Skewness and Kurtosis Trades

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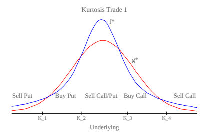


Motivation

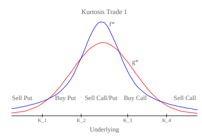
Recall from option pricing theory

$$\begin{aligned}\text{European put:}\quad P &= e^{-r\tau}\int_0^\infty \max(K_1 - S_T, 0)\,q(S_T)\,dS_T\\ \text{European call:}\quad C &= e^{-r\tau}\int_0^\infty \max(S_T - K_2, 0)\,q(S_T)\,dS_T,\end{aligned}$$

with time to maturity $\tau = T - t$, strike price K and risk–free interest rate r.

What is $q(S_T)$? – A state price density (SPD) of the underlying!

Black–Scholes world: $q(S_T)$ lognormal and unique.



Suppose there are two SPDs f^*, g^* with f^* more negatively skewed than g^* and a European OTM put respectively call.

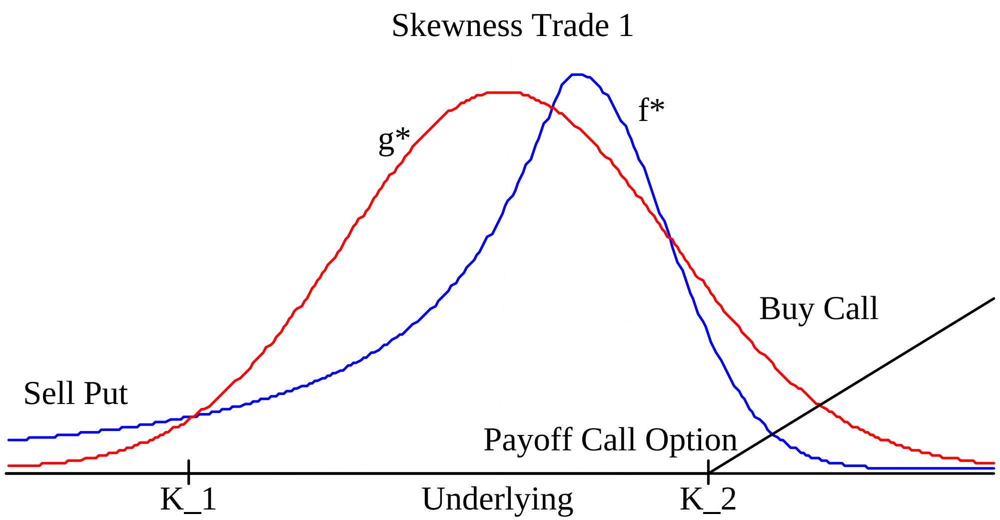
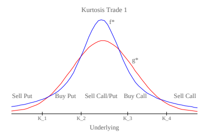


Figure 1: Skewness Trade 1.



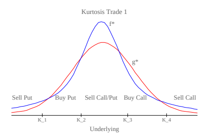
This tail situation implies for a European call option with strike K_2

$$C(f^*) \quad < \quad C(g^*)$$

$$\text{price computed with } f^* \quad < \quad \text{price computed with } g^*$$

If the call is priced using f^* but one regards g^* as a better approximation of the underylings' SPD, one would **buy** the call.

Analogously: Short a European OTM put with strike K_1.



This motivates a **skewness trade 1**: Portfolio of a short OTM put and a long OTM call, which is also called a Risk Reversal, Willmot (2002).

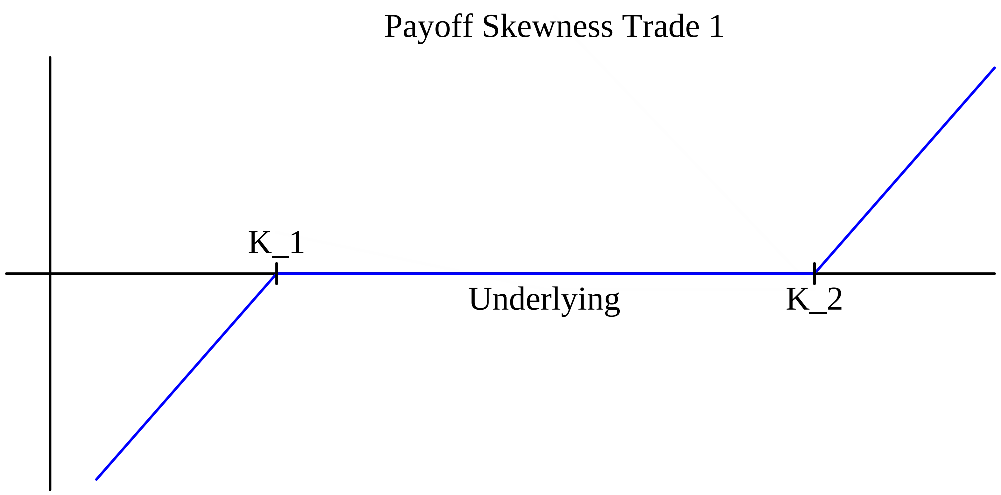


Figure 2: Payoff Skewness Trade 1

Comparing f^* and g^* leads also to **kurtosis trades**: Buy and sell calls and puts of different strikes.

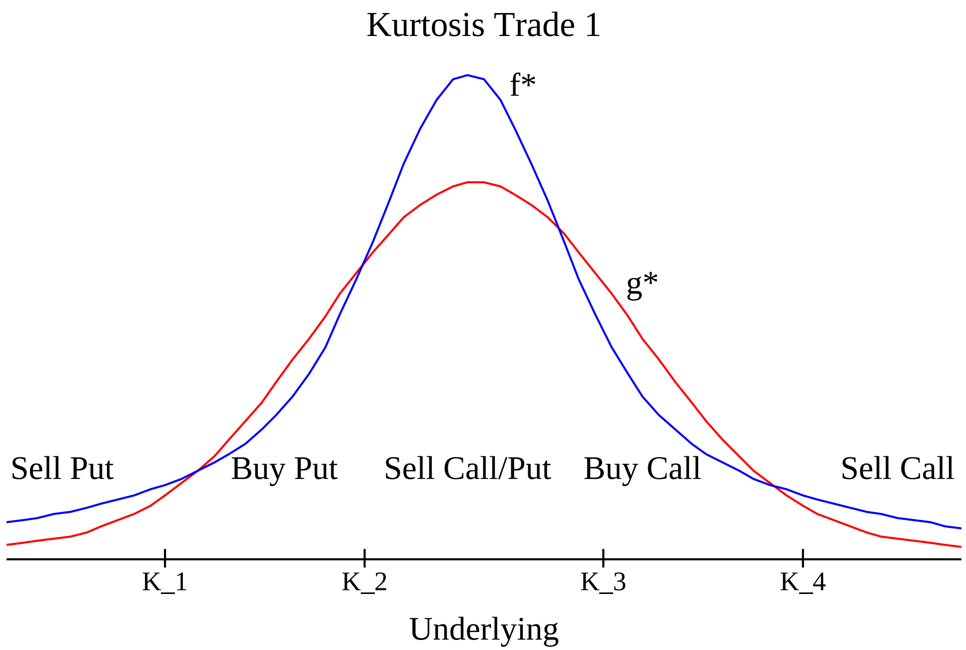
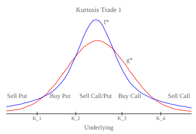


Figure 3: Kurtosis Trade 1.



The payoff profile at maturity is given in Figure 4, which is basically a modified butterfly.

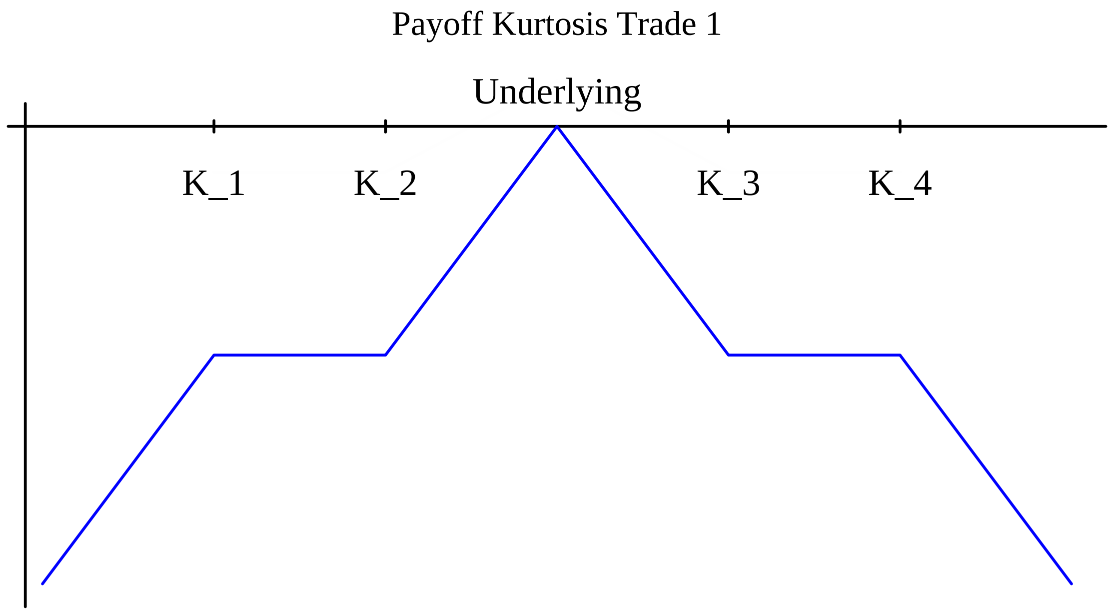


Figure 4: Payoff Kurtosis Trade 1

Data

daily DAX closing prices

daily EUREX DAX–option settlement prices

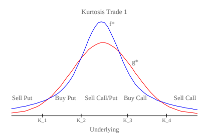
MD*BASE (http://www.mdtech.de) database

time period in this study: $04/97$ – $06/02$

General Comment

In an arbitrage free and complete market model exists exactly **one** risk–neutral density. If markets are not complete, for example when the volatility is stochastic, there are in general **many** risk–neutral measures.

Comparing **two** SPD's, as we do, amounts rather to compare two different models, and trades are initiated depending on the model in which one believes more.



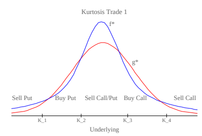
In this work, f^* is an option implied SPD and g^* is a (historical) time series SPD.

To compare implied to (historical) time series SPD's, we use Barle and Cakici's Implied Binomial Tree algorithm to estimate f^* whereas g^* is inferred from a combination of a non–parametric estimation from a historical time series of the DAX and a forward Monte Carlo simulation.

Later on we will specify in terms of moneyness $K/S_t e^{r\tau}$ where to buy or sell options.

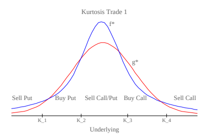
Within such a framework, is it profitable to trade skewness and kurtosis ?

Does a SPD comparison contain information about the stock market bubble that burst in March 2000?



Outline of the talk

✓ 1. Motivation
2. Estimation of the Implied SPD
3. Estimation of the Time Series SPD
4. Comparison of the Implied and Time Series SPD
5. Trading Strategies
6. Conclusion



Estimation of the Option–Implied SPD

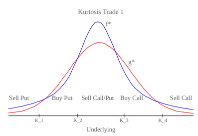
Implied Binomial Tree (IBT)

Numerical method to compute SPD adapted to volatility smile

Several approaches: Rubinstein (1994), Dupire (1994), Derman and Kani (1994) and Barle and Cakici (1998)

XploRe Quantlets compute Derman and Kani's `IBTdk` and Barle and Cakici's `IBTbc` IBT

Barle and Cakici's version proved to be more robust



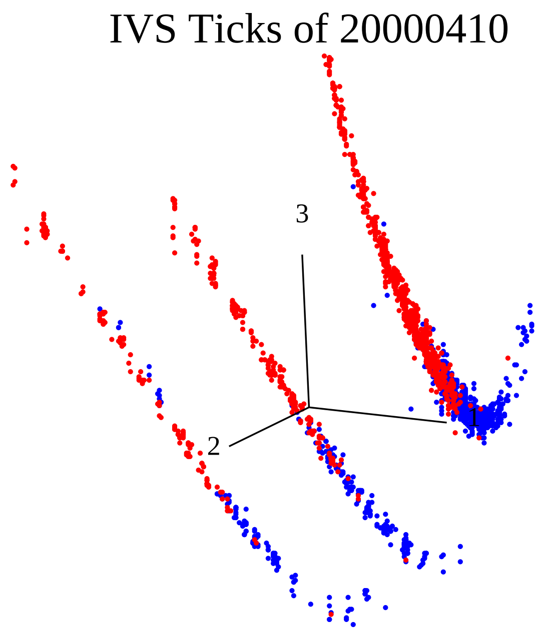
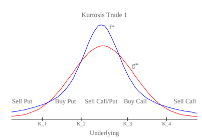


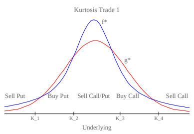
Figure 5: Implied volatility smile on 04/10/2000. Dimension 1: Time to Maturity, 2: Moneyness, 3: Implied Volatility.



(Implied) Binomial Tree

Each (implied) binomial tree consists of 3 trees (level n, node i):

- Tree of underlyings' values $s_{n,i}$
- Tree of transition probabilities $p_{n,i}$
- Tree of Arrow–Debreu (AD) prices $\lambda_{n,i}$
  Arrow–Debreu security: A financial instrument that pays off 1 EUR at node i at level n, and otherwise 0.



Example

$T = 1$ year, $\triangle t = 1/4$ year

smile structure: $\sigma_{imp}(K, t) = 0.15 - 0.0005K$

					119.91
				115.07	
			110.05		110.06
		105.13		105.14	
stock prices $s_{n,i}$	100.00		100.00		100.00
		95.12		95.11	
			89.93		89.93
				85.21	
					80.02

Example (2)

$T = 1$ year, $\triangle t = 1/4$ year

smile structure: $\sigma_{imp}(K, t) = 0.15 - 0.0005K$

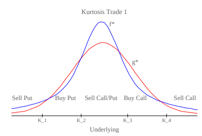
				0.596
			0.578	
		0.589		0.590
transition probabilities $p_{n,i}$	0.563		0.563	
		0.587		0.586
			0.545	
				0.589

Example (3)

$T = 1$ year, $\triangle t = 1/4$ year

smile structure: $\sigma_{imp}(K,t) = 0.15 - 0.0005K$

					0.111
				0.187	
			0.327		0.312
		0.559		0.405	
AD prices $\lambda_{n,i}$	1.000		0.480		0.342
		0.434		0.305	
			0.178		0.172
				0.080	
					0.033



Binomial Tree vs IBT

- **BT**: Discrete version of a diffusion process with constant volatility parameter:

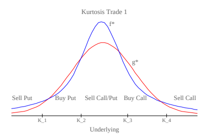
$$\frac{dS_t}{S_t} = \mu_t dt + \sigma dZ_t$$

Constant transition probabilities: $p_{n,i} = e^{\sigma\sqrt{\Delta t}}$ (with Δt fixed)

- **IBT**: Discrete version of diffusion process with a generalized volatility parameter:

$$\frac{dS_t}{S_t} = \mu_t dt + \sigma(S_t, t) dZ_t$$

Non constant transition probabilities $p_{n,i}$ (with Δt fixed)



IBT

Recombining tree divided into N equally spaced time steps of length $\Delta t = \tau/N$

IBT constructed on basis of observed option prices, i.e. takes the smile as an input

IBT–implied SPD: at final nodes assign

$$e^{r\tau}\lambda_{N+1,i} \text{ to } s_{N+1,i},\ i = 1, \ldots, N+1$$

where $\lambda_{N+1,i}$ denote the Arrow–Debreu prices

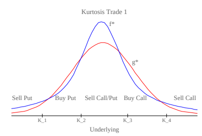
Application to EUREX DAX–Options

20 non overlapping periods from June 1997 to June 2002 ($\tau \approx 65/250$ fixed)

period from Monday following 3rd Friday to 3rd Friday 3 months later

Example: on Monday, $23/06/97$, we estimate f^* of Friday, $19/09/97$

- volsurf estimates implied volatility surface using:
  - Option data of preceeding 2 weeks (Monday, $09/06/97$, to Friday, $20/06/97$)
- IBTbc computes IBT with input parameters:
  - DAX on Monday June 23, 1997, $S_0 = 3748.79$
  - time to maturity $\tau = 65/250$ and interest rate $r = 3.12$



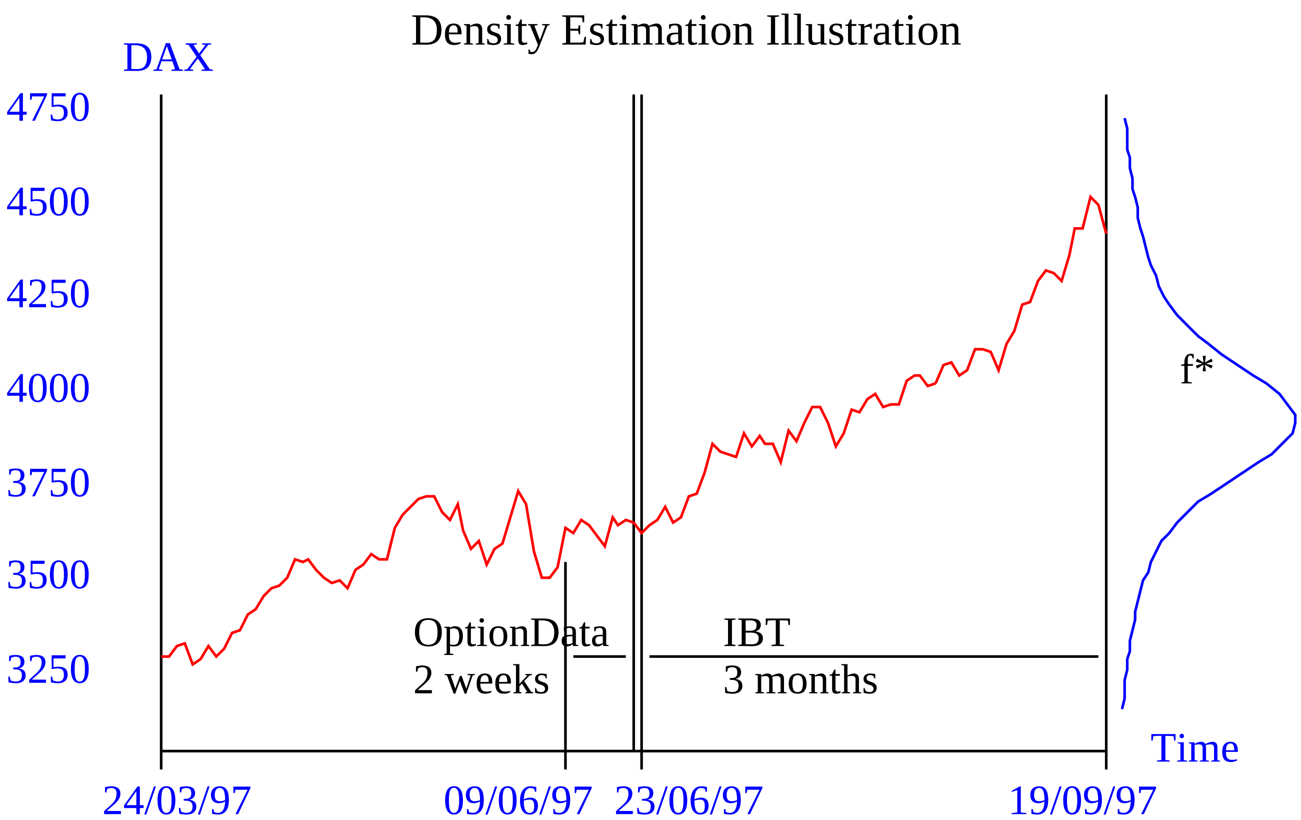
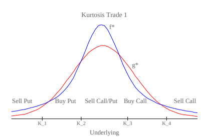


Figure 6: Procedure to estimate implied SPD of Friday, 19/09/97, estimated on Monday, 23/06/97, by means of 2 weeks of option data.



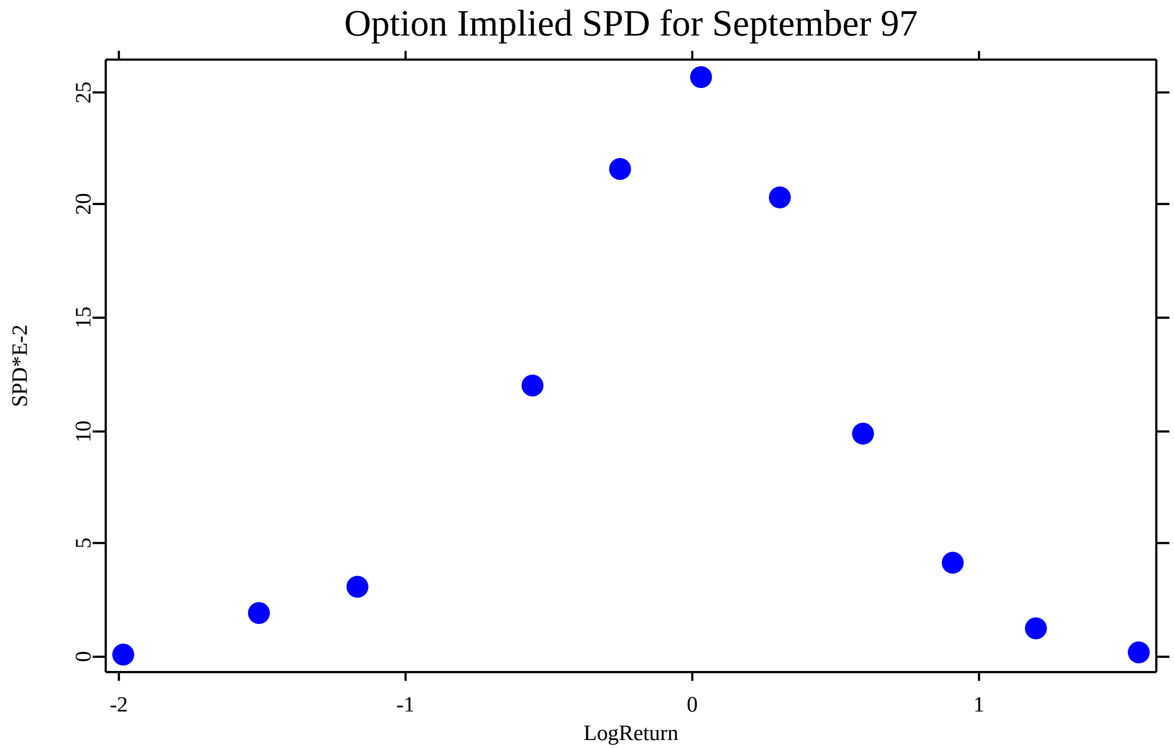


Figure 7: Implied SPD of Friday, 19/09/97, estimated on Monday, 23/06/97, by an IBT with $N = 10$ time steps, $S_0 = 3748.79$, $r = 3.12$ and $\tau = 65/250$.

Estimation of the Time Series SPD

The estimation of the (historical) time series SPD is based on Ait–Sahalia, Wang & Yared (2001).

S follows a diffusion process

$$dS_t \quad = \quad \mu(S_t)dt + \sigma(S_t)dW_t.$$

Further assume a flat yield curve and the existence of a risk–free asset B which evolves according to

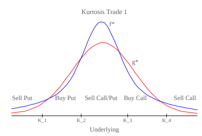
$$B_t \quad = \quad B_0 e^{rt}.$$

Estimation of the Time Series SPD (2)

Then the risk–neutral process follows from Itô's formula and Girsanov's theorem (giving a SPD g^* which will later be compared to the SPD f^*):

$$dS_t^* \quad = \quad rS_t^* dt + \sigma(S_t^*) dW_t^*$$

Drift adjusted but diffusion function is identical in both cases !



Estimation of the Diffusion Function

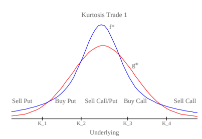
Florens–Zmirou (1993), Härdle & Tsybakov (1997) estimator for σ

$$\hat{\sigma}^2(S) \quad = \quad \frac{\sum_{i=1}^{N^*-1} K_\sigma(\frac{S_{i/N^*}-S}{h_\sigma}) N^* \{S_{(i+1)/N^*} - S_{i/N^*}\}^2}{\sum_{i=1}^{N^*} K_\sigma(\frac{S_{i/N^*}-S}{h_\sigma})}$$

K_σ kernel (in our simulation: Gaussian), h_σ bandwidth, N^* number of observed index values $(N^* \approx 65)$ in the time interval $[0, 1]$

$\hat{\sigma}$ consistent estimator of σ as $N^* \to \infty$

$\hat{\sigma}$ estimated using a 3 month time series of DAX prices



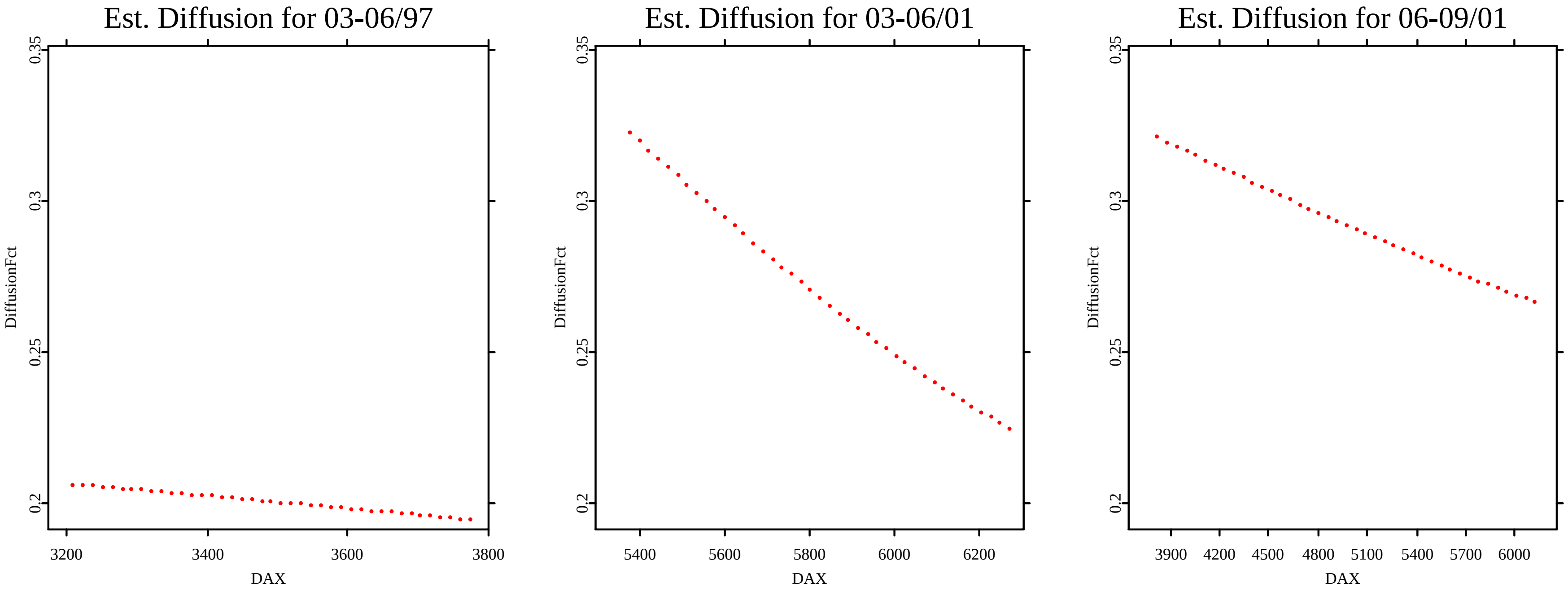
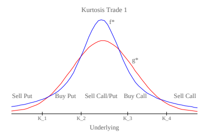


Figure 8: Estimated diffusion functions (”standardized” by $\hat{\sigma}/S_0$) for the periods 24/03/97–20/06/97 ($h_\sigma = 469.00$), 19/03/01–15/06/01 (460.00), 18/06/01–21/09/01 (1597.00) respectively.

regxest.xpl



Simulation of the Time Series SPD

- Use Milstein scheme given by

$$\begin{aligned} S^*_{i/N^*} &= S^*_{(i-1)/N^*} + rS^*_{(i-1)/N^*}\Delta t + \hat{\sigma}(S^*_{(i-1)/N^*})\Delta W^*_{i/N^*} + \\ & \frac{1}{2}\hat{\sigma}(S^*_{(i-1)/N^*})\frac{\partial \hat{\sigma}}{\partial S^*}(S^*_{(i-1)/N^*})\Big\{(\Delta W^*_{(i-1)/N^*})^2 - \Delta t\Big\}, \end{aligned}$$

  where $\Delta W^*_{i/N^*} \sim N(0, \Delta t)$ with $\Delta t = \frac{1}{N^*}$, drift set equal to r, $\frac{\partial \sigma}{\partial S^*}$ approximated by $\frac{\Delta \sigma}{\Delta S^*}$, $i = 1, \ldots, N^*$

- Simulate $M = 10000$ paths for time to maturity $\tau = \frac{N^*}{250}$
- Compute annualized log–returns for simulated paths:

$$u^*_{m,T=1} = \left\{\log(S^*_{m,T=1}) - \log(S^*_{t=0})\right\}\tau^{-1}, m = 1, \ldots, M$$

Simulation of the Time Series SPD (2)

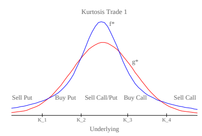
- SPD g^* obtained by means of nonparametric kernel density estimation $(t = 0)$:

$$
\begin{aligned}
g^*(S^*) &= \frac{\hat{p}_t^*\{\log(S^*/S_t^*)\}}{S^*}, \\
\hat{p}_t^*(u^*) &= \frac{1}{Mh_{p^*}} \sum_{m=1}^{M} K_{p^*}\Big(\frac{u_{m,t}^* - u^*}{h_{p^*}}\Big),
\end{aligned}
$$

  where K_{p^*} is a kernel (here: Gaussian) and h_{p^*} is a bandwidth.

- Note: $S_T^* \sim g^*$, then with $u^* = \ln(S_T^*/S_t^*)$ $\hat{p}_t^*$ is related to g^* by

$$
P_{p^*}(S_T^* \leq S^*) = P_{p^*}(u^* \leq \log(S^*/S_t^*)) = \int_{-\infty}^{\log(S^*/S_t^*)} p_t^*(u^*)du^* \, .
$$



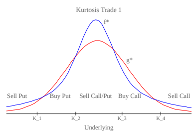
Application to DAX

20 periods from June 1997 to June 2002 ($\tau \approx 65/250$ fixed)

period from Monday following 3rd Friday to 3rd Friday 3 months later

Example: on Monday, 23/06/97, we estimate g^* of Friday, 19/09/97

- Friday, September 19, 1997, is the 3rd Friday
- $\hat{\sigma}$ estimated using DAX prices from Monday, March 23, 1997, to Friday, June 20, 1997
- Monte–Carlo simulation with parameters
  - DAX on Monday June 23, 1997, $S_0 = 3748.79$
  - time to maturity $\tau = 65/250$ and interest rate $r = 3.12$



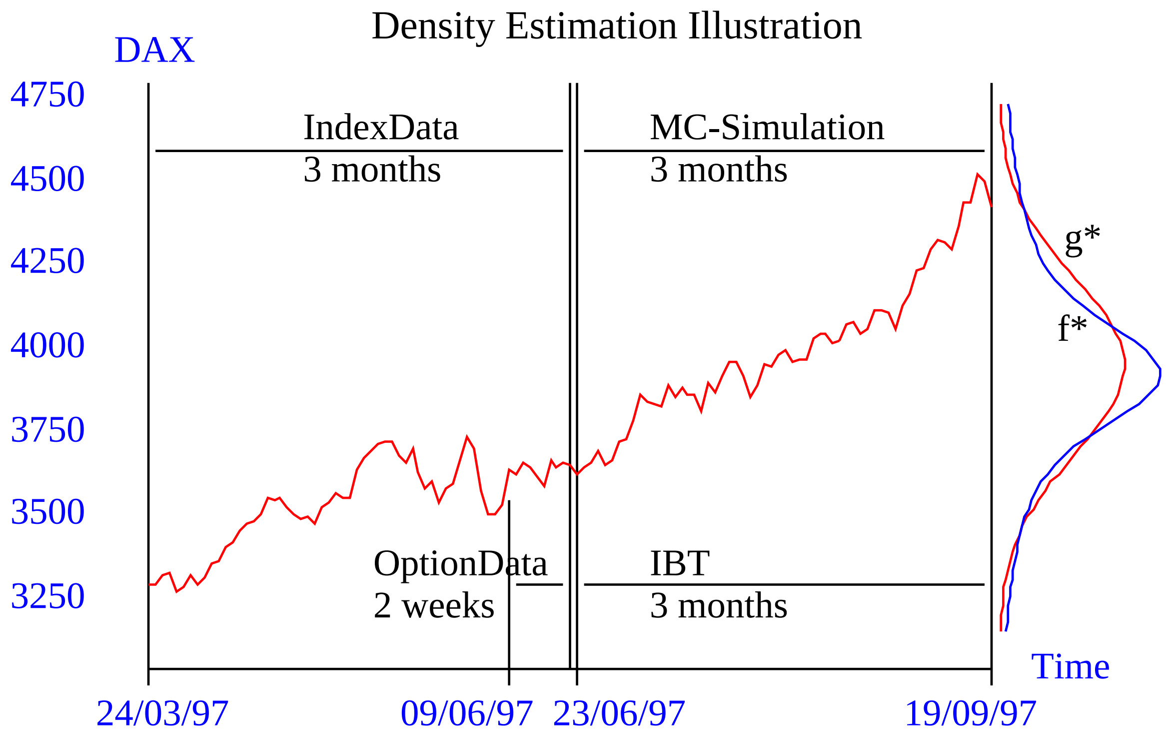
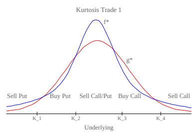


Figure 9: Comparison of procedures to estimate time series and implied SPD of Friday, $19/09/97$. SPD's estimated on Monday, $23/06/97$, by means of 3 months of index data respectively 2 weeks of option data.



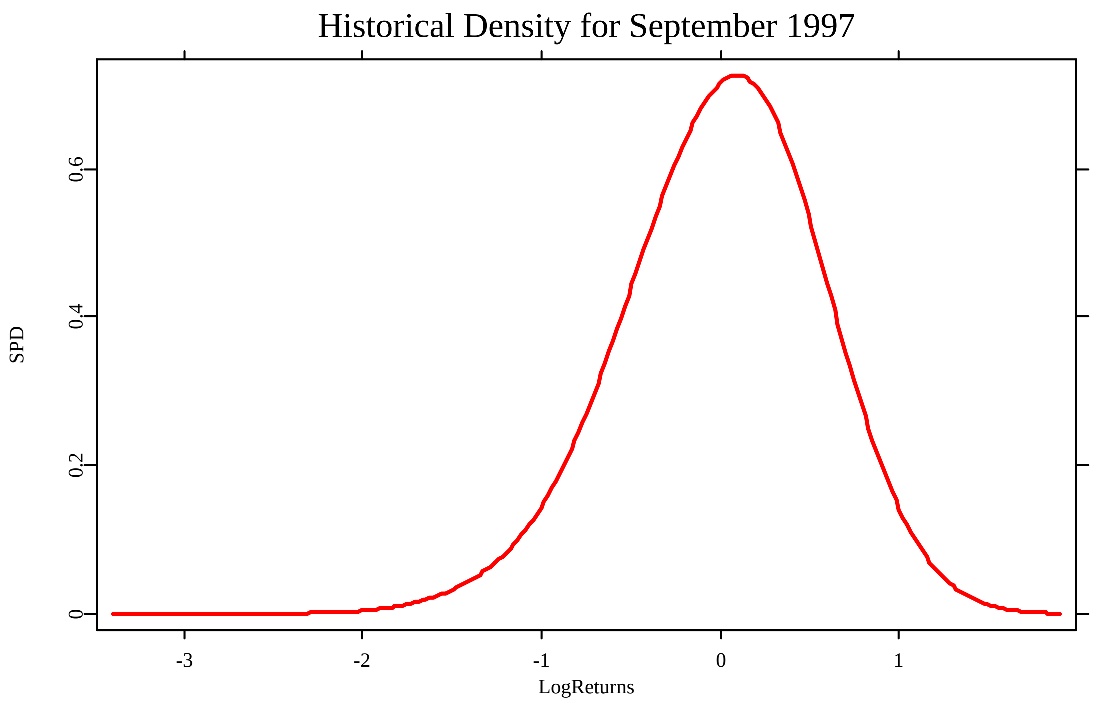


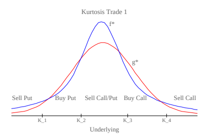
Figure 10: Estimated time series SPD of Friday, 19/09/97, estimated on Monday, 23/06/97. Simulated with $M = 10000$ paths, $S_0 = 3748.79$, $r = 3.12$ and $\tau = 65/250$.

Comparison of Implied and Time Series SPD

Comparison of 20 non–overlapping 3 months periods from June 1997 to June 2002.

SPDs estimated only for most liquidly traded option contracts maturing in March, June, September and December.

SPDs compared by looking at standard deviation, skewness and kurtosis.



Comparison of Standard Deviation

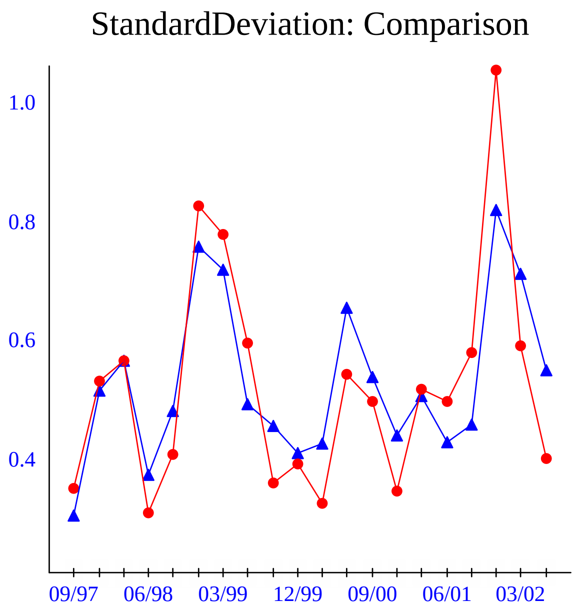
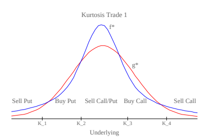


Figure 11: f^* denoted by a triangle and g^* denoted by a circle.

- standard deviation time series of f^* and g^* cross each other frequently
- it appears that standard deviations increase from 2000 on



Comparison of Skewness

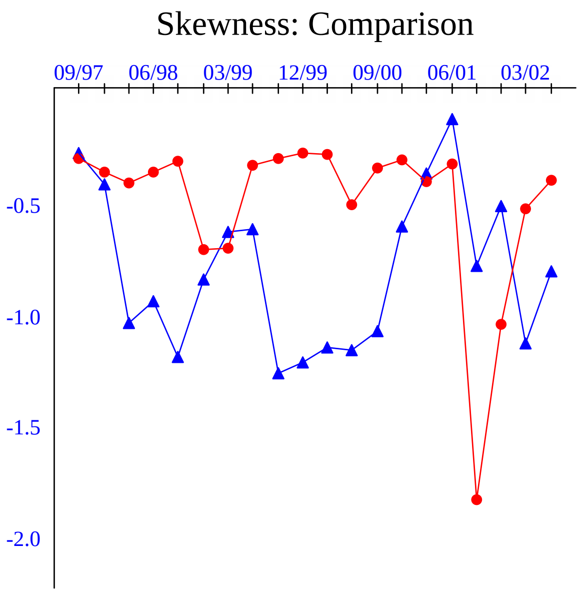
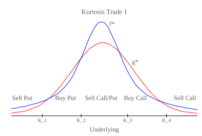
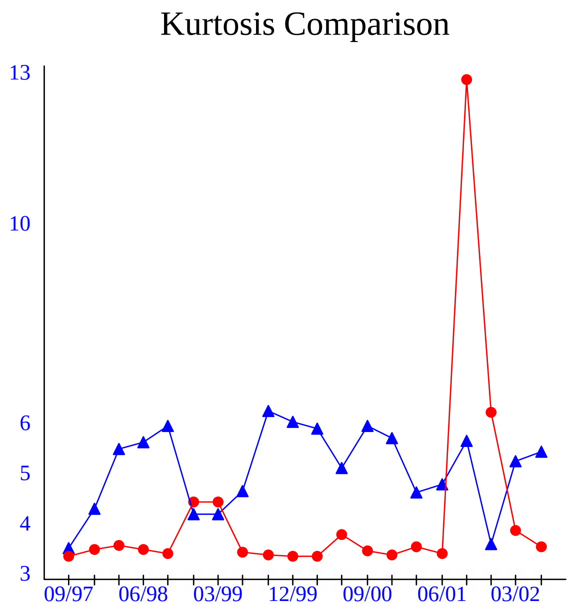


Figure 12: f^* denoted by a triangle and g^* denoted by a circle.

- f^* and g^* negatively skewed for all periods
- f^* more negatively skewed than g^*

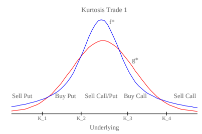


Comparison of Kurtosis



- f^* and g^* leptokurtic
- Kurt(f^*) $<$ Kurt(g^*)
- outlier of g^* in 09/01

Figure 13: f^* denoted by a triangle and g^* denoted by a circle.



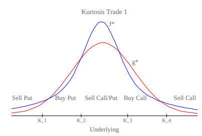
Trading Strategies

General interest:

- Is it possible to exploit the SPD comparison by means of a skewness and/or kurtosis trade?
- Is the strategy's performance consistent with the SPD comparison?

Strategy features:

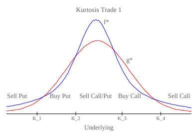
- Only European calls & puts with $\tau = \frac{65}{250}$ (3 months) considered
- All options are kept until expiration
- Buy/sell ONE option at each moneyness (strike) under consideration



What option to buy or to sell at the estimation day of f^* and g^* ?

Skewness Trade 1		Kurtosis Trade 1	
skew(f^*) $<$ skew(g^*)		kurt(f^*) $>$ kurt(g^*)	
Position	Moneyness	Position	Moneyness
short puts	< 0.95	short puts	< 0.90
		long puts	$0.90 - 0.95$
		short puts	$0.95 - 1.00$
		long calls	$1.00 - 1.05$
		short calls	$1.05 - 1.10$
long calls	> 1.05	long calls	> 1.10

Table 1: Definitions of moneyness $(K/S_t e^{r\tau})$ regions.



Performance Measurement

Return for each the 20 three month periods measured by:

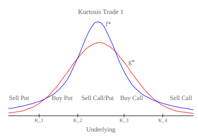
$$\text{portfolio return} = \frac{\text{net cash flow at } t = T}{\text{net cash flow at } t = 0} - 1.$$

Net EURO cash flow in $t = 0$ comprises:

- net cash flow from buying and selling puts and calls,
- for each short call sold buy one share of the underlying,
- for each put sold put the value of the puts' strike on a bank account.

Net EURO cash flow in $t = T$ results from:

- sum of options inner values,
- selling the underlying,
- receiving cash from the puts strikes.

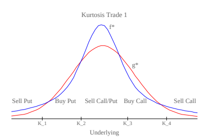


One DAX index point $= 1$ EURO. (As in a DAX option contract consisting of 5 options one index point has a value of 5 EURO.)

No interest rate between $t = 0$ and $t = T$ considered.

Remark: Buy/sell all options available in the moneyness region in question.

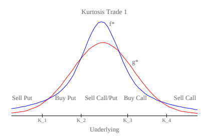
Note: This approach amounts to a careful performance measurement. Applying EUREX margin deposit requirements would decrease the cash outflow for each short option.



Performance S1 Trade

	Skewness Trade 1		
Period	06/97–03/00	06/00–03/02	Overall
Number of Subperiods	12	8	20
Total Return	4.85	-8.53	-2.05
Return Volatility	3.00	9.79	6.78
Minimum Return	-3.66	-25.78	-25.78
Maximum Return	7.65	7.36	7.65
Sharpe Ratio (Strategy)	0.10	-0.46	-0.24
Sharpe Ratio (DAX)	0.38	-0.35	0.02

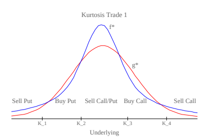
Table 2: Skewness Trade 1 Performance. Only Total Return is annualized. Returns are given in percentages.



Performance S1 Trade (2)

Month	CashFlow in $t = 0$	CashFlow in $t = T$	NetCashFlow
Jun97	541.80	37.88	579.68
Sep97	344.80	0.00	344.80
Dec97	516.90	3781.62	4298.52
Mar98	1042.00	942.68	1984.68
Jun98	1690.80	-6896.90	-5206.10
Sep98	-1559.50	0.00	-1559.50
Dec98	714.30	13.27	727.57
Mar99	923.90	286.41	1210.31
Jun99	964.60	0.00	964.60
Sep99	1019.40	3979.47	4998.87
Dec99	2259.60	2206.92	4466.52
Mar00	3537.40	-864.48	2672.92

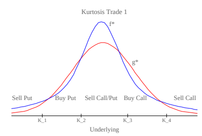
Table 3: Net option cash flows of S1 trade for 06/97-03/00 (no underlying, strikes considered). Cash flows are measured in EUROs.



Performance S1 Trade (3)

Month	CashFlow in $t = 0$	CashFlow in $t = T$	NetCashFlow
Jun00	197.80	0.00	197.80
Sep00	-283.20	-307.17	-590.37
Dec00	-321.90	-645.92	-967.82
Mar01	-420.10	0.00	-420.10
Jun01	293.60	-21616.16	-21322.56
Sep01	-5.80	4384.25	4378.45
Dec01	1003.30	0.00	1003.30
Mar02	813.50	-9499.20	-8685.70

Table 4: Net option cash flows of S1 trade for 06/00-03/02 (no underlying, strikes considered). Cash flows are measured in EUROs.



DAX evolution from 01/97 to 01/03

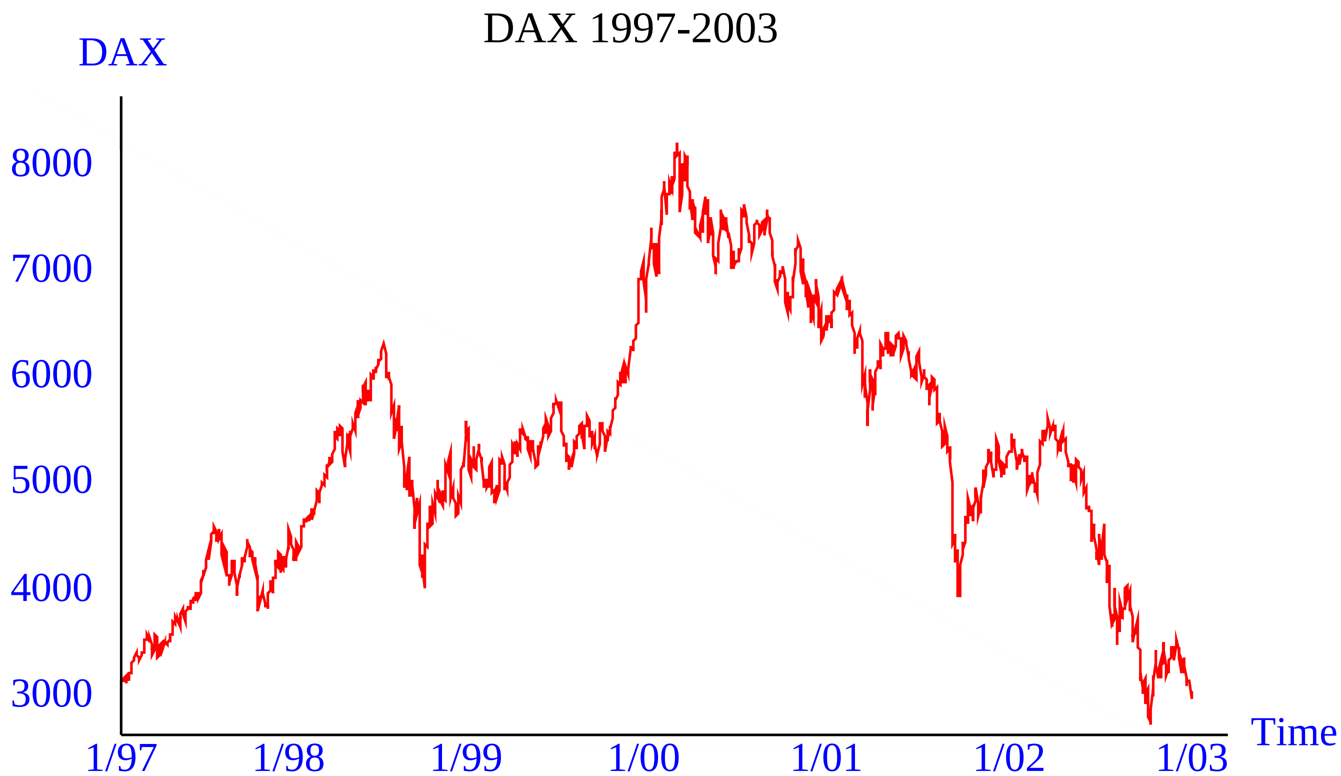
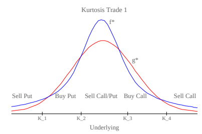


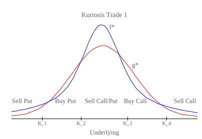
Figure 14: Evolution of DAX from 01/97 to 01/03



Performance K1 Trade

Kurtosis Trade 1			
Period	06/97–03/00	06/00–03/02	Overall
Number of Subperiods	12	8	20
Total Return	14.49	-7.48	2.01
Return Volatility	3.87	13.63	9.33
Minimum Return	-4.54	-28.65	-28.65
Maximum Return	8.79	18.14	18.14
Sharpe Ratio (Strategy)	0.55	-0.32	-0.05
Sharpe Ratio (DAX)	0.38	-0.35	0.02

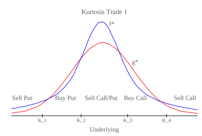
Table 5: Kurtosis Trade 1 Performance. Only Total Return is annualized. Returns are given in percentages.



Performance K1 Trade (2)

Month	CashFlow in $t = 0$	CashFlow in $t = T$	NetCashFlow
Jun97	1257.10	-989.40	267.70
Sep97	2047.20	-58.22	1988.98
Dec97	1345.70	-2451.94	-1106.24
Mar98	1793.90	-1744.51	49.39
Jun98	2690.30	-4612.70	-1922.40
Sep98	4758.10	-541.60	4216.50
Dec98	3913.40	-803.08	3110.32
Mar99	2233.50	-1190.94	1042.56
Jun99	1593.80	-338.32	1255.48
Sep99	1818.70	-3194.15	-1375.45
Dec99	2745.50	-3706.92	-961.42
Mar00	4940.50	-2419.08	2521.42

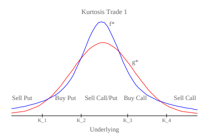
Table 6: Net option cash flows of K1 trade for 06/97-03/00 (no underlying, strikes considered). Cash flows are measured in EUROs.



Performance K1 Trade (3)

Month	CashFlow in $t = 0$	CashFlow in $t = T$	NetCashFlow
Jun00	2178.20	-555.30	1622.90
Sep00	2039.90	-2257.17	-217.27
Dec00	2477.90	-2957.40	-479.50
Mar01	1853.70	-502.50	1351.20
Jun01	1674.10	-12235.10	-10561.00
Sep01	2315.00	-2935.25	-620.25
Dec01	2190.80	-454.24	1736.56
Mar02	1464.90	-4976.00	-3511.10

Table 7: Net option cash flows of K1 trade for 06/00-03/02 (no underlying, strikes considered). Cash flows are measured in EUROs.



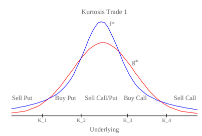
Conclusion

Trading performance positive in subperiods 06/97 – 03/00 and negative 06/00 – 03/02 for S1 as well as K1 trade. However, a SPD comparison does not produce any signal ex ante.

SPD comparison does not give information about the burst of stock market bubble.

SPD estimation methodology need to be fine tuned.

Strategy design (hedging) and performance measurement to be improved.



References

Ait–Sahalia, Y., Wang, Y. & Yared, F. (2001). Do Option Markets correctly Price the Probabilities of Movement of the Underlying Asset?, *Journal of Econometrics* **102**: 67–110.

Barle, S. & Cakici, N., (1998). How to Grow a Smiling Tree, *The Journal of Financial Engineering* **7**: 127–146.

Black, F. & Scholes, M., (1998). The Pricing of Options and Corporate Liabilities, *Journal of Political Economy* **81**: 637–659.

Blaskowitz, O. (2002). *Trading on Deviations of Implied and Historical Density*, in W. Härdle, T. Kleinow, G. Stahl: Applied Quantitative Finance, Springer Verlag, Heidelberg.

Breeden, D. & Litzenberger, R., (1978). Prices of State Contingent Claims Implicit in Option Prices, *Journal of Business*, **9, 4**: 621–651.

Cox, J., Ross, S. & Rubinstein, M. (1979). Option Pricing: A simplified Approach, *Journal of Financial Economics* **7**: 229–263.

Derman, E. & Kani, I. (1994). The Volatility Smile and Its Implied Tree,

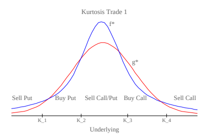
Dupire, B. (1994). Pricing with a Smile, *Risk* **7**: 18–20.

Florens–Zmirou, D. (1993). On Estimating the Diffusion Coefficient from Discrete Observations, *Journal of Applied Probability* **30**: 790–804.

Franke, J., Härdle, W. & Hafner, C. (2001). *Einführung in die Statistik der Finanzmärkte*, Springer Verlag, Heidelberg.

Härdle, W. & Simar, L. (2003). *Applied Multivariate Statistical Analysis*, Springer Verlag, Heidelberg (3-540-03079-4).

Härdle, W. & Tsybakov, A., (1997). Local Polynomial Estimators of the Volatility Function in Nonparametric Autoregression, *Journal of Econometrics*, **81**: 223–242.



Härdle, W. & Zheng, J. (2002). *How Precise Are Price Distributions Predicted by Implied Binomial Trees?*, in W. Härdle, T. Kleinow, G. Stahl: Applied Quantitative Finance, Springer Verlag, Heidelberg.

Kloeden, P., Platen, E. & Schurz, H. (1994). *Numerical Solution of SDE Through Computer Experiments*, Springer Verlag, Heidelberg.

Rubinstein, M. (1994). Implied Binomial Trees, *Journal of Finance* **49**: 771–818.

Willmot, P. (2002). *Paul Willmot Introduces Quantitative Finance*, Wiley.

Yatchew, A. & Härdle, W. (2003). *Dynamic Nonparametric State Price Density Estimation using Constrained Least Squares and the Bootstrap*, Journal of Econometrics, accepted.