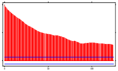
An Empirical Investigation of Long Range Dependence in Factors of Implied Volatility Dynamics

Julius Mungo
Wolfgang Härdle

CASE-Center for Applied Statistics and Economics
Institut für Statistik and Ökonometrie
Humboldt-Universität zu Berlin

Empirical data oriented perspective relates long memory to a high degree of persistence of the observed autocorrelations.
They show significant autocorrelation up to very long lags, often defined as "hyperbolic decay".
Many economic and financial time series show evidence of neither $I(0)$ nor $I(1)$.
This difficulty of distinguishing between stationarity and non-stationarity is exacerbated by "nearly non-stationarity".



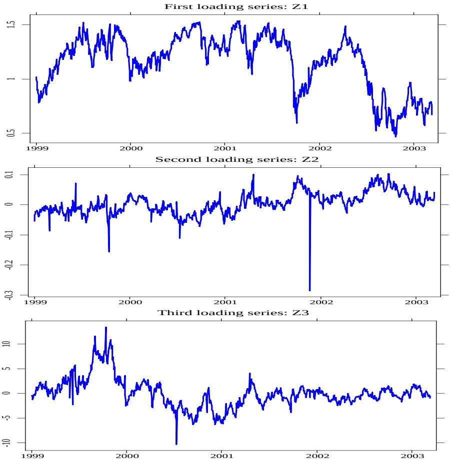
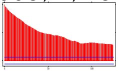


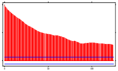
Figure 1: *Time series plots in levels of three loading series from a DSFM fit for the DAX-Option analyzed from 1999/4/1-2003/2/25*



A Dynamic Semiparametric Factor Model (DSFM):

$$Y_{t,j} = \sum_{l=0}^{L} z_{tl} m_l(X_{t,j}) + \varepsilon_{t,j} \qquad (1)$$

where $z_{t0} = 1$, $j = 1, \ldots, J_t$ $(t = 1, \ldots, T)$ is the number of IV observations on day t, L is the number of basis functions.
$X_{t,j}$ is a two-dimensional variable containing moneyness and maturity.
z_{tl} are time dependent factors or weights of the smooth basis function m_l, for $(l = 0, \ldots, L)$. [Borak, Härdle and Fengler (2005)]



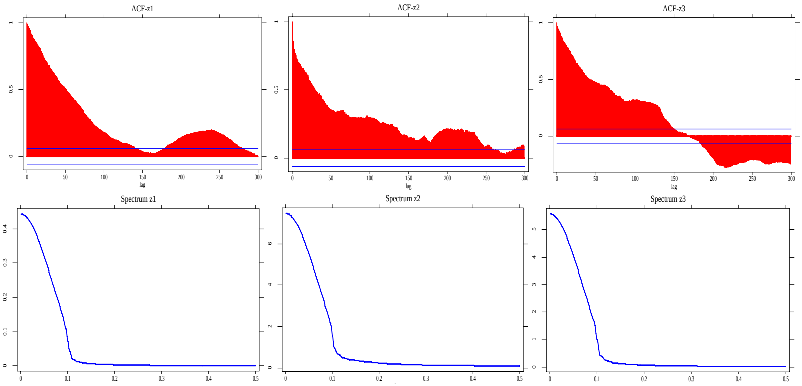
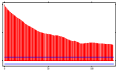


Figure 2: Plots of the sample autocorrelation functions with length 300 and spectrum of the loadings series in levels.



Fractional differencing

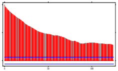
$$(1-L)^d y_t = \varepsilon_t \tag{2}$$

- $d = 0$, is a white noise process.
- $d = 1$, is a random walk.

In random shock form,

$$\begin{aligned} y_t &= (1-L)^{-d}\varepsilon_t \\ &= \left(1 + dL + \frac{1}{2!}d(d+1)L^2 + \dots\right)\varepsilon_t \\ &= \varepsilon_t + d\varepsilon_{t-1} + \frac{1}{2!}d(d+1)\varepsilon_{t-2} + \dots. \end{aligned}$$

(2) allows for stong persistence in a time series.
[Granger and Joyeux (1980)]

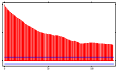


Fractional integration

$\{y_t\}_{t=0}^{\infty}$ is $I(d)$ if differenced d times to induce stationarity.

$$\begin{aligned}(1-L)^d y_t &= y_t - dy_{t-1} + \frac{1}{2!}d(d-1)y_{t-2} + \dots \\ &+ \frac{(-1)^j}{j!}d(d-1)\dots(d-j+1)y_{t-j} + \dots\end{aligned} \tag{3}$$

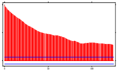
For $0 < d < 1$, current values of y_t are influenced not only by the immediate past values but values from previous time periods. Such series is said to have Long Memory.
Equation 3 is highly nonlinear in $d \Rightarrow$ estimation of d is problematic.



Long Range Dependence (LRD)

	Series	Memory	Mean reversion	Variance	ACF
$-0.5 < d < 0$	fractional integrated	antipersistent	$\checkmark$	finite	hyperbolic
$d = 0$	stationary	short	$\checkmark$	finite	exponential
$0 < d < 0.5$	fractional integrated	long	$\checkmark$	finite	hyperbolic
$0.5 \leq d < 1$	fractional integrated	long	$\checkmark$	infinite	hyperbolic
$d = 1$	integrated	infinite	x	infinite	linear

Table 1: *Time series long memory characteristics.*



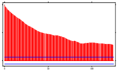
Long Range Dependence (LRD)

A stationary process X_t, $t \in N$ exhibit long-memory if the correlation function ρ_k behaves for $k \to \infty$ as:

$$\lim_{k \to \infty} \frac{\rho(k)}{c_\rho k^{2d-1}} = 1 \tag{4}$$

$k = 1, 2, \ldots,\ c_\rho > 0$ and $d \in (0, 0.5)$ is the memory parameter.[Beran (1994)]

The correlations decay slowly with a hyperbolic rate and consequently are not summable, $\lim_{T \to \infty} \sum_{j=-T}^{T} |\rho(j)| = \infty$
T is number observations. [McLeod and Hipel (1978)]

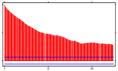


Long Range Dependence (LRD)

A stationary process X_t with covariances $\gamma_k = cov(X_t, X_{t+k}),\ \ t \in N$ exhibit LRD if the spectral density $f(\lambda) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty} e^{(tk\lambda)}\gamma(k)$ behaves for $\lambda \to 0$ as

$$\lim_{\lambda\to 0} \frac{f(\lambda)}{c_f\,|\lambda|^{2d}} = 1 \tag{5}$$

$\Rightarrow$ as k $\to \infty$, γ_k are proportional to k^{2d-1} and hence they are not summable. [Mandelbrot (1983)]



Research evidence

[Bollerslev and Mikkelsen (1996)]: Daily squared returns r_t^2 for S&P500

[Lobeto and Savin (1998)]: Daily squared and absolute returns for S&P500

[Liu 2000]: Daily squared returns for S&P500

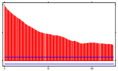
[Giraitis, Kokoszka, Leipus (2001)]: Daily squared returns r_t^2 in Pounds per US dollar exchange rate.

[Sibbertsen (2001)]: Daily absolute returns $|r_t|$ for DAX.

Why fractional integrated processes?

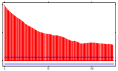
- They allow for substantially more flexibility than the extreme assumption of unit root.
- Their implication of the complete persistence of a shock.
- The slow decay of shocks implied by the $I(d)$ process and the eventual adjustment to equilibrium.

Empirical success on modelling volatility of asset prices and power transformation of returns.



Overview

1. Motivation ✓
2. Detection LRD and Estimation
3. Data and Empirical Analysis
4. Models specifications and Estimation
5. Conclusion



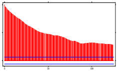
Rescale Variance method: V/S

By centering of the KPSS statistic based on the partial sum of the deviations from the mean.

$$V/S(q) = \frac{1}{T^2 \hat{\sigma}_T^2(q)} \left[\sum_{k=1}^{T} \left(\sum_{j=1}^{k} (Y_j - \overline{Y}_T) \right)^2 - \frac{1}{T} \left(\sum_{k=1}^{T} \sum_{j=1}^{k} (Y_j - \overline{Y}_T) \right)^2 \right] \tag{6}$$

$S_k = \sum_{j=1}^{k} (Y_j - \overline{Y}_T)$ are the partial sums of the observations and $\hat{\sigma}_T^2(q) = \hat{\gamma}_0 + 2 \sum_{j=1}^{q} \left(1 - \frac{j}{1+q} \right) \hat{\gamma}_j, \;\; q < T$ is the [Newey and West (1994)] Heteroscedastic and autocorrelation consistent estimator of the variance at truncation lag q.
We reject $I(0)$ e.g at 5% significance level if $V/S(q) > 0.1869$

Giraitis, Kokoszka and Leipus (1998)

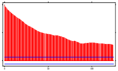


Lobato and Robinson (1998)

Based on $\lim_{\lambda_i \to 0^+} f(\lambda_i) = C\lambda_i^{-2d}$

$$t_{LR} = \sqrt(m)\frac{\widehat{C}_1}{\widehat{C}_0} \tag{7}$$

with $\widehat{C}_k = \frac{1}{m}\sum_{j=1}^{m} \zeta_j^k I(\lambda_j)$ and $\zeta_j = log(j) - \frac{1}{m}\sum_{i=1}^{m} log(i)$,
where
$I(\lambda) = \frac{1}{2\pi T}\left|\sum_{t=1}^{T} Z_t e^{it\lambda}\right|^2$ is the periodogram estimated for degenerate band of Fourier frequencies
$\lambda_j = \frac{2\pi j}{T}, \quad j = 1, \dots, m << [T/2]$ with bandwidth parameter m.
Under null hypothesis of I(0), t_{LR} is asymptotically normally distributed.



Log-periodogram Regression (GPH)

$$f(\lambda) = C\left\{4sin^2(\lambda_j/2)\right\}^{-d}$$

Estimate d with the spectral regression:

$$log\left\{I(\lambda_j)\right\} = logC - d\log\left\{4sin^2(\lambda_j/2\right\} + log\varepsilon_j \qquad (8)$$

at harmonic frequencies.

$\lambda_j = \frac{2\pi j}{T}$ is the j^{th} Fourier frequency, with $j \in (l; m]$, l is a trimming parameter discarding the lowest frequencies and m is a bandwidth parameter. [Geweke and Porter-Hudax]

Semiparametric Gaussian estimator

Based on

$$\lim_{\lambda_i \to 0^+} f(\lambda_i) = C\lambda_i^{-2d}$$

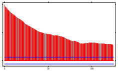
d is obtained by solving the minimization

$$\left\{\widehat{C}, \widehat{d}\right\} = \arg\min_{C,d} L(C,d) = \frac{1}{m}\sum_{j=1}^{m}\left\{log(C\lambda_j^{-2d}) + \frac{I(\lambda_j)}{C\lambda_j^{-2d}}\right\} \tag{9}$$

the estimator $\widehat{d}$ is equal to:

$$\widehat{d} = \arg\min_{d}\left\{log\left(\frac{1}{m}\sum_{j=1}^{m}\frac{I(\lambda_j)}{C\lambda_j^{-2d}}\right) - \frac{2d}{m}\sum_{j=1}^{m} log(\lambda_j)\right\}$$

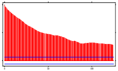
[Robinson (1995)]



Unit root test statistics

Series	ADF-AIC	$\hat{p}$	ADF-HQ	$\hat{p}$	ERS-AIC	$\hat{b}$	ERS-HQ	$\hat{b}$
z_{t1}	-1.98 [0.29]	6	-2.24 [0.19]	2	3.78^{*}	6	2.95^{**}	6
z_{t2}	-3.36^{**} [0.01]	8	-4.21^{***} [0.01]	4	5.29	8	3.34^{*}	4
z_{t3}	-2.87^{**} [0.05]	7	-2.87^{**} [0.05]	7	1.45^{***}	7	1.45^{***}	7

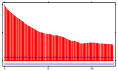
Table 2: *ADF-AIC and ADF-HQ refer to ADF tests using AIC and HQ criteria respectively to estimate lag length p. ERS-AIC and ERS-SC criteria used, refer to the lag length b chosen for the estimation regression of the autoregressive spectral density estimator. Critical values for ADF test are* -2.57 *(10%),* -2.86 *(5%), and* -3.44 *(1%) (see, [MacKinnon (1991)]). The p-values for the ADF tests are given in brackets. Critical values for ERS test (see, [Elliot, Rothenberg and Stock (1996)]) are 4.48 (10%),* 3.26 *(5%) and* 1.99 *(1%).* *** , ** *and* * *denote significance at 1%, 5%, and 10% level respectively.*



Model independent tests for $I(0)$ against $I(d)$

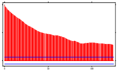
Z1	level				
q	5	7	10	20	50
V/S	2.22	1.68	1.24	0.68	0.32
LoRob	-0.80	-0.86	-1.11	-3.57	-10.62
Z2	level				
V/S	1.99	1.54	1.15	0.65	0.31
LoRob	-1.11	-1.40	-2.04	-4.07	-11.05
Z3	level				
V/S	1.82	1.38	1.02	0.57	0.27
LoRob	0.13	-0.46	-1.34	-3.48	-9.87

Table 3: *Model independent tests for $I(0)$ against $I(d)$ for levels with the rescaled variance V/S and semiparametric LoRob tests. For the V/S test, short-memory process is rejected at the 5% significance level if the statistic is greater than the critical value, 0.1869. For the LoRob test, if the value of the test is in the lower tail of the standard normal distribution, the null hypothesis of I(0) is rejected against the alternative that the series displays long-memory. If the value of the test is in the upper tail of the standard normal distribution, the null hypothesis I(0) is rejected against the alternative that the series is antipersistent.*



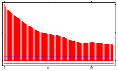
z1	r_t				
q	5	7	10	20	50
V/S	0.02	0.03	0.04	0.05	0.06
$LoRob$	-0.79	0.29	1.38	1.26	0.96
Z2	r_t				
V/S	0.01	0.01	0.01	0.02	0.03
$LoRob$	0.56	1.07	1.76	2.41	1.48
Z3	r_t				
V/S	0.02	0.02	0.03	0.03	0.04
$LoRob$	0.82	0.45	0.13	1.38	1.62

Table 4: *Model independent tests for $I(0)$ against $I(d)$ for returns r_t with the rescaled variance V/S and semiparametric LoRob tests. For the V/S test, short-memory process is rejected at the 5% significance level if the statistic is greater than the critical value, 0.1869.*



Z1	$\lvert r_t \rvert$				
q	5	7	10	20	50
V/S	0.19	0.15	0.13	0.09	0.07
LoRob	0.93	0.70	0.71	0.38	-1.89
Z2	$\lvert r_t \rvert$				
V/S	0.05	0.04	0.05	0.04	0.04
LoRob	0.29	0.49	0.57	.91	0.38
Z3	$\lvert r_t \rvert$				
V/S	0.72	0.64	0.56	0.44	0.28
LoRob	-0.93	-1.27	-2.10	-4.06	-6.82

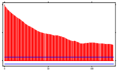
Table 5: *Model independent tests for $I(0)$ against $I(d)$ for loading absolute returns $|r_t|$, using the rescaled variance V/S and the semiparametric LoRob tests. For the V/S test, short-memory process is rejected at the 5% significance level if the statistic is greater than the critical value, 0.1869.*



Fractional parameter Estimation

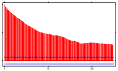
GPH(d)	z1		
bandwidth (b)	level	r_t	$\|r_t\|$
33	0.92	-0.23	0.25
66	0.95	-0.16	0.17
132	0.91	-0.16	0.30
261*	0.97	-0.10	0.27
263	0.96	-0.10	0.28
GPH(d)	z2		
33	0.69	-0.57	0.01
66	0.87	-0.33	-0.06
132	0.75	-0.38	-0.05
261*	0.70	-0.39	-0.01
263	0.70	-0.39	-0.01
GPH(d)	z3		
33	0.90	-0.28	0.34
66	0.89	-0.24	0.34
132	0.84	-0.24	0.19
261*	0.85	-0.22	0.17
263	0.85	-0.22	0.17

Table 6: *Log periodogam estimates of d based on [Geweke and Porter-Hudax] GPH for levels, returns and absolute returns.* $b_{opt} = T^{\frac{4}{5}} = 261^*$ *represent the optimal number of frequencies (bandwidth) used for the estimation [Hurvich et al.,(1998)].*



RobWhittle(d)	z1		
bandwidth (b)	level	r_t	$\lvert r_t \rvert$
33	0.82	-0.23	0.18
66	0.95	-0.11	0.28
132	0.90	-0.17	0.32
261*	0.93	-0.12	0.27
263	0.93	-0.12	0.27
RobWhittle(d)			z2
33	0.96	-0.58	-0.066
66	0.76	-0.33	-0.03
132	0.67	-0.40	-0.02
261	0.67	-0.38	0.04
263	0.66	-0.37	0.03
RobWhittle(d)			z3
33	0.78	-0.35	0.39
66	0.83	-0.24	0.34
132	0.83	-0.21	0.20
261	0.85	-0.17	0.17
263	0.85	-0.17	0.17

Table 7: *Semiparametric estimates of d for levels, returns and absolute returns based on [Robinson (1995)] RobWhittle. Bandwidth b is chosen such that $b = \frac{T}{2}, \frac{T}{4}, \frac{T}{8}, \frac{T}{16}$ where T is the sample size.*



ARFIMA(p,d,q) model

$$\Phi(L)(1-L)^d z_t = \Theta(L)\varepsilon_t \tag{10}$$

where $\varepsilon_t \sim WN(0,\sigma^2)$.

$$\Phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \ldots, \phi_1 L^p$$
$$\Theta(L) = 1 - \theta_1 L - \theta_2 L^2 - \ldots, \theta_1 L^q$$

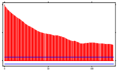
are AR and MA lag polynomials respectively with roots outside the unit circle.
We consider all possible model up to an order ARFIMA(5,d,5) and report the best fitted model for each series z_1, z_2 and z_3 in levels and absolute returns using the AIC criterion.

[Doornik and Ooms (1999)]

Parameter Estimation

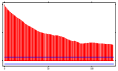
ARFIMA	$z1$ (5, d, 4)	$z2$ (2, d, 1)	$z3$ (5, d, 3)
$d(ML)$	0.13 (1.00)	0.45 (6.94)	0.01 (0.05)
ϕ_1	0.56 (0.99)	0.87 (10.60)	0.47 (1.78)
ϕ_2	-0.05 (-0.07)	0.05 (1.02)	0.26 (4.31)
ϕ_3	0.83 (1.84)		0.92 (4.39)
ϕ_4	0.05 (0.06)		-0.52 (-2.12)
ϕ_5	-0.44 (-1.08)		-0.14 (-2.48)
θ_1	0.34 (0.62)	-0.85 (-10.9)	0.23 (2.25)
θ_2	0.29 (1.59)		0.13 (1.02)
θ_3	-0.54 (-2.05)		-0.80 (-6.54)
θ_4	-0.53 (-1.15)		
const.	-0.1		
logL	1888.48	2761.45	3628.33
AIC	-3752.97	-5510.90	-7234.66

Table 8: *Maximum likelihood estimation of ARFIMA model for loading levels z_1, z_2 and z_3 from 04.01.199 to 25.02.2003. t-value of the estimated parameters in brackets, logL is the log-likelihood and (AIC) Akaike Information Criterion.*



ARFIMA	z1 (5, d, 4)	z2 (2, d, 1)	z3 (5, d, 3)
$d(NLS)$	0.29 (1.77)	0.57 (6.40)	-0.15 (-1.98)
ϕ_1	0.59 (3.13)	0.81 (10.70)	0.07 (0.27)
ϕ_2	0.07 (0.31)	0.12 (1.89)	0.11 (2.04)
ϕ_3	0.29 (1.84)		0.91 (20.00)
ϕ_4	0.50 (2.71)		-0.04 (-0.16)
ϕ_5	-0.47 (-3.92)		-0.07 (-1.38)
θ_1	0.19 (0.94)	-0.91 (-13.3)	0.83 (3.28)
θ_2	0.04 (0.29)		0.78 (3.15)
θ_3	-0.17 (-1.37)		-0.16 (-0.69)
θ_4	-0.58 (-4.78)		
constant	-0.1		
logL	1892.33	2765.53	3628.18
AIC	-3760.67	-5519.07	-7234.37

Table 9: *Non-linear least squares estimation of ARFIMA model for loading levels z_{t1}, z_{t1} and z_{t1} from 04.01.1999 to 25.02.2003. t-value of the estimated parameters in brackets, logL is the log-likelihood and (AIC) Akaike Information Criterion.*



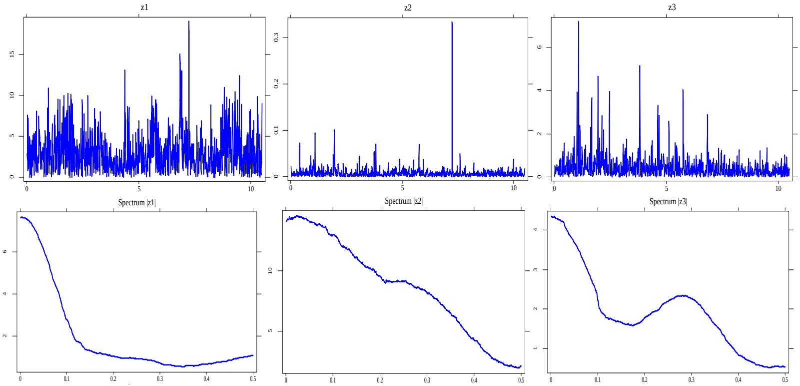
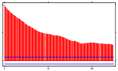
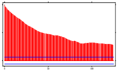


Figure 3: *A time plot of the Absolute return values of the loading series and spectrum from a DSFM for DAX from 04.01.1999-25.02.2003.*



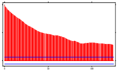
ARFIMA	z_{t1} (2, d, 2)	z_{t2} (1, d, 5)	z_{t3} (1, d, 2)
$d(ML)$	0.29 (4.96)	-0.32 (-0.77)	0.24 (2.51)
ϕ_1	-0.81 (-4.57)	0.92 (10.80)	0.56 (2.51)
ϕ_2	-0.02 (-0.09)		
θ_1	0.64 (3.01)	0.02 (0.07)	-0.36 (-1.83)
θ_2	-0.19 (-0.93)	-0.46 (-2.99)	-0.27 (-7.15)
θ_3		-0.09 (-0.96)	
θ_4		-0.05 (-0.67)	
θ_5		-0.01 (-0.15)	
const.			
logL	2381.19	2914.40	3926.66
AIC	-4.57	-5.59	-7.54

Table 10: *Maximum likelihood estimation of ARFIMA model for absolute returns of the factor loadings z_{t1}, z_{t1} and z_{t1} from 04.01.199 to 25.02.2003. t-value of the estimated parameters in brackets, logL is the log-likelihood and (AIC) Akaike Information Criterion.*



ARFIMA	z_{t1} (2, d, 2)	z_{t2} (1, d, 5)	z_{t3} (1, d, 2)
$d(NLS)$	0.31 (4.73)	-0.21 (-0.71)	0.27 (2.34)
ϕ_1	-0.79 (-4.65)	0.89 (5.89)	0.57 (4.28)
ϕ_2	0.01 (0.02)		
θ_1	0.61 (2.90)	0.06 (-0.31)	-0.40 (-2.11)
θ_2	-0.22 (-1.09)	-0.48 (-5.59)	-0.28 (-8.40)
θ_3		-0.06 (-0.61)	
θ_4		-0.03 (-0.40)	
θ_5		-0.00 (-0.07)	
$const.$	0.3		
$logL$	2381.53	2913.99	3927.21
AIC	-4.57	-5.59	-7.54

Table 11: *Non-linear least squares estimation of ARFIMA model for absolute returns of the factor loadings z_{t1}, z_{t1} and z_{t1} from 04.01.1999 to 25.02.2003. t-value of the estimated parameters in brackets, logL is the log-likelihood and (AIC) Akaike Information Criterion.*



ARFIMA model forecasting

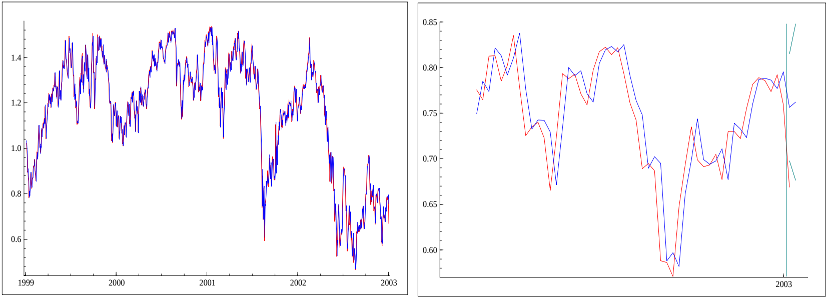
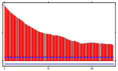


Figure 4: Left panel: actual series (red) and in-sample prediction (blue) of the *ARFIMA*$(5, 0.13, 4)$, for levels of $z1$ and a one-step ahead forecast (right panel)



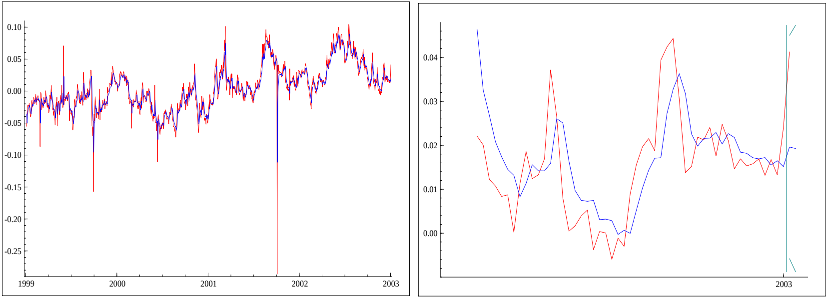


Figure 5: Left panel: actual series (red) and in-sample prediction (blue) of the *ARFIMA*(2, 0.45, 1) for levels of *z*2 with a one-step ahead forecast (right panel)

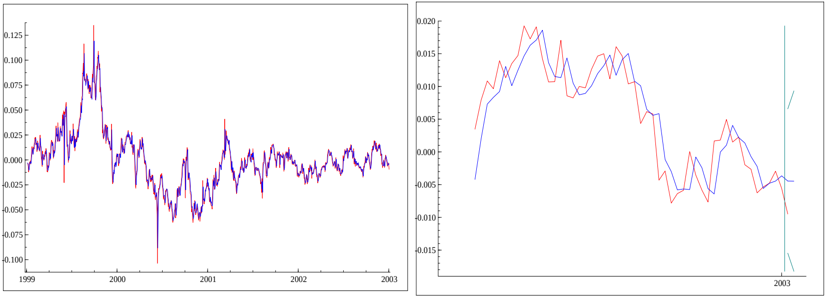
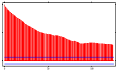


Figure 6: Left panel: actual series (red) and in-sample prediction (blue) of the $ARFIMA(5, 0.01, 3)$ for levels of $z3$ and a one-step ahead forecast (right panel



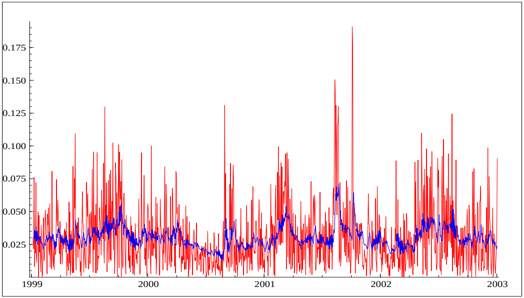
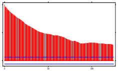


Figure 7: *Absolute returns forecast from ARFIMA*$(2, 0.29, 2)$ *for* $z1$. *Actual series (red) and in-sample prediction (blue). Sample period from 04.01.1999 - 25.02.2003.*



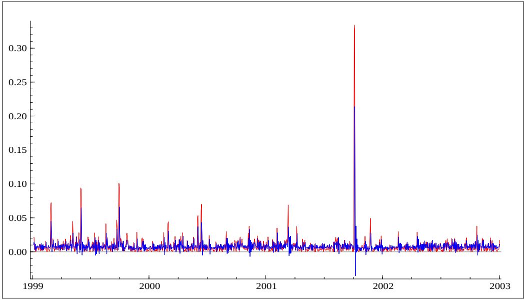
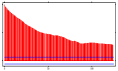


Figure 8: *Absolute returns forecast from ARFIMA*$(1, -0.32, 5)$ *for z2. Actual series (red) and in-sample prediction (blue). Sample period from 04.01.1999 - 25.02.2003.*



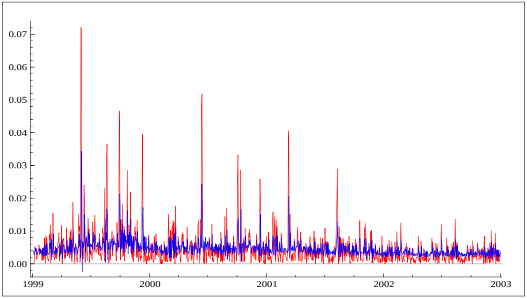
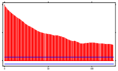
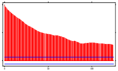


Figure 9: *Absolute returns forecast from ARFIMA$(1, 0.24, 2)$ for z3. Actual series (red) and in-sample prediction (blue). Sample period from 04.01.1999 - 25.02.2003.*



Conclusion

- Factors of Implied volatility dynamics exhibit long range dependence in levels and absolute returns.
- The class of fractional integrated model can better describe the long-run behavior of the loading series in a flexible way.
- For the factors of DSFM, long memory present an information source for long range volatility forecast that should be useful to regulators, derivative market participants and practitioners whose success depend on the ability to reasonably forecast stock market movements.



Unit root tests

The Augmented Dickey-Fuller (ADF) test refers to the regression equation

$$\Delta z_{t,k} = \phi z_{t-1,k} + \sum_{i=1}^{p} \alpha_i \Delta z_{t-i,k} + \varepsilon_{t,k}, \tag{11}$$

where p is the number of lags of $\Delta z_{t,k}$ by which the regression equation (11) is augmented in order to get residuals free of autocorrelation. The size of the test is better when p is large but causes the test to lose power.

Under H_0, the unit root the parameter ϕ should be zero. Hence, the t-statistic of the OLS estimator of ϕ is used as the ADF test statistic.

The limiting distribution of the test statistic is nonstandard. Critical or p-values have to be derived by the help of simulation methods.

The critical values (Mackinnon, J.G 1991) are -2.57 (10%), -2.86 (5%), and -3.44 (1%). Lag order p is determined by the AIC, HQ, and SC information criteria.

- ADF test suffers from low power, therefore may fail to detect a stationary time series

Point-optimal unit root test: (ERS) Elliot, Rothenberg and Stock (1996).
Superior to ADF in case of processes affected by conditional heteroscedasticity.
Test is based on quasi-differences of $z_{t,k}$ which are defined by

$$d(z_{t,k}|a) = \begin{cases} 1 & \text{if } t = 1 \\ z_{t,k} - a z_{t-1,k} & \text{if } t > 1, \end{cases}$$

a is the point alternative against which the null of a unit root is tested. Following the suggestion of Elliot et al. (1996), we use $a = \bar{a} = 1 - 7/T$ since only a constant term is considered.

Let $\hat{e}_t$ be the residuals from a regression of the time series on a quasi-differenced constant and let $S(\bar{a})$ and $S(1)$ be the sums of squared residuals for the cases $a = \bar{a}$ and $a = 1$ respectively. Then the test is defined by

$$ERS = (S(a) - aS(1))/\hat{\omega}_b, \tag{12}$$

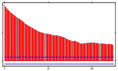
where $\hat{\omega}_b$ is the spectral density estimator of $\hat{e}_t$ at frequency zero. We apply the autoregressive spectral density estimator as proposed by Elliot et al. (1996).
Critical values are 4.48 (10%), 3.26 (5%) and 1.99 (1%).

Beran, J.
Statistics of long-memory processes
Chapman & Hall, New York, 1994.

Bollerslev, T. and Mikkelsen, H.O.
Modeling and pricing long memory in stock market volatility,
Journal of Econometics , **73**: 151–184, 1996

Borak, S. and Härdle, W. & and Fengler, M.
DSFM fitting of Implied Volatility Surfaces,
Proceedings 5th International Conference on Intelligent System Design and Applications, IEEE Computer Soceity Number P2286, Library of Congress Number 2005930524.

Diebold, F. X. and Inoue, A.
Long memory and structural change
Stern School of Business discussion paper , 1999



Doornik, J.A. and Ooms, M.
A package for estimating, forecasting and simulating ARFIMA models: Arfima package 1.0 for Ox,
Discussion paper, Nuffield College, Oxford.

Elliot, G., Rothenberg, T. J & Stock, J. H.
Efficient tests for an autoregressive unit root.
Econometrica , **64**: 813–836, 1996

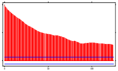
Geweke, J.F. and Porter-Hudax, S.
The estimation and application of long memory time series models.
Journal of Time Series Analysis, **4**(4):221–238, 1983.

Giraitis, L., Kokoszka, P. and Leipus, R.
Testing for long memory in the presence of a general trend.
Journal of Applied Probability, **38**:1033–1054, 2001.

Granger, C.W.J and Joyuex, R.
An introduction to long memory time series models and fractional differencing.
Journal of Time Series Analysis, **1**:15–39, 1980.

Hall, P., Härdle, W., Kleinow, T. and Schmidt, P.
On Semiparametric Bootstrap Approach to Hypothesis Tests and Confidence Intervals for Hurst Coefficients,
Statistics of Stochastic Processes, 3: 263-276, 2000.

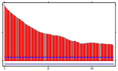
Hurvich, C., Doe, R. and Brodsky, J.
The Mean Square Error of Geweke and Porter-Hudak's estimator of the Memory Parameter of a Long-Memory time series , *Journal of Time Series Analysis*, 19: 19-46, 1998.



Kwiatkowski, D., Peter C. B. Phillips, Peter Schmidt and Yongcheol Shin
Testing the Null Hypothesis of Stationarity Against the Alternative of a Unit Root
Journal of Econometrics, **54**:159–178, 1992.

Liu, M.
Modelling long memory in stock market volatility,
Journal of Econometrics, **99**: 139–171, 2000.

Lobeto, I. and Savin, N.E
Real and Spurious Long-Memory Properties of Stock Market Data,
Journal of Business and Economic Statistics, **16**: 261–283, 1998.

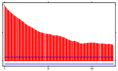


MacKinnon, J. G.
Critical values for cointegration tests, *in* Granger, C. W. J. and Engle, R. F. (eds), *Long-Run Economic Relationships*, Oxford University Press, Oxford, 1991.

Mandelbrot, B.B.
The fractal geometry of nature,
Freeman, New York, 1983.

McLeod, A.I. and Hipel, K.W.
Preservation of the rescaled adjusted range, 1. A reassessment of the Hurst phenomenon.
Water Resources Research, **14**:159–178, 1978.

Newey, W. and West, K.
Automatic Lag Selection in Covariance Matrix Estimation
Review of Economic Studies, **61**:631–653, 1994.



Robinson, P.M.
Gaussian Semiparametric Estimation of Long-Range Dependence,
The Annals of Statistics, **23**:1630–1661, 1995.

Sibbertsen, P.
Long-memory in volatilities of German stock returns
SFB,University of Dortmund, 2001.