Risk Patterns and Correlated Brain Activities

Alena Myšičková
Piotr Majer
Song Song
Peter N. C. Mohr
Wolfgang K. Härdle
Hauke R. Heekeren

C.A.S.E. Centre for Applied Statistics and Economics
Humboldt-Universität zu Berlin
Freie Universität Berlin
Max Planck Institute for Molecular Genetics
http://lvb.wiwi.hu-berlin.de
http://www.languages-of-emotion.de
http://www.molgen.mpg.de

Risk Perception

- Which part is activated during *risk related decisions* ?
- Can statistical analysis help to detect this area?
- Response curve (to stimuli)? classify "risky people"?

Risk Perception

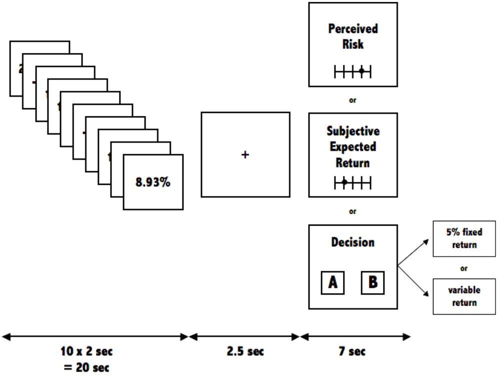
- Survey conducted by Max Planck Institute
- 22 young, native German, right-handed and healthy volunteers

  3 subjects with extensive head movements ($> 5mm$)

  2 subjects with different stimulus frequency

  $n = 22 - (3 + 2) = 17$
- Experiment
  - Risk Perception and Investment Decision (RPID) task ($\times 45$)
  - fMRI images every 2.5 sec.

Risk Perception



Returns Pause Decision

Risk Perception – Thermodynamics

Theoretical framework

- Risk-return model
  Mohr et al., 2010

Empirical evidence

- fMRI analysis

- Mechanical Equivalent of Heat
  the first law of thermodynamics
  Mayer, 1841

- Experiments "Joule apparatus"
  Joule, 1843

Risk Perception

- functional Magnetic Resonance Imaging

- Measuring Blood Oxygenation Level Dependent (BOLD) effect every 2-3 sec
  High-dimensional, high frequency & large data set

Risk Perception

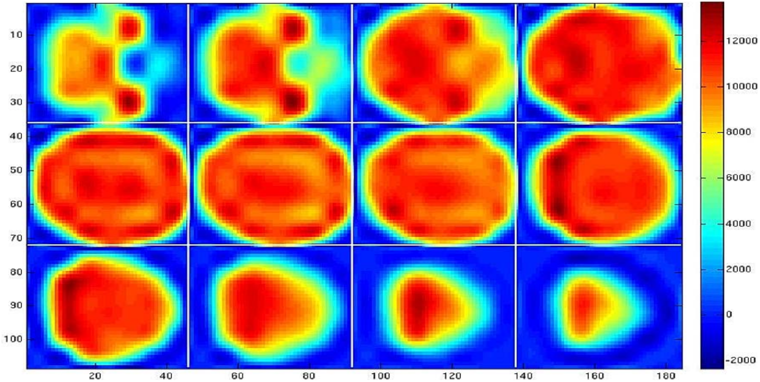
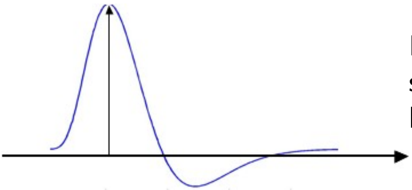


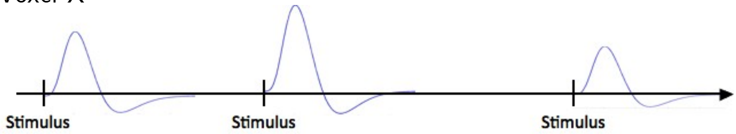
Figure 1: Example of a fMRI image at fixed time point, 12 horizontal slices of the brain's scan. fMRI

fMRI



Is there a significant reaction to specific stimuli in the hemodynamic response?

Voxel X



fMRI methods

- Voxel-wise GLM ▸ Voxel-wise GLM
  - linear model for each voxel separately
  - strong a priori hypothesis necessary
- Dynamic Semiparametric Factor Model (DSFM)
  - Use a “time & space” dynamic approach
  - Separate low dim time dynamics from space functions
  - Low dim time series exploratory analysis

Outline

1. Motivation ✓
2. DSFM
3. Results vs. Subject's Behaviour
4. Conclusion
5. Future Perspectives

Notation

$$\underbrace{(X_{1,1}, Y_{1,1}), \ldots, (X_{J,1}, Y_{J,1})}_{t=1}, \ldots, \underbrace{(X_{1,T}, Y_{1,T}), \ldots, (X_{J,T}, Y_{J,T})}_{t=T},$$

$X_{j,t} \in \mathbb{R}^d$, $Y_{j,t} \in \mathbb{R}$
T - the number of observed time periods
J - the number of the observations in a period t
$\mathsf{E}(Y_t|X_t) = F_t(X_t)$

What is $F_t(X_t)$? How does it move?

Dynamic Semiparametric Factor Model

$$\mathsf{E}(Y_t|X_t) = \sum_{l=0}^{L} Z_{t,l} m_l(X_t) = Z_t^\top m(X_t) = Z_t^\top A^* \Psi$$

$Z_t = (\mathbf{1}, Z_{t,1}, \ldots, Z_{t,L})^\top$ low dim (stationary) time series

$m = (m_0, m_1, \ldots, m_L)^\top$, tuple of functions

$\Psi = \{\psi_1(X_t), \ldots, \psi_K(X_t)\}^\top$, $\psi_k(x)$ space basis

A^* : $(L+1) \times K$ coefficient matrix

DSFM Estimation

$$Y_{t,j} = \sum_{l=0}^{L} Z_{t,l} m_l(X_{t,j}) + \varepsilon_{t,j} = Z_t^\top A^* \psi(X_{t,j}) + \varepsilon_{t,j}$$

- $\psi(x) = \{\psi_1(x), \dots, \psi_K(x)\}^\top$ tensor B-spline basis

$$(\widehat{Z}_t, \widehat{A^*}) = \arg \min_{Z_t, A^*} \sum_{t=1}^{T} \sum_{j=1}^{J} \left\{ Y_{t,j} - Z_t^\top A^* \psi(X_{t,j}) \right\}^2 \qquad (1)$$

- Minimization by Newton-Raphson algorithm

B-Splines

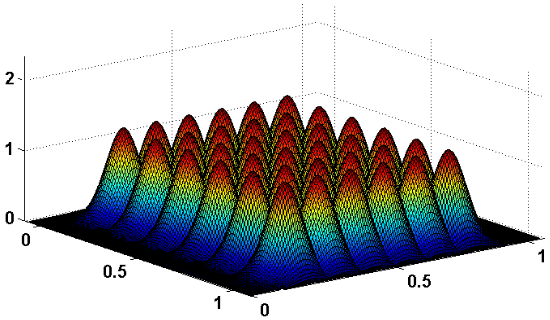


Figure 2: B-splines basis functions; order of B-splines: quadratic; number of knots: 36

DSFM Estimation

- Selection of L by explained variance

$$EV(L) = 1 - \frac{\sum_{t=1}^{T} \sum_{j=1}^{J} \left\{ Y_{t,j} - \sum_{l=0}^{L} Z_{t,l} m_l(X_{t,j}) \right\}^2}{\sum_{t=1}^{T} \sum_{j=1}^{J} \left\{ Y_{t,j} - \bar{Y} \right\}^2}$$

number of B-splines (equally spaced) knots: $12 \times 14 \times 14$

$L = 2$	$L = 4$	$L = 5$	$L = 10$	$L = 20$
92.07	92.25	92.29	93.66	95.19

Table 1: EV in percent of the model with different numbers of factors L, averaged over all 17 analyzed subjects.

Panel DSFM

$$Y_{t,j}^{i} = \sum_{l=0}^{L} (Z_{t,l}^{i} + \alpha_{t,l}^{i}) m_l(X_{t,j}) + \varepsilon_{t,j}, \quad 1 \leq j \leq J,\ 1 \leq t \leq T,$$

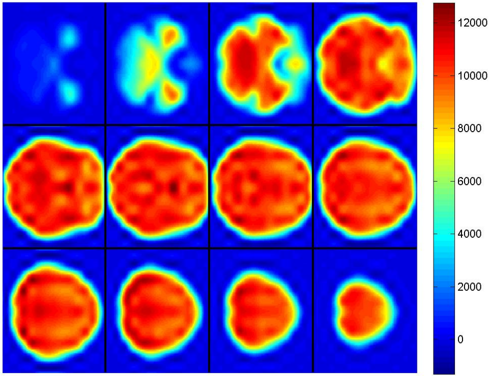
- $n = 17$ weakly/strongly risk-averse subjects
- $Y_{t,j}$ - BOLD signal; X_j voxel's index
  $\alpha_{t,l}^{i}$ - fixed individual effect
- Identification condition: $\mathsf{E}\left\{\sum_{i=1}^{n}\sum_{l=0}^{L} \alpha_{t,l}^{i} m_l(X_{t,j}) | X_{t,j}\right\} = 0$

Panel DSFM Estimation

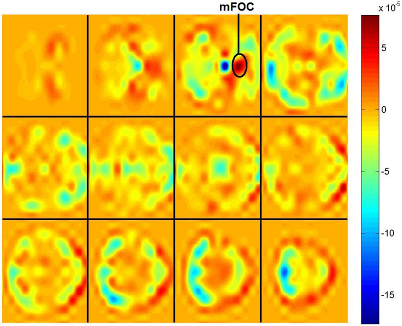
1. Average $Y^i_{t,j}$ over subjects i to obtain $\bar{Y}_{t,j}$
2. Estimate factors m_l for the "average brain" (via one step of 1)
3. Given $\widehat{m}_l$, for i, estimate $Z^i_{t,l}$

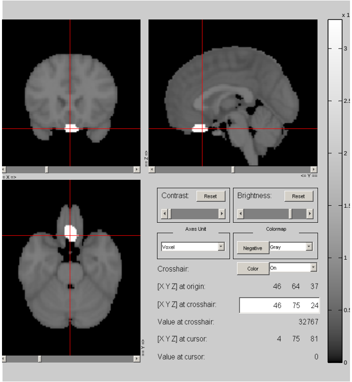
$$Y^i_{t,j} = \sum_{l=0}^{L} Z^i_{t,l} \widehat{m}_l(X_{t,j}) + \varepsilon^i_{t,j}$$

- 26h - estimation time; CPU - 2×2.8GHz; data set of size 24.31 GB

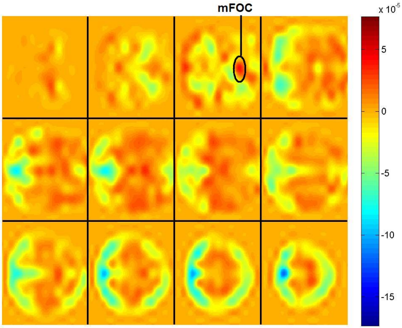


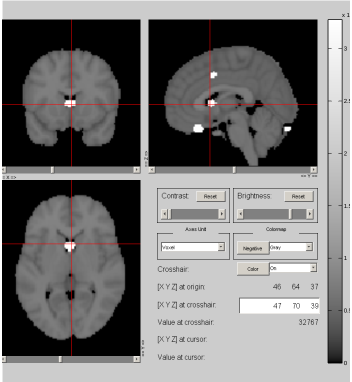
Estimated constant factor $\widehat{m}_0 = \sum_{k=1}^{K} \widehat{a}_{0,k}\psi(X)$ with $L = 20$



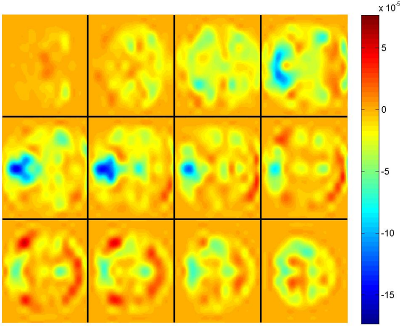


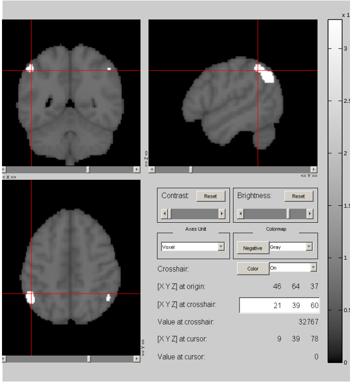
Estimated factor $\widehat{m}_5 = \sum_{k=1}^{K} \widehat{a}_{5,k} \psi(X)$ with $L = 20$
(MOFC = Medial orbitofrontal cortex)



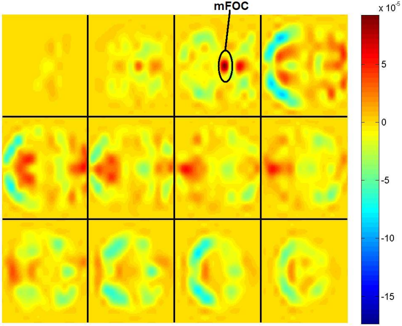


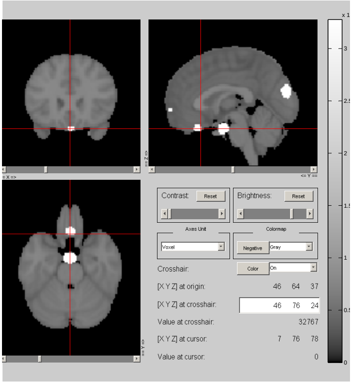
Estimated factor $\widehat{m}_9 = \sum_{k=1}^{K} \widehat{a}_{9,k} \psi(X)$ with $L = 20$



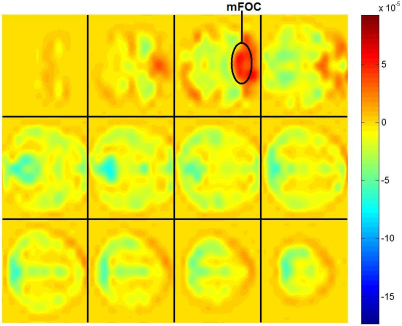


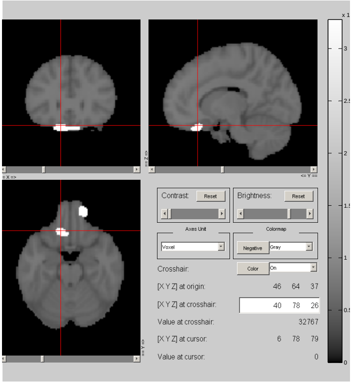
Estimated factor $\widehat{m}_{12} = \sum_{k=1}^{K} \widehat{a}_{12,k}\psi(X)$ with $L = 20$



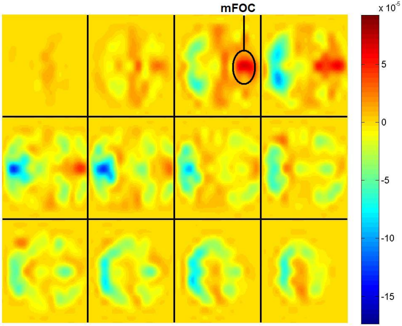


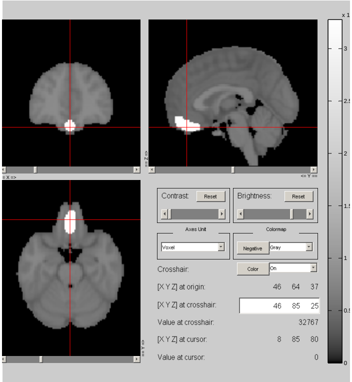
Estimated factor $\widehat{m}_{16} = \sum_{k=1}^{K} \widehat{a}_{16,k}\psi(X)$ with $L = 20$





Estimated factor $\widehat{m}_{17} = \sum_{k=1}^{K} \widehat{a}_{17,k}\psi(X)$ with $L = 20$





Estimated factor $\widehat{m}_{18} = \sum_{k=1}^{K} \widehat{a}_{18,k}\psi(X)$ with $L = 20$

Estimated Factor Loading $\widehat{Z}_5$

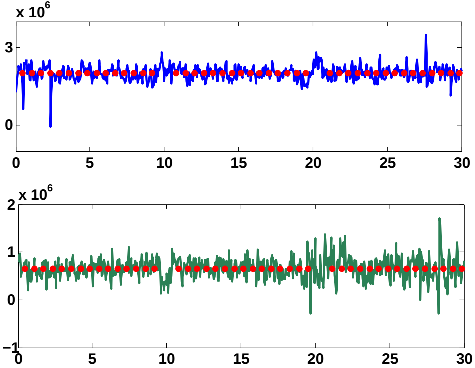


Figure 3: Estimated factor loading $\widehat{Z}_5$ for subjects within 30 minutes: 12 (lower panel) and 19 (upper panel) with $L = 20$; red dots denote stimulus

Estimated Factor Loading $\widehat{Z}_9$

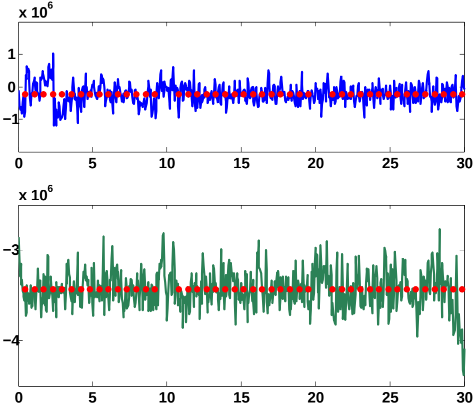


Figure 4: Estimated factor loading $\widehat{Z}_9$ for subjects within 30 minutes: 12 (lower panel) and 19 (upper panel) with $L = 20$; red dots denote stin

Estimated Factor Loading $\widehat{Z}_{12}$

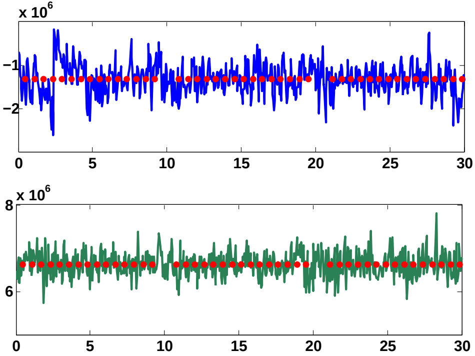


Figure 5: Estimated factor loading $\widehat{Z}_{12}$ for subjects within 30 minutes: 12 (lower panel) and 19 (upper panel) with $L = 20$; red dots denote stimulus

Estimated Factor Loading $\widehat{Z}_{16}$

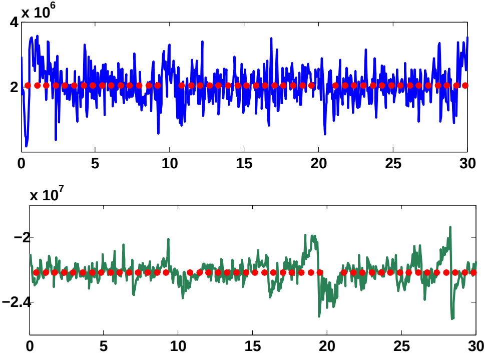


Figure 6: Estimated factor loading $\widehat{Z}_{16}$ for subjects within 30 minutes: 12 (lower panel) and 19 (upper panel) with $L = 20$; red dots denote stimulus

Estimated Factor Loading $\widehat{Z}_{17}$

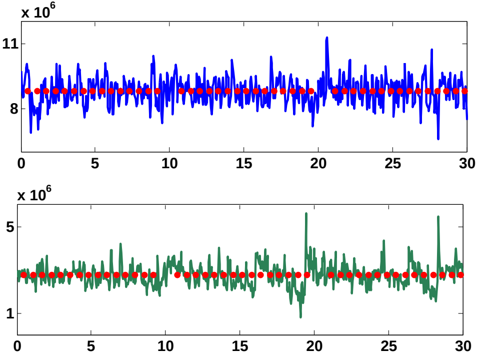


Figure 7: Estimated factor loading $\widehat{Z}_{17}$ for subjects within 30 minutes: 12 (lower panel) and 19 (upper panel) with $L = 20$; red dots denote stimulus

Estimated Factor Loading $\widehat{Z}_{18}$

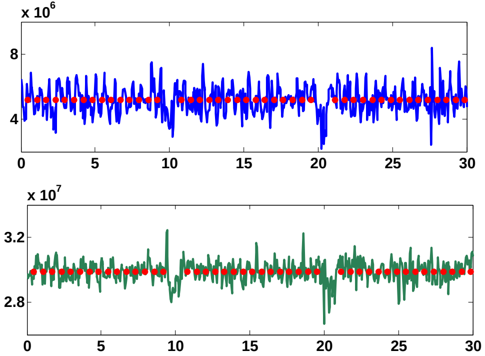


Figure 8: Estimated factor loading $\widehat{Z}_{18}$ for subjects within 30 minutes: 12 (lower panel) and 19 (upper panel) with $L = 20$; red dots denote stimulus

Reaction to the stimulus

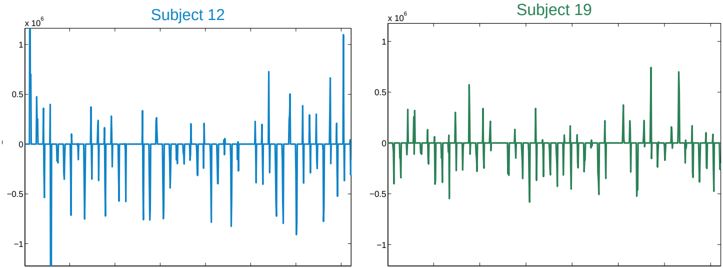


Figure 9: Reaction to stimulus for factors loadings $\widehat{Z}_{t,12}$ for subjects 12 (left) and 19 (right) during the whole experiment (45 stimuli).

Risk attitude

- Subject's risk perception - risk metrics
  - standard deviation
  - empirical frequency of loss (negative return)
  - difference between highest an lowest return (range)
  - coefficient of range (range/mean)
  - empirical frequency of ending below 5%
  - coefficient of variation (standard deviation/mean)

- Different subject - different risk perception

  fitted by correlation between risk metrics of return streams and answers for 1 task, $N = 27$

Risk attitude

- Subjective expected return - return ratings
  - recency (higher weights on later returns)
  - primacy (higher weights on earlier returns)
  - below 0% (higher weights on returns below 0%)
  - below 5% (higher weights on returns below 5%)
  - mean

- Selecting return ratings for each subject individually

  best model by one-leave-out cross validation procedure, $N = 27$

Risk attitude

- Risk-return choice model

$$V_i = m_i - \beta_i R_i, \quad 1 \leq i \leq n,$$

m_i-subjective expected return, R_i - perceived risk, V_i - subjective value, 5% - risk free return

- β Risk attitude parameter

Risk attitude

- Estimation of individual risk attitude by logistic regression

$$
\begin{aligned}
\mathrm{P}\left\{\text{risky choice}|(m,R)\right\} &= \frac{1}{1+\exp(m-\beta R-5)}\\
\mathrm{P}\left\{\text{sure choice}|(m,R)\right\} &= 1-\frac{1}{1+\exp(m-\beta R-5)}
\end{aligned}
$$

risky choice - unknown return, sure choice - fixed, 5% return

- $\widehat{\beta}$ derived by maximum likelihood method

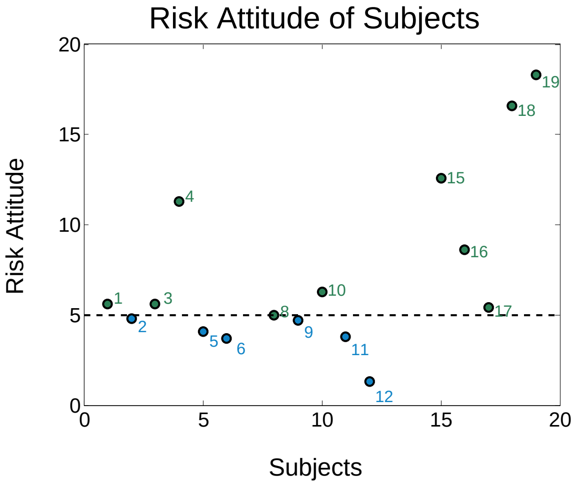


Figure 10: Risk attitude for 16 subjects; modeled by the softmax function from individuals' decisions, estimated by ML method ‣ Mohr et. al.

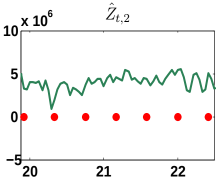
SVM Classification Analysis

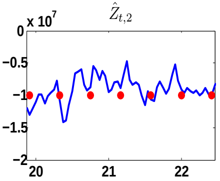
- Support Vector Machines (SVM)
  17 subjects, 20 factor loadings per subject
- Leave-one-out method to train and estimate classification rate
  SVM with Gaussian kernel; (R, C) chosen to maximize classification rate
- Weakly/strongly risk-averse subjects have larger/smaller differences of $\widehat{Z}^i_{t,l}$ inside each trial

SVM Classification Analysis

- Reaction to the RPID corresponds to dynamics of $\widehat{Z}^{i}_{t,l}$, $l = 5, 9, 12, 16, 17, 18$
- First 3 observations (7.5 sec.) after stimulus

  Decison time - 7 sec.





SVM Classification Analysis

1. factors attributed to risk patterns: $l = 5, 9, 12, 16, 17, 18$
2. only "Decision under Risk" ($Q3$) stimulus
3. $\Delta\widehat{Z}^i_{t,l} \stackrel{\text{def}}{=} \widehat{Z}^i_{s+t,l} - \widehat{Z}_{s,l}$, s is the time of stimulus
4. average reaction to s stimulus $\overline{\Delta}\widehat{Z}^i_{s,l} = \frac{1}{3}\sum_{\tau=1}^{3} \Delta\widehat{Z}^i_{s+\tau,l}$

SVM input data: volatility of $\overline{\Delta}\widehat{Z}^i_{s,l}$ over all $Q3$

Std		Estimated Strongly	Estimated Weakly
Data	Strongly	1.00	0.00
	Weakly	0.14	0.86

Table 2: Classification rates of the SVM method, **without** knowing the subject's estimated risk attitude.

Conclusion

- Factors $\widehat{m}$ identify activated areas, neurological reasonable
- Estimated factor loadings show differences for individuals with different risk attitudes (e.g. 12 vs. 19)
- SVM classification analysis of measurements in $\widehat{Z}_{t,l}$, $l = 5, 9, 12, 16, 17, 18$ after stimulus, can distinguish weakly/strongly risk-averse individuals with high classification rate, **without** knowing the subject's answers

Future Perspectives

- Comparison with the PCA/ICA (PARAFAC) approach
- Analysis of the second part of the experiment (under assumption of independency) to "generate" larger number of subjects
- Improvement of the classification criterion
- Penalized DSFM with seasonal effects

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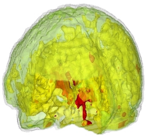

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Voxel-wise GLM

fMRI methods

- FEAT - FMRI Expert Analysis Tool by Department of Clinical Neurology, University of Oxford

- GLM framework

$$Y = XB + \eta,$$

  Y - single voxel **BOLD** time series, X - design matrix (regressors, i.e. visual, auditory)

- Significant, active areas (B) selected by *z-scores*