Time Varying Independent Component Analysis

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Source extraction and dimension reduction

High dimensional and complex financial time series are neither Gaussian distributed nor stationary.

Multivariate Data Analysis (MDA)

Let $\mathbf{X}_t \in \mathbb{R}^p$ denote the returns of financial assets.

- Principal component analysis: $\mathbf{X}_t = \Gamma \times \mathrm{PC}_t$,
- Factor analysis: $\mathbf{X}_t = \Gamma\Lambda^{1/2}F_t + U_t$,

Jolliffe (2002), Härdle and Simar (2012)

Under Gaussianity, cross-uncorrelatedness indicates independence.

Jacobian transformation for a linear transformation $X = AZ$:

$$f_Z(z) = \prod_{j=1}^{p} f_{Z_j}(z_j), \quad f_X(x) = abs(|A|^{-1}) \cdot f_Z(A^{-1}X)$$

Fact: Financial time series are heavy-tailed distributed.

Independent Component Analysis (ICA)

Let $\mathbf{X}_t \in \mathbb{R}^p$ denote the returns of financial assets:

$$
\begin{aligned}
\text{IC}_t &= B\mathbf{X}_t = (b_1, \cdots, b_p)^\top \mathbf{X}_t \\
\begin{pmatrix} \text{IC}_{1t} \\ \vdots \\ \text{IC}_{pt} \end{pmatrix} &= \begin{pmatrix} b_{11} & \cdots & b_{1p} \\ . & \cdots & . \\ b_{p1} & \cdots & b_{pp} \end{pmatrix} \begin{pmatrix} x_{1t} \\ \vdots \\ x_{pt} \end{pmatrix} \\
\text{equivalently } \mathbf{X}_t &= A \times \text{IC}_t
\end{aligned}
$$

where B is a nonsingular filter matrix: $B^{-1} = A$.

How to find ICs?

$$\mathbf{X}_t = A \times \mathrm{IC}_t$$

Jones and Sibson (1987): projection pursuit

Hyvärinen and Oja (1997): FastICA

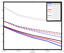
Hyvärinen, Karhunen and Oja (2001): MLE and others

Chen, Guo, Härdle and Huang (2011): COPICA

The observed series as well the ICs are assumed to be stationary.

The filter A (or B) is constant over time.

Fact: Turbulences in financial markets indicate nonstationary.



Demonstration

Log returns of HD, HPQ and IBM.

$$\mathbf{X}_t = \begin{cases} A_1 \mathrm{IC}_t & t \in [1, 300] \\ A_2 \mathrm{IC}_t & t \in [301, 600] \end{cases}$$

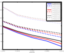
where IC_t are NIG distributed, see Barndorff-Nielson (1997).

Two ICA filters are:

$$A_1 = 10^{-3} \begin{pmatrix} 0.6 & 13.0 & 6.2 \\ 3.8 & 2.7 & 13.0 \\ 7.9 & 5.9 & 4.8 \end{pmatrix}, \qquad A_2 = 10^{-3} \begin{pmatrix} -0.1 & 0.8 & 5.3 \\ 7.0 & 1.9 & 1.6 \\ 0.1 & 4.2 & 1.1 \end{pmatrix}.$$

2008/09/03 – –2009/08/31, (a period with market turbulence)

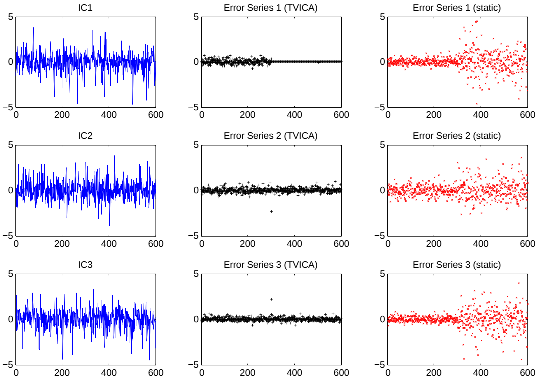
2004/07/30 – –2006/12/29 (a relatively quiet period)

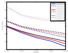


Demonstration (Continued)

Static ICA: average value of RMSEs is 0.886 (1.196 after change)

Time varying ICA: average value of RMSEs is 0.201 (0.160 after change)



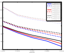


Literature review

Matteson and Tsay (2009): allow the mixing matrix B to vary over time via a smooth function of other transition variables.

- Volatility and co-volatility literature, see e.g. Baillie and Morana (2009), Scharth and Medeiros (2009),
- Incorporate changes via Markov-Switching or mixture of multiplicative error specifications,
- Need a globally given mechanism for this time variation.

Mercurio and Spokoiny (2004) use a local change point (LCP) approach: completely data driven approach.

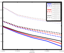


TVICA

Let $\mathbf{X}_t \in \mathbb{R}^p$ denote the returns of financial assets, TVICA model:

$$\mathbf{X}_t = A_t \mathrm{IC}_t$$

- Time varying independent source extraction,
- For each time point t, LCP identifies a "trust interval" $I_t = [t - m_t, t]$, over which the filter A_t $\approx$const.,
- Neither prior information (on say states of the market) nor distributional assumption is required. Data-driven and applicable for various kinds of breaks (macroeconomic or political changes).



Outline

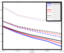
1. Motivation ✓
2. TVICA and estimation
3. Simulation study
4. Real data analysis
5. Conclusion

TVICA

Let $\mathbf{X}_t \in \mathbb{R}^p$ denote the returns of financial assets,
$\mathbf{Z}_t = \{z_1(t), \cdots, z_p(t)\}^\top$ are cross independent.

$$\text{TVICA model:} \quad \mathbf{X}_t = A_t \mathbf{Z}_t, \quad \mathbf{Z}_t = B_t^{-1} \mathbf{X}_t$$

Local Homogeneity: for any particular time point t there exists a past time interval $I_t = [t - m_t, t]$, over which the linear filter A_t is approximately constant, i.e. $A_s \approx A, \forall s \in I_t$.



Estimation: under homogeneity

Suppose that at time point t, an interval of **homogeneity** $I_t = [t - m_t, t)$ is given with m_t indicating the length of the interval.

The log-likelihood function on the interval I_t is:

$$L(I_t, B_t) = \sum_{s=t-m_t}^{t} \sum_{j=1}^{r} \log\{f_j(b_{jt}^{\top} \mathbf{X}_s)\} + (m_t + 1) \log |\det B_t|, \quad (1)$$

where $f_j(z_j)$ is the pdf of IC z_j, $j = 1, \cdots, p$. MLE is $\tilde{B}_t$.

Estimation: under local homogeneity

Small modeling bias: divergence of a time varying model (local homogeneity) to a static model (homogeneity) is small, Spokoiny (2011).
For $r, \rho > 0$, the fitted log likelihood with $B_t = B^*$ satisfies:

$$\mathsf{E}_{B^*}|L(I_k, \tilde{B}_t^{(k)}, B^*)|^r = \mathsf{E}_{B^*}|L(I_k, \tilde{B}_t^{(k)}) - L(I_k, B^*)|^r \le R_r(B^*), \tag{2}$$

where $R_r(B^*) = \max_{k \le K} \mathsf{E}_{B^*}|L_{I_k}(\tilde{B}_k, B^*)|^r$.

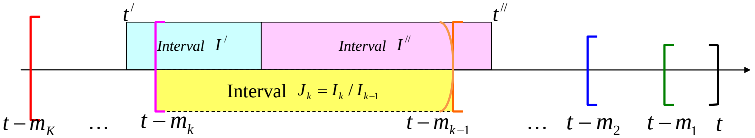
Goal: For any time point t and nested intervals, $I_0 \subset I_1 \subset \cdots \subset I_{K-1} \subset I_K$, LCP method finds the longest interval of local homogeneity.
The identification of the trust interval is done via a sequential testing algorithm.

LCP algorithms

H_0 : I_k is a local homogeneous interval *given that I_{k-1} was not rejected.*

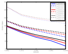
Initialization: I_0 is accepted $\hat{B}_t^{(0)} = \tilde{B}_t^{(0)}$.

Next for $k = 1, \cdots, K$, screen $J_k = I_k \setminus I_{k-1} = [t - m_k, t - m_{k-1})$ and check for a change point.



$$T_{I,t} = \max_{B'', B'} \{L_{I''}(B'') + L_{I'}(B')\} - \max_{B} L_I(B), \quad (3)$$

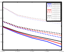
$$T_k = \max_{t \in J_k} T_{I,t} \begin{cases} \leq \eta_k & H_0 \text{ is not rejected: } \hat{B}_t^{(k)} = \tilde{B}_t^{(k)} \\ > \eta_k & H_0 \text{ is rejected, terminate} \end{cases} \quad (4)$$



LCP parameters

Set of intervals: $I_k = [t - m_k, t]$ with $m_k = m_0 a^k$.

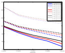
- The starting value m_0 should be sufficiently small to provide a reasonable local homogeneity.
- The coefficient $a > 1$ controls the increasing speed of the candidate intervals.



LCP parameters

Critical values $\{\eta_k\}$ are calculated under H_0.

- MC: generate homogeneous series $\mathbf{X}_t = (B^*)^{-1}\mathrm{IC}_t$.
- The final estimate $\hat{B} = \hat{B}_K$ depends on the critical values $\{\eta_k\}_{k=1}^{K}$.
- Small modeling bias: $\mathrm{E}_{B^*}|L(I_k, \tilde{B}_t^{(k)}, \hat{B})|^r \le \rho R_r(B^*)$,
  - B^* is the MLE over I_0.
  - The hyperparameter r specifies the loss function that measures the divergence of a time varying model to a static model.
  - The hyperparameter ρ is similar to the test level parameter.
  - Given the values of r and ρ, $R_r(B^*)$ can be computed straightforwardly.



Find ICs

Pre-whitening: use the Mahalanobis transformation $\tilde{\Sigma}_x^{-1/2}\mathbf{X}_t$.

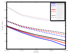
Quasi maximum likelihood estimation: for leptokurtic sources

$$\log f_j(x_j) = \alpha_1 - 2\log\cosh(x_j) = \alpha_1 - 2\log\{\frac{1}{2}(e^{x_j} + e^{-x_j})\}.$$

The first derivative of $\log f_j$:

$$g_j(x_j) = -2\tanh(x_j) = -\frac{2\{\exp(2x_j) - 1\}}{\exp(2x_j) + 1}, \quad \forall\, j = 1, \ldots, p,$$

A small misidentification in the density doesn't affect the consistency of the QMLE, Hyvärinen and Oja (1999).



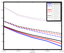
Data

$\mathbf{X}_t \in \mathbb{R}^{10}$: log returns of HD, HPQ, IBM, INTC, JNJ, JPM, KFT, KO, MCD and MMM over a stationary time period: 2010/01/14–2010/10/28. Fit IC_t under NIG assumption. Generate 10 independent univariate series, with 610 sample points for each series and with 1000 replications.

Homogeneity scenario (HOMO): $\mathbf{X}_t = A_t \mathrm{IC}_t$ with $A_t = I_{10}$,
Jump scenario (JPLM and JPEM): a sudden change after $t = 250$.
Smooth change scenario (SLEM): interval with changes: $[220, 380]$

Investigate detection power and location of the change point.

Analyze impact of the hyperparameters (r, ρ) on the LCP algorithm.

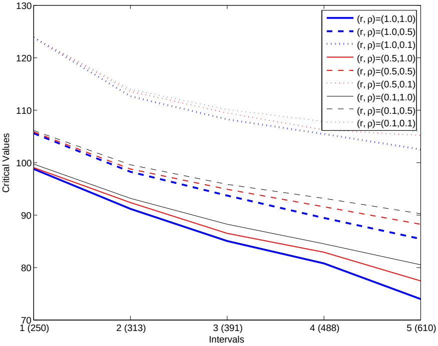


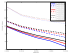
Critical values

Set of intervals: $m_k = m_0 a^k$ with $m_0 = 200$, $a = 1.25$ and $K = 5$

$$I_0 = 200,\ I_1 = 250,\ I_2 = 313,\ I_3 = 391\ I_4 = 488,\ I_5 = 610,$$

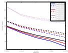
r and ρ are assigned to be $1, 0.5$ and 0.1





Result: rejection ratio and location

ρ	@	$r = 0.1$ l_1	l_2	l_3	l_4	$r = 0.5$ l_1	l_2	l_3	l_4	$r = 1.0$ l_1	l_2	l_3	l_4
0.1	HOMO	— 0.6 —				— 0.6 —				— 0.7 —			
	JPLF	–	–	100	–	–	–	100	–	–	–	100	–
	JPEM	–	–	99.2	0.8	–	–	99.4	0.6	–	–	99.4	0.6
	SLEM	–	5.9	93.1	1.0	–	6.8	92.4	0.8	–	7.9	91.3	0.8
0.5	HOMO	— 4.9 —				— 5.9 —				— 8.3 —			
	JPLF	0.1	–	99.9	–	0.1	0.1	99.8	–	0.1	0.1	99.8	–
	JPEM	–	0.1	99.5	0.4	–	0.2	99.5	0.3	–	0.2	99.6	0.2
	SLEM	0.2	32.4	67.4	–	0.2	34.4	65.4	–	0.2	36.1	63.7	–
1.0	HOMO	— 15.3 —				— 20.3 —				— 26.8 —			
	JPLF	0.2	0.4	99.4	–	0.2	0.4	99.4	–	0.2	0.7	99.1	–
	JPEM	–	0.4	99.5	0.1	–	0.6	99.4	–	–	0.8	99.2	–
	SLEM	0.2	49.5	50.3	–	0.2	52.6	47.2	–	0.4	56.4	43.2	–



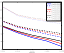
Data and experiments

$\mathbf{X}_t \in \mathbb{R}^{10}$: log returns of HD, HPQ, IBM, INTC, JNJ, JPM, KFT, KO, MCD and MMM.

The set of intervals: $m_k = m_0 a^k$ with $m_0 = 200$, $a = 1.25$ and $K = 5$.

The parameters $(r, \rho) = (0.5, 0.5)$ and $(r, \rho) = (0.1, 0.1)$ are considered respectively.

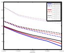
B^*: MLE over I_0 or identity matrix.



Data and experiments

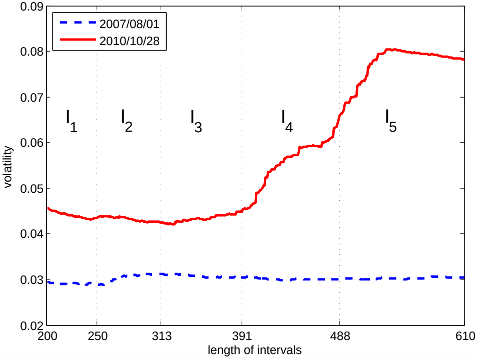
The first experiment considers the time interval 2005/03/01–2007/08/01, during which no influential economic or financial events occurred.
The second experiment considers the time interval 2008/05/30–2010/10/28, during which the stock market crash occurred in 2008.

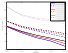
Does the proposed method detect intervals of local homogeneity? Can we identify an interval in a post-financial crisis world that indicates a relatively stationary period?



Empirical evidence

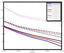
Realized volatility recursively computed for the 1st August 2007 and the 28th October 2010. The set of intervals with $m_0 = 200$, $a = 1.25$ and $K = 5$ is marked in the plot to highlight the underlying pattern across the intervals.





Results: CVs and test statistics

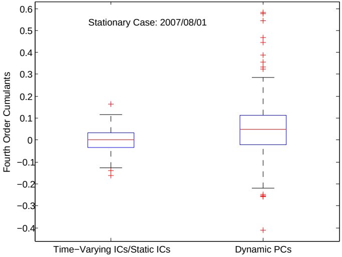
	2005/03/01-2007/08/01					2008/05/30-2010/10/28				
	CV				T_I	CV				T_I
(r, ρ)	$(0.5, 0.5)$		$(0.1, 0.1)$			$(0.5, 0.5)$		$(0.1, 0.1)$		
B^*	MLE	Identity	MLE	Identity		MLE	Identity	MLE	Identity	
I_1	107.23	102.84	122.37	120.89	74.36	108.87	105.85	126.51	123.74	69.81
I_2	98.40	98.45	117.43	113.21	76.62	101.71	98.67	116.86	113.95	81.97
I_3	93.15	92.35	112.30	108.44	66.86	96.32	94.92	113.91	110.05	**265.35**
I_4	89.64	88.81	109.53	105.57	77.52	92.59	91.57	111.18	107.80	**469.99**
I_5	86.28	85.74	106.82	103.01	72.79	88.72	88.21	108.99	105.85	**205.60**



Results: Independence under homogeneity

Fourth order cross-cumulant is used as a measure of statistical independence:

$$\mathrm{cum}(z_i, z_j, z_k, z_l) = \mathrm{E}(z_i z_j z_k z_l) - \mathrm{E}(z_i z_j)\,\mathrm{E}(z_k z_l) - \mathrm{E}(z_i z_k)\,\mathrm{E}(z_j z_l) - \mathrm{E}(z_i z_l)\,\mathrm{E}(z_j z_k),$$



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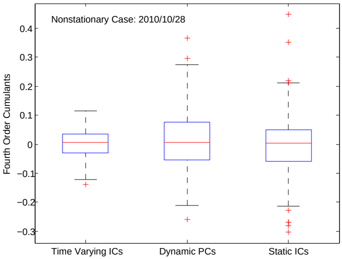

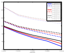

Conclusion

- Develop a time varying modeling for independent source extraction, ✓
- For each time point t, LCP approach helps to identify a "trust interval" $I_t = [t - m_t, t)$, over which the linear filter A_t (or B_t) is approximately const., ✓
- Simulation study and real data analysis show that the TVICA method is data driven. It provides a stable performance for different parameter selection and works well, ✓
- A universal statistical MDA method that is applicable for non-Gaussian and non-stationary financial time series.

Appendix

HD: The Home Depot

HPQ: Hewlett-Packard

IBM: International Business Machines

INTC: Intel

JNJ: Johnson & Johnson

JPM: JPMorgan Chase

KFT: Kraft Foods

KO: Coca-Cola

MCD: McDonald's

MMM: 3M

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