Adaptive Pointwise Estimation in Conditional-Heteroscedasticity Models

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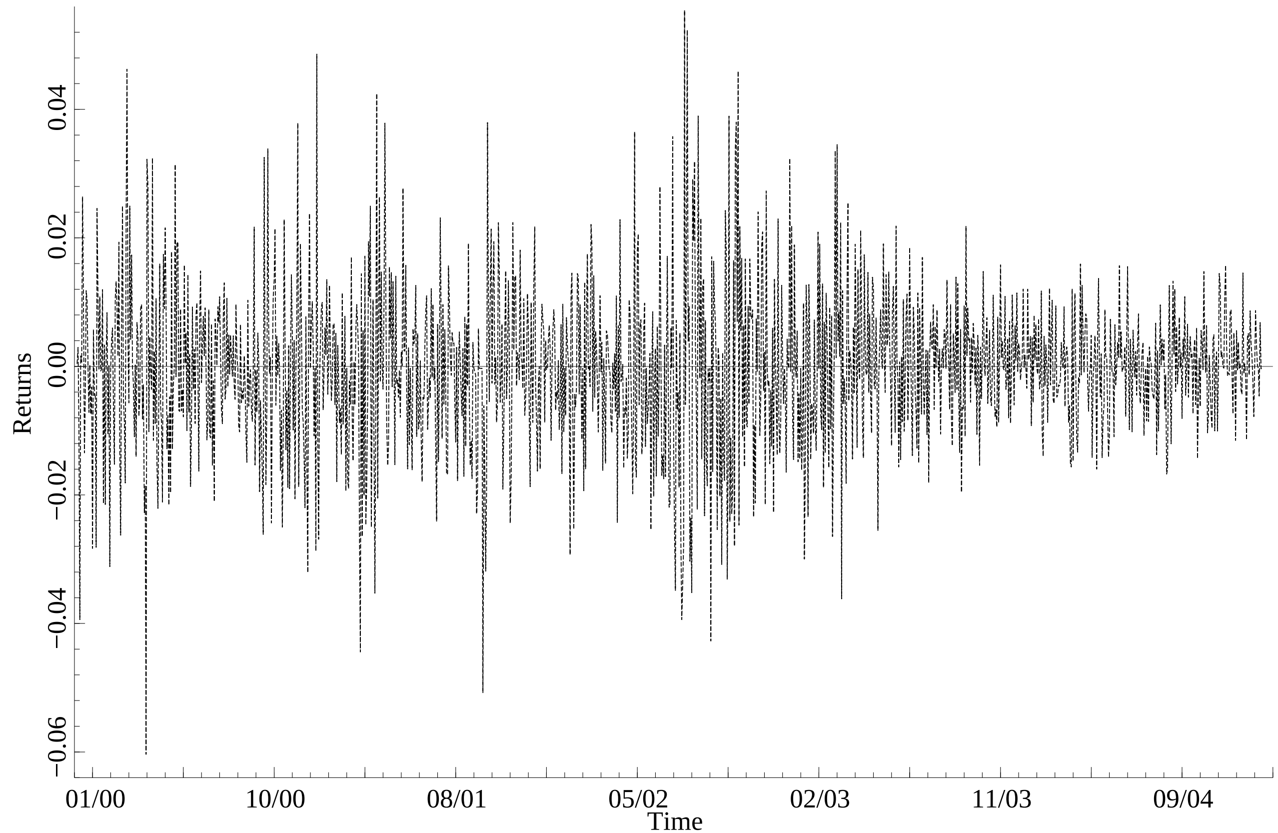
Time series regression models

- Classical parametric approach: data follow one model
- Structural breaks and prediction in time series

(S&P500 stock index from January, 2000 to October, 2004)



Time-homogeneity vs. structural breaks

Conditional heteroscedasticity models

- ARCH, GARCH models (Engle, 1982; Bolerslev, 1986)
- stochastic-volatility models (Taylor, 1986)
- time-homogeneous structure

Structural breaks (change points) in financial time series (Andreou & Ghysels, 2002; Herwatz & Reimers, 2002)

- Breakpoint detection (Mikosch & Starica, 2001)
  – supremum tests (supLR) for GMM (Andrews, 1993)
  – CUSUM-ARCH (Lee et al., 2003)
- Nonparametric time-varying coefficient models
  – parameters smooth in time (Cai, Fan, & Li, 2000)
  – pointwise adaptive (Mercurio & Spokoiny, 2004)

Extend pointwise adaptive estimation to data-hungry models (e.g., GARCH family)

Local parametric modeling

- Assumption: time series $Y_t, t = 1, \ldots, T$, follows *locally* some parametric model ($f_t \in \mathcal{F}$; e.g., $f_t = f[\theta(t)]$)

$$Y_t = f_t(Y_{t-1}, \ldots, Y_{t-p}, \varepsilon_t, \ldots, \varepsilon_{t-q}) \tag{1}$$

- Example of GARCH(1,1)
  – time-homogeneous

$$\begin{aligned} Y_t &= 0 + \varepsilon_t, \\ E(\varepsilon_t|\varepsilon_{t-1}) &= 0, \\ h_t^2(\theta) = Var(\varepsilon_t|\varepsilon_{t-1}) &= \omega + \alpha Y_{t-1}^2 + \beta h_{t-1}^2(\theta), \end{aligned}$$

  – time-heterogeneous ($\theta(t) = (\omega(t), \alpha(t), \beta(t))$):

$$h_t^2(\theta(t)) = Var(\varepsilon_t|\varepsilon_{t-1}) = \omega(t) + \alpha(t) Y_{t-1}^2 + \beta(t) h_{t-1}^2(\theta(t))$$

Adaptive pointwise estimation

Find the most recent structural break and estimate model on the longest interval of homogeneity "(break, now)"

1. **Initialization:** Start with a short interval $(t_s, t_e = T)$
2. **SupLR test:** Perform the supLR test (critical values!)

$$supLR_{\mathcal{F}}(t_s, t_e) = \sup_{t_s + \Delta L < t_b < t_e - \Delta L} LR_{\mathcal{F}}(t_b; t_s, t_e)$$

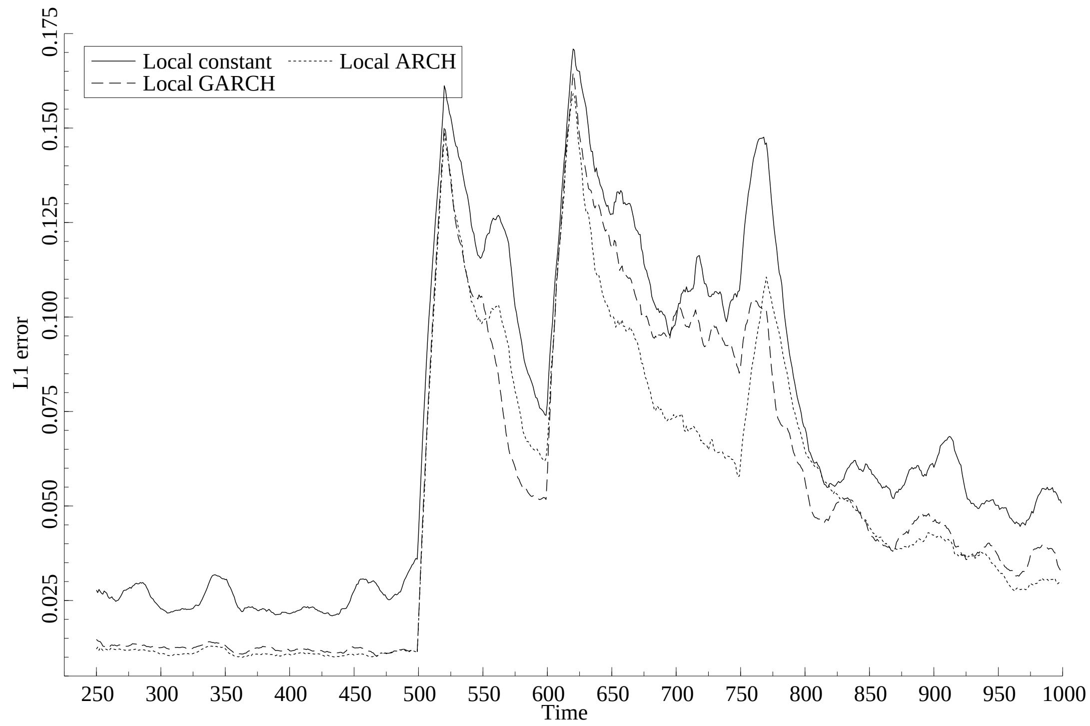
3. **Extension:** No break detected
   $\Rightarrow$ prolong (t_s, t_e), for example by 50%, and go to (2)
4. **Detection:** Break detected at $t_b \in (t_s, t_e)$
   $\Rightarrow$ estimate the model on $I = (t_b, T)$

Submodel hierarchy

- construct sequence of submodels $M_0 \subset M_1 \subset \ldots \subset M_S$
- search interval I_k for model $M_k, k = 0, 1, \ldots, S,$
- select best model by forecasting performance

Simulations from change-point GARCH

- 1000 observations, 100 replications, 1 period ahead
- Structural breaks in volatility at 500, 600, 750
- L_1 prediction error averaged over 20 observations
- GARCH(c,0.2,0.1): ARCH and GARCH perform equally



- GARCH(c,0.2,0.7): GARCH prevails (short horizon only)

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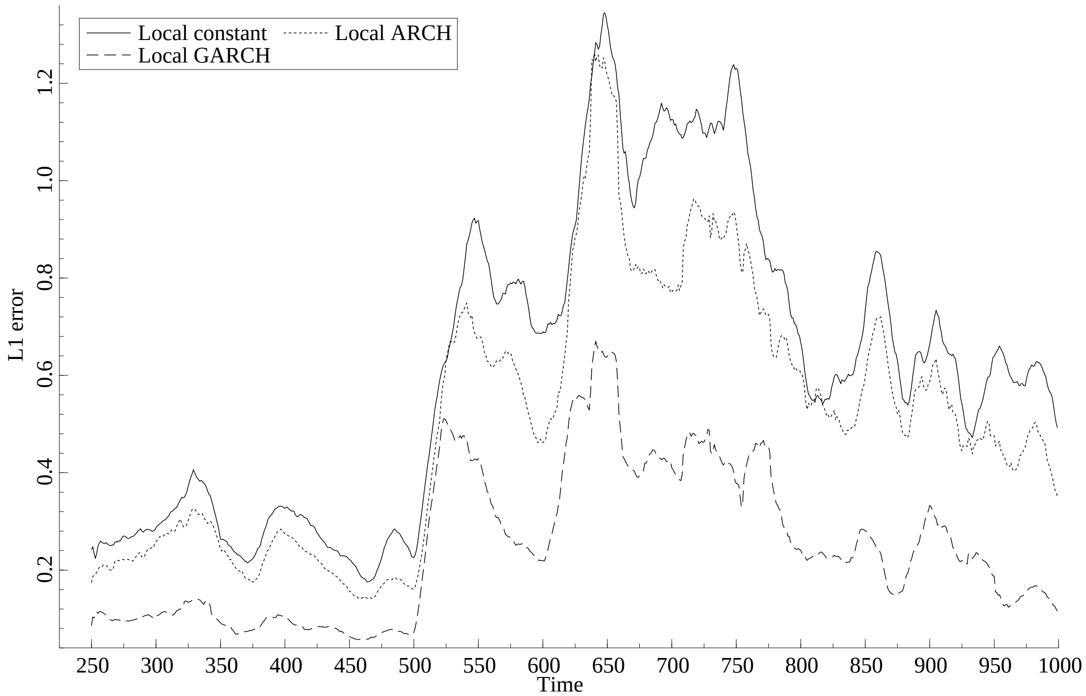
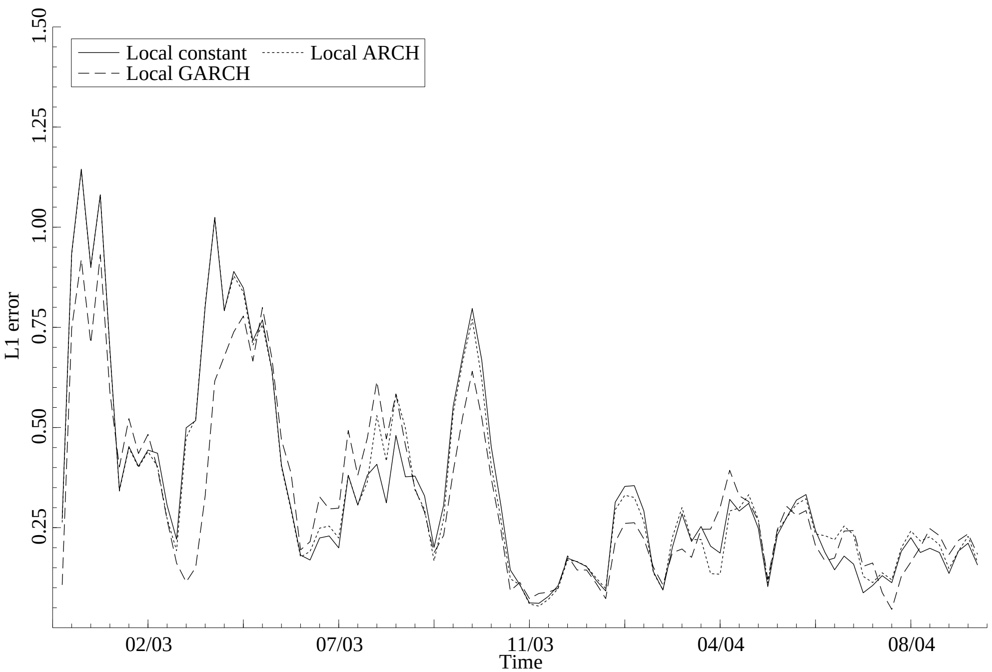


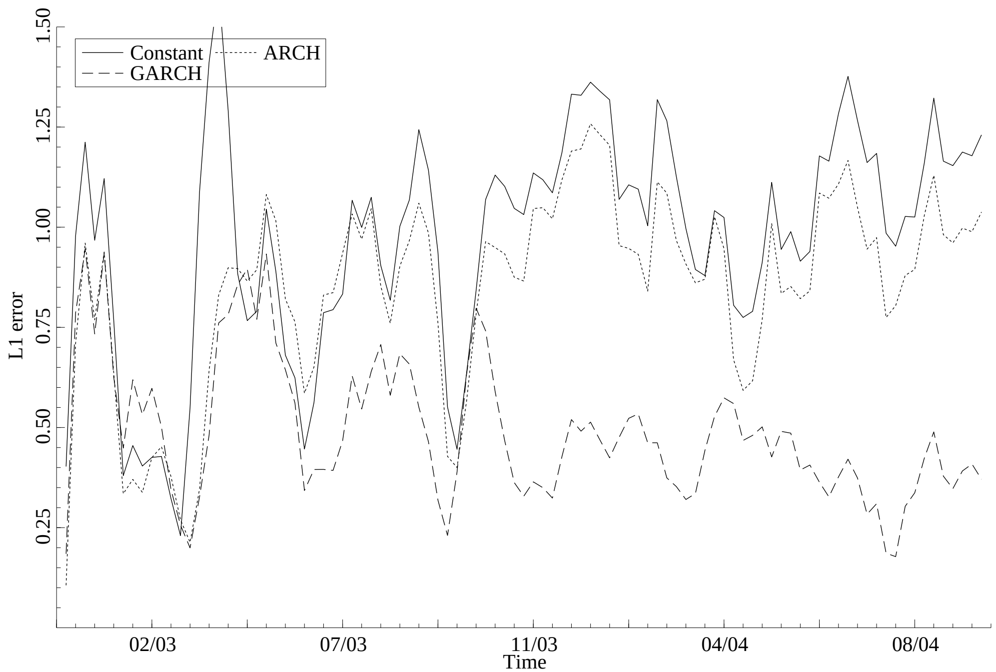
Standard & Poors 500 index

- S&P500 stock index from January 2000 to October 2004
- Prediction horizon: one day ahead
- Prediction period: January 2003 to October 2004
- L_1 prediction error averaged over 20 observations (one month)

Adaptive methods

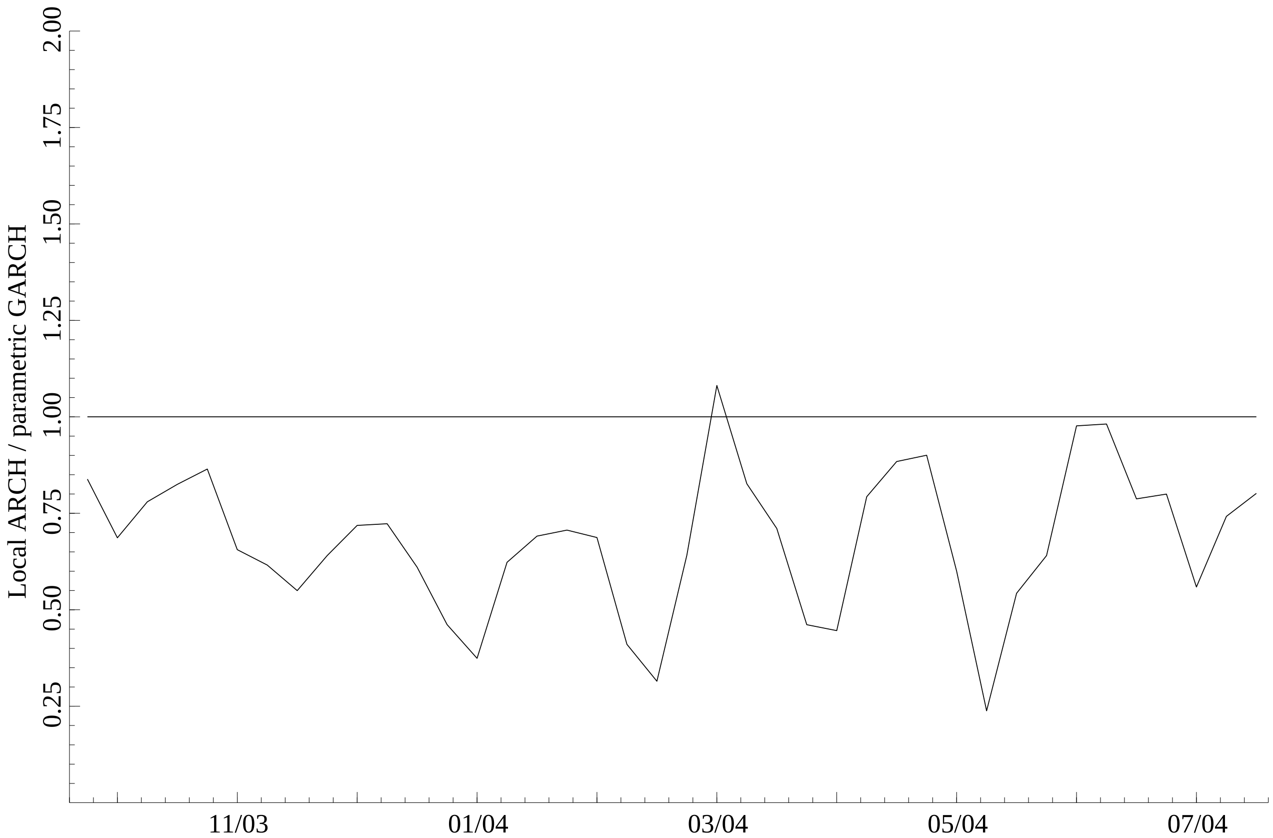


Parametric methods



Adaptive ARCH vs. parametric GARCH

- S&P500 index from January 2000 to August 2004
- Prediction: one day, September 2003 to August 2004
- Ratio of L_1 prediction errors: adaptive ARCH / GARCH



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Conclusion

- Pointwise adaptive estimation for GARCH
  - search for the longest interval of homogeneity possible using a submodel hierarchy
  - detect breaks in stock index series (10/1997, 3/2003, ...)
  - outperforms standard GARCH
- Pointwise adaptive estimation for ARCH
  - can perform as well as GARCH
  - less sensitive to choice of critical values
  - other model hierarchies reasonable (e.g., ARCH(0) – ARCH(1) – ... – ARCH(p))