Portfolio Credit Risk Contribution

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The Concept of Central Counterparty (CCP)

Central Counterparty interposes itself between counterparties and becomes the buyer to every seller and the seller to every buyer.

Figure 1: Legal process of replacing original OTC contract to central counterparty (novation)
**Risk Reserve Architecture of CCP**

- **Membership Requirements**: e.g. minimum requirement of rating, minimum capital requirement, …
- **Variation margin**: Margin based on daily changes in market value of the cleared product
- **Initial margin**: Margin based on potential future exposure (via stress test, e.g. largest 5 days decline)
- **Default Funds**: Funds based on loss given default of single largest clearing member or simultaneous defaults of second and third largest

Risk Measurement of CDS Portfolios
Motivation

Credit Default Swap Spread

Figure 2: Cash-flow structure of CDS until credit event in $\tau$

$$S_{t_0} = \frac{E^Q_{t_0} \left[ \exp \left\{ - \int_{t_0}^{\tau} r(\ell) d\ell \right\} \{1 - R(\tau)\} \mathbf{1}_{\{\tau < T\}} \right]}{E^Q_{t_0} \left[ \sum_{i=1}^{n} \exp \left\{ - \int_{t_0}^{t_i} r(\ell) d\ell \right\} \mathbf{1}_{\{\tau > t_i\}} \right]}$$

Risk Measurement of CDS Portfolios
Risk measures

- Value at Risk (VaR)

\[ \text{VaR}_t^\alpha = \inf \{ x \in \mathbb{R} : P(X_{t+d} \leq x \mid \mathcal{F}_t) \geq \alpha \} \quad (1) \]

where \( X_t \) denotes the spread returns.

- Expected shortfall (ES)

\[ \text{ES}_t^\alpha = -E \left( X_{t+d} \mid X_{t+d} \leq \text{VaR}_t^\alpha \right) \quad (2) \]

Expected shortfall is the expected return given the return exceeds its \( \text{VaR}_t^\alpha \) value.
CCP’s credit risk contribution from each member?

Question: What is the portfolio’s credit risk and credit risk contribution of each constituent?
Objectives

- VaR and CoVaR calculation under consideration of CDS spreads as indicator for credit risk and market variables
- Indirect spillover effect via CoVaR calculation under consideration of market variables
- Semi-parametric quantile regression modelling
- Significance of market variables in predicting CDS spreads
Outline

1. Motivation ✓
2. Linear quantile regression
3. Partial linear quantile regression
4. CDS spreads data
5. Linear versus nonparametric quantile regression model
6. Research outlook
CoVaR

Adrian & Brunnermeier (2011): linear quantile regressions

\[
X_{i,t} = \alpha_i + \gamma_i^\top M_{t-1} + \epsilon_{i,t}, \\
X_{j,t} = \alpha_{j|i} + \beta_{j|i} X_{i,t} + \gamma_{j|i}^\top M_{t-1} + \epsilon_{j,t}.
\]

\(M_t\): state variables. \(F^{-1}_{\epsilon_{i,t}}(\tau|M_{t-1}) = 0\) and \(F^{-1}_{\epsilon_{j,t}}(\tau|M_{t-1}, X_{i,t}) = 0\).

\[
\widehat{\text{VaR}}_{i,t} = \hat{\alpha}_i + \hat{\gamma}_i^\top M_{t-1}, \\
\widehat{\text{CoVaR}}_{j|i,t} = \hat{\alpha}_{j|i} + \hat{\beta}_{j|i} \widehat{\text{VaR}}_{i,t} + \hat{\gamma}_{j|i}^\top M_{t-1}.
\]
Quantile Regression in Risk Calibration

- Chao, Härdle & Wang (2013): partial linear quantile regression:

\[ X_{i,t} = \alpha_i + \gamma_i^\top M_{t-1} + \varepsilon_{i,t}; \]
\[ X_{j,t} = \tilde{\alpha}_{j|i} + \tilde{\beta}_{j|i}^\top M_{t-1} + l_{j|i}(X_{i,t}) + \varepsilon_{j,t}. \]

- \( l \): a general function.
- \( M_t \): state variables.
- \( F_{\varepsilon_{i,t}}^{-1}(\tau|M_{t-1}) = 0 \)
- and \( F_{\varepsilon_{j,t}}^{-1}(\tau|M_{t-1}, X_{i,t}) = 0 \).

\[ \widehat{\text{VaR}}_{i,t} = \hat{\alpha}_i + \hat{\gamma}_i^\top M_{t-1}, \]
\[ \widehat{\text{CoVaR}}_{j|i,t} = \hat{\alpha}_{j|i} + \hat{\gamma}_{j|i}^\top M_{t-1} + \hat{l}_{j|i}(\widehat{\text{VaR}}_{i,t}). \]
State variables

$M_t$: 7 state variables suggested by AB and further extension:

1. VIX
2. Short term liquidity spread
3. Change in the 3M T-bill rate
4. Change in the slope of the yield curve
5. Change in the credit spread between 10 years BAA-rated bonds and the T-bond rate
6. S&P500 returns
7. Dow Jones U.S. Real Estate index returns
8. Constituent’s specific stock returns
9. Constituent’s specific stock volatility returns
Data:

- G14 FI: daily CDS spreads of 14 biggest derivative dealers
- Overall data period: Sept 2002 - Dec 2011 ($N = 2228$)
- Segregation into three sub-periods
  - pre-crisis: Sept 2002 - June 2007
  - crisis: July 2007 - March 2009
  - post-crisis: April 2009 - Dec 2011

Risk Measurement of CDS Portfolios
## Characteristics of CDS spreads data

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<tr>
<th>G14 FI</th>
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<tr>
<td></td>
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<tr>
<td>Citi</td>
<td>7.44</td>
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<td></td>
<td>µ</td>
<td>σ</td>
<td>µ</td>
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<tr>
<td>Citi</td>
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<tr>
<td>CS</td>
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<td>31.42</td>
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<tr>
<td>MS</td>
<td>33.88</td>
<td>14.03</td>
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QR of MS daily spread returns and VIX

Figure 3: Left: overall period; right: pre-crisis period; y-axis = MS spread returns; x-axis = VIX. Locally linear quantile estimation. Linear quantile regression line. 95% asymptotic CB, dash: bootstrap CB.
QR of MS daily spread returns and VIX

Figure 4: Left: crisis period; right: post-crisis period; y-axis = MS spread returns; x-axis = VIX. Locally linear quantile estimation. Linear quantile regression line. 95% asymptotic CB, dash: bootstrap CB.
QR of Morgan Stanley daily spread returns and daily stock returns

Figure 5: Left: overall period; right: pre-crisis period; y-axis = MS spread return; x-axis = MS stock return. Locally linear quantile estimation. Linear quantile regression line. 95% asymptotic CB, dash: bootstrap CB.

Risk Measurement of CDS Portfolios
QR of Morgan Stanley daily spread returns and daily stock returns

Figure 6: Left: crisis period; right: post-crisis period; y-axis = MS spread returns; x-axis = MS stock return. Locally linear quantile estimation. Linear quantile regression line. 95% asymptotic CB, dash: bootstrap CB.

Risk Measurement of CDS Portfolios
Confidence band violation area

Area between confidence band and linear regression line:

\[ D = \int_{-\infty}^{0} \{ |y_o(x) - l(x)| \} 1 \{ y_o(x) < l(x) \} dx + \int_{-\infty}^{0} \{ |y_u(x) - l(x)| \} 1 \{ y_u(x) > l(x) \} dx. \]

Area calculated for Morgan Stanley quantile regression:

<table>
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<tr>
<th>State variables</th>
<th>2002-2011</th>
<th>pre-crisis</th>
<th>crisis</th>
<th>post-crisis</th>
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<tr>
<td>VIX</td>
<td>0.051</td>
<td>0.010</td>
<td>0.053</td>
<td>0.841</td>
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<tr>
<td>diffRepoTB3M</td>
<td>0.148</td>
<td>0.001</td>
<td>1.863</td>
<td>0.001</td>
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<tr>
<td>diffSlopeYieldCurve</td>
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<td>0.074</td>
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<td>diffCDSSpread</td>
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<td>0.003</td>
<td>0.003</td>
<td>0.004</td>
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<td>diffEquityReturn</td>
<td>13.507</td>
<td>0.781</td>
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<tr>
<td>diffRealEstateReturn</td>
<td>3.557</td>
<td>1.092</td>
<td>1.251</td>
<td>60.255</td>
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<tr>
<td>Stock returns</td>
<td>37.102</td>
<td>0.001</td>
<td>0.007</td>
<td>0.004</td>
</tr>
</tbody>
</table>
Preliminary Conclusion

- Dependence behaviour during overall period differs from sub-period analysis.
- Dependence between spread and market variables seems to increase in crisis period.
- Low significance for non-linearity for some state variables in certain period.
Next steps

- Significance test based on confidence band violation area
- Additive model (Dynamic SPM) for VaR and CoVaR calculation
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Partial Linear Model (PLM)

The partial linearity observation implies:

\[ X_{i,t} = \alpha_i + \gamma_i^\top M_{t-1} + \varepsilon_{i,t}; \]
\[ X_{j,t} = \tilde{\beta}_{j|i}^\top M_{t-1} + l_{j|i}(X_{i,t}) + \varepsilon_{j,t}. \]  \hspace{1cm} (3)

\( l \): a general function. \( M_t \): state variables. \( F_{\varepsilon_{i,t}}^{-1}(\tau| M_{t-1}) = 0 \)
and \( F_{\varepsilon_{j,t}}^{-1}(\tau| M_{t-1}, X_{i,t}) = 0. \)

Advantages

- Capturing nonlinear asset dependence
- Avoid curse of dimensionality
Estimation of Partial Linear Model

- PLM model: Liang, Härdle and Carroll (1999) and Härdle, Ritov and Song (2012)

\[ Y_t = \beta^\top M_{t-1} + l(X_t) + \varepsilon_t. \]

- Consider \([0, 1]\) (standard rank space). Dividing \([0, 1]\) into \(a_n\) equally divided subintervals \(I_{nt}, a_n \uparrow \infty\). On each subinterval, \(l(\cdot)\) is roughly constant.
Estimation of PLM QR

1. Linear element $\beta$:

$$\hat{\beta} = \arg\min_{\beta} \min_{l_1, \ldots, l_n} \sum_{t=1}^{n} \rho_{\tau} \left\{ Y_t - \beta^\top M_{t-1} - \sum_{m=1}^{a_n} l_m 1(X_t \in I_{nt}) \right\}$$

2. Nonlinear element $l(\cdot)$: With data $\{(X_t, Y_t - \hat{\beta}^\top M_{t-1})\}_{t=1}^{n}$, applying LLQR.
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