mathcal{T}$, i.e. the smallest m such that $\mathcal{T}$ is the union of m independent subsets.
- $\mathcal{X}^*(\mathcal{T})$ is the minimum of $\sum_j w_j$ over all proper fractional covers $\{(\mathcal{T}_j, w_j)\}_j$.

$\mathcal{X}^*(\mathcal{T})$: measure of dependence; $\mathcal{X}^*(\mathcal{T}) = 1$ (independent).



Assumption 3

With a high probability p, Ψ_t, U_t *and* ε_t *are such that*

$$
\begin{aligned}
(J^{-1}\sum_{k=1}^{K}\sum_{j=1}^{J}\Psi_{tkj}\varepsilon_{tj}U_{tr})^2 &\leqslant b_t^2 \\
\mathrm{E}(JT)^{-1}\Big\{\sum_{t=1}^{T}(\sum_{k=1}^{K}\sum_{j=1}^{J}\Psi_{tkj}\varepsilon_{tj}U_{tr})^2\Big\}^{1/2} &\leqslant \frac{C'}{\sqrt{T}}.
\end{aligned}
$$

for $\forall\ r$ *and some constants* $b_t, C' > 0, t = 1, \ldots, T$.

Assumptions

B1 $X_{1,1}, \ldots, X_{T,J}$, $\varepsilon'_{1,1}, \ldots, \varepsilon'_{T,J}$, and $Z_{0,1}, ..., Z_{0,T}$ are independent.

B2 $X_{t,1}, \ldots, X_{t,J}$ are identically distributed, support $[0,1]^d$ and a density f_t that is bounded from below and above on $[0,1]^d$, uniformly over $t = 1, \ldots, T$.

B3 We assume that $\mathrm{E}\,\varepsilon'_{t,j} = 0$ for $1 \leq t \leq T, 1 \leq j \leq J$, and for $c > 0$ small enough $\sup_{1 \leq t \leq T, 1 \leq j \leq J} \mathrm{E} \exp\{c(\varepsilon'_{t,j})^2\} < \infty$.

B4 The vector of functions $m = (m_1, \ldots, m_L)^\top$ can be approximated by Ψ_k, i.e.

$$\delta_K \stackrel{\text{def}}{=} \sup_{x \in [0,1]^d} \inf_{A \in \mathbb{R}^{L \times K}} \|m(x) - A\Psi(x)\| \to 0$$

as $K \to \infty$. We denote A that fulfills $\sup_{x \in [0,1]^d} \|m(x) - A\Psi(x)\| \leq 2\delta_K$ by A^*.

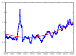
B5 There exist constants $0 < C_L < C_U < \infty$ such that all eigenvalues of the matrix $T^{-1}\sum_{t=1}^{T} Z_{0t}Z_{0,t}^{\top}$ lie in the interval $[C_L, C_U]$ with probability tending to one.

B6 The minimization (3) runs over all values β with

$$\sup_{x\in[0,1]^d} \max_{1\leqslant t\leqslant T} \|Z_{0,t}^{\top}A\Psi(x)\| \leqslant M_T,$$

where the constant M_T fulfils $\max_{1\leqslant t\leqslant T}\|Z_{0,t}\| \leqslant M_T/C_m$ (with probability tending to one) for a constant C_m such that $\sup_{x\in[0,1]^d}\|m(x)\| < C_m$.

B7 It holds that $\rho^2 = (K+T)M_T^2\log(JTM_T)/(JT) \to 0$. The dimension L is fixed.



Assumptions

C1 $Z_{0,t}$ is a strictly stationary sequence with $\mathrm{E}(Z_{0,t}) = 0$, $\mathrm{E}(\|Z_{0,t}\|^{\gamma}) < \infty$ for some $\gamma > 2$. It is strongly mixing with $\sum_{i=1}^{\infty} \alpha(i)^{(\gamma-2)/\gamma} < \infty$. The matrix $\mathrm{E}\, Z_{0,t} Z_{0,t}^{\top}$ has full rank. The process $Z_{0,t}$ is independent of $X_{11}, \ldots, X_{TJ}, \varepsilon'_{11}, \ldots, \varepsilon'_{TJ}$.

C2 It holds that

$$[\log(KT)^2\{(KM_T/J)^{1/2} + T^{1/2} M_T^4 J^{-2} + K^{3/2} J^{-1} + K^{4/3} J^{-2/3} T^{-1/6}\} + 1] T^{1/2}(\rho^2 + \delta_K^2) = \mathcal{O}(\rho^2 + \delta_K^2)$$