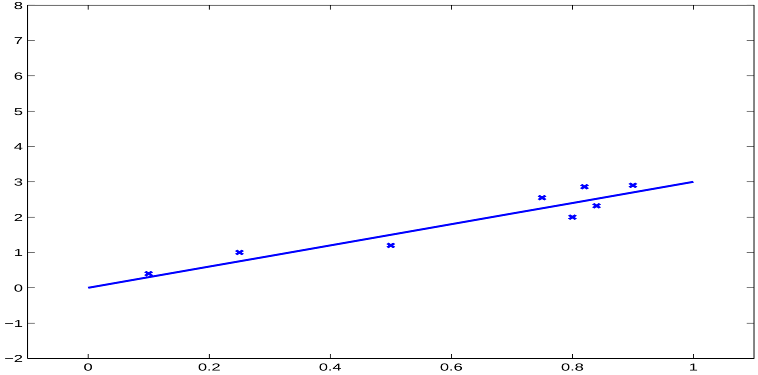
Dynamic Semiparametric Factor Models

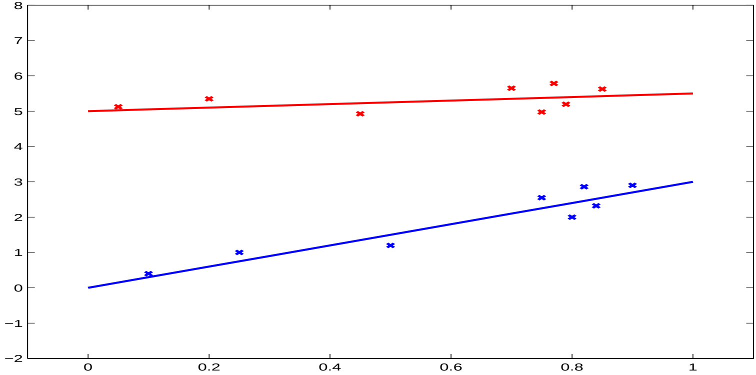
Szymon Borak
Wolfgang K. Härdle
Enno Mammen
Beyong Park

CASE-Center for Applied Statistics and Economics
Humboldt-Universität zu Berlin
Universität Mannheim
and
Seoul National University

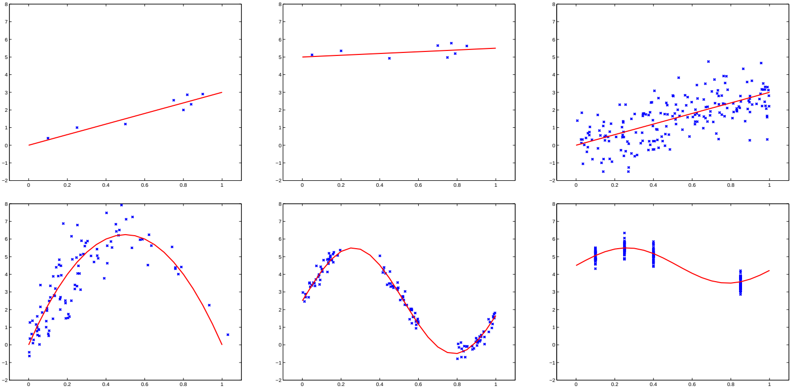
One Regression Line



Two Regression Lines



More Complex Regressions



Implied Volatility Dynamics

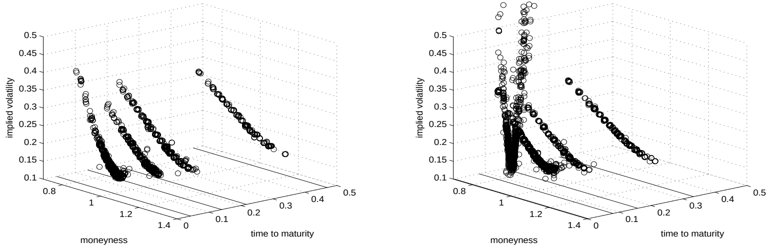


Figure 1: *The typical IV data design on two different days. Bottom solid lines indicate the observed maturities, which move towards the expiry. Left panel: observations on 20040701. Right panel: observations on 20040819.*

$$\underbrace{(X_{1,1}, Y_{1,1}), \ldots, (X_{J_1,1}, Y_{J_1,1})}_{t=1} \underbrace{(X_{1,2}, Y_{1,2}), \ldots}_{t=2} \ldots \underbrace{\ldots (X_{J_T,T}, Y_{1,T})}_{t=T}$$

where:

$X_{j,t} \in \mathbb{R}^d$

$Y_{j,t} \in \mathbb{R}$

T - the number of observed time periods (days)

J_t - the number of the observations in (day) t

$E(\mathbf{Y}_t|\mathbf{X}_t) = F_t(\mathbf{X_t})$.

What is $F_t(\mathbf{X_t})$?

- neglecting dependency on t then $F(\mathbf{X})$ is a usual regression problem based on the pooled data
- analyzing F_t separately for each t leads to T regression problems, common structure is lost
- one needs some compromise between the common structure and time dependency

Example

- panel data models, one observes J_t records through time

$$E(Y_{jt}|X_{jt}) = \sum_{s=1}^{d} Z_s X_{jt}^{(s)} + V_j + \Lambda_t$$

$X_{jt}^{(s)}$ is s-th coordinate of the vector X_{jt}
V_j represents individual specifics, (random or non-random)
Λ_t reflects external time effects

Example

- time varying linear regression (Hansen et al. (2004) for Risk Theory)

$$E(\mathbf{Y}_t|\mathbf{X}_t) = Z_{t,0} + \sum_{s=1}^{d} Z_{t,s} X_t^{(s)}$$

- time varying nonlinear regression (Connor and Linton (2007) for stock returns)

$$E(\mathbf{Y}_t|\mathbf{X}_t) = Z_{t,0} + \sum_{s=1}^{d} Z_{t,s} g_s(X_t^{(s)})$$

$X_t^{(s)}$ is s-th coordinate of the vector $\mathbf{X}_t$, g_s are known or unknown.

Example

- parametric factor approach (Nelson and Siegel (1987) for yield curves) - m_l known

$$E(\mathbf{Y}_t|\mathbf{X}_t) = m_0(\mathbf{X}_t) + \sum_{l=1}^{L} Z_{t,l} m_l(\mathbf{X}_t)$$

- multivariate time series dimension reduction techniques

$$E(\mathbf{Y}_t|\mathbf{X}_t) \approx \widehat{F}_t(\mathbf{X}_t) \approx m_0(\mathbf{X}_t) + \sum_{l=1}^{L} Z_{t,l} m_l(\mathbf{X}_t)$$

where $\widehat{F}_t$ is a multivariate time series representation of F_t. (Ramsay and Silverman (1997) in functional data analysis context)

DSFM

The **dynamic semiparametric factor model (DSFM)** assumes the following form:

$$E(\mathbf{Y}_t|\mathbf{X}_t) = m_0 + \sum_{l=1}^{L} Z_{t,l} m_l(\mathbf{X}_t) = Z_t^\top m(\mathbf{X}_t)$$

$m(\cdot)$ is a tuple of functions $(m_0, m_1, \ldots, m_L)^\top$
$Z_t = (1, Z_{t,1}, \ldots, Z_{t,L})^\top$ is a multivariate time series with a certain dynamic structure.

DSFM

- m reflects the time invariant (factor) structure
- $\widehat{m}$ is nonparametric estimator obtained **directly** from the data
- Z_t describes the dynamic behavior
- the dynamics is analyzed via estimates $\widehat{Z}_t$

What is the difference of the inference based on the $\widehat{Z}_t$ instead of Z_t?

The inference on Z_t is essential for:

- forecasting Y_t distribution (eg. pricing exotic options, risk management)
- cointegration with external variables (eg. macroeconomic variables)
- studying dynamics of related objects (eg. empirical pricing kernel)

Overview

1. Motivation✓
2. Model Formulation
3. Asymptotic Results
4. Simulations
5. Application
6. Conclusion

Model

The model has the form:

$$Y_{t,j} = Z_t^\top m(X_{t,j}) + \varepsilon_{t,j} \tag{1}$$

For simplicity of notation:
$t = 1, \ldots, T$,
$j = 1, \ldots, J$ $(J_t = J)$,
$X_{t,j} \in [0,1]^d$,
$m_j : [0,1]^d \to \mathbb{R}$

Implied Volatility Surface

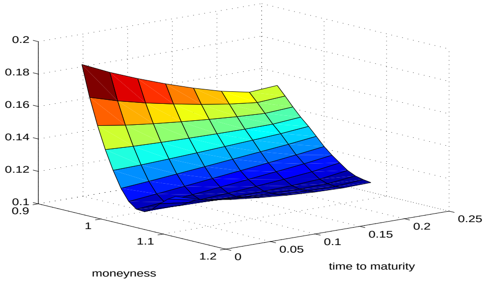


Figure 2: *The implied volatility surface estimated on 20050629 using a two-dimensional local linear estimator.*

Kernel Estimator

Fengler et al. (2007) propose to minimize:

$$\sum_{t=1}^{T}\sum_{j=1}^{J_t}\int\left\{Y_{t,j}-\sum_{l=0}^{L}\widehat{Z}_{t,l}\widehat{m}_l(u)\right\}^2 K_h(u-X_{t,j})\,du, \qquad (2)$$

where K_h denotes a two dimensional product kernel, $h=(h_1,h_2)$, $K_h(u)=k_{h_1}(u_1)\times k_{h_2}(u_2)$, with a one-dimensional kernel $k_{h_\cdot}(v)= = h_\cdot^{-1}k(h_\cdot^{-1}v)$. A kernel smoothing procedure can be equivalently replaced by a series estimator.

Series Estimator

$$Z_t^\top m(X) = \sum_{l=0}^{L} Z_{t,l} \sum_{k=1}^{K} a_{l,k}\psi_k(X) = Z_t^\top A\psi(X)$$

where $\psi(X) = (\psi_1, \ldots, \psi_K)^\top(X)$ is a vector of known basis functions, $A \in \mathbb{R}^{(L+1)\times K}$ is a coefficient matrix. K plays a role of the bandwidth h in (2). Define the least squares estimators $\widehat{Z}_t = (\widehat{Z}_0, ..., \widehat{Z}_L)^\top$ and $\widehat{A} = (\widehat{a}_{l,k})_{l=0,...,L;k=1,...,K}$

$$(\widehat{Z}_t, \widehat{A}) = \arg\min_{Z_t, A} \sum_{t=1}^{T}\sum_{j=1}^{J}\left\{Y_{t,j} - \widehat{Z}_t^\top \widehat{A}\psi(X_{t,j})\right\}^2 \qquad (3)$$

Identification Issues

The minimization problem (3) has no unique solution. If $(\widehat{Z}_t, \widehat{A})$ is a minimizer then also

$$(\widetilde{B}^\top \widehat{Z}_t, \widetilde{B}^{-1}\widehat{A})$$

is a minimizer. Here $\widetilde{B}$ is an arbitrary matrix of the form

$$\widetilde{B} = \begin{pmatrix} 1 & 0 \\ 0 & B \end{pmatrix}$$

for an invertible matrix B.

Smoothing parameters

- L - the dimension of the time series
- K - number of the series expansion functions
- ψ - type of the basis functions (here B-splines)

Inference

The differences in the inference based on $\widehat{Z}_t$ instead of (true unobservable) Z_t are asymptotically negligible.

This asymptotic equivalence carries over to estimation and testing procedures in the framework of fitting a vector autoregresssive model.

Therefore it is justified to fit vector autoregressive model and proceed as if $\widehat{Z}_t$ were observed.

Suppose that the model (1) holds and that $(\widehat{Z}_t, \widehat{A})$ is defined by the minimization problem (3). Define a random matrix $\widetilde{B}$

$$\widetilde{B} = \left(T^{-1} \sum_{t=1}^{T} Z_t \widehat{Z}_t^\top \right)^{-1} T^{-1} \sum_{t=1}^{T} Z_t Z_t^\top, \tag{4}$$

and

$$\widetilde{Z}_t = \widetilde{B}^\top \widehat{Z}_t,$$

Theorem

Under regularity assumptions (see Appendix) for $h \geq 0$

$$\frac{1}{T}\sum_{t=h+1}^{T}\left(\tilde{Z}_t - \overline{\tilde{Z}}\right)\left(\tilde{Z}_{t-h} - \overline{\tilde{Z}}\right)^{\top} - \frac{1}{T}\sum_{t=h+1}^{T}\left(Z_t - \overline{Z}\right)\left(Z_{t-h} - \overline{Z}\right)^{\top} = \mathcal{O}_P(T^{-1/2})$$

and

$$\overline{\tilde{Z}} - \overline{Z} = \mathcal{O}_P(T^{-1/2}),$$

where $\overline{\tilde{Z}} = \frac{1}{T}\sum_{t=1}^{T}\tilde{Z}_t$ *and* $\overline{Z} = \frac{1}{T}\sum_{t=1}^{T} Z_t$

VAR processes

Consider Z_t as a VAR(p) process:

$$Z_t = \mathcal{A}_1 Z_{t-1} + \ldots + \mathcal{A}_p Z_{t-p} + U_t,$$

where U_t is white noise and $\mathcal{A}_i$ is a coefficient matrix.
Define $\theta = (\mathcal{A}_1, \ldots, \mathcal{A}_p)$ then $\widehat{\theta}$ is a function of the autocovariance matrices (Yule-Walker equations) and

$$\sqrt{T}(\widehat{\theta} - \theta) = \mathcal{O}_P(1).$$

Adopting similar notation for $\widetilde{Z}_t$ and $\widetilde{\theta}$ one obtains:

$$\begin{aligned}\sqrt{T}\left(\widehat{\widetilde{\theta}}-\theta\right) &= \sqrt{T}\underbrace{\left(\widehat{\widetilde{\theta}}-\widehat{\theta}\right)}_{\text{DSFM}}+\sqrt{T}\left(\widehat{\theta}-\theta\right)\\ &= \underbrace{\mathcal{o}_P(1)}_{\text{Theorem}}+\mathcal{O}_P(1)=\mathcal{O}_P(1)\end{aligned}$$

The asymptotic error of the DSFM estimation is of smaller order than the error of parameter estimation in the VAR framework!

Simulation Setting

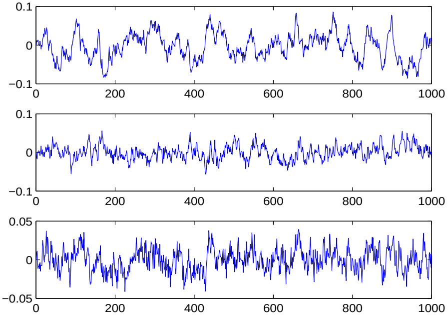
- Simulate Z_t from a VAR(1) model
- Simulate design points $X_{t,j}$ from uniform distribution $[0, 1]^2$
- Set some known functions m and generate $Y_{t,j}$ from Z_t, m and $X_{t,j}$
- Estimate $\widetilde{Z}_t$ and compare autocovariance matrices of $\widetilde{Z}_t$ and Z_t

VAR Model

$$Z_t = \mathcal{A} Z_{t-1} + U_t$$

where:

$$U_t \sim N(0, \Sigma_U)$$



$$\mathcal{A} = \begin{pmatrix} 0.95 & -0.2 & 0 \\ 0 & 0.8 & 0.1 \\ 0.1 & 0 & 0.6 \end{pmatrix} \quad \Sigma_U = \begin{pmatrix} 10^{-4} & 0 & 0 \\ 0 & 10^{-4} & 0 \\ 0 & 0 & 10^{-4} \end{pmatrix}.$$

The following tuple of 2-dimensional functions are considered:

$$\begin{pmatrix} m_0 \\ m_1 \\ m_2 \\ m_3 \end{pmatrix} (x_1, x_2) = \begin{pmatrix} 1 \\ 3.46(x_1 - \frac{1}{2}) \\ 9.45\left\{(x_1 - \frac{1}{2})^2 + (x_2 - \frac{1}{2})^2\right\} - 1.6 \\ 1.41 \sin(2\pi x_2) \end{pmatrix}. \tag{5}$$

The coefficients in (5) were chosen so that m_1, m_2, m_3 are close to orthogonal.

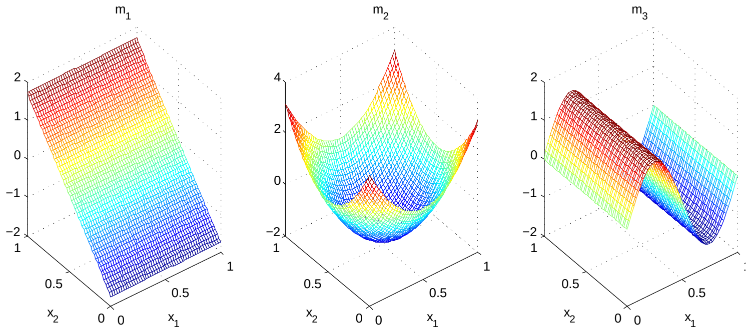


Figure 3: True functions m_1, m_2, m_3 from which the data were generated. $(m_0 = 1)$

For the basis functions ψ we choose tensor B-splines on the equidistance knots in each direction.

T	500	1000	2000
J	100	250	1000
K	36	49	64

for each setting the simulations were repeated 250 times

The one-dimensional linear B-splines $\widetilde{\psi}_k$ are defined on a consecutive equidistant knots x^k, x^{k+1}, x^{k+2} by

$$\widetilde{\psi}_k(x) = \begin{cases} (x - x^k)/(x^{k+1} - x^k), & x \in (x^k, x^{k+1}], \\ (x^{k+2} - x)/(x^{k+2} - x^{k+1}), & x \in (x^{k+1}, x^{k+2}], \\ 0, & \text{otherwise.} \end{cases}$$

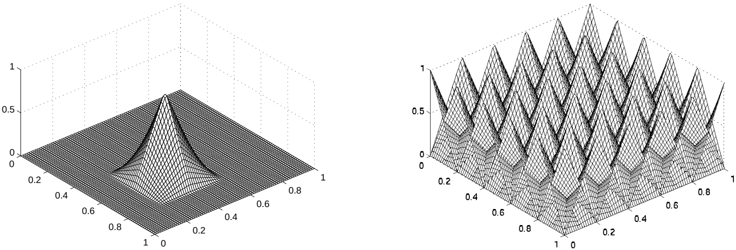


Figure 4: *Tensor linear B-spline basis used in the estimation. Left panel: one particular basis function* ψ_k*. Right panel: the whole set of basis functions for* $K = 36$*.*

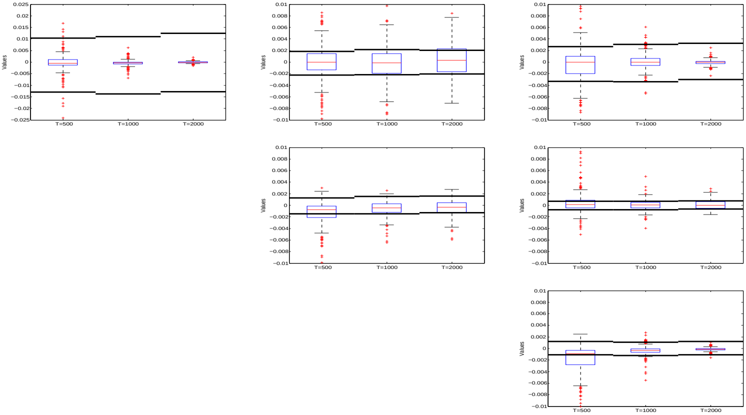


Figure 5: The boxplots based on 250 differences of the elements of the scaled covariance matrices. The bold line represents 95% and 5% quantiles of "true" differences.

The differences of the scaled covariance matrices (boxplots):

$$\frac{1}{\sqrt{T}}\left\{\sum_{t=1}^{T}\left(\tilde{Z}_t-\overline{\tilde{Z}}\right)\left(\tilde{Z}_t-\overline{\tilde{Z}}\right)^{\top}-\sum_{t=1}^{T}\left(Z_t-\overline{Z}\right)\left(Z_t-\overline{Z}\right)^{\top}\right\}$$

The "true" differences (bold line):

$$\frac{1}{\sqrt{T}}\left\{\sum_{t=1}^{T}\left(Z_t-\overline{Z}\right)\left(Z_t-\overline{Z}\right)^{\top}-\Gamma\right\},$$

where Γ is the true covariance matrix of the simulated VAR process.

Implied Volatility

Volatility $\hat{\sigma}$ as *implied* by observed market prices $\tilde{C}_t$:

$$\hat{\sigma} : \quad \tilde{C}_t - C_t^{BS}(S_t, K, \tau, r, \hat{\sigma}) = 0 \, .$$

Unlike assumed in the Black-Scholes (BS) model, $\hat{\sigma}_t(K, \tau)$ exhibits **distinct, time-dependent** functional patterns across K (**smile or smirk**), and a **term-structure** $T - t$: Thus $\hat{\sigma}_t(K, \tau)$ is interpreted as a **random surface**: the implied volatility surface (IVS).

Degenerated Design

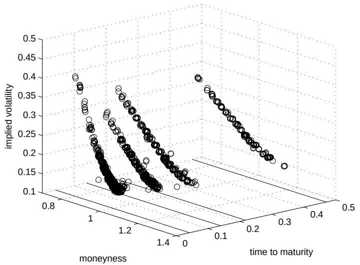


Figure 6: *Left panel: IV strings observed on 20040701. Right panel: kernel density estimator of the design points from 20040701 to 20050629*

Data Overview

	Min.	Max.	Mean	Median	Stdd.	Skewn.	Kurt.
T. to mat.	0.03	0.50	0.120	0.088	0.088	1.787	6.282
Moneyness.	0.70	1.20	0.989	0.994	0.055	-0.708	5.324
IV	0.03	1.61	0.159	0.153	0.040	1.615	14.621

Table 1: *Summary statistics from 20040701 to 20050629. Source: EUREX, ODAX, stored in the SFB 649 FEDC.*

Data Preprocessing

In order to avoid problems with very skewed design we transform data with marginal empirical distribution functions.

For the tensor B-splines we place equidistant knots in each direction (10 knots for splines of order 3 in moneyness direction (X_1), 5 knots for splines of order 2 in time to maturity direction (X_2)).

We estimate $L = 2$ basis functions.

$$RV(L) = \frac{\sum_t^T \sum_j^{J_t} \{Y_{t,j} - \sum_{l=0}^{L} \widehat{Z}_{t,l} \widehat{m}_l(X_{t,j})\}^2}{\sum_t^T \sum_j^{J_t} (Y_{t,j} - \bar{Y})^2}$$

No. Factors	$1 - RV(L)$
L=1	0.848
L=2	0.969
L=3	0.976
L=4	0.978
L=5	0.980

At the last step of the estimation we **orthogonalize** the functions and **order** them in such a way that the explained variation by the first function is maximal.

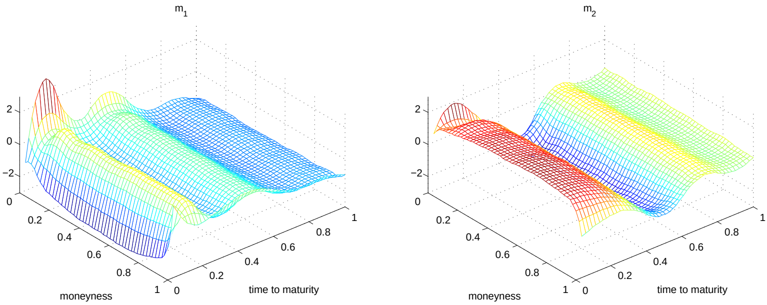


Figure 7: *Dynamic basis functions* $\widehat{m}_1$ *and* $\widehat{m}_2$

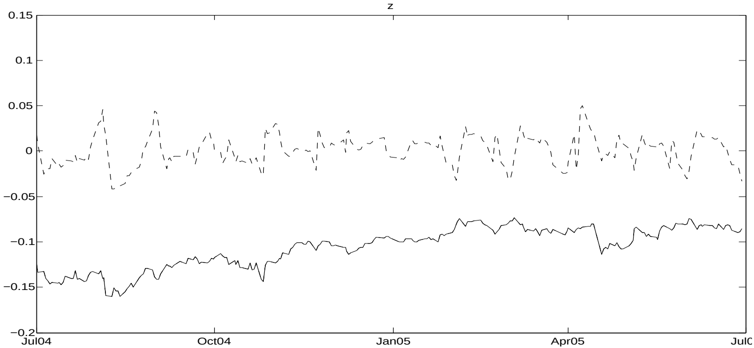


Figure 8: *Time series of weights $\widehat{Z}_{t,1}$ (lower) and $\widehat{Z}_{t,2}$ (upper).*

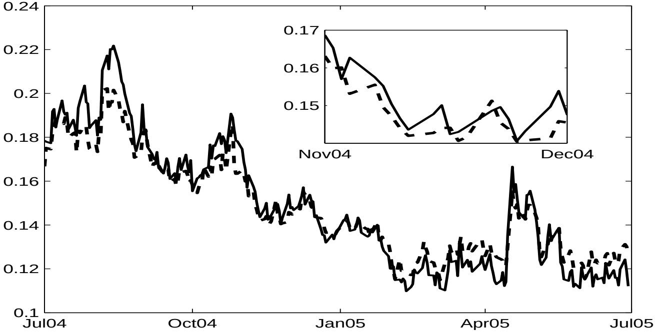


Figure 9: *VDAX from 20040701 to 20050629 (solid) and the dynamics of the corresponding IV given by the sub-model* $\widehat{m}_0 + \widehat{Z}_{t,1}\widehat{m}_1$ *(dashed).*

VAR modelling of $\widehat{Z}_t$

- We fit the VAR(1) (Schwarz and Hannan-Quinn criteria) and VAR(2) (AIC criterion) models for $\widehat{Z}_t$.
- Roots lay inside the unit root circle.
- Autocorrelation tests (Portmanteau and LM) cannot be rejected.

VAR parameters

	$\widehat{Z}_{t-1,1}$	$\widehat{Z}_{t-1,2}$	$\widehat{Z}_{t-2,1}$	$\widehat{Z}_{t-2,2}$	c
	VAR(1)				
$\widehat{Z}_{t,1}$	0.984	-0.029			-0.001
$\widehat{Z}_{t,2}$	0.055	0.739			0.005
	VAR(2)				
$\widehat{Z}_{t,1}$	0.913	-0.025	0.071	-0.004	-0.001
$\widehat{Z}_{t,2}$	0.124	0.880	-0.065	-0.187	0.006

Table 2: The estimated parameters for VAR(1) and VAR(2) models.

Conclusion

- asymptotic convergence of the covariance matrix of $\widetilde{Z}_t$ to the covariance matrix of Z_t
- confirmed by the simulations
- inference on $\widehat{Z_t}$ is justified
- DSFM could be used for the analysis of the IVS dynamics

Terra Incognita

External Variable:

$$E(\mathbf{Y}_t|\mathbf{X}_t) = Z_t^\top m(\mathbf{X}_t) + G(S_t)$$

Constraints:

$$E(\mathbf{Y}_t|\mathbf{X}_t) = Z_t^\top m(\mathbf{X}_t) \in \mathcal{G}$$

Multi DSFM:

$$E(\mathbf{Y}_t^{(i)}|\mathbf{X}_t^{(i)}) = Z_t^\top m^{(i)}(\mathbf{X}_t^{(i)})$$

Reference

Black, F. and M. Scholes
The pricing of options and corporate liabilities
Journal of Political Economy, 81:637–654, 1973.

Borak, S., Härdle, W., Mammen, E. and Park, B.
Time Series Modelling with Sempiparametric Factor Dynamics
SFB 649 Discussion Paper, 2007-023, 2007.

Connor, G. and Linton, O.
Semiparametric Estimation of a Characteristic-based Factor Model of Common Stock Returns
Journal of Empirical Finance, 2007.

For Further Reading

- Fengler, M., Härdle, W. and Mammen, E.
  A Dynamic Semiparametric Factor Model for Implied Volatility String Dynamics
  Journal of Financial Econometrics, 5(2):189–218, 2007.
- Hansen, L., Nielsen, B. and Nielsen, J.
  Two sided analysis of variance with a latent time series
  Nuffield College Economic Working Paper, 2004-W25, 2004.
- Nelson, C. R. and Siegel, A. F.
  Parsimonoius Modelling of Yield Curves
  Journal of Business, 60:473–489, 1987.

For Further Reading

Ramsay, J. O. and Silverman, B. W.
Functional Data Analysis
Springer, 1997

Appendix A

(A1) The variables $X_{1,1},\dots,X_{T,J}$, $\varepsilon_{1,1},\dots,\varepsilon_{T,J}$ are independent.

(A2) For $t = 1, \dots, T$ the variables $X_{t,1},\dots,X_{t,J}$ are identically distributed, have support $[0,1]^d$ and a density f_t that is bounded from below and above on $[0,1]^d$, uniformly over $t = 1, \dots, T$.

(A3) We assume that

$$\begin{aligned} \mathsf{E}[\varepsilon_{t,j}] &= 0 \text{ for } t = 1, \dots, T, j = 1, \dots, J, \\ \sup_{t=1,\dots,T, j=1,\dots,J} \mathsf{E}[\varepsilon_{t,j}^2] &< \infty. \end{aligned}$$

(A4) The functions ψ_k are normed: $\int_{[0,1]^d} \psi_k^2(x)\, dx = 1$

(A5) The components $m_0,\ldots,m_L$ can be approximated by $\psi_1, \ldots, \psi_K$, i.e.

$$\delta_K = \sup_{x \in [0,1]^d} \inf_{A \in \mathbb{R}^{(L+1)\times K}} |m(x) - A\psi(x)| \to 0$$

for $l = 0, \ldots, L$ and $K \to \infty$. We denote a matrix that fulfills $\sup_{x \in [0,1]^d} |m(x) - A\psi(x)| \leq 2\delta_K$ by A. We assume that $\delta_K = \mathcal{O}(K^{1/2} J^{-1/2})$ for $K, J \to \infty$.

(A6) There exist constants $0 < C_L < C_U < \infty$ such that all eigenvalues of the random matrix $T^{-1} \sum_{t=1}^{T} Z_t Z_t^\top$ lie in the interval $[C_L, C_U]$ with probability tending to one.

(A7) It holds that $(K \log K)/J \to 0$ and $\log T/J \to 0$.

(A8) It holds $\max_{1\leq t\leq T}\|Z_t\| \leq M_T/C_m$ with a constant $C_m > \sup_{x\in[0,1]}\|m(x)\|$ and $M_T^2(K\log K/J) \to 0$ and $M_T^2(\log T/J) \to 0$.

(A9) The bound $\max_{1\leq t\leq T}\|Z_t\| \leq M_T$ holds with probability tending to one and it holds that $M_T^2\{(K\log K)/J\} \to 0$ and $M_T^2(\log T/J) \to 0$.

(A10) Z_t is strictly stationary with $\mathrm{E}(Z_t) = 0$ and $\mathrm{E}\|Z_t\|^\gamma < \infty$ for some $\gamma > 2$. It is strongly mixing with $\sum_{i=1}^\infty \alpha(i)^{(\gamma-2)/\gamma} < \infty$. The matrix $\mathrm{E}Z_tZ_t^\top$ has full rank. The process Z_t is independent of $X_{11},\ldots,X_{TJ},\varepsilon_{11},\ldots,\varepsilon_{TJ}$.

(A11) The functions $m_0, ..., m_L$ are linearly independent. In particular, no function is equal to 0. Furthermore, it holds that $\sup_{x\in[0,1]} \|\psi(x)\| = \mathcal{O}(K^{1/2})$.

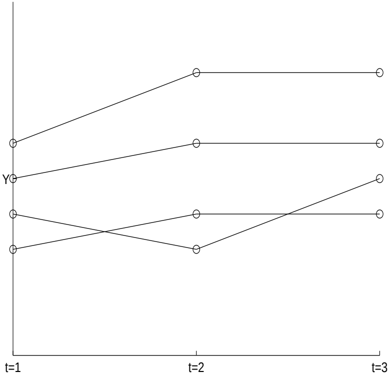
(A12) It holds that $K/J + \delta_K = \scriptstyle\mathcal{O}(T^{-1/2})$, $\log T = \scriptstyle\mathcal{O}(K)$, $K^5 J^{-4} (\log K)^2 = \scriptstyle\mathcal{O}(T^{-1})$, and $K^7 J^{-5} (\log K)^2 = \scriptstyle\mathcal{O}(T^{-1})$.

Appendix B

How to model dynamics of multidimensional phenomena

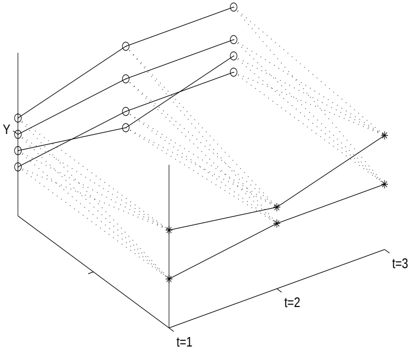
Time Series

- $Y_t = (Y_1, \ldots, Y_d)_t^\top$ is d dimensional time series.
- The components $Y_1, \ldots, Y_d$ are not linked together - each permutation defines same series



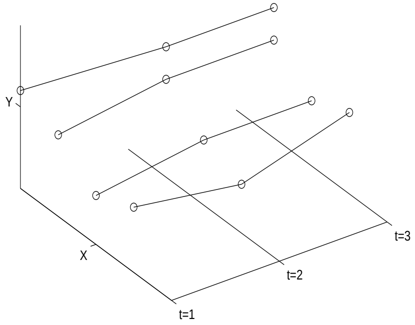
Factor Analysis for Times Series

- For dimension reduction one may consider factor model $Y_t = MZ_t$
- The dimension of Z_t is much smaller than dimension of Y_t



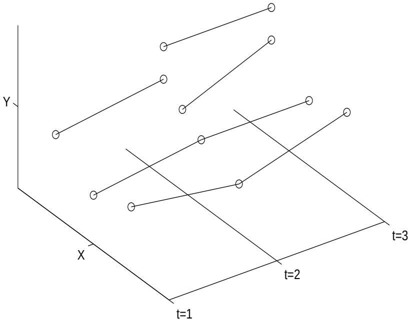
Balanced Panel

- For each individual i in each time point t one observes external variable X_{it}
- For each time point t one has a regression $E(Y_t|X_t) = F_t(X_t)$



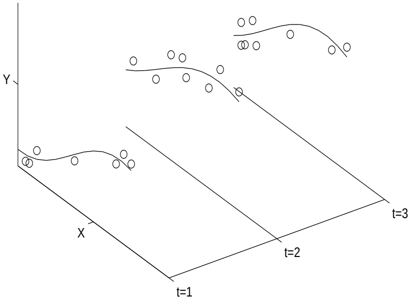
Unbalanced Panel

- Not every individual has to be observed for the whole time range
- The regression structure is kept but the model can no longer be recognized as a classical multidimensional time series



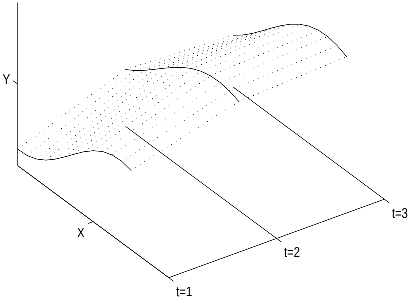
Dynamic Regression

- There can be no direct link among observations through the time
- One observes evolution of regression $E(Y_t|X_t) = F_t(X_t)$ problems but it is not a panel any more



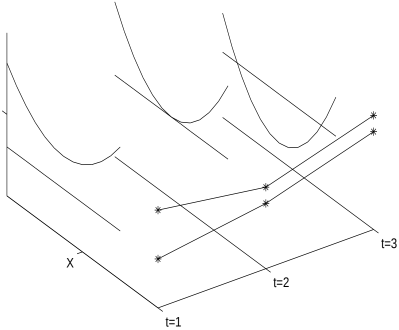
Functional Data Approach

- A possible solution: smooth the data and obtain a balanced panel (multivariate time series)
- Points are linked according to external variable X_t



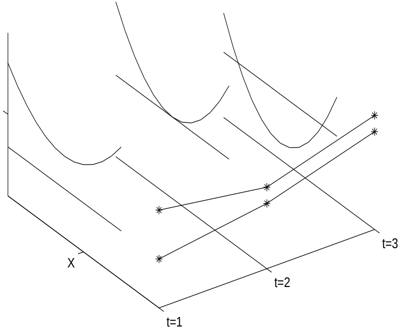
Factor Models for Dynamic Regression

- For dimension reduction one may consider factor models $E(Y_t|X_t) = m^\top(X_t)Z_t$ where m is a tuple of functions
- m and Z_t can be obtained in different ways



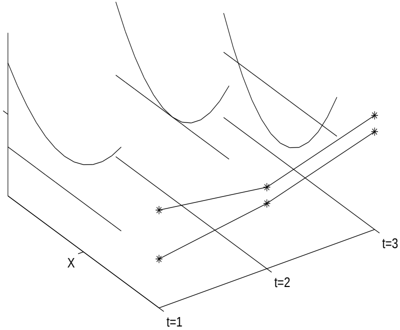
Factor Models for Dynamic Regression

- Z_t observed - varying coefficient model
- m and Z_t estimated through some techniques applied to multidimensional time series (functional approach) + projection on X_t



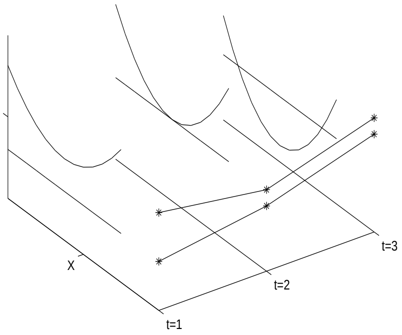
Factor Models for Dynamic Regression

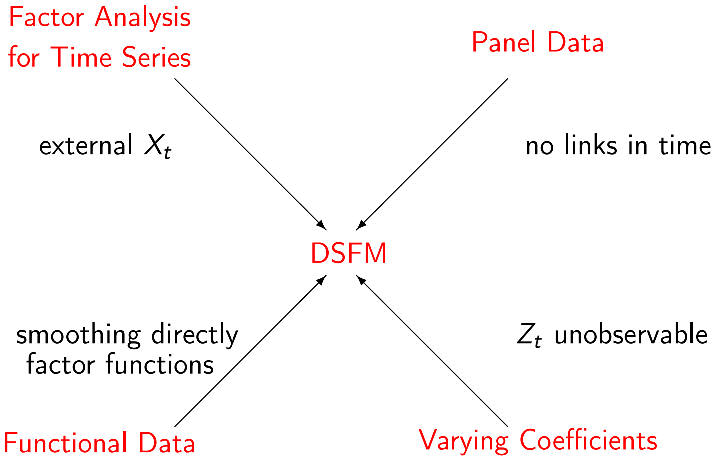
- m may be specified parametrically and estimated directly from the data through pooled least squares - special case unbalanced panel

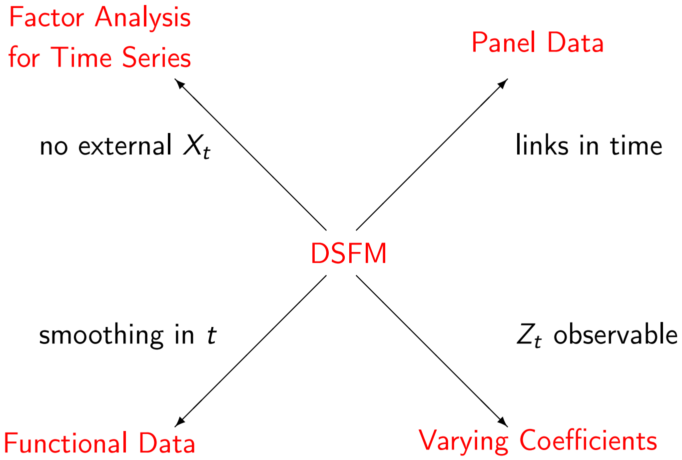


Factor Models for Dynamic Regression

- m and Z_t estimated directly from the data without parametric assumptions about m - DSFM




Factor Analysis
for Time Series
Panel Data
external X_t
no links in time
DSFM
smoothing directly
factor functions
Z_t unobservable
Functional Data
Varying Coefficients



Factor Analysis for Time Series
Panel Data
no external X_t
links in time
DSFM
smoothing in t
Z_t observable
Functional Data
Varying Coefficients